



KU LEUVEN

**Mplus Workshop: Theory into Practice
(FLAMES in cooperation with SOKA, KU Leuven)**

Day 1 (March 9, 2015)

Path and Growth Models



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Who are you?

- What experience have you with Structural Equation Modeling (SEM) so far?
 - 12% no experience; 36% beginners; 40% occasional users; 12% experienced users
- Have you used Mplus before?
 - 13% no; 87% yes
- Have you used Mplus before to analyze longitudinal data?
 - 48% no; 52% yes
- What is your main learning objective today?
 - 21% curious about SEM and Mplus; 74% learning SEM and Mplus for longitudinal data; 10% my advisor requires me; 36% complex data; 10% latent variable stuff

With special thanks to...

Luc Goossens



Tod Little



Patrick Curran



Karl Jöreskog

Bengt & Linda Muthén
(no pics available)

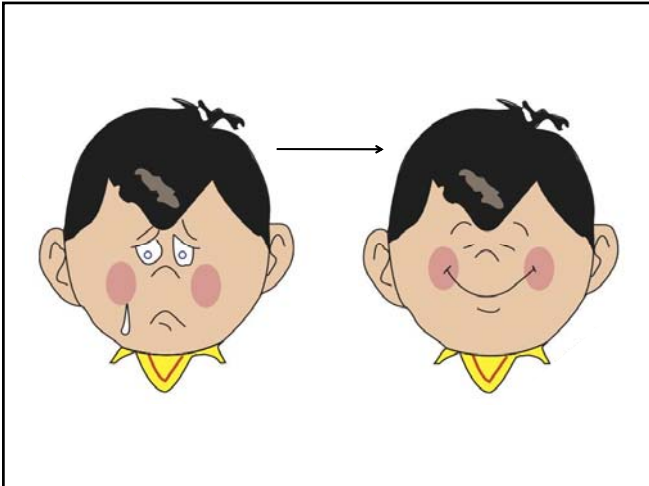
Change is inevitable. Change is constant.
(Benjamin Disraeli)

Change is the nursery of music, joy, life, and eternity.
(John Donne)

When you're finished changing, you're finished.
(Benjamin Franklin)

We are restless because of incessant change, but we would be frightened if change were stopped.
(Lyman Bryson)

Change is a measure of time.
(Edwin Way Teale)



DEVELOPMENT

- General measurement and design issues
 - Time & Intervals
 - Differential growth
 - Missing values
 - Classic methods and disadvantages
 - Difference score
 - Repeated ANOVA
 - Autoregression
 - Cross-lagged models
 - New(er) methods
 - LCM
 - LGCM
 - LCGA
 - LGMM
 - LTA
 - Exercise !
- General measurement and design issues
 - Time and intervals
 - Differential growth
 - Missing values
 - Classic methods and disadvantages
 - Absolute change: the difference score
 - Absolute change+: repeated measures ANOVA
 - Relative change: autoregression
 - Relative change+: cross-lagged models
 - Newer and better methods
 - With 2+ waves: LCM
 - With 3+ waves: LGCM + extensions (e.g. MI)
 - With 3+ waves: LCGA
 - With 4+ waves: LGMM
 - With 3/4+ waves: LTA (mover-stayer)

The example DATA

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- N = 405 adolescents + mothers

- From three cohorts

- 1: M_{age} = 13 at Time 1
- 2: M_{age} = 15 at Time 2
- 3: M_{age} = 17 at Time 3

- Measures:

Time 1:

- **Gender:** 203 boys (1) and 202 girls (0) (A-report)
- **Support** from mother (A-report) 1-6 3.00 (0.80)
- **Structure** by mother (A-report) 1-6 3.29 (0.96)
- **Shaming** by mother (A-report) 1-6 2.49 (0.82)

Time 1-2-3-4 (yearly measurement):

- **Antisocial behavior** (M-report) 0-10 1.66 - 1.83 - 2.03 - 2.06
- **School GPA on PE-class** (A-report) 0-10 2.52 - 4.08 - 5.00 - 5.77

- Missing data (coded 9999):

- dropout and nonresponse from T3 onwards!
- from 7% (ANTI-3) to 34% (GPA-4)
- 11% overall

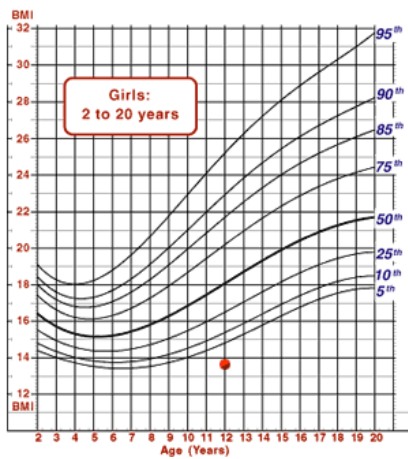
[SEMDATA.SAV](#)
[SEMDATA.DAT](#)
[SEMDATA.XLS](#)

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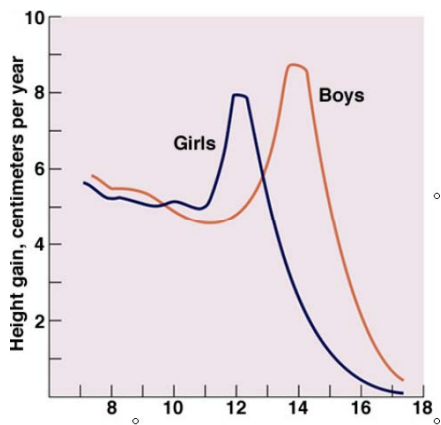
- **When measuring CHANGE, how can we define TIME?**
 - **Age** in years, months, days.
 - **Experiential time:** Amount of time something is experienced
 - Years of schooling (grade), length of relationship, amount of practice
 - Calibrate on beginning of event, measure time experienced
 - **Episodic time:** Time of onset of a life event
 - Age, toilet trained, driver license, puberty, birth of child, retirement
 - Early onset, on-time, late onset: used to classify or calibrate
 - Time since onset or time from normative or expected occurrence.
- **What measurement Intervals should we take?**
 - **How fast** is the developmental process?
 - **Intervals** must be equal to or less than expected processes of change (e.g., schooling studies at half-year intervals)
 - **If too short:** too sensitive to measurement error
 - **If too long:** insensitive to change and variability in change

