The ‘Structural After Measurement’ (SAM) approach to SEM

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literature

Rosseel, Y., & Loh, W.W. (2024). A structural after measurement approach to structural equation modeling. *Psychological Methods, 29*(3), 561–588.
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Levy, R. (2023). Precluding interpretational confounding in factor analysis with a covariate or outcome via measurement and uncertainty preserving parametric modeling. *Structural Equation Modeling: A Multidisciplinary Journal.*
https://doi.org/10.1080/10705511.2022.2154214

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https://doi.org/10.1111/bmsp.12227 (Open Access)

Kuha, J., & Bakk, Z. (arxiv.org). Two-step estimation of latent trait models.
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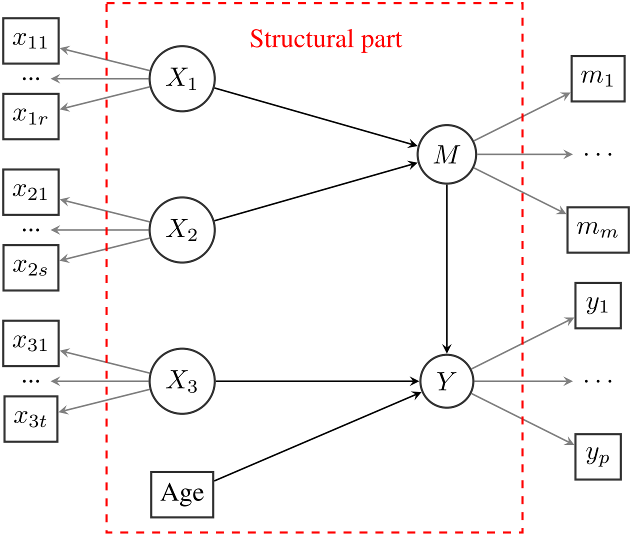
Vermunt, J. (2024). Stepwise latent variable modeling: An overview of approaches.
https://jeroenvermunt.nl/stepwiseLVM2024.pdf

the setting: structural equation models



the setting (2)

- structural equation models
- we focus on 'large' models with many (say, > 100) parameters:
  - many constructs (motivation, ability, personality traits, ...)
  - each construct is measured by a set of (observed) indicators
  - many 'background' variables (age, gender, ...)
  - multilevel data, missing data, ...
- we are mostly interested in the structural part of the model:
  - regression model: variables are either dependent or independent
  - path analysis model: includes mediating effects, perhaps non-recursive
- assumption: the measurement instruments for the latent variables are well established, and usually fit (reasonably) well
- BUT: the sample size is not large (say, $N = 150$)

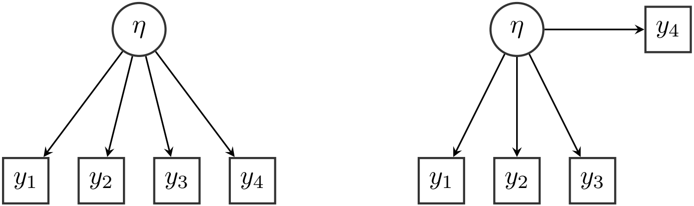
model parameters

- the majority of the model parameters (in SEM) are related to the measurement part of the model:
  - factor loadings
  - residual variances for the indicators
  - factor (co)variances
- only a small portion ($< 10\%$) of the model parameters is related to the structural part of the model:
  - regression coefficients (usually the focus of the analysis)
  - (residual) (co)variances

the standard estimation approach in SEM: ‘system-wide’ estimation

- all parameters (measurement and structural) are estimated jointly
- frequentist: typically using an iterative optimization approach (e.g., ML); Bayesian: typically using MCMC
- advantages:
  - one-step, and therefore efficient (in terms of sampling variability)
  - inference is straightforward (standard errors, hypothesis testing)
  - (relatively) easy to handle constraints, missing data, ...
- works very well if the following conditions are met:
  - correctly specified model, large sample size
  - (normally distributed data)
- but under less ideal circumstances, system-wide estimation does not (always) work well (bias, instability, nonconvergence, improper solutions, ...)
- in addition: combining ‘structural’ and ‘measurement’ is not a good idea

example (stolen from Roy Levy, 2023)



