Score-based bibliometric rankings of authors

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Abstract

Scoring rules form a family of bibliometric rankings of authors such that authors are ranked according to the sum over all their publications of some partial scores. Many of these rankings are widely used (e.g., number of publications, weighted or not by the impact factor, by the number of authors or by the number of citations,). We present an axiomatic analysis of the family of all scoring rules and of some particular cases within this family.

1 Introduction

Many indices can be found in the literature for quantifying the scientific production (in terms of publications) of researchers, departments or universities. These indices are then often used to derive rankings of authors or departments. Since a few year, we witness a dramatic increase in the number of such indices or rankings. Many researchers, analyzing previously existing indices, find that they have some drawback and then propose an adapted version of the incriminated index or a brand new one, supposedly better than the older one. Unfortunately, the reasoning of the proponents of such new indices is often *ad hoc*: they propose a new index, not suffering the same drawback as the older one that they analyzed, but nothing guarantees that the new index does not have many other weaknesses.

In this paper (like in Marchant [to appear]), instead of using an *ad hoc* reasoning, we try to construct a theory of bibliometric rankings. In this theory, we do not focus on a particular advantage or drawback of a ranking; we completely characterize a ranking by some properties that we call axioms. In other words, given a ranking under scrutiny, we look for a few properties that are satisfied by this ranking and only by this one. In a practical application, ideally, it is then possible to identify some axioms that appear as compelling in that context and to select the unique ranking characterized by those axioms. If there is no such ranking, we can then select a ranking satisfying most of them. A similar approach has been followed by, among others, Woeginger [to appear] for

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indices evaluating authors and by Palacios-Huerta and Volij [2004] for indices evaluating journals.

As in Marchant [to appear], we emphasize that there is no *right* ranking. A ranking is used by a person (or an organization) pursuing a goal in some context. Depending on the person, the goal and the context, different rankings can be used. Let us illustrate this by an example. Suppose a scientific society wants to rank departments according to merit, in function of their publications. This society might rank them according to the total number of citations (eventually weighted by impact factor) divided by the size of the department (because size is not relevant for evaluating merit). So, a department of 50 people with 2000 citations might be outranked by a department of 5 with 250 citations. Suppose now a graduate student is offered a grant in different departments to prepare a Ph.D. thesis. For him, the size does matter. In a larger department, he will have more opportunities. So, he might rank the small department below the larger one, even if the number of citations per capita slightly favours the smaller one. So, anybody willing to use a ranking should select some axioms that seem relevant, given the context and his/her goal and then look for a ranking characterized by these axioms.

After a section devoted to notation, we will introduce a family of rankings that we call scoring rules (Section 3). In this family, each publication has a score, depending on the journal, the number of citations and the number of authors. The score of an author is then the sum of the score of all his publications. In Section 4, we will characterize this family. In the next section, we will then analyze some subsets of rankings within this family, before concluding.

2 Notation and definitions

Let $J = \{j, k, l, \dots\} \subset \mathbb{N}$ represent the set of journals. We represent an author by a mapping f from $J \times \mathbb{N} \times \mathbb{N}$ to $\mathbb{N}$ and we interpret $f(j, x, a)$ as the number of publications of author f in journal j with exactly x citations and a coauthors (the number of authors being $a + 1$). Let X be the set of all mappings f from $J \times \mathbb{N} \times \mathbb{N}$ to $\mathbb{N}$ such that $\sum_{j \in J} \sum_{x \in \mathbb{N}} \sum_{a \in \mathbb{N}} f(j, x, a)$ is finite. This set is called the set of authors. The elements of X are usually denoted by $f, f', g, \dots$ In this paper, we will investigate how we can construct a ranking (a complete and transitive binary relation[1]) $\succsim$ on X. The statement '$x \succsim y$' is interpreted as 'given their publication/citation records, author x is at least as good as author y.' When $x \succsim y$ and $y \not\succsim x$, we write $x \succ y$ (x is strictly better than y). When $x \succsim y$ and $y \succsim x$, we write $x \sim y$ (x and y are equivalent).

