FACULTY OF ENGINEERING AND ARCHITECTURE

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ELECTROMAGNETICS RESEARCH GROUP

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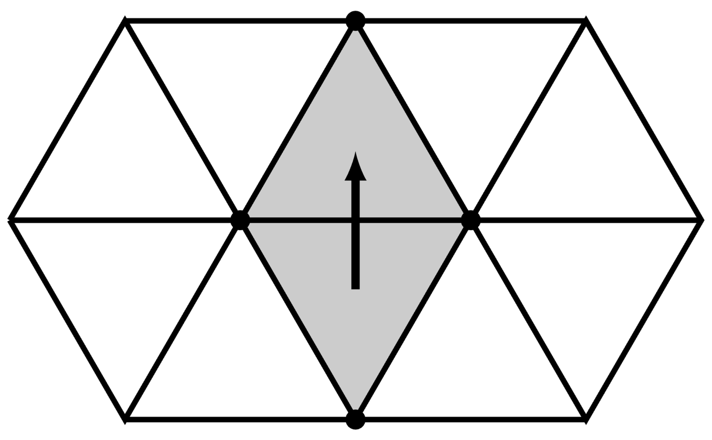
A YUKAWA-CALDERON TIME-DOMAIN COMBINED FIELD INTEGRAL EQUATION FOR ELECTROMAGNETIC SCATTERING

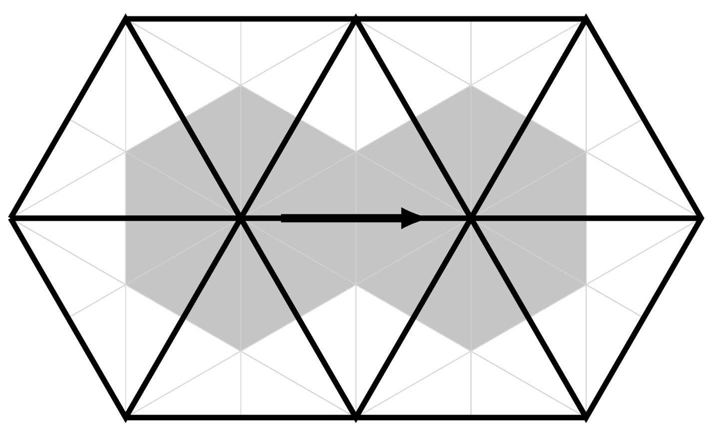
Introduction

This contribution presents a Yukawa-Calderon time-domain combined field integral equation for scattering by a perfect electric conductor, which has the following properties:

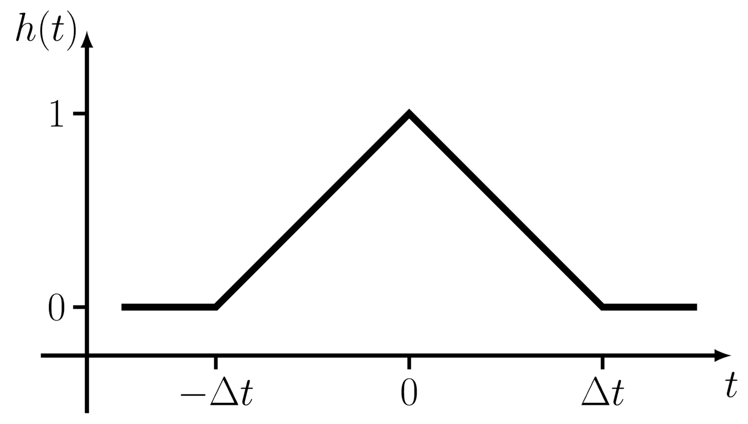
- immune to resonant frequencies
- well-conditioned when the mesh is fine
- well-conditioned at large time steps
- free from late time (dc) instability.