- left panel: only measurement
- right panel: measurement + structural
- conceptually very different
- mathematically identical (in system-wide SEM)

generate some data

```
> library(lavaan)
> Sigma <- matrix(c(2.0, 1.0, 1.0, 1.0, 1.5,
                    1.0, 2.0, 1.0, 1.0, 0.1,
                    1.0, 1.0, 2.0, 1.0, 0.1,
                    1.0, 1.0, 1.0, 2.0, 0.1,
                    1.5, 0.1, 0.1, 0.1, 2.0), nrow = 5, ncol = 5)
> rownames(Sigma) <- colnames(Sigma) <- c("y1", "y2", "y3", "y4", "z")
> Sigma
```

```
    y1  y2  y3  y4   z
y1 2.0 1.0 1.0 1.0 1.5
y2 1.0 2.0 1.0 1.0 0.1
y3 1.0 1.0 2.0 1.0 0.1
y4 1.0 1.0 1.0 2.0 0.1
z  1.5 0.1 0.1 0.1 2.0
```

```
> set.seed(3)
> Data <- MASS::mvrnorm(n = 200L, mu = rep(0, 5), Sigma = Sigma)
```

R code left panel (model1)

```
> model1 <- '
     f =~ y1 + y2 + y3 + y4
 '
> fit1 <- sem(model1, data = Data)
> summary(fit1)

lavaan 0.6-20.2234 ended normally after 25 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                         8

  Number of observations                           200

Model Test User Model:

  Test statistic                                 0.140
  Degrees of freedom                                 2
  P-value (Chi-square)                           0.932

Parameter Estimates:

  Standard errors                             Standard
  Information                                 Expected
  Information saturated (h1) model          Structured
```

```
Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  f =~
    y1                1.000
    y2                0.996    0.123    8.122    0.000
    y3                0.894    0.116    7.689    0.000
    y4                0.911    0.120    7.608    0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .y1                1.096    0.148    7.388    0.000
   .y2                0.837    0.127    6.601    0.000
   .y3                0.987    0.128    7.694    0.000
   .y4                1.081    0.138    7.820    0.000
    f                 1.031    0.208    4.959    0.000
```

R code right panel (model2)

```
> model2 <- '
    f =~ y1 + y2 + y3
    y4 ~ f
 '
> fit2 <- sem(model2, data = Data)
> summary(fit2)

lavaan 0.6-20.2234 ended normally after 26 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                         8

  Number of observations                           200

Model Test User Model:

  Test statistic                                 0.140
  Degrees of freedom                                 2
  P-value (Chi-square)                           0.932

Parameter Estimates:

  Standard errors                             Standard
  Information                                 Expected
  Information saturated (h1) model          Structured
```

```
Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  f =~
    y1                1.000
    y2                0.996    0.123    8.122    0.000
    y3                0.894    0.116    7.689    0.000

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  y4 ~
    f                 0.911    0.120    7.608    0.000

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .y1                1.096    0.148    7.388    0.000
   .y2                0.837    0.127    6.601    0.000
   .y3                0.987    0.128    7.694    0.000
   .y4                1.081    0.138    7.820    0.000
    f                 1.031    0.208    4.959    0.000
```

change outcome variable (y4 becomes z) (model3)

```
> model3 <- '
     f =~ y1 + y2 + y3
     z ~ f
 '
> fit3 <- sem(model3, data = Data)
> summary(fit3)

lavaan 0.6-20.2234 ended normally after 58 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                         8

  Number of observations                           200

Model Test User Model:

  Test statistic                                50.659
  Degrees of freedom                                 2
  P-value (Chi-square)                           0.000

Parameter Estimates:

  Standard errors                             Standard
  Information                                 Expected
  Information saturated (h1) model          Structured
```

```
Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  f =~
    y1                1.000
    y2                0.138    0.053    2.601    0.009
    y3                0.133    0.051    2.591    0.010

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  z ~
    f                 0.276    0.087    3.156    0.002

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .y1               -4.169    1.801   -2.315    0.021
   .y2                1.740    0.173   10.071    0.000
   .y3                1.699    0.168   10.086    0.000
   .z                 1.677    0.211    7.953    0.000
    f                 6.296    1.715    3.671    0.000
```

interpretational confounding

- replacing y_4 by z (= changing the structural part) also changes the parameters of the measurement model
- if the resulting parameters of the measurement model imply a different 'meaning' of the latent variable than was intended by the researcher, we have a problem
- this problem was coined "interpretational confounding" by Burt (1976)

  Burt, R.S. (1976). Interpretational confounding of unobserved variables in structural equation models. *Sociological Methods & Research, 5*(1), 3–52.