For all $j \in J$ and $x, a \in \mathbb{N}$, we denote by $\mathbf{1}_{j,x,a}$ the author such that $\mathbf{1}_{j,x,a}(j', x', a') = 0$ whenever $j' \neq j$ or $x' \neq x$ or $a' \neq a$ and $\mathbf{1}_{j,x,a}(j, x, a) = 1$. So, $\mathbf{1}_{j,x,a}$ represents an author with exactly one publication and such that this publication is in journal j, is cited x times and has $a + 1$ authors. An author

[1] A binary relation $\succsim$ on a set X is transitive if, $\forall x, y, z \in X$, $x \succsim y$ and $y \succsim z$ imply $x \succsim z$. It is complete if, $\forall x, y \in X$, $x \succsim y$ or $y \succsim x$.

without publication is represented by $\mathbf{0}$. We now present some desirable properties that should definitely be satisfied by any sensible bibliometric ranking. These properties will be called axioms.

A 1 Non-Triviality. *There are f and g such that $f \succ g$.*

This axiom just expresses the fact that we do not want a complete tie; we want to discriminate among authors.

A 2 CDNH. *For all $j \in J$ and all $x, x', a \in \mathbb{N}$, $x \geq x'$ implies $\mathbf{1}_{j,x,a} \succsim \mathbf{1}_{j,x',a}$.*

The name CDNH stands for 'Citations Do Not Harm'. In other words, if two authors have a single publication each, in the same journal and with the same number of coauthors, then the author that has more citations cannot be ranked in a lower position than the other one. Actually, condition CDNH is a monotonicity condition, but very weak since it applies only to authors with a single publication. Since these two axioms are so compelling, we do not want to consider rankings that do not satisfy them. That is why we include them in the next definition.

Definition 1 *A bibliometric ranking is a complete and transitive relation on X satisfying Non-Triviality and CDNH.*

We are now ready to present scoring rules.

3 Scoring rules

We say that a bibliometric ranking $\succsim$ is a scoring rule if there is a mapping $u : J \times \mathbb{N} \times \mathbb{N} \to \mathbb{R} : (j, x, a) \to u(j, x, a)$ such that

$$f \succsim g \iff \sum_{j \in J} \sum_{x \in \mathbb{N}} \sum_{a \in \mathbb{N}} f(j, x, a) u(j, x, a) \geq \sum_{j \in J} \sum_{x \in \mathbb{N}} \sum_{a \in \mathbb{N}} g(j, x, a) u(j, x, a).$$

In this expression, $u(j, x, a)$ represents the value or the score of one publication in journal a, with x citations and a coauthors. The triple sum represents the total score of an author. For the sake of brevity, we will often write

$$\sum_{j \in J} \sum_{x \in \mathbb{N}} \sum_{a \in \mathbb{N}} f(j, x, a) u(j, x, a) = U(f).$$

Many popular bibliometric rankings are scoring rules. For instance, if we choose u equal to a positive constant, we obtain the ranking based on the number of publications. If we define u by $u(j, x, a) = x$ for all $j \in J, x, a \in \mathbb{N}$, we obtain the ranking based on the number of citations. If we define u by $u(j, x, a) = 0$ for all $j \in J, x, a \in \mathbb{N}$ with $x < \alpha$ and $u(j, x, a) = 1$ for all $j \in J, x, a \in \mathbb{N}$ with $x \geq \alpha$, we obtain a ranking based on the number of publications with at least α citations, used by Chapron and Husté [2006]. If we define u by $u(j, x, a) = IF(j)$

for all $j \in J, x, a \in \mathbb{N}$, where $IF(j)$ is the impact factor of journal j, we obtain a ranking based on the sum of the impact factors, used e.g. by Fava and Ottolini [2000]. If we define u by $u(j, x, a) = x/(a+1)$ for all $j \in J, x, a \in \mathbb{N}$, we obtain a ranking based on the total number of citations, weighted by the number of authors, used e.g. by Pijpers [2006]. Of course, many other rankings can be obtained by an appropriate choice of the mapping u.

Some rankings do not belong to the family of scoring rules: for instance, the ranking based on the h-index [Hirsch, 2005], the ranking based on the maximal number of citations [Eto, 2003], the ranking based on the average number of citations [van Raan, 2006].

4 Characterization of scoring rules

We will need two axioms to characterize the family of all scoring rules.