RWG basis function $\boldsymbol{f}_n(\boldsymbol{r})$



BC basis function $\boldsymbol{g}_n(\boldsymbol{r})$



Temporal basis function $h_i(t)$

A. Time-domain integral equations

Let Γ be the closed boundary of a perfect electric conductor. The surface current density $\boldsymbol{j}$ on Γ satisfies the time-domain EFIE and MFIE

$$(\mathcal{T}\boldsymbol{j})(\boldsymbol{r},t) = -\mathbf{n}\times\boldsymbol{e}^{in}(\boldsymbol{r},t),$$

$$\left(\frac{1}{2}I + \mathcal{K}\right)\boldsymbol{j}(\boldsymbol{r},t) = \mathbf{n}\times\boldsymbol{h}^{in}(\boldsymbol{r},t).$$

Here, I is the identity operator, $(\boldsymbol{e}^{in},\boldsymbol{h}^{in})$ is the incident electromagnetic wave, and

$$\mathcal{T}\boldsymbol{j} = \frac{\eta}{c}(\mathcal{T}^s\boldsymbol{j}) + \eta c\,(\mathcal{T}^h\boldsymbol{j}),$$

$$\mathcal{T}^s\boldsymbol{j} = -\mathbf{n}\times\int_\Gamma \frac{\partial_t \boldsymbol{j}(\boldsymbol{r}',\tau)}{4\pi R}\,\mathrm{d}s',$$

$$\mathcal{T}^h\boldsymbol{j} = \mathbf{n}\times\mathbf{grad}_{\mathbf{x}}\int_{-\infty}^{\tau}\int_\Gamma \frac{\mathrm{div}'_\Gamma\,\boldsymbol{j}(\boldsymbol{r}',t)}{4\pi R}\,\mathrm{d}s'\mathrm{d}t,$$

$$\mathcal{K}\boldsymbol{j} = -\mathbf{n}\times\mathbf{curl}_{\mathbf{x}}\int_\Gamma \frac{\boldsymbol{j}(\boldsymbol{r}',\tau)}{4\pi R}\,\mathrm{d}s',$$

where $R = |\boldsymbol{r}-\boldsymbol{r}'|$ and $\tau = t - R/c$.

Numerical issues:

- Resonant and late-time instabilities
- Dense discretization and large-time step breakdowns.

B. Yukawa-type integral operators

The "frequency-domain" operators of wave number $-j\kappa$

$$T_{-j\kappa}\boldsymbol{j} = \eta\kappa\,(T^s_{-j\kappa}\boldsymbol{j}) + \frac{\eta}{\kappa}(T^h_{-j\kappa}\boldsymbol{j}),$$

$$T^s_{-j\kappa}\boldsymbol{j} = \mathbf{n}\times\int_\Gamma \frac{e^{-\kappa R}}{4\pi R}\boldsymbol{j}(\boldsymbol{r}')\mathrm{d}s',$$

$$T^h_{-j\kappa}\boldsymbol{j} = -\mathbf{n}\times\mathbf{grad}_{\mathbf{x}}\int_\Gamma \frac{e^{-\kappa R}}{4\pi R}\mathrm{div}'_\Gamma\,\boldsymbol{j}(\boldsymbol{r}')\mathrm{d}s',$$

$$K_{-j\kappa}\boldsymbol{j} = -\mathbf{n}\times\mathbf{curl}_{\mathbf{x}}\int_\Gamma \frac{e^{-\kappa R}}{4\pi R}\boldsymbol{j}(\boldsymbol{r}')\;\mathrm{d}s'.$$

Advantage: do not exhibit resonances.

C. Yukawa-Calderon time-domain CFIE

The Yukawa-Calderon (YC) TD-CFIE reads as

$$\left(-T_{-j\kappa}\mathcal{T} + \alpha\left(\frac{1}{2}I - K_{-j\kappa}\right)\left(\frac{1}{2}I + \mathcal{K}\right)\right)\boldsymbol{j}(\boldsymbol{r},t)$$

$$= T_{-j\kappa}(\mathbf{n}\times\boldsymbol{e}^{in}) + \alpha\left(\frac{1}{2}I - K_{-j\kappa}\right)(\mathbf{n}\times\boldsymbol{h}^{in}).$$

Optimal parameters:

- Wave number: $\kappa = (c.\Delta t)^{-1}$
- Coupling parameter: $\alpha = \eta^2$.

