DISTRIBUTIVE LATTICES AND ACYCLIC DOMAINS OF LINEAR ORDERS : SURVEY AND NEW RESULTS

Bernard MONJARDET[1]

[1]CES, Université Paris 1

An acyclic domain of linear orders is a set of linear orders defined on a n-set where the use of majority rule never induces an "effet Condorcet" : the (strict) majority relation of every profile of linear orders taken in an acyclic domain is always a (linear or not) order. Then, the majority rule applied on on such a domain defines an aggregation rule satisfying all the good properties (transitivity of the collective preference, anonymity, symmetry, independence, monotonicity, unanimity, . . .). The first acyclic domain was discoverd by Black (1948) whereas many others were discovered and/or characterized by Guilbaud (1952), Ward (1965), Frey (1971), Blin (1973), Romero (1978), Kim and Roush (1980), Abello (1984,1985), Arrow and Raynaud (1986), Chameni-Nembua (1989), Craven (1992) or Fishburn (1992,1997,2002). An important problem raised in these researches was to find the maximum size of an acyclic domain on a n-set (during a long time it was conjectured to be 2^{n-1}, a very wrong conjecture). Lnear orders defined on a n-set are one-to-one with the permutations of this set. The set of these permutations endowed with the weak Bruhat order is the permutoèdre lattice S_n (Guilbaud et Rosenstiehl, [7]). So an acyclic domain is a subset of this lattice. From 1989 Chameni-Nembua ([4]) showed that every distributive sublattice of S_n preserving the covering relation of this lattice is an acyclic domain. Galambos et Reiner ([6]) have recently shown that almost all the acyclic domains already obtained are distributive sublattices of S_n preserving the covering relation, a result unifying many previous results. For example this kind of acyclic domain can be obtained from a maximal chain of the lattice S_n ([1], [2]) and is characterized by the exclusion of some ordered triples ([5], [6]) . In our talk we will first present the main results on the acyclic domaines of this kind and we will give some clues on their proofs (which can be direct or to rely on the theory of higher Bruhat orders ([9]). Then we will specify these results for three particular cases : the domains formed by a maximal chain of S^n, those given by Fishburn's alternating scheme ([5]) and those discovered by Black ([3]). We will also give the

state of the question and some conjectures on the -indeed very difficult- problem of the mamimum size of an acyclic domain.

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