A 3 Independence. *For all* $f, g \in X$*, all* $j \in J$*, all* $x, a \in \mathbb{N}$*,* $f \succsim g$ *iff* $f + \mathbf{1}_{j,x,a} \succsim g + \mathbf{1}_{j,x,a}$.

In the statement of this axiom, $f + \mathbf{1}_{j,x,a}$ is the sum of two functions. It is therefore a function and represents also an author. Intuitively, Independence can be understood as follows. Suppose an author f is at least as good as g. Suppose also both of them publish one additional paper in the same journal, with the same number of citations and the same number of coauthors. So, both make the same improvement. Then these two authors (now represented by $f + \mathbf{1}_{j,x,a}$ and $g + \mathbf{1}_{j,x,a}$) should compare in the same way as previously, i.e., $f + \mathbf{1}_{j,x,a}$ is at least as good as $g + \mathbf{1}_{j,x,a}$.

Independence is quite a mild condition. It is easy to check that it is satisfied by all scoring rules. It is also satified, for instance, by the lexicographic ranking (not a scoring rule) defined by

- $f \sim g$ iff $f = g$ and
- $f \succ g$ iff $\sum_{j \in J} \sum_{a \in \mathbb{N}} f(j, x, a) > \sum_{j \in J} \sum_{a \in \mathbb{N}} g(j, x, a)$ for some x and $\sum_{j \in J} \sum_{a \in \mathbb{N}} f(j, y, a) = \sum_{j \in J} \sum_{a \in \mathbb{N}} g(j, y, a)$ for all $y > x$.

Independence is not satisfied by the ranking based on the maximal number of citations nor is it by the ranking based on the h-index.

One might argue that Independence is not always a desirable condition. Suppose for example that f and g are two authors such that $f \sim g$ and suppose that both of them publish an additional paper in the same journal j, with 10 citations and no coauthor. So, after the change, these authors are represented by $f + \mathbf{1}_{j,10,0}$ and $g + \mathbf{1}_{j,10,0}$. According to Independence, we should have $f + \mathbf{1}_{j,10,0} \sim g + \mathbf{1}_{j,10,0}$. Yet, if we have some reasons to think that the additional paper of author f is better than the additional paper of author g, then we might expect that $f \succ g$, thereby contradicting Independence. But, since we are working in a setting where a paper is completely described by a journal, a number of citations and a number of coauthors, there can be no reason to think that the additional paper of author f is better than the additional paper of author g. So, this cannot be an argument against Independence, in this setting, but it is of course an argument against the setting of this paper. In reality, a paper is not completely described by a journal, a number of citations and a number of coauthors. The the type of paper (review or not) is also relevant, as well as the 'sign' of the citations (positive or negative) and some other characteristics. But our goal is not to support or criticize a setting or a ranking. We just want to analyze rankings that are used in practice in a context where the type of a paper and the sign of the citations are not available.

The second condition that we will need in order to characterize scoring rules is Archimedeanness.

A 4 Archimedeanness. *For all $f, g, h, e \in X$ with $f \succ g$, there is an integer n such that $e + n \cdot f \succsim h + n \cdot g$.*

In this condition, $n \cdot g$ is the standard product of a function by a number; it is a new function and also represents an author. Let us try to explain the intuitive content of this condition. Suppose $f \succ g$ and $e \prec h$. Let us add f and e, on one hand, and g and h on the other hand. It can happen that the difference between f and g is so large that it compensates the difference between e and h. In that case, we have $f + e \succsim g + h$. Suppose now this is not the case. Then repeating the same operation, we might have $f + f + e \succsim g + g + h$. Suppose this is still not the case. Then perhaps $f + f + f + e \succsim g + g + g + h$. The Archimedean condition says that keeping adding f and g will necessarily lead to $f + \ldots + f + e \succsim g + \ldots + g + h$ because the difference between $f + \ldots + f$ and $g + \ldots + g$ gets larger and larger.

All scoring rules clearly satisfy Archimedeanness. The lexicographic ranking just introduced violates Archimedeanness. So do the ranking based on the maximal number of citations and the ranking based on the h-index. In order to help the reader better understand Archimedeannes, we shortly prove our last assertion.