A great example article:
 Reuter & Conger (1998). doi: 10.1037/0012-1649.34.6.1470

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DROPOUT

X (wave 1)	Y (wave 2)			
	Compleet	MCAR	MAR	MNAR
130	101	101	101	101
145	155			
136	140	140	140	
146	134	134		134
111	129		111	129
134	124		124	124
153	112			112
137	122	122	122	122
118	118	118	118	118

- How to test for?
 - Not really possible. But...
 - MVA (SPSS) →
- References:
 - Little & Rubin (2002). *Statistical analysis with missing data*. Wiley
 - Schafer & Graham (2002); doi: 10.1037/1082-989X.7.2.147

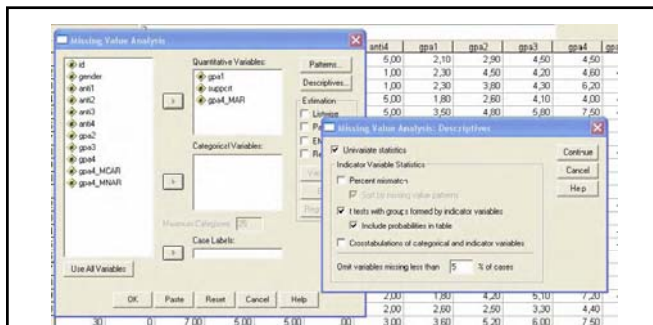


Table with **Separate Variance t Test** in output, with in rows all variables with (+5%) missings, and in columns all variables in dataset. Cell contain a t-value (+ p) indicating whether or not missingness in the row variable is correlated significantly with the values of the column variable, and therefore is selective. Check patterns of significant t-values. If not clear pattern, MAR is very likely!

When selecting EM in the 'Estimation' -menu, **Little's MCAR test** is provided (= summary of t-tests above). If not significant: MCAR! If X^2/df (normed X^2) < 2: MAR.

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PREVENTING!

- **Dillman (1978)**
 - ✓ Intensive follow-up and tracking of subjects
 - ✓ Repeated invitations to participation, reminders
 - ✓ Repeated sending of the measurements
 - ✓ Do everything to prevent large dropout!
- **Planned missingness**
 - ✓ Do not measure all variables in all participants at all times.
- **Cohort-sequential design!!**
 - ✓ Let new persons come in at each wave of the study,
 - ✓ This way you create different patterns of missingness, not only dropout!

= Different ways to increase the chances of MAR or MCAR!

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CURING!

- **Purpose is NOT to fill in empty cells in the data!**
- **Purpose IS to estimate the population parameters as good as possible, using a sample with missing data!**

Which methods can help us in this challenging task?

BAD ways to deal with missing data

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- **List-wise Deletion**
 - Variances biased, means biased
 - *Acceptable only if power is not an issue and the incomplete data is MCAR*
- **Pair-wise Deletion**
 - Variances biased, means biased
 - *Acceptable only if power is not an issue and the incomplete data is MCAR*
- **Sample-wise Mean Substitution**
 - Variances reduced, correlations biased
 - *Never acceptable!*
- **Subject-wise Mean Substitution**
 - Depends on homogeneity of the items used
 - *Acceptable only if set of items is homogeneous and only few missings!*

QUESTIONABLE ways to deal with missing data

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- **Regression Imputation**
 - All subjects with same values on IV get the same estimated value on the DV.
 - Variances reduced
 - *Assumes MCAR*
- **Stochastic Regression Imputation**
 - Same as above but a random error component is added to reduce the loss in variance
 - *Still assumes MCAR*

GOOD ways to deal with missing data

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- **But only if enough variables related to missingness are included in analysis (MAR), or missingness is MCAR**
- **EM Imputation**
 - Imputes the missing data values in an iterative way, starting with the E step
 - The E(stimation)-step is a stochastic regression-based imputation for each variable.
 - The M(aximization)-step is to calculate a complete covariance matrix based on the estimated values.
 - The E-step is repeated for each variable but the regression is now on the covariance matrix estimated in the previous M-step.
 - The EM-steps are repeated until the imputed estimates don't differ from one iteration to the other

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- **But only if enough variables related to missingness are included in analysis (MAR), or missingness is MCAR**
- **Multiple Imputation**
 - Estimate N (e.g., 5) datasets using the EM algorithm
 - Each dataset is based on a kind of resampling of the original sample (equivalent to a random selection of a different sample from the population)
 - Possible way 1:
 - Run the analyses N times
 - Summarize the results of these N analyses using the formulas of Rubin (1987)
 - Possible way 2:
 - Collapse the N samples to one dataset and do the analyses.

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- **But only if enough variables related to missingness are included in analysis (MAR), or missingness is MCAR, but even in cases of MNAR!**
- **Full Information Maximum Likelihood**
 - Sufficient statistics (means, covariances) are estimated with the Expectation Maximization (EM) algorithm
 - Those estimates then serve as the start values for the Maximum Likelihood model estimation
 - Does not impute the missing values.
 - Can only be used when testing a SEM-model.
 - Available in Lisrel, AMOS, Mplus, EQS, etc.

