FACULTY OF ENGINEERING AND ARCHITECTURE

DEPARTMENT OF INFORMATION TECHNOLOGY
ELECTROMAGNETICS RESEARCH GROUP

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A DC STABLE, WELL-CONDITIONED AND LOW-FREQUENCY REGULARIZED TIME-DOMAIN PMCHWT EQUATION*

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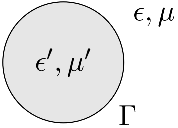
IDLab INTERNET & DATA LAB

*A joint work with F. P. Andriulli and K. Cools.

Time-domain PMCHWT equation

$$\begin{pmatrix} \eta\mathcal{T} + \eta'\mathcal{T}' & \mathcal{K} + \mathcal{K}' \\ -\mathcal{K} - \mathcal{K}' & \frac{1}{\eta}\mathcal{T} + \frac{1}{\eta'}\mathcal{T}' \end{pmatrix} \begin{pmatrix} \boldsymbol{j} \\ \boldsymbol{m} \end{pmatrix} = - \begin{pmatrix} \mathbf{n} \times \boldsymbol{e}^{in} \\ \mathbf{n} \times \boldsymbol{h}^{in} \end{pmatrix},$$



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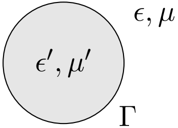

where $\eta = \sqrt{\mu/\epsilon}$, $c = 1/\sqrt{\mu\epsilon}$, and

$$\mathcal{T}\boldsymbol{j} = \partial_t \mathcal{T}^s \boldsymbol{j} + \partial_t^{-1} \mathcal{T}^h \boldsymbol{j}, \qquad \mathcal{K}\boldsymbol{j} = -\mathbf{n} \times \mathbf{curl}_{\mathbf{x}}(G * \boldsymbol{j}),$$

$$\mathcal{T}^s \boldsymbol{j} = -\frac{1}{c}\mathbf{n} \times (G * \boldsymbol{j}), \qquad \mathcal{T}^h \boldsymbol{j} = c\,\mathbf{n} \times \mathbf{grad}_{\mathbf{x}}(G * \mathrm{div}_\Gamma \boldsymbol{j}).$$

$$(G * \boldsymbol{j})(\boldsymbol{x}, t) := \int_\Gamma \frac{\boldsymbol{j}(\boldsymbol{y}, t - |\boldsymbol{x} - \boldsymbol{y}|/c)}{4\pi |\boldsymbol{x} - \boldsymbol{y}|} \, \mathrm{d}s_{\boldsymbol{y}}.$$

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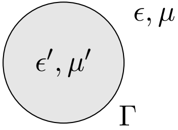

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Why Time-domain? TD allows coupling to non-linear systems.

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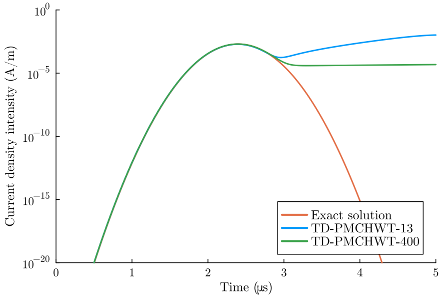
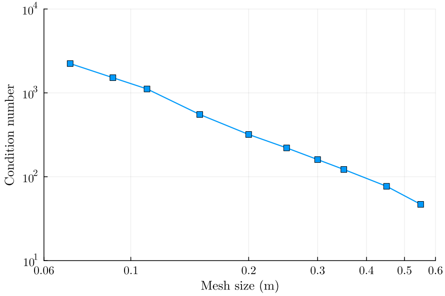
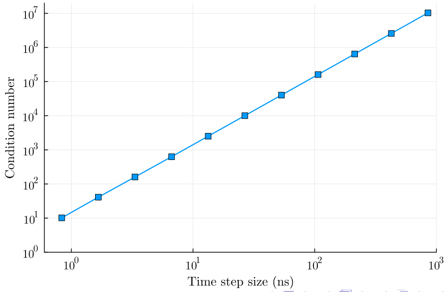
Numerical issues