Consider first the ranking based on the maximal number of citations and let $f = \mathbf{1}_{j,3,0}$ and $g = \mathbf{1}_{j,1,0}$. We have $f \succ g$. Let $e = \mathbf{1}_{j,3,0}$ and $h = \mathbf{1}_{j,6,0}$. For any integer n, the maximal number of citations of $e + n \cdot f$ is 3 while the maximal number of citations of $h + n \cdot g$ is 6. Hence $e + n \cdot f \prec h + n \cdot g$, thereby contradicting Archimedeanness.

Consider now the ranking based on the h-index and let $f = 2 \cdot \mathbf{1}_{j,2,0}$ and $g = \mathbf{1}_{j,1,0}$. We have $f \succ g$. Let $e = \mathbf{1}_{j,1,0}$ and $h = 3 \cdot \mathbf{1}_{j,3,0}$. For any integer n, the h-index of $e + n \cdot f$ is 2 while the h-index of $h + n \cdot g$ is 3. Hence $e + n \cdot f \prec h + n \cdot g$, thereby contradicting Archimedeanness.

We now provide an example of a ranking satisfying Archimedeanness but not Independence: for all $f \neq \mathbf{0}$, $f \succ \mathbf{0}$ and $f \sim \mathbf{1}_{j,0,0}$ for some $j \in J$. We have $\mathbf{1}_{j,0,0} \succ \mathbf{0}$ but $2 \cdot \mathbf{1}_{j,0,0} \sim \mathbf{0} + \mathbf{1}_{j,0,0}$, thereby violating Independence.

Our first result shows that Independence and Archimedeanness are not only necessary conditions for scoring rules but that they are also sufficient.

Theorem 1 *A bibliometric ranking $\succsim$ satisfies Independence (A3) and Archimedeanness (A4) if and only if it is a scoring rule, with $u \not\equiv 0$ and u non-decreasing in its second argument. Futhermore, the mapping u is unique up to a positive affine transformation.*

Before proving this theorem, we recall a standard theorem in Measurement Theory [Roberts and Luce, 1968].

Theorem 2 *Let R be a binary relation on a set A. The asymmetric (resp. symmetric) part of R is denoted by P (resp. I). Let $\circ$ be a closed binary operation on A. For all $a \in A$ and $n > 1$, define $a(1) = a$ and $a(n) = a(n-1) \circ a$. The triple $(A, R, \circ)$ satifies*

(i) *R is transitive and complete;*

(ii) $\forall a, b, c \in A, a \; R \; b$ *iff* $a \circ c \; R \; b \circ c$;

(iii) $\forall a, b, c \in A, a \circ (b \circ c) = (a \circ b) \circ c$;

(iv) $\forall a, b, c, d \in A$ *with* $a \; P \; b$, *there is an integer* n *such that* $a(n) \circ c \; R \; b(n) \circ d$;

(v) *there is* $\epsilon \in A$ *such that, for all* $a \in A, \epsilon \circ a \; I \; a$;

if and only if there is a mapping $\phi : A \to \mathbb{R}$ such that, for all $a, b \in A$,

- $a \; R \; b$ *iff* $\phi(a) \geq \phi(b)$,
- $\phi(\epsilon) = 0$ *and*
- $\phi(a \circ b) = \phi(a) + \phi(b)$.

Furthermore, the mapping ϕ is unique up to a multiplication by a positive constant.

Proof of Theorem 1. It is easy to show that Independence and Archimedeanness are necessary conditions for scoring rules. In order to prove the sufficiency, we first show that the triple $(X, \succsim, +)$ satisfies all conditions of Theorem 2. Condition (i) is clearly satisfied because $\succsim$ is transitive and complete. Condition (ii) holds because of Independence. Indeed, suppose $f \succsim g$ and suppose e is an author with k publications. If we apply k times Independence, we find $f + e \succsim g + e$. Condition (iii) is satisfied because the binary operation $+$ on X is associative. Condition (iv) holds because of Archimedeanness. Finally, it is easy to see that $\mathbf{0}$ is an identity for the operation $+$, just like ϵ is an identity for $\circ$, so that condition (v) is verified.

