H2020 Marie Skłodowska-Curie innovative training network

Workshop:
Missing Data & Longitudinal Models in Mplus

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Who are you?

- What experience have you with Structural Equation Modeling (SEM) so far?
  - 12% no experience; 36% beginners; 40% occasional users; 12% experienced users

- Have you used Mplus before?
  - 13% no; 87% yes

- Have you used Mplus before to analyze longitudinal data?
  - 48% no; 52% yes

- What is your main learning objective today?
  - 21% curious about SEM and Mplus; 74% learning SEM and Mplus for longitudinal data; 10% my advisor requires me; 36% complex data; 10% latent variable stuff

With special thanks to...

Luc Goossens       Tod Little      Patrick Curran

Karl Jöreskog       Bengt & Linda Muthén
(no pics available)
Change is inevitable. Change is constant.
(Benjamin Disraeli)

Change is the nursery of music, joy, life, and eternity.
(John Donne)

When you’re finished changing, you’re finished.
(Benjamin Franklin)

We are restless because of incessant change, but we would be frightened if change were stopped.
(Lyman Bryson)

Change is a measure of time.
(Edwin Way Teale)
• General measurement and design issues
  – Time & Intervals
  – Differential growth
  – Missing values

• Classic methods and disadvantages
  – Difference score
  – Repeated ANOVA
  – Autoregression
  – Cross-lagged models

• New(er) methods
  – LCM
  – LGCM
  – LGMM
  – LTA

• Exercise!

The example DATA

- N = 405 adolescents + mothers
- From three cohorts
  - 1: M age = 13 at Time 1
  - 2: M age = 15 at Time 2
  - 3: M age = 17 at Time 3

Classic methods and disadvantages
- Difference score
- Repeated ANOVA
- Autoregression
- Cross-lagged models

New(er) methods
- LCM
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Exercise!

When measuring CHANGE, how can we define TIME?
- Age in years, months, days.
- Experiential time: Amount of time something is experienced
  - Years of schooling (grades), length of relationship, amount of practice
  - Estimate on beginning of event, measure time experienced
- Episodic time: Time of onset of a life event
  - Age, toilet trained, driver license, puberty, birth of child, retirement
  - Early onset, on-time, late onset; used to classify or calibrate
  - Time since onset or time from normative or expected occurrence

What measurement intervals should we take?
- How fast is the developmental process?
- Intervals must be equal to or less than expected processes of change (e.g., schooling studies at half-year intervals)
- If too short: Too sensitive to measurement error
- If too long: Inensitive to change and variability in change

A great example article:

SEMDATA.SAV
SEMDATA.DAT
SEMDATA.XLS
General measurement and design issues
- Time & Intervals
- Differential growth
- Missing values

Classic methods and disadvantages
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Exercise!

Missing data
- What is the problem?
- Types of missing data?
- Preventing
- Curing
  - Bad methods
  - Questionable methods
  - Good methods
DROPOUT/MISSING VALUES

- Aim of statistic analyses =
  - based on sample data, draw conclusions about a population
  - estimate the population parameters as good as possible, based on the sample data

- What if we have incomplete data?
  - Can we still estimate correctly the population parameters?
  - Can we still draw correct conclusions about the population?

Æ Missing values!

- occur in about very empirical study
- particularly in longitudinal research (dropout)

TYPES of MISSING VALUES

- Missing Completely at Random (MCAR)
  - No relationship of missingness with non-observed (missing) data, and no relationship with observed data (= completely a-selective dropout)

- Missing at Random (MAR)
  - No relationship of missingness with non-observed (missing) data, but possibly (and preferably) a relationship with observed data

- Missing Not at Random (MNAR)
  - A relationship of missingness with non-observed (missing) data (and possibly also with observed data)
• DROPOUT

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<th>MAR</th>
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• How to test for?
  – Not really possible. But...
  – MVA (SPSS)

• References:
  – Little & Rubin (2002). Statistical analysis with missing data. Wiley

Table with Separate Variance t Test in output, with in rows all variables with (+5%) missings, and in columns all variables in dataset. Cell contain a t-value (+ p) indicating whether or not missingness in the row variable is correlated significantly with the values of the column variable, and therefore is selective. Check patterns of significant t-values. If not clear pattern, MAR is very likely!

When selecting EM in the 'Estimation'-menu, Little’s MCAR test is provided (= summary of t-tests above). If not significant: MCAR! If \( \chi^2/df \) (normed \( \chi^2 \)) < 2: MAR.