- Late-time (DC) instabilities
- Ill-conditioning at dense meshes
- Ill-conditioning at low frequencies







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Dense-mesh preconditioner

The "static EFIE" operator

$$T_0 = T_0^s - T_0^h,$$

where

$$(T_0^s \boldsymbol{j})(\boldsymbol{x}) = \mathbf{n} \times \int_\Gamma \frac{\boldsymbol{j}(\boldsymbol{y})}{4\pi R} \, \mathrm{d}s'_{\boldsymbol{y}},$$

$$(T_0^h \boldsymbol{j})(\boldsymbol{x}) = \mathbf{n} \times \mathbf{grad}_{\mathbf{x}} \int_\Gamma \frac{\mathrm{div}_\Gamma \, \boldsymbol{j}(\boldsymbol{y})}{4\pi R} \, \mathrm{d}s'_{\boldsymbol{y}}.$$

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Properties of T_0:

- ✓ is independent of time
- ✓ is an elliptic operator $\Rightarrow$ free from resonant frequencies
- ✓ can serve as a preconditioner at dense meshes
- ☐ does not precondition at low frequencies.

Dense-mesh preconditioner (cont.)

The bilinear form of T_0

$$\langle \boldsymbol{g}, T_0 \boldsymbol{f} \rangle := (\mathbf{n} \times \boldsymbol{g}, T_0 \boldsymbol{f})_{\mathrm{L}^2(\Gamma)},$$

can be Helmholtz decomposed into loop and star components

$$\langle \boldsymbol{g}, T_0 \boldsymbol{f} \rangle = \begin{matrix} & \mathbf{P}^L & \mathbf{P}^S \\ \mathbf{P}^L & T_0^s & T_0^s \\ \mathbf{P}^S & T_0^s & T_0^s - T_0^h \end{matrix} \implies \begin{matrix} & \mathbf{P}^L & \mathbf{P}^S \\ \mathbf{P}^L & T_0^s & 0 \\ \mathbf{P}^S & 0 & -T_0^h \end{matrix} = \langle \boldsymbol{g}, Z_0 \boldsymbol{f} \rangle,$$

✓ Z_0 inherits all properties of T_0, but it is "diagonal".

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Dense-mesh preconditioner (cont.)

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✓ Z_0 inherits all properties of T_0, but it is "diagonal".

The preconditioned TD-PMCHWT formulation

$$Z_0 \begin{pmatrix} \eta \mathcal{T} + \eta' \mathcal{T}' & \mathcal{K} + \mathcal{K}' \\ -\mathcal{K} - \mathcal{K}' & \frac{1}{\eta} \mathcal{T} + \frac{1}{\eta'} \mathcal{T}' \end{pmatrix} \begin{pmatrix} \boldsymbol{j} \\ \boldsymbol{m} \end{pmatrix} = -Z_0 \begin{pmatrix} \mathbf{n} \times \boldsymbol{e}^{in} \\ \mathbf{n} \times \boldsymbol{h}^{in} \end{pmatrix} .$$

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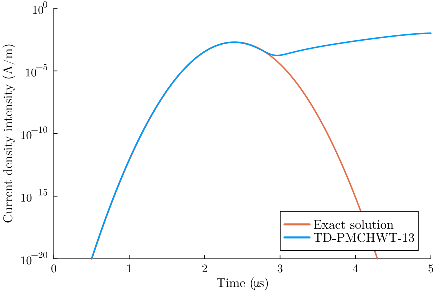
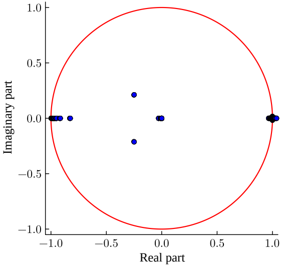
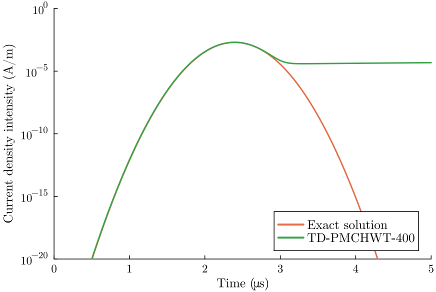
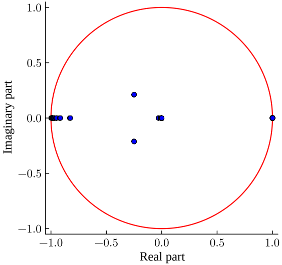
Late-time (DC) instabilities









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Late-time (DC) instabilities (cont.)

