FACULTY OF ENGINEERING AND ARCHITECTURE

DEPARTMENT OF INFORMATION TECHNOLOGY
ELECTROMAGNETICS RESEARCH GROUP

ICEAA - IEEE APWC 2023 Conference

A YUKAWA-CALDERÓN TIME-DOMAIN COMBINED FIELD INTEGRAL EQUATION FOR ELECTROMAGNETIC SCATTERING*

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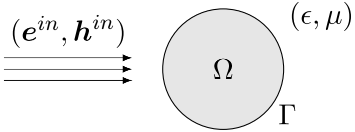
IDLab INTERNET & DATA LAB

*A joint work with P. Cordel, F. P. Andriulli and K. Cools.

Time-domain integral equations

TD-EFIE: $\mathcal{T}\boldsymbol{j} = -\mathbf{n} \times \boldsymbol{e}^{in},$

TD-MFIE: $\left(\frac{1}{2}\mathcal{I} + \mathcal{K}\right)\boldsymbol{j} = \mathbf{n} \times \boldsymbol{h}^{in},$



where $\mathcal{I}$ is the identity operator, $\eta = \sqrt{\mu/\epsilon}, c = 1/\sqrt{\mu\epsilon}, R = |\boldsymbol{r} - \boldsymbol{r}'|$, $\tau = t - R/c$,

$$(\mathcal{T}\boldsymbol{j})(\boldsymbol{r}, t) = (\mathcal{T}^s\boldsymbol{j})(\boldsymbol{r}, t) + (\mathcal{T}^h\boldsymbol{j})(\boldsymbol{r}, t),$$

$$(\mathcal{T}^s\boldsymbol{j})(\boldsymbol{r}, t) = -\frac{\eta}{c}\,\mathbf{n} \times \int_\Gamma \frac{\partial_t \boldsymbol{j}(\boldsymbol{r}', \tau)}{4\pi R}\,\mathrm{d}s',$$

$$(\mathcal{T}^h\boldsymbol{j})(\boldsymbol{r}, t) = c\eta\,\mathbf{n} \times \mathbf{grad}_{\mathbf{x}} \int_\Gamma \int_{-\infty}^{\tau} \frac{\mathrm{div}_\Gamma\,\boldsymbol{j}(\boldsymbol{r}', t')}{4\pi R}\,\mathrm{d}t'\,\mathrm{d}s',$$

$$(\mathcal{K}\boldsymbol{j})(\boldsymbol{r}, t) = -\mathbf{n} \times \mathbf{curl}_{\mathbf{x}} \int_\Gamma \frac{\boldsymbol{j}(\boldsymbol{r}, \tau)}{4\pi R}\,\mathrm{d}s'.$$

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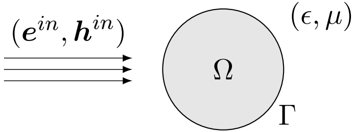

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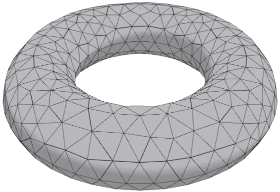
TD vs FD: TD allows coupling to non-linear systems.

Motivations

Numerical issues:

- Late-time (dc) instability
- Dense discretization breakdown
- Large-time step breakdown
- Inaccuracy of sulution
- Nontrivial nullspaces of static MFIE operators on toroidal surfaces
- Resonant instability.



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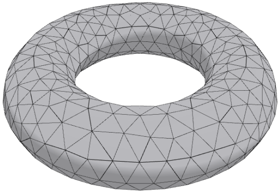

Goal of this talk: to introduce a TD formulation that is immune to all above issues.

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Time-domain and Frequency-domain

Time-domain $\Rightarrow$ **Frequency-domain**: Fourier transform w.r.t. angular frequency $\omega = \kappa c$.

- FD-EFIE operator

$$(T_\kappa \boldsymbol{j})(\boldsymbol{r}) = (T_\kappa^s \boldsymbol{j})(\boldsymbol{r}) + (T_\kappa^h \boldsymbol{j})(\boldsymbol{r}),$$

$$(T_\kappa^s \boldsymbol{j})(\boldsymbol{r}) = -j\kappa\eta\,\mathbf{n} \times \int_\Gamma \frac{\exp(-j\kappa R)}{4\pi R}\,\boldsymbol{j}(\boldsymbol{r}')\,\mathrm{d}s',$$

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Frequency-domain $\Rightarrow$ **Time-domain**: inverse Fourier transform w.r.t. ω.