EXAMPLES comes with LGC and other models.

CHANGE_{t4} = ANTI_{t4} - ANTI_{t1}

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- Though many problems with it, still popular (e.g., intervention, pretest-posttest, or clinical studies)
- Most cited problem: Unreliability of the difference score
 - when the measures comprising the difference are only modestly reliable and positively correlated →
 - which is typically the case in longitudinal research!
- And therefore also: lack of validity
- + Change is measured without taking level into account!

[difference score.inp](#)

```
DEFINE: changel4 = anti4-antil;
MODEL: changel4 ON support;
```

MODEL RESULTS				
	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
CHANGE14 ON SUPPORT	-0.038	0.183	-0.207	0.836

r_1		0.60	0.70	0.80	0.90	1.00
		$r_1 = 0.60$				
r_{12}	0.60	0.00	0.13	0.25	0.38	0.50
	0.70	-0.33	-0.17	0.00	0.17	0.33
	0.80	-1.00	-0.75	-0.50	-0.25	0.00
	0.90	-	-	-	-	-1.00
	0.95	-	-	-	-	-
		$r_1 = 0.70$				
	0.60	0.13	0.25	0.38	0.50	0.63
	0.70	-0.17	0.00	0.17	0.33	0.50
	0.80	-0.75	-0.50	-0.25	0.00	0.25
	0.90	-	-	-	-1.00	-0.50
	0.95	-	-	-	-	-
		$r_1 = 0.80$				
	0.60	0.25	0.38	0.50	0.63	0.75
	0.70	0.00	0.17	0.33	0.50	0.67
	0.80	-0.50	-0.25	0.25	0.50	0.75
	0.90	-	-	-	-1.00	-0.50
	0.95	-	-	-	-	-
		$r_1 = 0.90$				
	0.60	0.38	0.50	0.63	0.75	0.88
	0.70	0.17	0.33	0.50	0.67	0.83
	0.80	-0.25	0.00	0.25	0.50	0.75
	0.90	-	-	-1.00	0.00	0.50
	0.95	-	-	-	-1.00	0.00
		$r_1 = 1.00$				
	0.60	0.50	0.63	0.75	0.88	1.00
	0.70	0.33	0.50	0.67	0.83	1.00
	0.80	0.00	0.25	0.50	0.75	1.00
	0.90	-	-1.00	0.00	0.50	1.00
	0.95	-	-	-1.00	0.00	1.00

CHANGE_{t4} = ANTI_{t4} - ANTI_{t1}

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- Exercise!

Though many problems with it, still popular (e.g., intervention, pretest-posttest, or clinical studies)

Most cited problem: Unreliability of the difference score

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- which is typically the case in longitudinal research!

And therefore also: lack of validity

+ Change is measured without taking level into account!

[difference score.inp](#)

```
DEFINE: changel4 = anti4-anti1;
MODEL: changel4 ON support;
```

MODEL RESULTS				
	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
CHANGE14 ON SUPPORT	-0.098	0.189	-0.207	0.836

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CHANGE = effect of TIME in a repeated ANOVA

- SPSS output: [R ANOVA.spv](#)
- So, a good method
 - To describe and test an overall mean change function, and test for the form of it (linear, quadratic, cubic, etc.)
 - To test for the effect of covariates on the change function (e.g., support): Time x Support interactions!
 - To test for the effect of between-subject factors on the change function (e.g., gender): Time x Gender interactions!
- But
 - Only tests mean change over time in the whole sample and not deviations from that mean change
 - And... group statistics (e.g., mean) represent everyone, and no one!
 - Equal intervals between measurements are necessary!
 - Change is an outcome of the repeated measures (time) and cannot be used as a predictor of outcomes.

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• With observed variables: [autoregression.inp](#)

• With latent variables! [autoregression latent.ino](#)

• But...

- implicit assumption of decreasing correlations across time!
- indifferent to the functional form of change!
- only RELATIVE change in terms of rank order!
- Interpretation?
 - Does ANTI1 predict (.86) an increase in ANTI4?
 - Increase only relative to others in the sample!
 - Even if everyone decreases!
- No trajectories of individual change over time!

Extensions: mediation: [autoregressionb.inp](#)
 moderation: [autoregression-multigroup.inp](#)

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• A multivariate extension of the autoregressive model of change: [crosslagged.inp](#)

• Interesting for examining direction of effects, by comparing cross-lagged coefficients (easily done in SEM)

• But

- Same problem with interpretation of change as in AR model!
- Also shares all other problems of the AR model!

+ Statistical drawbacks:
 If X is (much) more stable than Y, then Y will have a stronger effect on X than vice versa!
 And, the less reliable X is, the less Y can explain it.

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So,

What do we want exactly when we try to assess change?

RELIABLE estimates of change!
 Estimates of ABSOLUTE change!
 INDIVIDUAL estimates of change!

With 2 WAVES of data

Latent Change Models (LCM)
(McArdle & Nesselroade, 1994; Hertzog & Nesselroade, 2003)

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- Change in **latent** variables, using (CFA)
- First step = Longitudinal invariant factor model (multiple indicators), resulting in **reliable** scores for Anti-1 and 4. →
- Second step = Restructuring Anti-1 and Anti-4 in latent Level and Change factors, using a very simple equation:

$$\text{Anti-1} = 1 \times \text{Anti-Level}$$

$$\text{Anti-4} = 1 \times \text{Anti-Level} + 1 \times \text{Anti-Change}$$
- As a consequence:

$$\text{Anti-Change} = \text{Anti-4} - \text{Anti-1}$$

$$\text{Anti-Level} = \text{Anti-1}$$

↓

a reliable difference score !
- [lcm_ANTLine](#)

Observed variables?