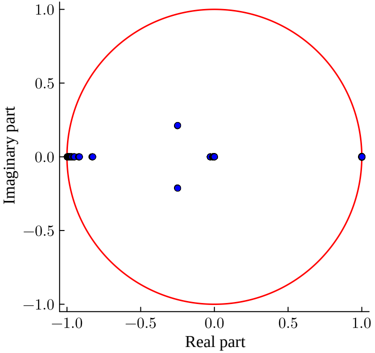
The TD-PMCHWT operator

$$\mathcal{Q} := \begin{pmatrix} \eta\mathcal{T} + \eta'\mathcal{T}' & \mathcal{K} + \mathcal{K}' \\ -\mathcal{K} - \mathcal{K}' & \frac{1}{\eta}\mathcal{T} + \frac{1}{\eta'}\mathcal{T}' \end{pmatrix}.$$

The origin of DC instabilities:

- In theory, there is no sourceless nullspace
- In practice, it is observed (on sphere)

$$N_{\text{poles at 1}} = 2N_{\text{sol}}.$$



Hypotheses:

- ✓ The impacts of $\mathcal{K}$ and $\mathcal{K}'$ are insignificant
- ✓ DC instabilities of TD-PMCHWT are of TD-EFIE.

Rescaling procedure

The preconditioned TD-PMCHWT operator

$$Z_0\,\mathcal{Q} := Z_0 \begin{pmatrix} \eta\mathcal{T} + \eta'\mathcal{T}' & \mathcal{K} + \mathcal{K}' \\ -\mathcal{K} - \mathcal{K}' & \frac{1}{\eta}\mathcal{T} + \frac{1}{\eta'}\mathcal{T}' \end{pmatrix}.$$

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Rescaling procedure

$$\left\langle \left(\mathbf{P}^L + \partial_t^{-1} \, \mathbf{P}^S \right) \boldsymbol{g}, \, Z_0 \, \mathcal{Q} \left(\mathbf{P}^L + \partial_t \, \mathbf{P}^S \right) \boldsymbol{f} \right\rangle.$$

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Rescaling procedure

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Rescaling procedure

$$\left\langle \left(\mathbf{P}^L + \partial_t^{-1}\,\mathbf{P}^S\right)\boldsymbol{g},\, Z_0\,\mathcal{Q}\left(\mathbf{P}^L + \partial_t\,\mathbf{P}^S\right)\boldsymbol{f}\right\rangle.$$

The diagonal blocks

$$\begin{array}{cc} & \begin{array}{cc} \mathbf{P}^L & \partial_t\,\mathbf{P}^S \end{array} \\ \begin{array}{c} \mathbf{P}^L \\ \partial_t^{-1}\,\mathbf{P}^S \end{array} & \begin{bmatrix} Z_0\mathcal{T} & Z_0\mathcal{T} \\ Z_0\mathcal{T} & Z_0\mathcal{T} \end{bmatrix} \end{array} = \begin{array}{cc} & \begin{array}{cc} \mathbf{P}^L & \mathbf{P}^S \end{array} \\ \begin{array}{c} \mathbf{P}^L \\ \mathbf{P}^S \end{array} & \begin{bmatrix} T_0^s\,\mathbf{P}^L\,\partial_t\mathcal{T}^s & T_0^s\,\mathbf{P}^L\left(\partial_t^2\mathcal{T}^s + \mathcal{T}^h\right) \\ -T_0^h\,\mathbf{P}^S\,\mathcal{T}^s & -T_0^h\,\mathbf{P}^S\,\partial_t\mathcal{T}^s \end{bmatrix} \end{array}$$