PREVENTING DROPOUT & MNAR!

• Dillman (1978)
  ✓ Intensive follow-up and tracking of subjects
  ✓ Repeated invitations to participation, reminders
  ✓ Repeated sending of the measurements
  ✓ Do everything to prevent large dropout!

• Planned missingness
  ✓ Do not measure all variables in all participants at all times.

• Cohort-sequential design!!
  ✓ Let new persons come in at each wave of the study,
  ✓ This way you create different patterns of missingness, not only dropout!

= Different ways to increase the chances of MAR or MCAR!
**CURING!**

- Purpose is NOT to fill in empty cells in the data!
- Purpose IS to estimate the population parameters as good as possible, using a sample with missing data!

**Which methods can help us in this challenging task?**

- **General measurement and design issues**
  - Time & Intervals
  - Differential growth
  - Missing values
- **Classic methods and disadvantages**
  - Difference score
  - Repeated ANOVA
  - Autoregression
  - Cross-lagged models
- **New(er) methods**
  - LCM
  - LGCM
  - LGPA
  - LGMM
  - LTA
- **Exercise!**

**BAD ways to deal with missing data**

- **List-wise Deletion**
  - Variances biased, means biased
  - Acceptable only if power is not an issue and the incomplete data is MCAR
- **Pair-wise Deletion**
  - N varies for each correlation
  - Variances biased, means biased
  - Sometimes estimation problems!
  - Acceptable only if power is not an issue and the incomplete data is MCAR
- **Sample-wise Mean Substitution**
  - For long time very popular method!
  - Variances reduced, correlations biased
  - Never acceptable!
- **Subject-wise Mean Substitution**
  - Depends on homogeneity of the items used
  - Acceptable only if set of items is homogeneous and only few missings!

**QUESTIONABLE ways to deal with missing data**

(single imputation methods)

- **Regression Imputation**
  - All subjects with same values on IVs get the same estimated value on the DV.
  - Variances reduced
  - Assumes MCAR
- **Stochastic Regression Imputation**
  - Same as above but a random error component is added to reduce the loss in variance
  - Still assumes MCAR

**Exercise!**
GOOD ways to deal with missing data

- But only if enough variables related to missingness are included in analysis (MAR), or missingness is MCAR

- EM Imputation
  - Imputes the missing data values in an iterative way, starting with the E step
  - The E(stimation)-step is a stochastic regression-based imputation for each variable.
  - The M(aximization)-step is to calculate a complete covariance matrix based on the estimated values.
  - The E-step is repeated for each variable but the regression is now on the covariance matrix estimated in the previous M-step.
  - The EM-steps are repeated until the imputed estimates don’t differ from one iteration to the other

EM in SPSS (onder assumptie MCAR)

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<th>Variable</th>
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<th>Covariance</th>
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<th>SD</th>
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<td>2.5</td>
<td>0</td>
<td>50</td>
<td>5</td>
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Ook beschikbaar in Lisrel (Prelis), SAS (PROC MI, met OUTEM= optie), e.a.

GOOD ways to deal with missing data

- But only if enough variables related to missingness are included in analysis (MAR), or missingness is MCAR

- Multiple Imputation
  - Estimate N (e.g., 5) datasets using the EM algorithm
  - Each dataset is based on a kind of resampling of the original sample (equivalent to a random selection of a different sample from the population)
  - Possible way 1:
    - Run the analysis N times
    - Summarize the results of these N analyses using the formulas of Rubin (1987)
  - Possible way 2:
    - Collapse the N samples to one dataset and do the analyses.

General measurement and design issues
- Time & Intervals
- Differential growth
- Missing values

Classic methods and disadvantages
- Difference scores
- Autocorrelation
- Cross-lagged models

New(er) methods
- LCM
- LCGA
- LGM
- LTA

Exercise!
MI in SPSS

Further info at www.appliedmissingdata.com/spss-multiple-imputation.pdf

MI in Mplus

GOOD ways to deal with missing data

- But only if enough variables related to missingness are included in analysis (MAR), or missingness is MCAR, but even in cases of MNAR!
- Full Information Maximum Likelihood (FIML)
  - Sufficient statistics (means, covariances) are estimated with the Expectation Maximization (EM) algorithm
  - Those estimates then serve as the start values for the Maximum Likelihood model estimation
  - Does not impute the missing values.
  - Can only be used when testing a SEM-model.
  - Available in Lisrel, AMOS, Mplus, EQS, etc.
- EXAMPLES comes with LGC and other models.