The problem of measurement error
DATA = MODEL + ERROR

- True variance: correlated
- Error variance: not correlated
- Total covariance: underestimated!

Latent variables!

Solution for measurement error!

SEM = analysis with latent variables!

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- As a consequence:

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$$\text{Anti-Level} = \text{Anti-1}$$

↓

a reliable difference score !
- [lcm_ANTLIine](#)

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- Main advantages:
 - Only two waves of data needed!
 - Can be extended to multiple successive change factors!
 - Reliable estimates of change!
 - Change is assessed as a latent factor!
 - With a mean: Mean change in the total sample!
 - With a variance: Individual differences in change!
 - That can be predicted and used as a predictor!
 - Latent factor scores can be estimated!
- But
 - Only linear change function!

With 3+ WAVES of data

Latent Growth Curve Models (LGCM)
(Duncan et al., 1994; McArdle & Nesselroade, 2002; Willet & Sayer, 1994)

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- Questions:
 - Does an individual characteristic (e.g., antisocial behavior) change over time?
 - Which trajectory is followed?
 - Interindividual differences?
- Step 1: Within-Person
 - Equation for every subject in the sample:

$$\text{anti} = \text{intercept} + (\text{slope} \times \text{Time}) + \text{error} \quad (\text{regression})$$
 - Growth can be non-linear too!

$$\text{anti} = \text{intercept} + (\text{slope} \times \text{Time}) + (\text{curve} \times \text{Time}^2) + \text{error}$$
 - Assumption: Individuals share the shape of the change function (e.g., linear), but can differ in the amount or rate of change (individual growth parameters: intercept, slope, etc.)
- Step 2: Between-Person
 - Means (fixed) & variances (random) of intercepts, slopes
 - Predictors of change (conditional growth models).

With 3+ WAVES of data

Latent Growth Curve Models (LGCM)
 (Duncan et al., 1994; McArdle & Nesselroade, 2002; Willet & Sayer, 1994)

time (age)	anti
0 (13)	3
1 (14)	3
2 (15)	5
3 (16)	5

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 (Duncan et al., 1994; McArdle & Nesselroade, 2002; Willet & Sayer, 1994)

time (age)	anti
0 (13)	2
1 (14)	3
2 (15)	5
3 (16)	6

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With 3+ WAVES of data

Latent Growth Curve Models (LGCM)
 (Duncan et al., 1994; McArdle & Nesselroade, 2002; Willet & Sayer, 1994)

anti = 2 + (1.5 x time) + error

time (age)	anti
0 (13)	2
1 (14)	3
2 (15)	5
3 (16)	6

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 - Difference score
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- Exercise !

With 3+ WAVES of data

Latent Growth Curve Models (LGCM)
(Duncan et al., 1994; McArdle & Nesselroade, 2002; Willet & Sayer, 1994)

$anti = 1 + (1.5 \times time) + (.5 \times time^2) + error$

- General measurement and design issues
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Two Approaches to GCM

Multilevel	SEM: LGCM
- regression approach	- factor approach
- fixed & random effects	- means & variances
- flexible with missing values & individual-varying intervals	- not that flexible with individual-varying intervals
- less flexible in prediction	- very flexible in prediction
$length = b_{10} + b_{11} \times gender + error_{i0} + (b_{10} + b_{11} \times gender + error_{i1}) \times time + error$	
- little attention for model fit	- overload of fit indices

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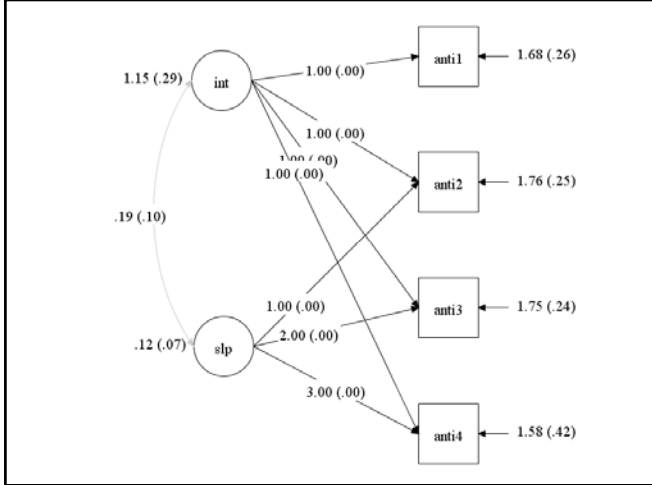
Latent Growth Curve Models (LGCM)
(Duncan et al., 1994; McArdle & Nesselroade, 2002; Willet & Sayer, 1994)

PARAMETERS in the model

- Mean intercept / fixed effect intercept
= Mean initial level of all individuals
- Variance intercept / random effect intercept
= Interindividual differences in initial level
- Mean slope / fixed effect slope
= Mean rate of growth across individuals
- Variance slope / random effect slope
= Interindividual differences in rate of change

[lgcm_anti.inp](#) (with missing data, and FIML)

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= Interindividual differences in rate of change

[lgcm_anti.inp](#) (with missing data, and FIML)
[lgcm_anti + predictors.inp](#) (gender and support as predictors)
[lgcm_anti + predictors + interaction.inp](#) (support X level anti)
[lgcm_anti - piecewise.inp](#) (piecewise model with 2 slopes)

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• Can be extended to a multivariate LGCM!
[lgcm_anti + gpa.inp](#)

- Correlated intercepts = cross-sectional association
- Correlated intercept & slope = level of IV is predicting rates of change in DV!
- Correlated slopes = common underlying growth in two constructs = change associated with change (causality?)