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Truncating the infinite tail

The off-diagonal blocks

$$\begin{array}{c} \\ \mathbf{P}^L \\ \partial_t^{-1}\,\mathbf{P}^S \end{array} \begin{array}{c} \begin{array}{cc} \mathbf{P}^L & \partial_t\,\mathbf{P}^S \end{array} \\ \begin{bmatrix} Z_0\mathcal{K} & Z_0\mathcal{K} \\ Z_0\mathcal{K} & Z_0\mathcal{K} \end{bmatrix} \end{array} = \begin{array}{c} \\ \mathbf{P}^L \\ \mathbf{P}^S \end{array} \begin{array}{c} \begin{array}{cc} \mathbf{P}^L & \mathbf{P}^S \end{array} \\ \begin{bmatrix} Z_0\mathcal{K} & Z_0\,\partial_t\mathcal{K} \\ Z_0\,\partial_t^{-1}\mathcal{K} & Z_0\mathcal{K} \end{bmatrix} \end{array}$$

which give rise to the infinite tail.

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Truncating the infinite tail

The off-diagonal blocks

$$\begin{array}{c} \mathbf{P}^L \\ \partial_t^{-1}\,\mathbf{P}^S \end{array} \overset{\begin{array}{cc} \mathbf{P}^L & \partial_t\,\mathbf{P}^S \end{array}}{\begin{bmatrix} Z_0\mathcal{K} & Z_0\mathcal{K} \\ Z_0\mathcal{K} & Z_0\mathcal{K} \end{bmatrix}} = \begin{array}{c} \mathbf{P}^L \\ \mathbf{P}^S \end{array} \overset{\begin{array}{cc} \mathbf{P}^L & \mathbf{P}^S \end{array}}{\begin{bmatrix} Z_0\mathcal{K} & Z_0\,\partial_t\mathcal{K} \\ Z_0\,\partial_t^{-1}\mathcal{K} & Z_0\mathcal{K} \end{bmatrix}}$$

which give rise to the infinite tail. Fortunately, the tail has the property

$$\mathbf{P}^S\,Z_0\,K_0\,\mathbf{P}^L = -\,\mathbf{P}^S\,T_0^h\,\mathbf{P}^S\,K_0\,\mathbf{P}^L = 0,$$

where

$$(K_0\boldsymbol{j})(\boldsymbol{x}) = -\mathbf{n}\times\mathbf{curl}_{\mathbf{x}}\int_\Gamma \frac{\boldsymbol{j}(\boldsymbol{y})}{4\pi R}\,\mathrm{d}s'_{\boldsymbol{y}}.$$

✓ The infinite tail can be truncated.

Low-frequency ill-conditioning

$$\langle \boldsymbol{g},\, \mathcal{Q}\boldsymbol{f} \rangle \quad \Longrightarrow \quad \left\langle \left(\mathbf{P}^L + \partial_t^{-1}\mathbf{P}^S\right)\boldsymbol{g},\, Z_0\,\mathcal{Q}\,\left(\mathbf{P}^L + \partial_t\,\mathbf{P}^S\right)\boldsymbol{f} \right\rangle$$

$$\| \qquad\qquad \|$$

$$\mathcal{O}\left(\begin{array}{ccc|ccc} \omega & \omega & \omega & \omega^2 & \omega^2 & 1 \\ \omega & \omega & \omega & \omega^2 & 1 & 1 \\ \omega & \omega & \omega^{-1} & 1 & 1 & 1 \\ \hline \omega^2 & \omega^2 & 1 & \omega & \omega & \omega \\ \omega^2 & 1 & 1 & \omega & \omega & \omega \\ 1 & 1 & 1 & \omega & \omega & \omega^{-1} \end{array}\right) \quad \Longrightarrow \quad \mathcal{O}\left(\begin{array}{ccc|ccc} 1 & 1 & \omega & \omega & \omega & 1 \\ \omega & \omega & 1 & 1 & 1 & \omega \\ \omega & \omega & 1 & 1 & 1 & \omega \\ \hline \omega & \omega & 1 & 1 & 1 & \omega \\ 1 & 1 & \omega & \omega & \omega & 1 \\ 1 & 1 & \omega & \omega & \omega & 1 \end{array}\right).$$

✓ The rescaled TD-PMCHWT equation is well-conditioned at low frequencies.