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Yukawa-Calderón frequency-domain CFIE

A Yukawa-Calderón FD-CFIE formulation:

$$\left(-T_{-j\kappa}T_\kappa + \alpha\left(\frac{1}{2}I - K_{-j\kappa}\right)\left(\frac{1}{2}I + K_\kappa\right)\right)\boldsymbol{j}(\boldsymbol{r})$$
$$= T_{-j\kappa}\left(\mathbf{n}\times\boldsymbol{e}^{in}(\boldsymbol{r})\right) + \alpha\left(\frac{1}{2}I - K_{-j\kappa}\right)\left(\mathbf{n}\times\boldsymbol{h}^{in}(\boldsymbol{r})\right).$$

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Properties:

- ✓ free from resonant frequencies
- ✓ well-conditioned at all regimes.

1 A. Merlini, Y. Beghein, K. Cools, E. Michielssen, and F. P. Andriulli, Magnetic and combined field integral equations based on the quasi-Helmholtz projectors. *IEEE Trans. Antennas Propag.*, vol. 68, no. 5, pp. 3834-3846, 2020.

2 V. C. Le and K. Cools, A well-conditioned combined field integral equation for electromagnetic scattering, *preprint*.

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Yukawa-Calderón frequency-domain CFIE

Special property

A Yukawa-Calderón FD-CFIE operator:

$$-T_{-j\kappa}T_{\kappa} + \alpha\left(\frac{1}{2}I - K_{-j\kappa}\right)\left(\frac{1}{2}I + K_{\kappa}\right).$$

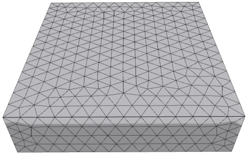
Given R^{Γ} the loop projection, the following special property holds

$$R^{\Gamma}\left(\frac{1}{2}I - K_0\right)\left(\frac{1}{2}I + K_0\right)R^{\Gamma} = 0.$$

Consequence: symmetrized MFIE operator is compact.

Properties:

- ✓ applicable for different domains
- ✓ gives accurate solutions.



Yukawa-Calderón time-domain CFIE

Formulation

Inverse Fourier transform of Yukawa-Calderón FD-CFIE: very complicated!

$$\left(-T_{-j\kappa}T_{\kappa} + \alpha\left(\frac{1}{2}I - K_{-j\kappa}\right)\left(\frac{1}{2}I + K_{\kappa}\right)\right)\boldsymbol{j}(\boldsymbol{r}) = T_{-j\kappa}\left(\mathbf{n}\times\boldsymbol{e}^{in}(\boldsymbol{r})\right) + \alpha\left(\frac{1}{2}I - K_{-j\kappa}\right)\left(\mathbf{n}\times\boldsymbol{h}^{in}(\boldsymbol{r})\right).$$

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Solution: $T_{\kappa}, K_{\kappa} \Rightarrow T_{\kappa'}, K_{\kappa'}$, with $\kappa' > 0$

$$\left(-T_{-j\kappa}T_{\kappa'} + \alpha\left(\frac{1}{2}I - K_{-j\kappa}\right)\left(\frac{1}{2}I + K_{\kappa'}\right)\right)\boldsymbol{j}(\boldsymbol{r}) = T_{-j\kappa}\left(\mathbf{n} \times \boldsymbol{e}^{in}(\boldsymbol{r})\right) + \alpha\left(\frac{1}{2}I - K_{-j\kappa}\right)\left(\mathbf{n} \times \boldsymbol{h}^{in}(\boldsymbol{r})\right).$$

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Yukawa-Calderón time-domain CFIE (cont.)

Formulation

The Yukawa-Calderón FD-CFIE:

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Applying the inverse Fourier transform w.r.t. $\omega' = \kappa' c$ gives the **Yukawa-Calderón TD-CFIE**

$$\left(-T_{-j\kappa}\mathcal{T} + \alpha\left(\frac{1}{2}I - K_{-j\kappa}\right)\left(\frac{1}{2}\mathcal{I} + \mathcal{K}\right)\right)\boldsymbol{j}(\boldsymbol{r},t) = T_{-j\kappa}\left(\mathbf{n}\times\boldsymbol{e}^{in}(\boldsymbol{r},t)\right) + \alpha\left(\frac{1}{2}I - K_{-j\kappa}\right)\left(\mathbf{n}\times\boldsymbol{h}^{in}(\boldsymbol{r},t)\right).$$

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Yukawa-Calderón time-domain CFIE (cont.)