D. Discretization

The unknown $\boldsymbol{j}$ is expanded as follows

$$\boldsymbol{j}(\boldsymbol{r},t) = \sum_{n=1}^{N_S}\sum_{i=1}^{N_T}[\mathrm{j}_i]_n\,\boldsymbol{f}_n(\boldsymbol{r})h_i(t).$$

The identity and Yukawa-type operators are discretized as

$$[\mathbb{G}]_{mn} = \langle \mathbf{n}\times\boldsymbol{g}_m, \boldsymbol{f}_n\rangle,$$

$$[\mathbb{Z}]_{mn} = \langle \mathbf{n}\times\boldsymbol{g}_m, T_{-j\kappa}\boldsymbol{g}_n\rangle,$$

$$[\mathbb{M}]_{mn} = \langle \mathbf{n}\times\boldsymbol{g}_m, K_{-j\kappa}\boldsymbol{f}_n\rangle.$$

The time-domain operators and sources are discretized as

$$[\mathrm{e}_i]_m = \langle \mathbf{n}\times\boldsymbol{f}_m, \mathbf{n}\times\boldsymbol{e}^{in}\rangle\big|_{t=i\Delta t},$$

$$[\mathrm{h}_i]_m = \langle \mathbf{n}\times\boldsymbol{g}_m, \mathbf{n}\times\boldsymbol{h}^{in}\rangle\big|_{t=i\Delta t},$$

$$[\mathbf{Z}_i]_{mn} = \langle \mathbf{n}\times\boldsymbol{f}_m, \mathcal{T}(\boldsymbol{f}_n h_i)\rangle\big|_{t=0},$$

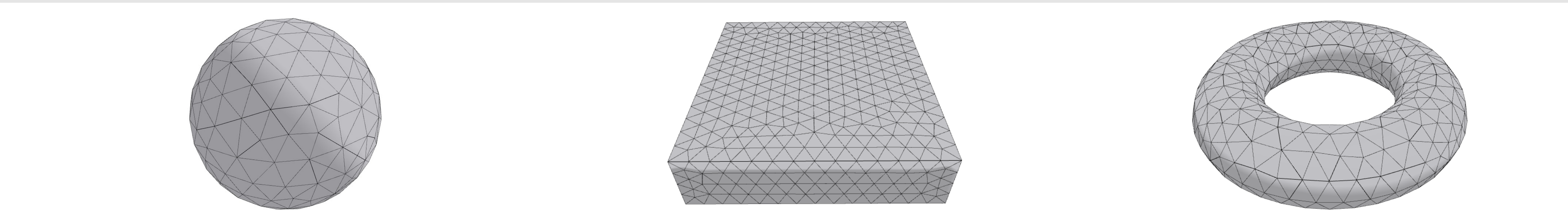
$$[\mathbf{M}_i]_{mn} = \langle \mathbf{n}\times\boldsymbol{g}_m, \mathcal{K}(\boldsymbol{f}_n h_i)\rangle\big|_{t=0}.$$

Marching-on-in-time system of the YC TD-CFIE reads as

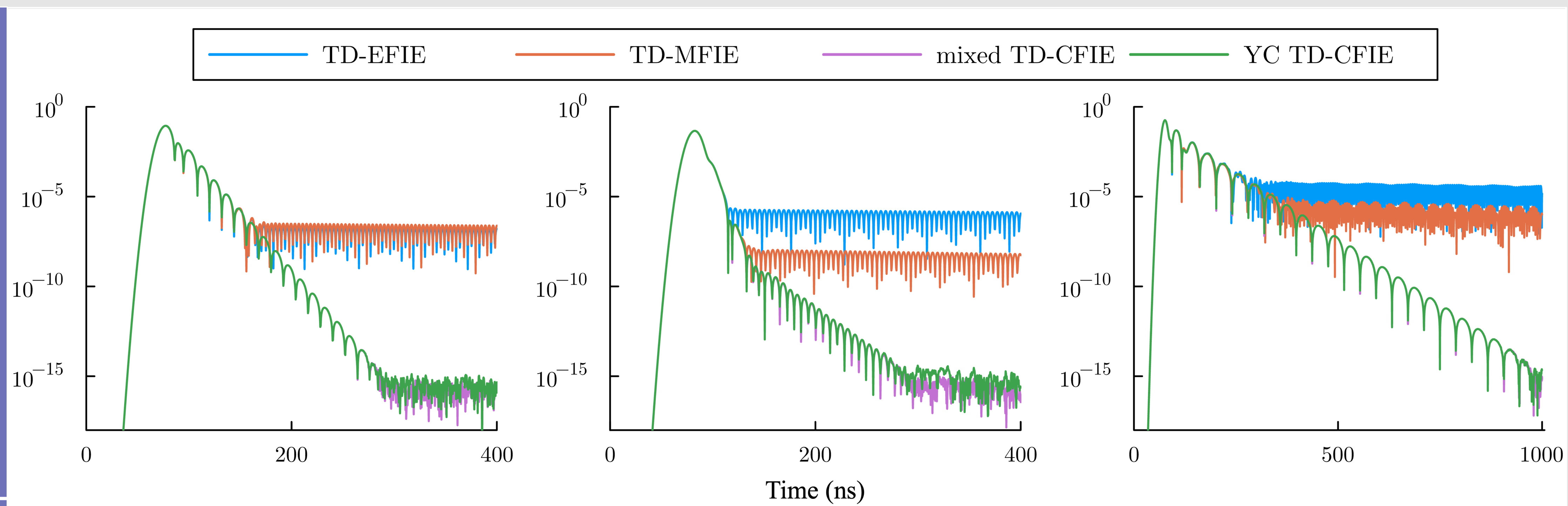
$$\left(\mathbb{Z}\mathbb{G}^{-\mathrm{T}}\mathbf{Z} + \eta^2\left(\frac{1}{2}\mathbb{G} - \mathbb{M}\right)\mathbb{G}^{-1}\left(\frac{1}{2}\mathbb{G} + \mathbf{M}\right)\right)\mathrm{j}$$