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The regularized TD-PMCHWT formulation

The standard TD-PMCHWT equation

$$\mathcal{Q}\begin{pmatrix} \boldsymbol{j} \\ \boldsymbol{m} \end{pmatrix} = -\begin{pmatrix} \mathbf{n} \times \boldsymbol{e}^{in} \\ \mathbf{n} \times \boldsymbol{h}^{in} \end{pmatrix}.$$

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The regularized TD-PMCHWT equation

$$\left\langle \left(\mathbf{P}^L + \partial_t^{-1}\,\mathbf{P}^S\right)\begin{pmatrix}\boldsymbol{\varphi}\\ \boldsymbol{\xi}\end{pmatrix}, Z_0\mathcal{Q}\left(\mathbf{P}^L + \partial_t\,\mathbf{P}^S\right)\begin{pmatrix}\boldsymbol{p}\\ \boldsymbol{q}\end{pmatrix}\right\rangle = -\left\langle \left(\mathbf{P}^L + \partial_t^{-1}\,\mathbf{P}^S\right)\begin{pmatrix}\boldsymbol{\varphi}\\ \boldsymbol{\xi}\end{pmatrix}, Z_0\begin{pmatrix}\mathbf{n}\times\boldsymbol{e}^{in}\\ \mathbf{n}\times\boldsymbol{h}^{in}\end{pmatrix}\right\rangle,$$

where

$$\begin{pmatrix}\boldsymbol{p}\\ \boldsymbol{q}\end{pmatrix} := \left(\mathbf{P}^L + \partial_t\,\mathbf{P}^S\right)^{-1}\begin{pmatrix}\boldsymbol{j}\\ \boldsymbol{m}\end{pmatrix} = \left(\mathbf{P}^L + \partial_t^{-1}\,\mathbf{P}^S\right)\begin{pmatrix}\boldsymbol{j}\\ \boldsymbol{m}\end{pmatrix}.$$

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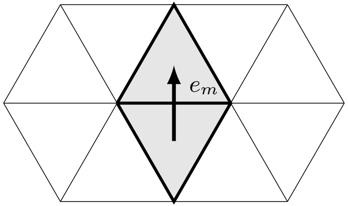
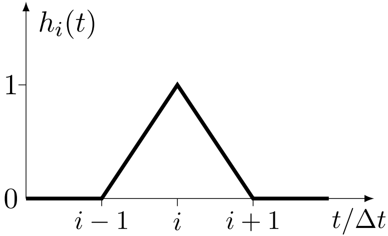
Discretization

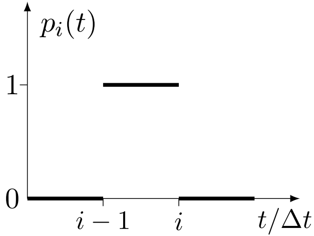
RWG function $\boldsymbol{f}_m(\boldsymbol{x})$



Temporal function $h_i(t)$



Temporal function $p_i(t)$



$$\boldsymbol{p}(\boldsymbol{x},t) \approx \sum_{i}^{N_T} \sum_{m}^{N_E} [\mathbf{p}_i]_m \left(\mathbf{P}^{\Lambda H}\, h_i(t) + \mathbf{P}^{\Sigma}\, p_i(t)\right) \boldsymbol{f}_m(\boldsymbol{x}),$$

$$\boldsymbol{q}(\boldsymbol{x},t) \approx \sum_{i}^{N_T} \sum_{m}^{N_E} [\mathbf{q}_i]_m \left(\mathbf{P}^{\Lambda H}\, h_i(t) + \mathbf{P}^{\Sigma}\, p_i(t)\right) \boldsymbol{f}_m(\boldsymbol{x}).$$