Formulation

The Yukawa-Calderón FD-CFIE:

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Optimal parameters: $\kappa = (c.\Delta t)^{-1}$ and $\alpha = \eta^2$.

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Yukawa-Calderón time-domain CFIE

Properties

The Yukawa-Calderón TD-CFIE

$$\left(-T_{-j\kappa}\mathcal{T} + \alpha\left(\frac{1}{2}I - K_{-j\kappa}\right)\left(\frac{1}{2}\mathcal{I} + \mathcal{K}\right)\right)\boldsymbol{j}(\boldsymbol{r}, t)$$
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Properties:

✓ Resonant stability

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Yukawa-Calderón time-domain CFIE

Properties

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Properties:

- ✓ Resonant stability
- ✓ Late-time (dc) stability
- ✓ Immune to nullspaces of static MFIE operators on toroidal surfaces

Yukawa-Calderón time-domain CFIE

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- ✓ Resonant stability
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- ✓ Free from dense discretization and large-time step breakdowns

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Properties:

- ✓ Resonant stability
- ✓ Late-time (dc) stability
- ✓ Immune to nullspaces of static MFIE operators on toroidal surfaces
- ✓ Free from dense discretization and large-time step breakdowns
- ✓ Accuracy of the solution.

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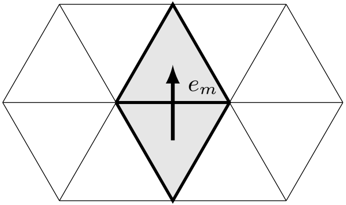
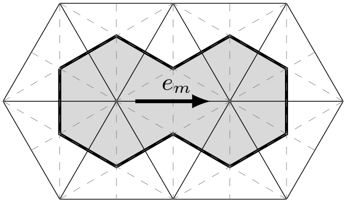
Discretization

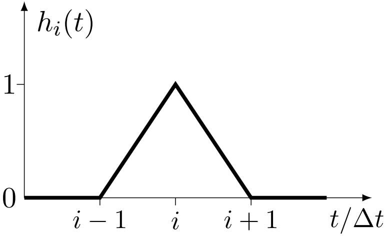
RWG function $\boldsymbol{f}_m(\boldsymbol{r})$



BC function $\boldsymbol{g}_m(\boldsymbol{r})$



Temporal function $h_i(t)$



$$\boldsymbol{j}(\boldsymbol{r},t) \approx \sum_{m=1}^{N_S} \sum_{i=1}^{N_T} [\mathrm{j}_i]_m \boldsymbol{f}_m(\boldsymbol{r}) h_i(t).$$

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Testing schemes

Frequency-domain operators	Time-domain operators
	$[\mathrm{e}_i]_m = \left\langle \mathbf{n} \times \boldsymbol{f}_m, \mathbf{n} \times \boldsymbol{e}^{in} \right\rangle\Big\|_{t=i\Delta t}$
$[\mathbb{G}]_{mn} = \langle \mathbf{n} \times \boldsymbol{g}_m, \boldsymbol{f}_n \rangle$	$[\mathrm{h}_i]_m = \left\langle \mathbf{n} \times \boldsymbol{g}_m, \mathbf{n} \times \boldsymbol{h}^{in} \right\rangle\Big\|_{t=i\Delta t}$
$[\mathbb{Z}]_{mn} = \langle \mathbf{n} \times \boldsymbol{g}_m, T_{-j\kappa} \boldsymbol{g}_n \rangle$	$[\mathbf{Z}_i]_{mn} = \langle \mathbf{n} \times \boldsymbol{f}_m, \mathcal{T}(\boldsymbol{f}_n h_i) \rangle\|_{t=0}$
$[\mathbb{M}]_{mn} = \langle \mathbf{n} \times \boldsymbol{g}_m, K_{-j\kappa} \boldsymbol{f}_n \rangle$	$[\mathbf{M}_i]_{mn} = \langle \mathbf{n} \times \boldsymbol{g}_m, \mathcal{K}(\boldsymbol{f}_n h_i) \rangle\|_{t=0}$

$$\langle \boldsymbol{f}, \boldsymbol{g} \rangle = \int_\Gamma \boldsymbol{f}(\boldsymbol{r}) \cdot \boldsymbol{g}(\boldsymbol{r}) \, \mathrm{d}s.$$

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Marching-on-in-time algorithm