- Burt (1976) already suggested the solution: first fit the measurement part of the model, and then fit the structural part of the model

solution: replace sem() by sam()

```
> fit3.sam <- sam(model3, data = Data)
> parameterEstimates(fit3.sam, remove.step1 = FALSE, ci = FALSE,
                     output = "text")

Latent Variables:
                      Step Estimate  Std.Err  z-value  P(>|z|)
  f =~
    y1                   1    1.000
    y2                   1    0.969    0.139    6.977    0.000
    y3                   1    0.884    0.127    6.979    0.000

Regressions:
                      Step Estimate  Std.Err  z-value  P(>|z|)
  z ~
    f                    2    0.540    0.121    4.471    0.000

Variances:
                      Step Estimate  Std.Err  z-value  P(>|z|)
   .y1                   1    1.067    0.170    6.287    0.000
   .y2                   1    0.865    0.151    5.723    0.000
   .y3                   1    0.983    0.142    6.900    0.000
   .z                    2    1.846    0.195    9.462    0.000
    f                    2    1.059    0.226    4.678    0.000
```

the structural-after-measurement (SAM) approach

- SAM is an umbrella term to describe many different (estimation) approaches that have the following in common:
  - first step: we estimate the parameters related to the measurement part
  - second step: we estimate the parameters related to the structural part
- the term SAM was used by Rosseel & Loh (2024), to avoid the overloaded terms 'two-step', 'two-stage', ...
- various SAM approaches have been suggested in the literature:
  - early references: Burt (1976), Hunter & Gerbing (1982), Lance, Cornwell & Mulaik (1988)
  - (uncorrected and bias-corrected) factor score regression (FSR)
  - SAM is the default approach in many other fields
- ... but they never received much attention in the SEM literature/community
- Rosseel & Loh (2024) proposed a special case: 'local SAM' (LSAM)

local SAM: rationale

- the measurement model:

$$\boldsymbol{y} = \boldsymbol{\nu} + \boldsymbol{\Lambda}\boldsymbol{\eta} + \boldsymbol{\epsilon}$$

- to solve this for $\boldsymbol{\eta}$, we proceed as follows:

$$\begin{aligned} \boldsymbol{\nu} + \boldsymbol{\Lambda}\,\boldsymbol{\eta} + \boldsymbol{\epsilon} &= \boldsymbol{y} \\ \boldsymbol{\Lambda}\,\boldsymbol{\eta} &= \boldsymbol{y} - \boldsymbol{\nu} - \boldsymbol{\epsilon} \\ \mathbf{M}\boldsymbol{\Lambda}\,\boldsymbol{\eta} &= \mathbf{M}\left[\boldsymbol{y} - \boldsymbol{\nu} - \boldsymbol{\epsilon}\right] \\ \boldsymbol{\eta} &= \mathbf{M}\left[\boldsymbol{y} - \boldsymbol{\nu} - \boldsymbol{\epsilon}\right] \end{aligned}$$

  where $\mathbf{M}$ is $M \times P$ mapping matrix such that $\mathbf{M}\boldsymbol{\Lambda} = \mathbf{I}_M$

- we assume $\mathrm{E}(\boldsymbol{\epsilon}) = \mathbf{0}$ and write $\mathrm{Var}(\boldsymbol{\epsilon}) = \boldsymbol{\Theta}$; it follows that

$$\begin{aligned} \mathrm{E}(\boldsymbol{\eta}) &= \mathbf{M}\left[\mathrm{E}(\boldsymbol{y}) - \boldsymbol{\nu}\right] \\ \mathrm{Var}(\boldsymbol{\eta}) &= \mathbf{M}\left[\mathrm{Var}(\boldsymbol{y}) - \boldsymbol{\Theta}\right]\mathbf{M}^T \end{aligned}$$