Missing values: Conclusions

- Missing values are part of every empirical study.
- Neglecting the missing data (listwise deletion) is a wrong approach.
- Different good methods are available to handle data that are MAR or MCAR, and give us correct population parameters!
- Even methods are available in case data are MNAR!

How to analyze change?

- Classic methods & disadvantages
- New(er) & better methods, using SEM
CHANGE_{
\text{14}} = \text{ANTI}_{4} – \text{ANTI}_{1}

• Though many problems with it, still popular (e.g., intervention, pretest-posttest, or clinical studies)
  • Most cited problem: Unreliability of the difference score
    – when the measures comprising the difference are only modestly reliable and positively correlated
    – which is typically the case in longitudinal research!
  • And therefore also: lack of validity
    + Change is measured without taking level into account!

difference score inp:
DEFINE: change14 = anti4-anti1;
MODEL: change14 ON support;

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  – LGM
  – LGCM
  – LGDAM
  – LTA
• Exercise!
CHANGE = effect of TIME in a repeated ANOVA

- SPSS output: R ANOVA.spv

- So, a good method
  - To describe and test an overall mean change function, and test for the form of it (linear, quadratic, cubic, etc.)
  - To test for the effect of covariates on the change function (e.g., support), Time x Support interactions!
  - To test for the effect of between-subject factors on the change function (e.g., gender), Time x Gender interactions!

- But
  - Only tests mean change over time in the whole sample and not deviations from that mean change
  - And, group statistics (e.g., mean) represent everyone, and no one!
  - Equal intervals between measurements are necessary!
  - Change is an outcome of the repeated measures (time) and cannot be used as a predictor of outcomes.

[General measurement and design issues]
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- Excercise 1

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- With observed variables: autoregression.inp
- With latent variables: autoregression latent.inp

- But...
  - implicit assumption of decreasing correlations across time!
  - indifferent to the functional form of change!
  - only RELATIVE change in terms of rank order!
  - Interpretation?
    - Does ANT1 predict (86) an increase in ANT4?
    - Increase only relative to others in the sample!
    - Even if everyone decreases!
  - No trajectories of individual change over time!

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- Excercise 1

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A multivariate extension of the autoregressive model of change: crosslagged.inp

- Interesting for examining direction of effects, by comparing cross-lagged coefficients (easily done in SEM)

- But
  - Same problem with interpretation of change as in AR model
  - Also shares all other problems of the AR model!
  - Statistical drawbacks:
    - If X is (much) more stable than Y, then Y will have a stronger effect on X than vice versa!
    - And, the less reliable X is, the less Y can explain it.

[General measurement and design issues]
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Antisocial behavior is 'causing' change in GPA!

[General measurement and design issues]
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- Excercise 1

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Stability in rank orders

Instability in rank orders

Indirect measure of individual differences in change

Stability in rank orders

Instability in rank orders

Excercise 1

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    - And, the less reliable X is, the less Y can explain it.
So,

What do we want exactly when we try to assess change?

RELIABLE estimates of change!

Estimates of ABSOLUTE change!

INDIVIDUAL estimates of change!

With 2 WAVES of data

Latent Change Models (LCM)


- Change in latent variables, using (CFA)
- First step = Longitudinal invariant factor model (multiple indicators), resulting in reliable scores for Anti-1 and Anti-4.
- Second step = Restructuring Anti-1 and Anti-4 in latent Level and Change factors, using a very simple equation:

\[ \text{Anti-1} = 1 \times \text{Anti-Level} \]
\[ \text{Anti-4} = 1 \times \text{Anti-Level} + 1 \times \text{Anti-Change} \]

- As a consequence:

\[ \text{Anti-Change} = \text{Anti-4} - \text{Anti-1} \]
\[ \text{Anti-Level} = \text{Anti-1} \]

\( a \) reliable difference score!

Observed variables?

The problem of measurement error

\[ \text{DATA} = \text{MODEL} + \text{ERROR} \]

- True variance: correlated
- Error variance: not correlated

\( \Rightarrow \) Total covariance: underestimated!
Latent variables!
Solution for measurement error!

\[ \begin{align*}
\epsilon_1 & \quad \text{error} \\
\epsilon_2 & \quad \text{error} \\
\epsilon_3 & \quad \text{error} \\
\text{True} & \quad \text{true} \\
\end{align*} \]

SEM = analysis with latent variables!