So, there is $\phi : X \to \mathbb{R}$ such that

$$f \succsim g \iff \phi(f) \geq \phi(g), \tag{1}$$

$$\phi(\mathbf{0}) = 0 \tag{2}$$

and

$$\phi(f + g) = \phi(f) + \phi(g). \tag{3}$$

Since any author f can be written as $\sum_{j \in J} \sum_{x \in \mathbb{N}} \sum_{a \in \mathbb{N}} f(j,x,a) \mathbf{1}_{j,x,a}$, using (3), we find

$$\phi(f) = \sum_{j \in J} \sum_{x \in \mathbb{N}} \sum_{a \in \mathbb{N}} f(j,x,a) \phi(\mathbf{1}_{j,x,a}).$$

If we now define $u(j,x,a) = \phi(\mathbf{1}_{j,x,a})$, we can rewrite (1) as

$$f \succsim g \iff \sum_{j \in J} \sum_{x \in \mathbb{N}} \sum_{a \in \mathbb{N}} f(j,x,a) u(j,x,a) \geq \sum_{j \in J} \sum_{x \in \mathbb{N}} \sum_{a \in \mathbb{N}} g(j,x,a) u(j,x,a).$$

Because of Non-Triviality, $u \not\equiv 0$ and because of CDNH, u is non-decreasing. This completes the proof. □

5 Some special cases

We now look at conditions that force the mapping u to take some of the forms that are used in the literature.

5.1 Scoring rules affine in the number of citations

Suppose an author has, among others, two publications in the same journal and with the same number of coauthors. Suppose one of these two papers gets one more citation. We might consider that it does not matter which one gets this new citation: in both cases, the rank of the author should improve in the same way. The next axiom is a weakening of this requirement because it applies only to authors with exactly two publications. It is strongly related to a condition named Additivity in [Marchant, to appear].

A 5 Transferability. *For all $j \in J$ and all $a, x, y \in \mathbb{N}$, $\mathbf{1}_{j,x,a} + \mathbf{1}_{j,y+1,a} \sim \mathbf{1}_{j,x+1,a} + \mathbf{1}_{j,y,a}$.*

As we show in our next result, a scoring rule satisfying Transferability has a score function u affine in the number of citations.

Theorem 3 *A bibliometric ranking $\succsim$ satisfies Independence (A3), Archimedeanness (A4) and Transferability (A5) if and only if it is a scoring rule and there are two mappings $\sigma, \rho : J \times \mathbb{N} \to \mathbb{R}$ such that, for all $j \in J$ and all $a \in \mathbb{N}$, $u(j,x,a) = \sigma(j,a) + x\,\rho(j,a)$.*

Proof. The bibliometric ranking $\succsim$ being independent and Archimedean, it must be a scoring rule (Theorem 1). Transferability therefore implies $u(j,x,a) + u(j,y+1,a) = u(j,x+1,a) + u(j,y,a)$. Letting $y = 0$, we find $u(j,x,a) + u(j,1,a) = u(j,x+1,a) + u(j,0,a)$. Equivalently, $u(j,x+1,a) = u(j,x,a) + u(j,1,a) - u(j,0,a)$. This clearly implies that $u(j,\cdot,a)$ must be an affine function of its second argument. □

We now turn our attention to publications without citations. One might argue that these should not count. The next condition is an extreme weakening of this requirement.

A 6 Condition Zero. *For all $j \in J$ and all $a \in \mathbb{N}$, there is f such that $f + \mathbf{1}_{j,0,a} \sim f$.*

Theorem 4 *A bibliometric ranking $\succsim$ satisfies Independence (A3), Archimedeanness (A4), Transferability (A5) and Condition Zero (A6) if and only if it is a scoring rule and there is a mapping $\rho : J \times \mathbb{N} \to \mathbb{R}$ such that, for all $j \in J$ and all $a \in \mathbb{N}$, $u(j, x, a) = x\,\rho(j, a)$.*

Proof. Let f be as in the statement of Condition Zero. Using the scoring rule representation, we find $U(f) + u(j, 0, a) = U(f)$. Hence, $u(j, 0, a) = 0$. From Theorem 3, we know that $u(j, x, a) = \sigma(j, a) + x\,\rho(j, a)$. So, $u(j, 0, a) = 0 = \sigma(j, a) + 0\,\rho(j, a)$. This yields $\sigma(j, a) = 0$. □

This result characterizes all bibliometric rankings such that the authors are ranked according to a score defined as the total number of citations, each one being weighted by $\sigma(j, a)$, in function of the number of authors and of the journal.