local SAM: estimation

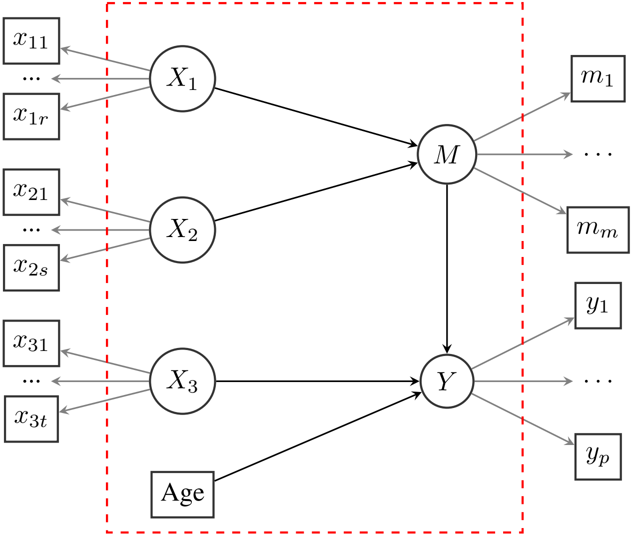
- first stage: estimation of the measurement part of the model (only)
- this results in estimates of:
  - $\mathrm{E}(\boldsymbol{\eta})$: the mean vector of the latent variables
  - $\mathrm{Var}(\boldsymbol{\eta})$: the variance-covariance matrix of the latent variables
  - $\Gamma(\boldsymbol{\eta})$ (‘Gamma’): capturing the sampling variability of these sample statistics
- second stage: a regression or path analysis is performed, using the sample statistics of the latent variables as input
  - twostep-corrected standard errors and fit measures
  - we can use ML, GLS, …or we can use noniterative estimators (OLS, TSLS)
- typical choice for the mapping matrix: the ‘ML/Bartlett’ matrix

$$\mathbf{M} = (\boldsymbol{\Lambda}^T \boldsymbol{\Theta}^{-1} \boldsymbol{\Lambda})^{-1} \boldsymbol{\Lambda}^T \boldsymbol{\Theta}^{-1}$$

example: generate population data mediation model

```
> library(lavaan)
> pop.model <- '
     # factor loadings
     Y  =~ 1*y1 + 1.2*y2  + 0.8*y3  + 0.5*y4
     M  =~ 1*m1 + 0.5*m2  + 0.5*m3  + 0.7*m4
     X1 =~ 1*x1 + 0.7*x2  + 0.6*x3  + 1.1*x4
     X2 =~ 1*x5 + 0.7*x6  + 0.6*x7  + 0.9*x8
     X3 =~ 1*x9 + 0.7*x10 + 0.6*x11 + 1.1*x12

     # covariances among exogenous X1-X3
     X1 ~~ 0.4*X2; X1 ~~ -0.2*X3; X2 ~~ 0.4*X3

     # regression part
     Y ~  0.25*X3 + 0.4*M + (-0.1)*Age
     M ~ -0.30*X1 + 1.1*X2
 '
> set.seed(1234)
> Data <- simulateData(pop.model, sample.nobs = 200L, empirical = TRUE)
```

example: diagram



example: fitting the model using traditional SEM

```
> model <- '
     # measurement part
     Y  =~ y1 + y2  + y3  + y4
     M  =~ m1 + m2  + m3  + m4
     X1 =~ x1 + x2  + x3  + x4
     X2 =~ x5 + x6  + x7  + x8
     X3 =~ x9 + x10 + x11 + x12

     # structural part
     Y ~ X3 + M + Age
     M ~ X1 + X2
 '
> fit.sem <- sem(model, data = Data, estimator = "ML")
> parameterEstimates(fit.sem, ci = FALSE, output = "text")[21:25,]
```

```
Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  Y ~
    X3                0.250    0.108    2.308    0.021
    M                 0.400    0.079    5.078    0.000
    Age              -0.100    0.083   -1.203    0.229
  M ~
    X1               -0.300    0.133   -2.258    0.024
    X2                1.100    0.165    6.670    0.000
```

example: model-implied variance-covariance matrix latent variables

```
> lavInspect(fit.sem, "cov.lv")

        Y      M     X1     X2     X3
Y   1.498
M   0.939  2.036
X1  0.006  0.140  1.000
X2  0.492  0.980  0.400  1.000
X3  0.450  0.500 -0.200  0.400  1.000
```