$$= -\mathbb{Z}\mathbb{G}^{-\mathrm{T}}\mathrm{e} + \eta^2\left(\frac{1}{2}\mathbb{G} - \mathbb{M}\right)\mathbb{G}^{-1}\mathrm{h}.$$

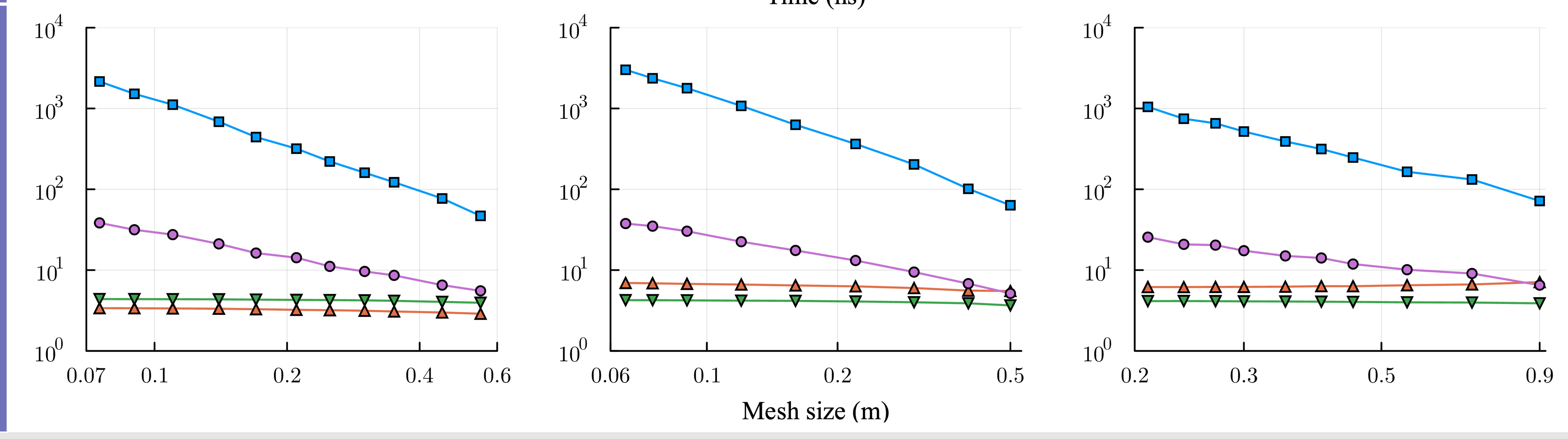
Geometries



Current density



Condition number



Conclusions

This contribution has introduced a Yukawa-Calderon TD-CFIE formulation for electromagnetic scattering by a perfect electric conductor. The proposed formulation is:

- immune to resonant and dc instabilities
- well-conditioned at all regimes
- applicable for different geometries.

Several numerical results have corroborated the superiority of the Yukawa-Calderon TD-CFIE.

Future works

In a forthcoming work, the proposed Yukawa-Calderon TD-CFIE should be combined with the quasi-Helmholtz projectors and a rescaling procedure to render a stabilized TD-CFIE formulation, which yields accurate solutions at moderately to extremely low frequencies.

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