5.2 Scoring rules inversely proportional to the number of authors

So far, we paid almost no attention to the number of authors. Yet, in many circumstances, publications with many authors should weigh less than publications with few authors. The following condition will help us to determine how the weight should vary with the number of authors.

A 7 Condition NRA. *For all $j \in J$ and all $x, m \in \mathbb{N}$ with $m > 1$, $\mathbf{1}_{j,x,0} \sim m \cdot \mathbf{1}_{j,x,m-1}$.*

The name NRA stands for 'No Reward for Association'. The rationale for this condition is the following. Suppose $f_1, f_2, \ldots$ represent m identical authors (clones) with exactly one publication in the same journal and without coauthors. Suppose now that, instead of publishing alone, these authors decide to form an association and to put each other's name on their papers. Then every author in this association has m publications, each with $m-1$ coauthors and is represented by $m \cdot \mathbf{1}_{j,x,m-1}$. Condition NRA states that such an 'artificial' inflation of the number of publications should have no effect. The next result analyzes the consequences of this condition when combined with the previously introduced conditions.

Theorem 5 *A bibliometric ranking $\succsim$ satisfies Independence (A3), Archimedeanness (A4) and Condition NRA (A7) if and only if it is a scoring rule and there is a mapping $\lambda : J \times \mathbb{N} \to \mathbb{R}$ such that, for all $j \in J$ and all $a, x \in \mathbb{N}$, $u(j, x, a) = \lambda(j, x)/(a + 1)$.*

If, in addition, $\succsim$ satisfies Transferability (A5) and Condition Zero (A6), then there is a mapping $\tau : J \to \mathbb{R}$ such that, for all $j \in J$ and all $a, x \in \mathbb{N}$, $u(j, x, a) = x\,\tau(j)/(a + 1)$.

Proof. From Theorem 1, we know that $\succsim$ is a scoring rule. Thanks to Condition NRA, we have $u(j, x, 0) = mu(j, x, m-1)$. So, $u(j, x, a) = u(j, x, 0)/(a+1)$. Defining $\lambda(j, x) = u(j, x, 0)$ completes the proof of the first part. The second part results from a simple application of Theorem 4. □

If we define $\tau(j) = IF(j)$, we then obtain a simple scoring rule ranking authors according to their number of citations, weighted by the number of authors and the impact factor. But defining $\tau(j) = IF(j)$ is certainly not the only possibility. Finding axioms that force $\tau(j)$ being equal to $IF(j)$ would be very interesting because it would help us understand what rationale lies behind that choice. Unfortunately, such axioms cannot be defined in our framework because the impact factor is computed for a given time window, but time does not make part of our setting. In addition, in order to compute the impact factor, we need to know the number of papers in each journal and the number of citations to each journal. This information is not available in our setting.

Another way of taking the number of authors into account consists in considering that an author with a coauthors wrote only $1/(a + 1)$ of the paper and should only be credited for that part. A difficulty with this approach is that it is not clear whether $1/(a + 1)$ is a fair share. When two authors write a paper together, one could argue that, because of the synergies, each one produces less than half the work he would do if alone. Or we could say that each one produces more than half the work because, in addition to writing one half of the paper, they also have to coordinate their work. So, instead of $1/(a + 1)$, we could for instance use $1/(a+1)^\gamma$ or some other real-valued function of a. But what is then the right value for γ? This is very difficult to know. Any value will probably be arbitrary. So, using condition NRA instead of entering the difficult problem of determining the fair share, we avoid these difficulties. Yet, note that, when assuming condition NRA, we indirectly impose $\gamma = 1$.

5.3 Scoring rules constant in the number of citations

We now introduce Condition NRC. Its name stands for No Reward for Citations.

A 8 Condition NRC. *For all $j \in J$ and all $x, a \in \mathbb{N}$, $\mathbf{1}_{j,x,a} \sim \mathbf{1}_{j,x+1,a}$.*

This condition clearly imposes that citations do not count : a paper with many citations is not worth more than a paper with few citations. Some will find this condition unreasonable but it may make sense when one judges the quality of a paper by the quality of the journal that publishes it and, more particularly, when the quality of a journal is based on the number of citations to this journal. Indeed, if one weighs a paper by the quality of the journal (e.g. impact factor) and by the number of citations of the paper, one then counts twice the citations.