example: fit measurement blocks, using $B = M$

```
> fit.Y  <- sem('Y  =~ y1 + y2  + y3  + y4',  data = Data)
> fit.M  <- sem('M  =~ m1 + m2  + m3  + m4',  data = Data)
> fit.X1 <- sem('X1 =~ x1 + x2  + x3  + x4',  data = Data)
> fit.X2 <- sem('X2 =~ x5 + x6  + x7  + x8',  data = Data)
> fit.X3 <- sem('X3 =~ x9 + x10 + x11 + x12', data = Data)

> # assemble Lambda and Theta
> Lambda <- matrix(0, 20, 5)
> Lambda[ 1:4,  1] <- lavInspect(fit.Y,  "est")$lambda
> Lambda[ 5:8,  2] <- lavInspect(fit.M,  "est")$lambda
> Lambda[ 9:12, 3] <- lavInspect(fit.X1, "est")$lambda
> Lambda[13:16, 4] <- lavInspect(fit.X2, "est")$lambda
> Lambda[17:20, 5] <- lavInspect(fit.X3, "est")$lambda

> Theta <- lav_matrix_bdiag(lavInspect(fit.Y,  "est")$theta,
                            lavInspect(fit.M,  "est")$theta,
                            lavInspect(fit.X1, "est")$theta,
                            lavInspect(fit.X2, "est")$theta,
                            lavInspect(fit.X3, "est")$theta)
```

example: compute ML version of the mapping matrix M

```
> Theta.inv <- solve(Theta)
> M <- solve(t(Lambda) %*% Theta.inv %*% Lambda) %*% t(Lambda) %*% Theta.inv

> # add age
> M     <- lav_matrix_bdiag(M,     matrix(1, nrow = 1L, ncol = 1L))
> Theta <- lav_matrix_bdiag(Theta, matrix(0, nrow = 1L, ncol = 1L))
> rownames(M) <- c("Y", "M", "X1", "X2", "X3", "Age")

> # compute (biased) sample covariance matrix 'S'
> N <- nrow(Data)
> S <- cov(Data) * (N - 1L)/N

> # compute Var(Eta)
> Var.eta <- M %*% (S - Theta) %*% t(M)
> round(Var.eta, 3)

         Y     M     X1    X2    X3  Age
Y    1.498 0.939  0.006 0.492  0.45 -0.1
M    0.939 2.036  0.140 0.980  0.50  0.0
X1   0.006 0.140  1.000 0.400 -0.20  0.0
X2   0.492 0.980  0.400 1.000  0.40  0.0
X3   0.450 0.500 -0.200 0.400  1.00  0.0
Age -0.100 0.000  0.000 0.000  0.00  1.0
```

example: second stage – using OLS

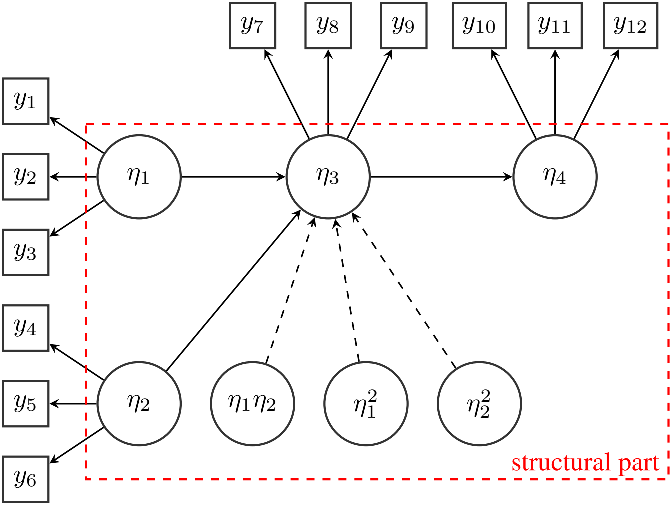
```
> # compute regression coefficients for M
> beta.M <- ( solve(Var.eta[c("X1", "X2"), c("X1", "X2")]) %*%
              Var.eta[c("X1", "X2"), "M", drop = FALSE] )
> round(beta.M, 3)

      M
X1 -0.3
X2  1.1
```

```
> # compute regression coefficients for Y
> beta.Y <- ( solve(Var.eta[c("X3", "M", "Age"), c("X3", "M", "Age")]) %*%
              Var.eta[c("X3", "M", "Age"), "Y", drop = FALSE] )
> round(beta.Y, 3)

        Y
X3   0.25
M    0.40
Age -0.10
```