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Testing scheme

The regularized TD-PMCHWT equation

$$\left\langle \left(\mathbf{P}^L + \partial_t^{-1}\mathbf{P}^S\right)\begin{pmatrix}\boldsymbol{\varphi}\\ \boldsymbol{\xi}\end{pmatrix}, Z_0 \mathcal{Q}\left(\mathbf{P}^L + \partial_t \mathbf{P}^S\right)\begin{pmatrix}\boldsymbol{p}\\ \boldsymbol{q}\end{pmatrix}\right\rangle = -\left\langle \left(\mathbf{P}^L + \partial_t^{-1}\mathbf{P}^S\right)\begin{pmatrix}\boldsymbol{\varphi}\\ \boldsymbol{\xi}\end{pmatrix}, Z_0 \begin{pmatrix}\mathbf{n}\times \boldsymbol{e}^{in}\\ \mathbf{n}\times \boldsymbol{h}^{in}\end{pmatrix}\right\rangle.$$

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Testing scheme

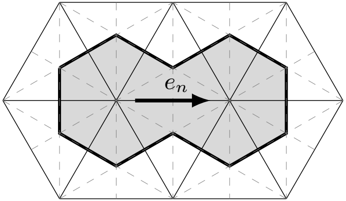
The regularized TD-PMCHWT equation

$$\left\langle \left(\mathbf{P}^L + \partial_t^{-1}\,\mathbf{P}^S\right) \begin{pmatrix} \boldsymbol{\varphi} \\ \boldsymbol{\xi} \end{pmatrix}, Z_0 \mathcal{Q} \left(\mathbf{P}^L + \partial_t\,\mathbf{P}^S\right) \begin{pmatrix} \boldsymbol{p} \\ \boldsymbol{q} \end{pmatrix} \right\rangle = -\left\langle \left(\mathbf{P}^L + \partial_t^{-1}\,\mathbf{P}^S\right) \begin{pmatrix} \boldsymbol{\varphi} \\ \boldsymbol{\xi} \end{pmatrix}, Z_0 \begin{pmatrix} \mathbf{n} \times \boldsymbol{e}^{in} \\ \mathbf{n} \times \boldsymbol{h}^{in} \end{pmatrix} \right\rangle.$$

Tesing functions

$$\boldsymbol{\varphi} = \boldsymbol{\xi} = \left(\frac{p_j(t)}{\Delta t} \mathbb{P}^{\Sigma H} + \delta_j(t) \mathbb{P}^{\Lambda} \right) \boldsymbol{g}_n(\boldsymbol{x}).$$

BC function $\boldsymbol{g}_n(\boldsymbol{x})$



Marching-on-in-time system:

$$\begin{pmatrix} \mathbf{L}_0 & & & \\ \mathbf{L}_1 & \mathbf{L}_0 & & \\ \vdots & \vdots & \ddots & \\ \mathbf{L}_{N_T-1} & \mathbf{L}_{N_T-2} & \dots & \mathbf{L}_0 \end{pmatrix} \begin{pmatrix} \mathrm{u}_1 \\ \mathrm{u}_2 \\ \vdots \\ \mathrm{u}_{N_T} \end{pmatrix} = \begin{pmatrix} \mathrm{r}_1 \\ \mathrm{r}_2 \\ \vdots \\ \mathrm{r}_{N_T} \end{pmatrix}.$$