And this is perhaps not reasonable. Our next result formally analyzes the consequences of this axiom.

Theorem 6 *A bibliometric ranking $\succsim$ satisfies Independence (A3), Archimedeanness (A4) and Condition NRC (A8) if and only if it is a scoring rule and there is a mapping $\sigma : J \times \mathbb{N} \to \mathbb{R}$ such that, for all $j \in J$ and all $a, x \in \mathbb{N}$, $u(j, x, a) = \sigma(j, a)$.*

If, in addition, $\succsim$ satisfies Condition NRA (A7), then there is a mapping $\tau : J \to \mathbb{R}$ such that, for all $j \in J$ and all $a, x \in \mathbb{N}$, $u(j, x, a) = \tau(j)/(a + 1)$.

Proof. From Theorem 1, we know that $\succsim$ is a scoring rule. Thanks to Condition NRC, we have $u(j, x, a) = u(j, x + 1, a)$. This obviously implies $u(j, x, a) = u(j, 0, a)$. Defining $\mu(j, a) = u(j, 0, a)$ completes the proof of the first part. The second part results from a simple application of Theorem 5. □

Note that Condition Zero is not compatible with the conditions of Theorem 6 (part 1) because it would force $\sigma(j, a) = 0$ for all $j \in J$ and $a \in N$ and the ranking would then violate Non-Triviality.

6 Conclusion

We presented an axiomatic analysis of some bibliometric rankings : the scoring rules. Within this family, we also analyzed some special cases. This does by no means imply that scoring rules are good or theoretically sound bibliometric rankings. Our analysis just helps better understand what hypotheses underlie these rankings. Our results should help anyone willing to use a ranking to choose one that more or less fits his problem, his context and his goal. The axioms characterizing scoring rules can under some circumstances be used as arguments in favor of scoring rules but under other circumstances as arguments against scoring rules. More research is needed in order to characterize a wide set of bibliometric rankings so that users can make an enlightened choice among these.

Note that this paper does also not support the use of bibliometric rankings. There are many good reasons for not using them [see e.g., Osterloh et al., 2008]. But, if one has to, then it is preferrable to know more about them.

In this paper, we characterized rankings and not on indices. Several indices can correspond to the same ranking. For instance, the number of publications and the squared number of publications are two indices yielding the same ranking. Actually, any strictly increasing transformation of an index yields the same ranking. So, in order to characterize indices [as in Woeginger, to appear], we need more axioms. Research in this field is also needed.

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References

G. Chapron and A. Husté. Open, fair, and free journal ranking for researchers. *Bioscience*, 56:558–559, 2006.

H. Eto. Interdisciplinary information input and output of a nano-technology project. *Scientometrics*, 58:5–33, 2003.

G. A. Fava and F. Ottolini. Impact factors versus actual citations. *Psychotherapy and Psychosomatics*, 69:285–286, 2000.

J. E. Hirsch. An index to quantify an individual's scientific research output. *Proceedings of the National Academy of Sciences*, 102:16569–16572, 2005.

T. Marchant. An axiomatic characterization of the ranking based on the h-index and some other bibliometric rankings of authors. *Scientometrics*, to appear.

M. Osterloh, B. S. Frey, and F. Homberg. Accountability in research and academia. Unpublished; a shorter version in French appeared in *Gérer et Comprendre*, vol. 91, March 2008, 2008.

I. Palacios-Huerta and O. Volij. Thhe measurement of intellectual influence. *Econometrica*, 72:963–977, 2004.

F. P. Pijpers. Performance metrics. *Astronomy and Geophysics*, 47:6.17–6.18, 2006.

F. S. Roberts and R. D. Luce. Axiomatic thermodynamics and extensive measurement. *Synthese*, 18:311–326, 1968.

A. F. J. van Raan. Comparison of the hirsch-index with standard bibliometric indicators and with peer judgment for 147 chemistry research groups. *Scientometrics*, 67:491–502, 2006.

G. J. Woeginger. An axiomatic characterization of the hirsch-index. *Mathematical Social Sciences*, to appear.