• General measurement and design issues
  – Measurement
  – Time & Intervals
  – Differential growth
  – Missing values

• Classic methods and disadvantages
  – Difference score
  – Repeated ANOVA
  – Autoregression
  – Cross-lagged models

• New(er) methods
  – LCM
  – LGCM
  – LCFA
  – LGM
  – LTA

• Exercise !

• Main advantages:
  – Only two waves of data needed!
  – Can be extended to multiple successive change factors!
  – Reliable estimates of change!
  – Change is assessed as a latent factor!
    - With a mean: Mean change in the total sample!
    - With a variance: Individual differences in change!
    - That can be predicted and used as a predictor!
  – Latent factor scores can be estimated!

• But
  – Only linear change function!

With 3+ WAVES of data

Latent Growth Curve Models (LGCM)
(Duncan et al., 1994; McArdle & Nesselroade, 2002; Willet & Sayer, 1994)

• Questions:
  – Does an individual characteristic (e.g., antisocial behavior) change over time?
  – Which trajectory is followed?
  – Interindividual differences?

• Step 1: Within-Person
  – Equation for every subject in the sample:
    \( \text{anti} = \text{intercept} + (\text{slope} \times \text{Time}) + \text{error} \) [regression]
  – Growth can be non-linear too!
    \( \text{anti} = \text{intercept} + (\text{slope} \times \text{Time}) + (\text{curve} \times \text{Time}^2) + \text{error} \)
  – Assumption: Individuals share the shape of the change function (e.g., linear), but can differ in the amount or rate of change (individual growth parameters: intercept, slope, etc.)

• Step 2: Between-Person
  – Means (fixed) & variances (random) of intercepts, slopes
  – Predictors of change (conditional growth models)
With 3+ WAVES of data
Latent Growth Curve Models (LGCM)
(Duncan et al., 1994; McArdle & Nesselroade, 2002; Willet & Sayer, 1994)

\[
\text{anti} = 2 + (1.5 \times \text{time}) + \text{error}
\]

\(0\) \(1\) \(2\) \(3\) \(4\) \(5\) \(6\) \(7\) \(8\) \(9\) \(10\)
\(0 \text{(13)} 1 \text{(14)} 2 \text{(15)} 3 \text{(16)}
\]

- General measurement and design issues
- Measurement
- Time & Intervals
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- Repeated ANOVA
- Autoregression
- Cross-lagged models
- New(er) methods
- LCM
- LGCM
- LCGA
- LGMM
- LTA
- Exercise !
Two Approaches to GCM

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<thead>
<tr>
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<th>Multilevel</th>
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<tr>
<td></td>
<td>factor approach</td>
<td>regression approach</td>
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<tr>
<td></td>
<td>means &amp; variances</td>
<td>fixed &amp; random effects</td>
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<td></td>
<td>not flexible with individual-varying intervals</td>
<td>flexible with missing values &amp; individual-varying intervals</td>
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<tr>
<td></td>
<td>very flexible in prediction</td>
<td>less flexible in prediction</td>
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<tr>
<td></td>
<td>overload of fit indices</td>
<td>little attention for model fit</td>
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- General measurement and design issues
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- Exercise!
With 3+ WAVES of data

Latent Growth Curve Models (LGCM)
(Duncan et al., 1994; McArdle & Nesselroade, 2002; Willet & Sayer, 1994)

PARAMETERS in the model
- Mean intercept / fixed effect intercept
  - Mean initial level of all individuals
- Variance intercept / random effect intercept
  - Interindividual differences in initial level
- Mean slope / fixed effect slope
  - Mean rate of growth across individuals
- Variance slope / random effect slope
  - Interindividual differences in rate of change

lgcm_anti.inp (with missing data, and FIML)
lgcm_anti + predictors.inp (gender and support as predictors)
lgcm_anti + predictors + interaction.inp (support X level anti)
lgcm anti - piecewise.inp (piecewise model with 2 slopes)
With 3+ WAVES of data

Latent Growth Curve Models (LGCM)
(Duncan et al., 1994; McArdle & Nesselroade, 2002; Willet & Sayer, 1994)

• Can be extended to a multivariate LGCM!
  - Correlated intercepts = cross-sectional association
  - Correlated intercept & slope = level of IV is predicting rates of change in DV!
  - Correlated slopes = common underlying growth in two constructs = change associated with change (causality?)

• But: Assumption: same shape of the growth function for all subjects; interindividual differences in change are modeled as deviations from that overall mean.