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Post-processing

Starting from

$$\begin{pmatrix} \boldsymbol{p} \\ \boldsymbol{q} \end{pmatrix} = \left(\mathbf{P}^{L} + \partial_t^{-1}\,\mathbf{P}^{S}\right)\begin{pmatrix} \boldsymbol{j} \\ \boldsymbol{m} \end{pmatrix}, \qquad \text{or} \qquad \begin{pmatrix} \boldsymbol{j} \\ \boldsymbol{m} \end{pmatrix} = \left(\mathbf{P}^{L} + \partial_t\,\mathbf{P}^{S}\right)\begin{pmatrix} \boldsymbol{p} \\ \boldsymbol{q} \end{pmatrix},$$

we get back the physical unknowns

$$\mathbf{j}_i = \mathbf{P}^{\Lambda H}\,\mathbf{x}_i + \Delta t^{-1}\,\mathbf{P}^{\Sigma}\left(\mathbf{p}_i - \mathbf{p}_{i-1}\right),$$

$$\mathbf{m}_i = \mathbf{P}^{\Lambda H}\,\mathbf{y}_i + \Delta t^{-1}\,\mathbf{P}^{\Sigma}\left(\mathbf{q}_i - \mathbf{q}_{i-1}\right).$$

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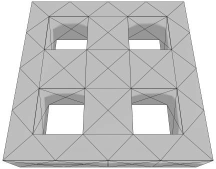
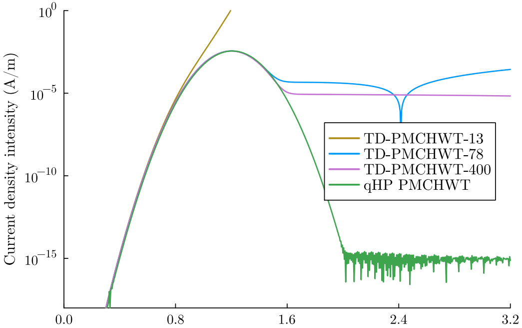
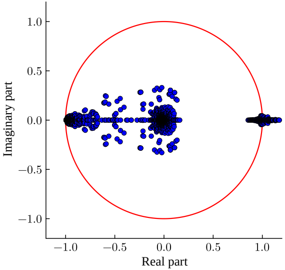
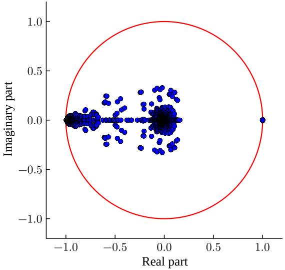
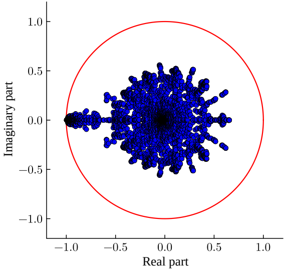
Numerical results



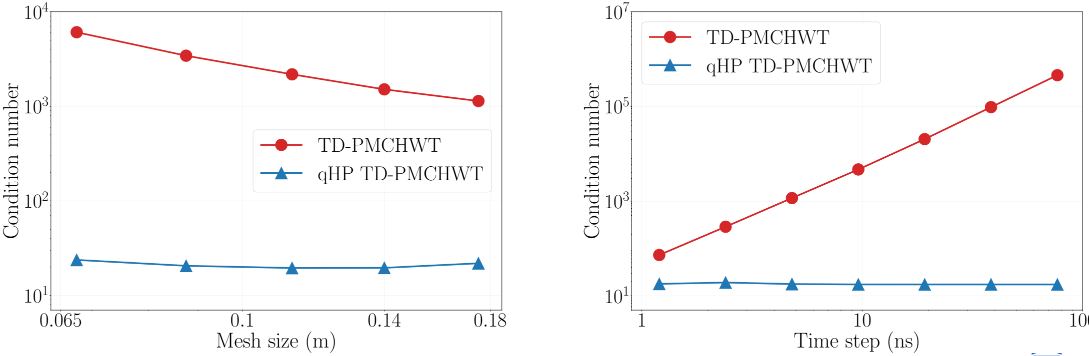








Numerical results



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Conclusions and future work

We have proposed a stable and well-conditioned time-domain PMCHWT equation, which is:

- ✓ late-time (DC) stable
- ✓ well-conditioned at all regimes
- ✓ free from infinite tail behavior
- ✓ applicable for different surfaces.

Future work: to further investigate the origin of late-time (DC) instabilities.

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