example: using the sam() function

```
> fit.lsam <- sam(model = model, data = Data)
> parameterEstimates(fit.lsam, ci = FALSE, output = "text")[1:5,]

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  Y ~
    X3                0.250    0.109    2.301    0.021
    M                 0.400    0.080    4.971    0.000
    Age              -0.100    0.083   -1.203    0.229
  M ~
    X1               -0.300    0.133   -2.251    0.024
    X2                1.100    0.176    6.235    0.000
```

application 1: adding latent quadratic and interaction terms



application 1: adding latent quadratic and interaction terms (2)

- in the joint setting, adding latent quadratic/interaction terms is not trivial
- two popular methods are the product-indicator (PI) approach, and the so-called 'Latent Moderated Structural Equations' (LMS) approach
- none of these scale well: they cannot handle many quadratic and latent interaction terms simultaneously
- but if you can decouple the measurement and structural part, this becomes feasible
- a very general SAM solution (allowing for polynomial relations between latent variables) was already described in Wall & Amemiya (2000)
- the local SAM approach: find an explicit expression for

$$\mathrm{E}(\boldsymbol{\eta} \otimes \boldsymbol{\eta}) \qquad \text{and} \qquad \mathrm{Var}(\boldsymbol{\eta} \otimes \boldsymbol{\eta})$$

where $\otimes$ denotes the tensor (or Kronecker) product

local SAM and interaction/quadratic terms

- creating the 'augmented' vector $\boldsymbol{\eta} \otimes \boldsymbol{\eta}$:
- several elements are duplicated
- based on the model, we select what we need

$$\boldsymbol{\eta} = \begin{pmatrix} 1 \\ \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix}$$

$$\boldsymbol{\eta} \otimes \boldsymbol{\eta} = \begin{pmatrix} 1 \\ \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_1 1 \\ \eta_1\eta_1 \\ \eta_1\eta_2 \\ \eta_1\eta_3 \\ \eta_2 1 \\ \eta_2\eta_1 \\ \eta_2\eta_2 \\ \eta_2\eta_3 \\ \eta_3 1 \\ \eta_3\eta_1 \\ \eta_3\eta_2 \\ \eta_3\eta_3 \end{pmatrix} = \begin{pmatrix} 1 \\ \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_1 \\ \eta_1^2 \\ \eta_1\eta_2 \\ \eta_1\eta_3 \\ \eta_2\eta_1 \\ \eta_2 \\ \eta_2^2 \\ \eta_2\eta_3 \\ \eta_3 \\ \eta_3\eta_1 \\ \eta_3\eta_2 \\ \eta_3^2 \end{pmatrix}$$

the augmented (latent) sample statistics

- augmented mean vector:

$$\mathrm{E}(\boldsymbol{\eta} \otimes \boldsymbol{\eta}) = \mathrm{vec}[\mathrm{Var}(\boldsymbol{\eta})] + \mathrm{E}(\boldsymbol{\eta}) \otimes \mathrm{E}(\boldsymbol{\eta})$$

- augmented variance-covariance matrix (simple version, assuming normality for the measurement error):

$$\mathrm{Var}(\boldsymbol{\eta}\otimes\boldsymbol{\eta}) \approx \mathrm{Var}(\mathbf{f}\otimes\mathbf{f}) - \left[\mathbf{Q} + \mathbf{K}_m\mathbf{Q} + \mathbf{Q}\mathbf{K}_m^T + \mathbf{K}_m\mathbf{Q}\mathbf{K}_m^T + \boldsymbol{\Gamma}_{22}^{\star(NT)}(\mathbf{r})\right]$$

where

$$\mathbf{Q} = \mathrm{Var}(\boldsymbol{\eta}) \otimes \mathrm{Var}(\mathbf{r}) + \mathrm{E}(\boldsymbol{\eta})\mathrm{E}(\boldsymbol{\eta})^T \otimes \mathrm{Var}(\mathbf{r})$$

and

$$\boldsymbol{\Gamma}_{22}^{\star(NT)}(\mathbf{r}) = \left(\mathbf{I}_{m^2} + \mathbf{K}_m\right)\left(\mathrm{Var}(\mathbf{r}) \otimes \mathrm{Var}(\mathbf{r})\right)$$