• But

- Assumption: same shape of the growth function for all subjects; interindividual differences in change are modeled as deviations from that overall mean.

With 3+ WAVES of data

Latent Growth Curve Models (LGCM)
(Duncan et al., 1994; McArdle & Nesselroade, 2002; Willet & Sayer, 1994)

Extension 1: Measurement invariance - curve of factors!

The diagram shows a latent growth curve model with three latent factors: *intercept*, *slope*, and three time-varying factors *FT1*, *FT2*, and *FT3*. *intercept* and *slope* are correlated. *intercept* and *slope* load onto all three *FT* factors. Each *FT* factor loads onto four observed variables: *FT1* onto *Y11*, *Y21*, *Y31*, *Y41*; *FT2* onto *Y12*, *Y22*, *Y32*, *Y42*; and *FT3* onto *Y13*, *Y23*, *Y33*, *Y43*. Error terms *e1* through *e12* are associated with each observed variable.

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With 3+ WAVES of data

Latent Growth Curve Models (LGCM)
(Duncan et al., 1994; McArdle & Nesselroade, 2002; Willet & Sayer, 1994)

Extension 1: Measurement invariance - curve of factors!

The diagram shows a latent growth curve model with three latent factors: *F_{T1}*, *F_{T2}*, and *F_{T3}*. *F_{T1}*, *F_{T2}*, and *F_{T3}* are correlated. Each *F_T* factor loads onto four observed variables: *F_{T1}* onto *Y₁₁*, *Y₂₁*, *Y₃₁*, *Y₄₁*; *F_{T2}* onto *Y₁₂*, *Y₂₂*, *Y₃₂*, *Y₄₂*; and *F_{T3}* onto *Y₁₃*, *Y₂₃*, *Y₃₃*, *Y₄₃*. Loadings are labeled λ_{1-1} , λ_2 , λ_3 , and λ_4 . Error terms *e1* through *e12* are associated with each observed variable.


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With 3+ WAVES of data

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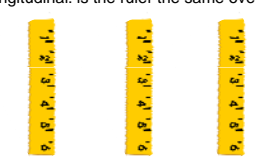
- Measurement invariance (e.g. Boy-girl)
- Longitudinal: is the ruler the same over time?
- if not: difficult to disentangle growth from change in the ruler!

With 3+ WAVES of data

Latent Growth Curve Models (LGCM)
(Duncan et al., 1994; McArdle & Nesselroade, 2002; Willet & Sayer, 1994)

Extension 1: Measurement invariance - curve of factors!

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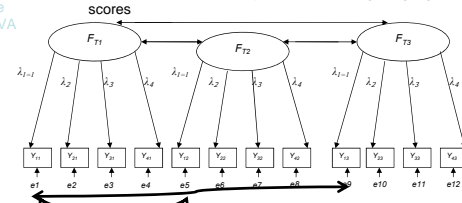
- Longitudinal: is the ruler the same over time?
- Elements?
- Procedure?

With 3+ WAVES of data

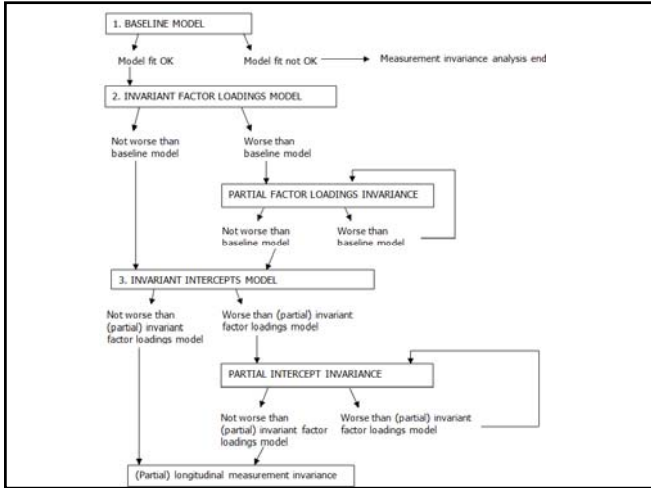
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- Note: Equal errors not required (⇔ MI boy-girl) → correlated over time



With 3+ WAVES of data

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- Models
 - [Antisocial baseline](#)
 - [Antisocial FL invariance](#)
 - [Antisocial FL+ I invariance](#)
 - [Antisocial FL+ I invariance partial](#)

With 3+ WAVES of data

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- Exercise !
- Factor loadings not invariant = problematic
- How far can you go in freeing intercepts?
 - Debate
 - Minimum 2 items for which all intercept (=number of time points) are equal

With 3+ WAVES of data

Latent Growth Curve Models (LGCM)
(Duncan et al., 1994; McArdle & Nesselroade, 2002; Willet & Sayer, 1994)

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- Measurement level of likert scales?
 - Nominal
 - Ordinal
 - Interval
 - Ratio
- Up to now: assumed interval
- Debate whether ordinal?!
 - More serious with lower number of scale points (e.g. 3 or less)