General measurement and design issues
– Measurement
– Time & Intervals
– Differential growth
– Missing values

Classic methods and disadvantages
– Difference score
– Repeated ANOVA
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Newer methods
– LGCM
– LGMM
– LTA

Exercise !
With 3+ WAVES of data

Latent Growth Curve Models (LGCM)  
(Duncan et al., 1994; Muthén & Nesselroade, 2002; Willet & Sayer, 1994)

Extension: Cohort sequential design!

Method 2: Data rearrangement (DATA COHORT)

- Estimate 1 intercept and 1 slope, using all available data
- Works out the time score based on birth and measurement year
- Idea = rearrange our cohort & time data to age data
- Then a growth curve for the complete age span!
- Only works with continuous variables!

cohort-sequential growth model anti_2.inp (linear)
cohort-sequential growth model anti_2b.inp (curvilinear)
cohort-sequential growth model anti_2_piecewise.inp

With 3/4+ WAVES of data

Latent Growth Mixture Modeling (LGMM)  
(Muthén, 2001, 2004)

and a special case of it: Latent Class Growth Analysis (LCGA)  
(Nagin, 1999, 2001)

- Ch. LGCM: heterogeneity in the sample is captured by variation around a mean growth function, i.e., it is assumed that all individuals are drawn from the same population.
- LGMM
  - relaxes this single population assumption to allow for parameter differences across unobserved subpopulations.
  - by using a combination of continuous and categorical latent variables (mixed)
  - by introducing a latent class variable C, a trajectory class variable, representing k unobserved subpopulations in the sample (note that in LGCM k = 1)
  - by estimating a separate growth model for each of the latent classes
  - and estimating latent class conditional probabilities (membership)
With 3/4+ WAVES of data
Latent Growth Mixture Modeling (LGMM)
Latent Class Growth Analysis (LCGA)

- **LGMM**: Class differences
  - Typically between mean intercepts between classes
  - Mean slopes in the classes
  - Variance in intercept and slope
  - Shape of the growth function
  - Influence of covariates

- **LGMM**: Extensions
  - Including covariates of change
  - Including outcomes that are predicted from growth

- **LCGA (Nagin)**
  - Very similar, only no variances within classes are estimated (therefore a semi-parametric approach)
  - Individuals within classes are treated as homogeneous

Estimation: Using the EM algorithm
- Estimation of each individual's probability of membership in each class (conditional probabilities)
- Measures of fit and classification quality:
  - BIC
    - Small values correspond to a good model with a large likelihood and not too many parameters
    - Look at the log-likelihood ratio statistic or one based on another index
    - Sensitivity to outliers of clearest
    - Less sensitive to differences in growth shape between classes
    - Large differences suggest two classes
  - LMRT test
    - Test of a solution with k-1 classes against a solution with k classes (e.g. 2 vs 1): low p-value indicates that solution k-1 should be rejected in favor of the solution with k classes.
  - Entropy
    - Measure of classification quality based on the individual class probabilities
    - High values (closer to 1) indicate good classification.

Examples with 2 classes:
- *lcga_anti2.inp* (Nagin approach)
- *lgmm_anti2.inp* (with equal variances across classes)
- *lgmm_anti2free.inp* (with free variances across classes)

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With 4+ WAVES of data
Latent Growth Mixture Modeling (LGMM)
Latent Class Growth Analysis (LCGA)

Antisocial data: fit statistics

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Excercise!
With 4+ WAVES of data
Latent Growth Mixture Modeling (LGMM)
Antisocial data: Best solution: 5 classes, LCGA, curvilinear

• General measurement and design issues
  – Measurement
  – Time & Intervals
  – Differential growth
  – Missing values

• Classic methods and disadvantages
  – Difference score
  – Repeated ANOVA
  – Autoregression
  – Cross-lagged models

• New(er) methods
  – LCM
  – LGCM
  – LCGA
  – LGMM
  – LTA

• Exercise!