- $\mathbf{K}_m$ is the commutation matrix
- $\boldsymbol{\Gamma}_{22}^{\star(NT)}(\mathbf{r})$ is the ‘Gamma’ matrix of the measurement error $(\mathbf{r})$

implementation in lavaan

```
> model <- '
     # measurement part
     f1 =~ y1 + y2 + y3
     f2 =~ y4 + y5 + y6
     f3 =~ y7 + y8 + y9

     # structural part
     f3 ~ f1 + f2 + f1:f1 + f2:f2 + f1:f2
 '
> fit <- sam(model, data = Data, se = "none") # or se = "bootstrap"
```

- no two-step analytic standard errors yet; but bootstrapping is possible
- forthcoming paper:

Rosseel, Y., Burghgraeve, E., Loh, W.W., Schermelleh-Engel, K. (in press). Structural after Measurement (SAM) approaches for accommodating latent quadratic and interaction effects. *Behavior Research Methods*.

application 2: noniterative SEM

- for CFA, many noniterative estimators are available; some (i.e., the multiple group method) perform better than ML in terms of mean squared error

  Dhaene, S. & Rosseel, Y. (2023). An Evaluation of Non-Iterative Estimators in Confirmatory Factor Analysis. *Structural Equation Modeling: A Multidisciplinary Journal*.

- we can use these noniterative estimators for the measurement part in SAM

  Dhaene, S., & Rosseel, Y. (2023). An Evaluation of Non-Iterative Estimators in the Structural after Measurement (SAM) Approach to Structural Equation Modeling (SEM). *Structural Equation Modeling: A Multidisciplinary Journal*, 30(6), 926–940

- “[the] local SAM approach outperforms traditional SEM in small to moderate samples (both in terms of convergence and MSE values), especially when reliability drops. ”

application 3: comparing structural relations across many groups

- reference (open access):

  Perez Alonso, A.F., Rosseel, Y., Vermunt, J.K., & De Roover, K. (in press). Mixture Multigroup Structural Equation Modeling: A Novel Method for Comparing Structural Relations Across Many Groups. *Psychological Methods*. https://doi.org/10.1037/met0000667

- relationships between latent variables are often different across groups (e.g., countries); but some groups may be similar in the sense that they have similar values for the regression coefficients

- we like to ‘discover’ these hidden clusters of similar groups

- in a first step, we estimated the measurement part across all groups (fixing the factor loadings to be the same across groups); this resulted in (model-implied) latent (co)variance matrices for all the groups

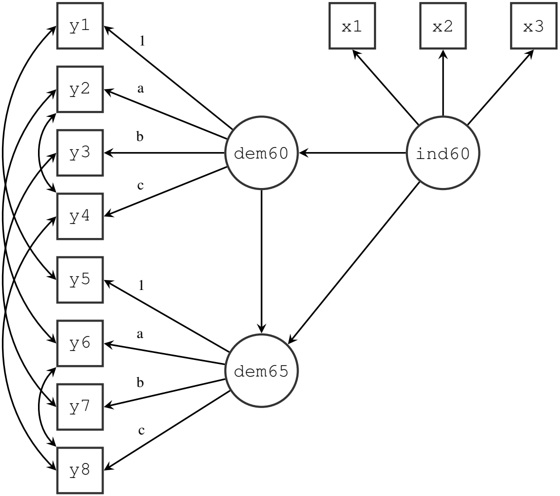
- in a second step, a mixture modeling approach is used to find homogeneous clusters that share similar regression coefficients