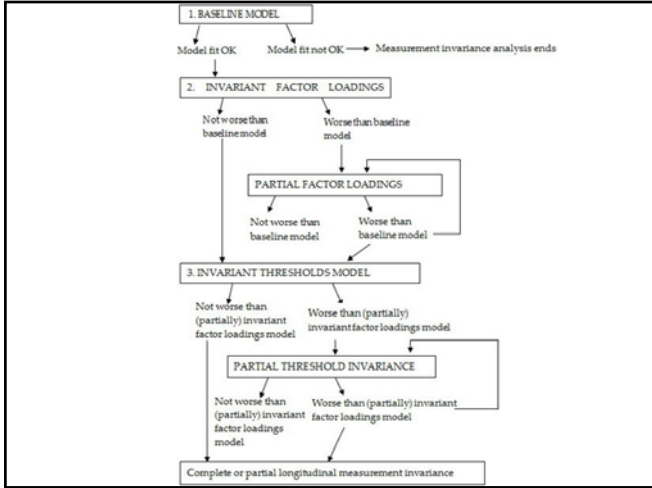
With 3+ WAVES of data

Latent Growth Curve Models (LGCM)
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- Consequence
 - Not 1 intercept but thresholds
 - Number of thresholds = likert points - 1
 - E.g. threshold 1= difficulty level of going from score 1 to score 2; threshold 4= difficulty level of going from score 4 to score 5
 - WLSMV estimator
 - can't easily compare Chi²
 - DIFFTEST



With 3+ WAVES of data

Latent Growth Curve Models (LGCM)
(Duncan et al., 1994; McArdle & Nesselroade, 2002; Willet & Sayer, 1994)

Extension 1: Measurement invariance - curve of factors!

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• Models

- [Antisocial_baselineWLSMV](#)
 - SAVEDATA: DIFFTEST IS deriv.dat
 - ANALYSIS: DIFFTEST IS deriv.dat
- [Antisocial_FL_invariance_WLSMV](#)
 - SAVEDATA: DIFFTEST IS deriv.dat
 - ANALYSIS: DIFFTEST IS deriv.dat
- [Antisocial_FL_invariance_WLSMV_2](#)
 - SAVEDATA: DIFFTEST IS deriv.dat
 - ANALYSIS: DIFFTEST IS deriv.dat
- [Antisocial_FL+Threshold_invariance_WLSMV](#)
 - SAVEDATA: DIFFTEST IS deriv.dat
 - ANALYSIS: DIFFTEST IS deriv.dat

With 3+ WAVES of data

Latent Growth Curve Models (LGCM)
(Duncan et al., 1994; McArdle & Nesselroade, 2002; Willet & Sayer, 1994)

Extension 2: Cohort sequential design!

= mixing cross-sectional & longitudinal design

- remember: CS-design → MAR

- Many longitudinal studies have multiple cohorts.

- Example: PhD of 4 years, but wanting to measure antisocial behavior from age 13 to 20 (8 years)!

Cohort	Time			
	T1	T2	T3	T4
1 (1985)	13	14	15	16
2 (1983)	15	16	17	18
3 (1981)	17	18	19	20

→ Linking adjacent segments of change from different cohorts to estimate a common growth curve

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With 3+ WAVES of data

Latent Growth Curve Models (LGCM)
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Extension 2: Cohort sequential design!

Method 1: Multigroup modeling

Cohort	Time			
	T1	T2	T3	T4
1 (1985)	13	14	15	16
2 (1983)	15	16	17	18
3 (1981)	17	18	19	20

- Growth curve for each cohort, but:

- Fix mean and variance of intercept and slope equal across cohorts
- Fix int-slp correlation equal across cohorts
- Adjust slope factor loadings of anti in line with cohort or birthyear
 - Cohort 1 (1985): 0 1 2 3
 - Cohort 2 (1983): 2 3 4 5
 - Cohort 3 (1981): 4 5 7 8

[cohort-sequential growth model anti.inp](#) (linear)

[cohort-sequential growth model antib.inp](#) (curvilinear)

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With 3+ WAVES of data

Latent Growth Curve Models (LGCM)
(Duncan et al., 1994; McArdle & Nesselroade, 2002; Willet & Sayer, 1994)

Extension 2: Cohort sequential design!

Method 2: Data rearrangement (DATA COHORT)

	Age							
Cohort	13	14	15	16	17	18	19	20
1 (1985)	X	X	X	X	MCAR	MCAR	MCAR	MCAR
2 (1983)	MCAR	MCAR	X	X	X	X	MCAR	MCAR
3 (1981)	MCAR	MCAR	MCAR	MCAR	X	X	X	X

- Estimate 1 intercept and 1 slope, using all available data
 - Works out the time score based on birth and measurement year
 - Idea = rearrange our cohort & time data to age data
 - Then a growth curve for the complete age span!
 - Only works with continuous variables!

[cohort-sequential growth model anti 2.inp](#) (linear)
[cohort-sequential growth model anti 2b.inp](#) (curvilinear)
 !? [cohort-sequential growth model anti 2_piecewise.inp](#)

With 3/4+ WAVES of data

Latent Growth Mixture Modeling (LGMM)
(Muthén, 2001, 2004)

and a special case of it:
Latent Class Growth Analysis (LCGA)
(Nagin, 1999, 2001)

- Cfr. LGCM: heterogeneity in the sample is captured by variation around a mean growth function, i.e., it is assumed that all individuals are drawn from the same population.
- LGMM
 - relaxes this single population assumption to allow for parameter differences across unobserved subpopulations
 - by using a combination of continuous and categorical latent variables (mixed)
 - by introducing a latent class variable C, a trajectory class variable, representing k unobserved subpopulations in the sample (note that in LGCM k = 1)
 - by estimating a separate growth model for each of the latent classes
 - and estimating latent class conditional probabilities (membership)

With 3/4+ WAVES of data

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With 3/4+ WAVES of data
Latent Growth Mixture Modeling (LGMM)
Latent Class Growth Analysis (LCGA)