The discretized linear system

$$\begin{pmatrix} \mathbf{L}_0 & & & \\ \mathbf{L}_1 & \mathbf{L}_0 & & \\ \vdots & \vdots & \ddots & \\ \mathbf{L}_{N_T-1} & \mathbf{L}_{N_T-2} & \dots & \mathbf{L}_0 \end{pmatrix} \begin{pmatrix} \mathrm{j}_1 \\ \mathrm{j}_2 \\ \vdots \\ \mathrm{j}_{N_T} \end{pmatrix} = \begin{pmatrix} \mathrm{r}_1 \\ \mathrm{r}_2 \\ \vdots \\ \mathrm{r}_{N_T} \end{pmatrix} \quad \Longrightarrow \quad \mathrm{j}_i = \mathbf{L}_0^{-1} \left(\mathrm{r}_i - \sum_{k=1}^{i-1} \mathbf{L}_k \, \mathrm{j}_{i-k} \right),$$

with

$$\mathbf{L}_i = \mathbb{Z}\mathbb{G}^{-\mathsf{T}} \, \mathbf{Z}_i + \eta^2 \left(\frac{1}{2}\mathbb{G} - \mathbb{M} \right) \mathbb{G}^{-1} \left(\frac{1}{2} \, \mathbf{G}_i + \mathbf{M}_i \right),$$

and

$$\mathrm{r}_i = -\mathbb{Z}\mathbb{G}^{-\mathsf{T}} \mathrm{e}_i + \eta^2 \left(\frac{1}{2}\mathbb{G} - \mathbb{M} \right) \mathbb{G}^{-1} \mathrm{h}_i .$$

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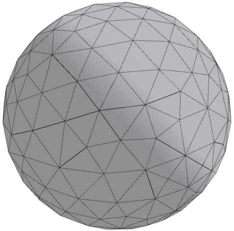
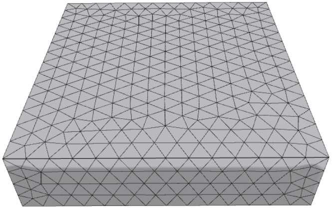
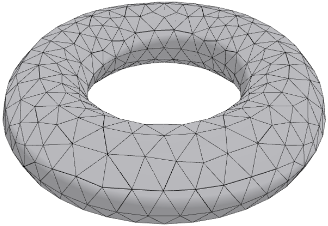
Numerical results

Geometries







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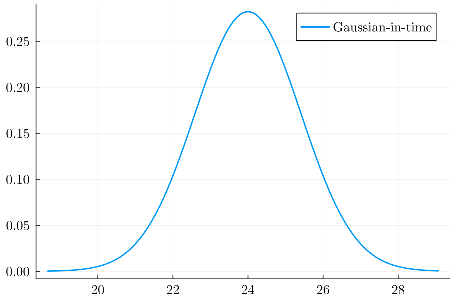
Numerical results

Experimental setting

Gaussian-in-time plane wave

$$\boldsymbol{e}^{in}(\boldsymbol{r}, t) = \frac{4A}{w\sqrt{\pi}} \boldsymbol{p} \exp\left(- \left(\frac{4}{w} \left(c(t - t_0) - \boldsymbol{k} \cdot \boldsymbol{r} \right) \right)^2 \right).$$



Numerical results are obtained using 4 formulations:

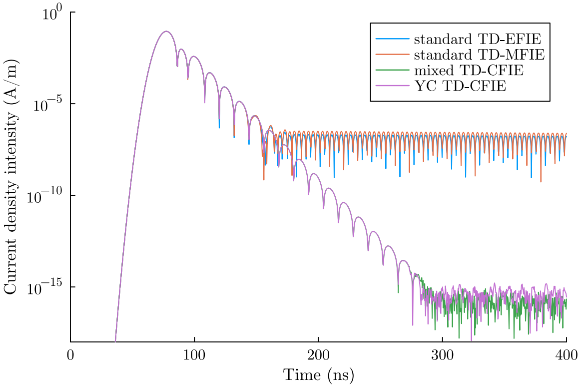
- the standard TD-EFIE;
- the standard TD-MFIE;
- the mixed TD-CFIE