SAM: software implementation

- the SAM approach has been implemented in the sam() function in the R package lavaan
- available methods:
  - `sam.method = "local"` (default)
  - `sam.method = "global"`
  - `sam.method = "fsr"` (using Bartlett factor scores)
- typical call:

```
> fit.sam <- sam(model, data = PoliticalDemocracy,
                 sam.method = "local",
                 # link measurement blocks
                 mm.list = list(ind = "ind60", dem = c("dem60", "dem65")),
                 # measurement options
                 mm.args = list(estimator = "ML"),
                 # structural options
                 struc.args = list(estimator = "GLS"),
                 # global options
                 meanstructure = FALSE)
```

diagram of the Political Democracy model



sam() output

```
> # just for illustration, we also show the estimated parameters
> # of the measurement blocks
> #
> summary(fit.sam, remove.step1 = FALSE)

This is lavaan 0.6-20.2234 -- using the SAM approach to SEM

  SAM method                                         LOCAL
  Mapping matrix M method                               ML
  Number of measurement blocks                           2
  Estimator measurement part                            ML
  Estimator  structural part                           GLS

  Number of observations                                75

Summary Information Measurement + Structural:

  Block      Latent Nind Chisq Df
      1       ind60    3  0.00  0
      2 dem60,dem65    8 15.32 16

  Model-based reliability latent variables:

  ind60 dem60 dem65
  0.966 0.868  0.87
```

```
  Summary Information Structural part:

  chisq df cfi rmsea srmr
      0  0   1     0    0

Parameter Estimates:

  Standard errors                                Twostep
  Information                                   Expected
  Information saturated (h1) model            Structured

Latent Variables:
                       Step Estimate  Std.Err  z-value  P(>|z|)
  ind60 =~
    x1                    1    1.000
    x2                    1    2.193    0.142   15.403    0.000
    x3                    1    1.824    0.153   11.883    0.000
  dem60 =~
    y1                    1    1.000
    y2          (a)       1    1.213    0.143    8.483    0.000
    y3          (b)       1    1.210    0.125    9.690    0.000
    y4          (c)       1    1.273    0.122   10.453    0.000
  dem65 =~
    y5                    1    1.000
    y6          (a)       1    1.213    0.143    8.483    0.000
    y7          (b)       1    1.210    0.125    9.690    0.000
    y8          (c)       1    1.273    0.122   10.453    0.000
```

```
Regressions:
                      Step Estimate  Std.Err  z-value  P(>|z|)
  dem60 ~
    ind60                2    1.454    0.389    3.741    0.000
  dem65 ~
    ind60                2    0.558    0.225    2.480    0.013
    dem60                2    0.871    0.076   11.497    0.000

Covariances:
                      Step Estimate  Std.Err  z-value  P(>|z|)
 .y1 ~~
   .y5                   1    0.577    0.364    1.585    0.113
 .y2 ~~
   .y4                   1    1.390    0.685    2.030    0.042
   .y6                   1    2.068    0.733    2.822    0.005
 .y3 ~~
   .y7                   1    0.727    0.611    1.190    0.234
 .y4 ~~
   .y8                   1    0.476    0.453    1.049    0.294
 .y6 ~~
   .y8                   1    1.257    0.583    2.156    0.031

Variances:
                      Step Estimate  Std.Err  z-value  P(>|z|)
   .x1                   1    0.084    0.020    4.140    0.000
   .x2                   1    0.108    0.074    1.455    0.146
   .x3                   1    0.468    0.091    5.124    0.000
   .y1                   1    1.879    0.431    4.355    0.000
```

```
.y2                 1   7.530   1.363   5.523   0.000
.y3                 1   4.966   0.966   5.141   0.000
.y4                 1   3.214   0.722   4.449   0.000
.y5                 1   2.499   0.518   4.824   0.000
.y6                 1   4.809   0.924   5.202   0.000
.y7                 1   3.302   0.699   4.722   0.000
.y8                 1   3.227   0.720   4.482   0.000
 ind60              2   0.446   0.087   5.135   0.000
.dem60              2   3.766   0.848   4.439   0.000
.dem65              2   0.189   0.224   0.843   0.399
```

last slide

- why should we decouple the measurement and structural part?
  - because we should (avoid interpretational confounding)
  - because we can (we can still do SEM)
  - this is what is done in most fields outside SEM
  - good performance in simulation studies (small/moderate sample sizes)
  - opens up modeling possibilities that were (computationally) difficult (if not impossible) in a joint estimation approach
- but still some obstacles:
  - analytic standard errors not always available (yet)
  - limitations (e.g., no higher-order measurement models)
  - more studies are needed to discover potential weaknesses
- the SAM approach deserves the (renewed) interest of the SEM community

Thank you!

(questions?)

https://lavaan.org

https://lavaan.ugent.be/about/donate.html

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