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- **LGMM**: Class differences
 - Typically between mean intercepts between classes
 - Mean slopes in the classes
 - Variance in intercept and slope
 - Shape of the growth function!
 - Influence of covariates!
- **LGMM**: Extensions
 - Including covariates of change
 - Including outcomes that are predicted from growth
- **LCGA** (Nagin)
 - Very similar, only no variances within classes are estimated (therefore a semi-parametric approach)
 - Individuals within classes are treated as homogeneous

With 3/4+ WAVES of data
Latent Growth Mixture Modeling (LGMM)
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- **Estimation**: Using the EM algorithm
 - Estimation of each individual's probability of membership in each class (conditional probabilities)
 - Measures of fit and classification quality:
 - BIC
 - Small values correspond to a good model with a large likelihood and not too many parameters
 - Look at the big drops in BIC from one solution to another!
 - Sensitive to the number of classes!
 - Less sensitive to differences in growth shape between classes
 - LMR-LR test
 - Test of a solution with k-1 classes against a solution with k classes (e.g. 2 vs 1); low p-value indicates that solution k-1 should be rejected in favor of the solution with k classes.
 - Entropy
 - Measure of classification quality based on the individual class probabilities.
 - High values (closer to 1) indicate good classification.
- **Examples with 2 classes**:
 - [lcga_anti2.inp](#) (Nagin approach)
 - [lgmm_anti2.inp](#) (with equal variances across classes)
 - [lgmm_anti2free.inp](#) (with free variances across classes)

With 4+ WAVES of data
Latent Growth Mixture Modeling (LGMM)
Latent Class Growth Analysis (LCGA)

Antisocial data: fit statistics

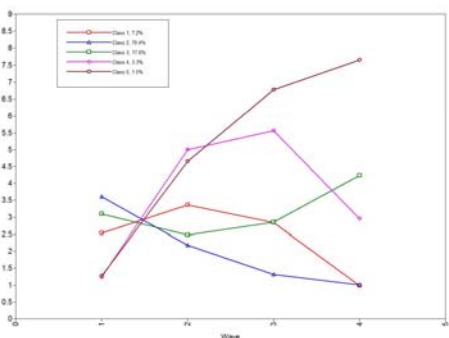
k	LCGA			LGMM		
	BIC	entropy	pLMRT	BIC	entropy	pLMRT
2i	4707.35	.97	.000	4653.78	.95	.002
3i	4681.69	.80	.030	4635.85	.79	.332
4i	4685.03	.81	.248	4625.88	.83	.154
2c	4533.67	.97	.000	4506.76	.99	.000
3c	4497.46	.82	.045	4487.26	.83	.411
4c	4476.20	.82	.014	4475.70	.85	.230
5c	4462.39	.83	.006			
6c	4460.83	.83	.426			

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With 4+ WAVES of data
Latent Growth Mixture Modeling (LGMM)
Latent Class Growth Analysis (LCGA)

Antisocial data: Best solution: 5 classes, LCGA, curvilinear



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With 4+ WAVES of data
Latent Growth Mixture Modeling (LGMM)

Antisocial data: Best solution: lcga_anti5c.dat

id	u1	u2	u3	u4	u5	u6	u7	u8	u9	u10	u11	u12	u13	u14	u15	u16	u17	u18	u19	u20
1	4.000	2.500	1.000	0.000	0.502	0.936	0.062	0.000	0.000	0.000	2.000									
2	1.000	2.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	1.000									
3	1.000	1.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000									
4	1.000	1.000	0.000	1.000	0.000	0.000	0.991	0.008	0.000	0.000	1.000									
5	2.250	1.000	0.000	0.000	0.000	0.978	0.022	0.000	0.000	0.000	2.000									
6	1.750	1.000	0.000	2.000	0.001	0.976	0.023	0.000	0.000	0.000	2.000									
7	1.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000									
8	1.750	2.833	0.000	0.000	0.000	0.198	0.803	0.001	0.000	0.000	2.000									
9	1.000	1.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000									
10	1.750	1.500	0.000	2.000	0.000	0.991	0.009	0.001	0.000	0.000	2.000									
11	1.750	1.000	1.000	0.000	0.001	0.999	0.000	0.000	0.000	0.000	2.000									
12	1.000	2.133	0.000	1.000	0.000	0.004	0.996	0.000	0.000	0.000	2.000									
13	1.000	2.187	0.000	1.000	0.001	0.999	0.001	0.000	0.000	0.000	2.000									
14	1.000	2.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000									
15	1.000	1.133	0.000	2.000	0.000	0.999	0.001	0.001	0.000	0.000	2.000									
16	1.000	1.187	0.000	1.000	0.001	0.991	0.009	0.000	0.000	0.000	2.000									
17	1.750	1.000	0.000	0.000	0.002	0.827	0.173	0.000	0.000	0.000	2.000									
18	1.000	2.133	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000									
19	1.750	1.887	1.000	2.000	0.000	0.991	0.009	0.000	0.000	0.000	2.000									
20	1.000	1.000	0.000	1.000	0.000	0.000	0.991	0.009	0.000	0.000	2.000									
21	1.750	4.187	1.000	1.000	0.000	0.930	0.070	0.000	0.000	0.000	1.000									
22	1.000	1.833	0.000	2.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000									
23	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000									
24	2.250	2.133	0.000	1.000	0.007	0.993	0.007	0.000	0.000	0.000	2.000									
25	1.750	2.000	0.000	2.000	0.018	0.982	0.018	0.000	0.000	0.000	2.000									
26	1.000	2.133	0.000	4.000	0.000	0.806	0.194	0.000	0.000	0.000	2.000									
27	1.750	1.833	0.000	1.000	0.000	0.999	0.001	0.000	0.000	0.000	2.000									
28	1.750	2.187	0.000	2.000	0.000	0.991	0.009	0.000	0.000	0.000	2.000									

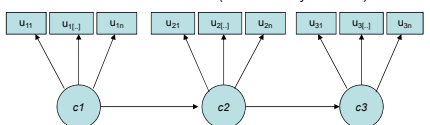
Observed scores Individual estimated class probabilities Classification



- General measurement and design issues
 - Measurement
 - Time & Intervals
 - Differential growth
 - Missing values
- Classic methods and disadvantages
 - Difference score
 - Repeated ANOVA
 - Autoregression
 - Cross-lagged models
- New(er) methods
 - LCM
 - LGCM
 - LCGA
 - LGMM
 - LTA
- Exercise !