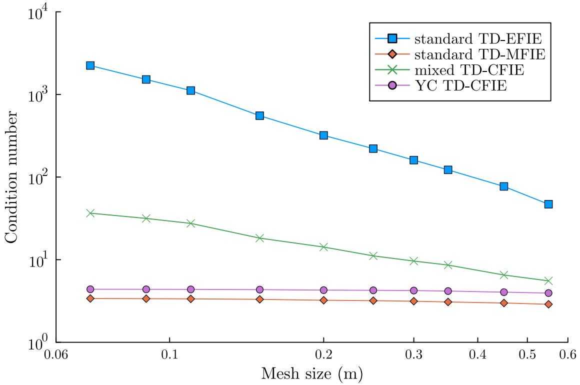
$$\mathbf{Z} + \eta\, \mathbb{G}_{xx} \mathbb{G}^{-1} \left(\frac{1}{2} \mathbf{G} + \mathbf{M} \right) = \mathrm{e} + \eta\, \mathbb{G}_{xx} \mathbb{G}^{-1} \mathrm{h};$$

- the Yukawa-Calderón (YC) TD-CFIE.

Numerical results

Sphere





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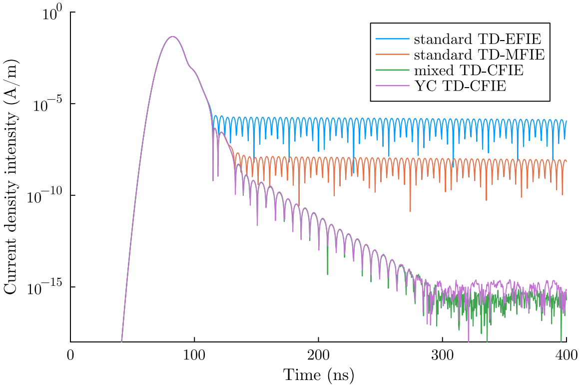
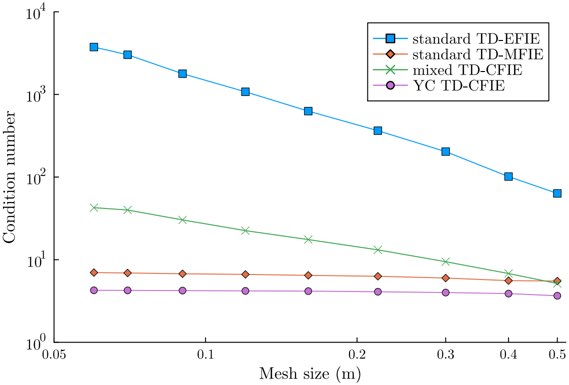
Numerical results

Cuboid





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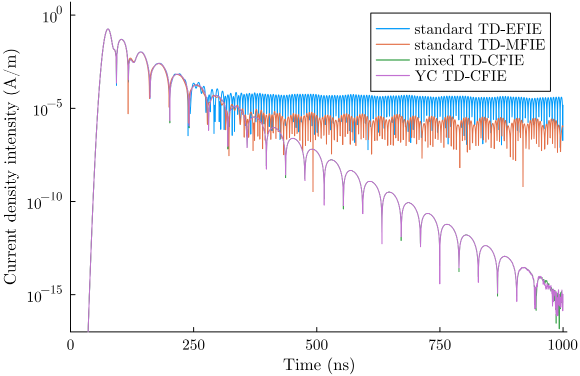
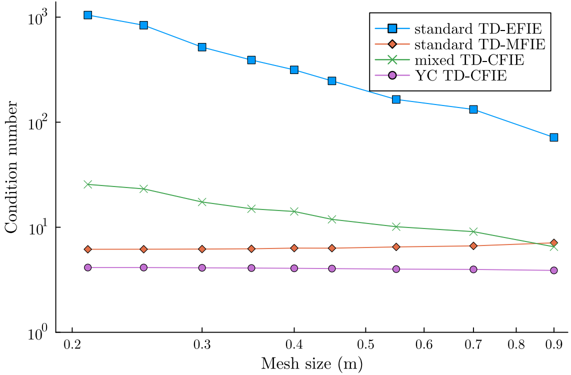
Numerical results

Torus





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Conclusions and future work

This contribution has introduced a Yukawa-Calderón TD-CFIE formulation, which:

- is immune to resonant instability and late-time instability
- is well-conditioned at all regimes
- gives accurate solutions
- is applicable for different domains
- is not affected by nullspaces of the static MFIE operators on toroidal surfaces.

Future work: to combine the Yukawa-Calderón TD-CFIE with the quasi-Helmholtz projectors to render a stabilized TD-CFIE formulation at low frequencies.

References and acknowledgment

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