With 4+ WAVES of data
Latent Growth Mixture Modeling (LGMM)
Antisocial data: Best solution: lcga_anti5c.dat

• With 3/4+ WAVES of data
Latent Transition Analysis (LTA)
(Kaplan, 2008; Meeus et al., 2010)

• A longitudinal extension of LCA or LPA
  – First, uses class-specific parameters (the continuous/ categorical observed multidimensional scores) as measurement parameters (≈ LCA or cluster analysis)
  – Invariant across time!
  – And, uses class probabilities as structural parameters to estimate the number of participants in each class
  – Then uses latent transition probabilities to calculate patterns of stability and change over time in the movement or transition between classes (≈ mover-stayer model)

• New(er) methods
  – LCM
  – LGCM
  – LCGA
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  – LTA

⇒ An ideal model for testing typological or person-oriented developmental theories!
• General measurement and design issues
  – Measurement
  – Time & Intervals
  – Differential growth
  – Missing values

• Classic methods and disadvantages
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• New(er) methods
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• Exercise!

With 3/4+ WAVES of data
Latent Transition Analysis (LTA)
(Kaplan, 2008; Meeus et al., 2010)

• Example: data on parenting across three waves
  – 659 adolescents, measured 3 times: 13, 15 and 17 years
  – psychological control, support/warmth, firm control (continuous)
  – scores standardized (z) for ease interpretation

• Selected output

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<th>k</th>
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<tbody>
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With 3/4+ WAVES of data
Latent Transition Analysis (LTA)
(Kaplan, 2008; Meeus et al., 2010)

• Selected output k = 5 solution

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<tbody>
<tr>
<td>Neglecting</td>
<td>-0.38</td>
<td>-0.11</td>
<td>-0.54</td>
<td>49%</td>
<td>29%</td>
</tr>
<tr>
<td>Permissive</td>
<td>-0.67</td>
<td>1.21</td>
<td>0.64</td>
<td>24%</td>
<td>18%</td>
</tr>
<tr>
<td>Rejecting</td>
<td>0.34</td>
<td>-1.17</td>
<td>-1.19</td>
<td>10%</td>
<td>8%</td>
</tr>
<tr>
<td>Controlling</td>
<td>1.00</td>
<td>-0.62</td>
<td>-0.57</td>
<td>15%</td>
<td>16%</td>
</tr>
<tr>
<td>Democratic</td>
<td>-1.66</td>
<td>1.36</td>
<td>1.23</td>
<td>1%</td>
<td>29%</td>
</tr>
</tbody>
</table>
To conclude

- Since two decades, we have interesting new methods to analyze change and development!

- *Mplus* provides a powerful tool to analyze change and development, and is constantly improving!

- And, as said before…
• General measurement and design issues
  – Measurement
  – Time & Intervals
  – Differential growth
  – Missing values

• Classic methods and disadvantages
  – Difference score
  – Repeated ANOVA
  – Autoregression
  – Cross-lagged models

• New(er) methods
  – LCM
  – LGCM
  – LCGA
  – LGMM
  – LTA

• Exercise!

  1. Check the direction of effects (using a cross-lagged model) between GPA and antisocial behavior, using data of Times 1 and 3. Check whether results are the same for boys and girls.
     
     DATA are SEMDATA.DAT; software = Mplus 8.0

  2. Estimate a LGCM of GPA, using the FIML approach for missing data. Check models with linear and curvilinear change. Interpret the parameters that are found.
     
     DATA are SEMDATA.DAT; software = Mplus 8.0

  3. Find the optimal LGCM/LGMM solution of the GPA data. Explain why this solution was chosen and interpret the different classes.
     
     DATA are SEMDATA.DAT; software = Mplus 7.3

  4. Evaluate the effect of mother support on development of GPA, using a conditional growth model and FIML.
     
     DATA are SEMDATA.DAT; software = Mplus 8.0

  5. Test the multivariate LGCM of ANTI & GPA, using the FIML approach for missing data. Search for the best fitting model. Interpret the all the estimated parameters in this model.

     - Setup and do the analyses using Mplus!
     - Ask for help while doing the analyses!
     - Present the results to the audience, using a single or two slides and explain the effects in words!

Informative websites

• www.statmodel.com: the Mplus site!

• http://davidakenny.net/cm/causalm.htm: great SEM page!

• https://stats.idre.ucla.edu/mplus/: online examples and videos on Mplus

• http://users.ugent.be/~wbeyers/workshop/index.html: the website for this workshop!
Literature

An introduction to SEM

Defining time and choosing intervals

Missing data and how to handle them

Latent Change Model (LCM)

Latent Growth Curve modeling (LGCM)
Literature

**Latent Class Growth Analysis (LCGA)**

**Latent Growth Mixture Modeling (LGMM)**

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**Longitudinal measurement invariance**
Literature

**Latent Transition Analysis**

Further literature

- **Model fit**

- **MLR (and related Chi-square test)**

- **Reporting about SEM**