With 3/4+ WAVES of data
Latent Transition Analysis (LTA)
 (Kaplan, 2008; Meeus et al., 2010)

- A longitudinal extension of LCA or LPA
 - first, uses **class-specific parameters** (the continuous/categorical observed multidimensional scores) as measurement parameters (~ LCA or cluster analysis)
 - invariant across time!
 - and, uses **class probabilities as structural parameters** to estimate the number of participants in each of the classes
 - then using **latent transition probabilities to calculate patterns of stability and change over time** in the movement or transition between classes (~ mover-stayer model)



→ An ideal model for testing typological or person-oriented developmental theories!



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• Exercise !

With 3/4+ WAVES of data
Latent Transition Analysis (LTA)
 (Kaplan, 2008; Meeus et al., 2010)

- Example: data on parenting across three waves
 - 659 adolescents, measured 3 times: 13, 15 and 17 years
 - psychological control, support/warmth, firm control (continuous)
 - scores standardized (z) to ease interpretation
- hypotheses: at least 4 latent classes (parenting styles) an gradually better parenting from 13 (peak of puberty years; reactive parenting) to 17 years
- estimated using Robust Maximum Likelihood estimation
- models with different # latent classes fitted and compared
- decision based on BIC, entropy, and interpretability

[lta parenting4.inp](#)

k	BIC	entropy
3	15308.59	.93
4	15016.72	.92
5	14982.35	.90
6	15324.43	.92

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• Exercise !

With 3/4+ WAVES of data
Latent Transition Analysis (LTA)
 (Kaplan, 2008; Meeus et al., 2010)

- Selected output k = 5 solution
 - Measurement part + latent class probabilities

	Psychological Control	Support	Firm Control	13	15	17
Neglecting	-0.38	-0.11	-0.54	49%	29%	30%
Permissive	-0.67	1.21	0.64	24%	18%	19%
Rejecting	0.34	-1.17	-1.19	10%	8%	9%
Controlling	1.00	-0.62	-0.07	15%	16%	16%
Democratic	-1.66	1.36	1.23	1%	28%	26%

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• Exercise !

With 3/4+ WAVES of data
Latent Transition Analysis (LTA)
 (Kaplan, 2008; Meeus et al., 2010)

- Selected output k = 5 solution
 - Transition probabilities

		Transition Probabilities into latent classes at T+1				
		Neglecting	Permissive	Rejecting	Controlling	Democratic
Neglecting 1		.574	.020	.000	.006	.400
Permissive 1		.000	.728	.000	.008	.264
Rejecting 1		.090	.000	.813	.000	.097
Controlling 1		.000	.000	.000	.974	.026
Democratic 1		.000	.000	.000	.272	.728
Neglecting 2		.971	.003	.000	.000	.026
Permissive 2		.000	.894	.000	.004	.102
Rejecting 2		.027	.000	.953	.000	.021
Controlling 2		.007	.000	.006	.870	.117
Democratic 2		.033	.094	.026	.086	.761

To conclude

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 - Measurement
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- Since two decades, we have interesting new methods to analyze change and development!
- *Mplus* provides a powerful tool to analyze change and development, and is constantly improving!
- And, as said before...

• Exercise !

Change is inevitable. Change is constant.
(Benjamin Disraeli)

Change is the nursery of music, joy, life, and eternity.
(John Donne)

When you're finished changing, you're finished.
(Benjamin Franklin)

We are restless because of incessant change, but we would be frightened if change were stopped.
(Lyman Bryson)

Change is a measure of time.
(Edwin Way Teale)

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1. Check the direction of effects (using a cross-lagged model) between GPA and antisocial behavior, using data of Times 1 and 3. check whether results are the same for boys and girls. DATA are SEMDATA.DAT; software = Mplus 7.3
2. Estimate a LGCM of GPA, using the FIML approach for missing data. Check models with linear and curvilinear change. Interpret the parameters that are found. DATA are SEMDATA.DAT; software = Mplus 7.3
3. Find the optimal LCGA/LGMM solution of the GPA data. Explain why this solution was chosen and interpret the different classes. DATA are SEMDATA.DAT; software = Mplus 7.3
4. Evaluate the effect of mother support on development of GPA, using a conditional growth model and FIML. DATA are SEMDATA.DAT; software = Mplus 7.3
5. Test a measurement invariant LGCM on the antisocial data, an evaluate whether this is better than a constant trend. If time allows also experiment/play with factor loadings of time, to model different intervals between waves. DATA are ANTISOCIAL.CSV; software = Mplus 7.3

- Setup and do the analyses using Mplus!
- Ask for help while doing the analyses!
- Present the results to the audience, using a single or two slides and explain the effects in words!

• Exercise !

Informative websites

- www.statmodel.com: thé *Mplus* site!
- <http://davidakenny.net/cm/causalm.htm>: great SEM page!
- <http://www.ats.ucla.edu/stat/seminars/>: online examples and videos on *Mplus*
- <http://users.ugent.be/~wbeyers/workshop/index.html>: the website for this workshop!

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