Valued relations aggregation with the Borda method.

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Abstract :

The Borda method is a well known method used to aggregate a profile of crisp binary relations into one weak order. We present a generalization of the Borda method, allowing it to aggregate a profile of valued relations. We give a characterisation of the Borda method in this context and we show some links with the PROMETHEE method.

Key words :

Borda, valued relation, characterisation, PROMETHEE.

I. Introduction

When one has to aggregate a profile of preferences expressed under the form of crisp binary relations, it is very common to use the Borda method, yielding a weak order (ranking method) or a choice set (choice method). When the profile to be aggregated is made of valued instead of crisp relations, we show in section III how the Borda method can be generalized in order to allow the aggregation.

Young (1974) characterized the Borda method as a choice method when the preferences are expressed by means of linear orders (crisp, asymmetric, complete and transitive relations). Nitzan and Rubinstein (1981) axiomatized it as a ranking method aggregating asymmetric and connected relations. Debord (1987) generalized Young's result when he presented a characterization of the Borda method as a choice method when the preferences are modelled by crisp binary relations belonging to a family *F*, that must contain all linear orders. He also characterized the Borda method as a ranking method for profiles of weak orders. In each characterization, the axioms are almost the same : neutrality, cancellation, consistency and faithfulness (monotonicity for Nitzan and Rubinstein). We don't present an exact formulation of these axioms because they are slightly different for each author, according to the context, but the idea is the following :

- *Neutrality* expresses the fact that the result of the method does not depend on the names or labels given to the alternatives or items to be compared.
- *Cancellation* : if for any pair of alternatives there are as much voters in favour of the first alternative as in favour of the second one, then all alternatives tie.
- If the method is applied to two groups of voters and if the result is the same for both, *consistency* implies that the method applied to a group of voters made of the two previous groups must yield the same result.
- When there is only one voter, if the relation that he uses to express his preferences is so simple that one result seems the only reasonable one, the result of the method must be that one. This is what *faithfulness* says.

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- If one voter improves the position of one alternative in the relation expressing his preferences, then *monotonicity* implies that the position of the alternative in the result of the method cannot worsen. Faithfulness and monotonicity are very much related : Debord (1987) showed that under some conditions, strict monotonicity and faithfulness are equivalent.

We use the same axioms in section III to generalize the previously mentioned works : we present a characterization of the Borda method as a ranking (or choice) method applied to a profile of valued relations belonging to a set *D*, with some restrictions on the set *D*. Section IV presents various kinds of valued relation for which our result is valid, i.e. satisfying the restrictions on the set *D*. The last section is devoted to the application of this result to the PROMETHEE method.

II. Definitions.

Let *X* be a finite set of alternatives. The alternatives will be denoted $x, y, z, \ldots$ or $x_1, x_2, \ldots$ Let $V = \{v, w, \ldots\}$ or $\{v_1, v_2, \ldots\}$ denote the set of the voters or criteria. We will consider that the set of voters is not fixed a priori. *V* can vary in size. A valued relation S is a mapping : $X^2 \rightarrow [0,1] : (x,y) \rightarrow S(x,y) = S_{xy}$. We shall write $\bar{S}$ a valued relation such that $\forall x, y \in X, \bar{S}_{xy} = S_{yx}$. A set *D* of valued relations is said stable by transposition iff for any relation $S \in D$, $\bar{S} \in D$. Let σ be a permutation on the alternative set *X*. σ(S) will denote the valued relation obtained from S by relabelling the elements of *X* according to σ. Thus $\forall x, y \in X\ \sigma(S_{xy}) = S_{\sigma(x)\sigma(y)}$. A set *D* of valued relations is said stable by permutation iff for any relation $S \in D$ and for any σ, $\sigma(S) \in D$. Obviously most sets *D* encountered in practice are stable by transposition and permutation. Let $H = \{h_1, h_2, \ldots\}$ be the set of all weak orders i.e. strongly complete and transitive binary relations (Roubens and Vincke, 1985). A profile is a mapping *p* from *V* to a set *D* containing *H* and stable by transposition and permutation. $P = \{p, q, \ldots\}$ or $\{p_1, p_2, \ldots\}$ denotes the set of all profiles.

We will consider an ordering function (OF) as a mapping $\gtrsim : P \rightarrow H : p \rightarrow \gtrsim(p)$. The expressions *ordering function* and *ranking method* cover the same concept, but we introduce here the first one, with a precise definition, to distinguish it from the second one, more fuzzy, which was used in the introduction to describe different methods.

$S_{xy}(p,v)$ denotes the value associated with the arc (x,y) in the graph that represents the valued relation of voter *v* in profile *p*.

$x \gtrsim(p) y$ means that $(x,y) \in \gtrsim(p)$, i.e. *x* is at least as good as *y* in the weak order result of the aggregation. $x > (p) y$ means that $(x,y) \in \gtrsim(p)$ and $(y,x) \notin \gtrsim(p)$, i.e. *x* is at strictly better than *y* in the weak order result of the aggregation. Finally, $x \sim (p) y$ means that (x,y) and $(y,x) \in \gtrsim(p)$, i.e. *x* and *y* are equivalent in the weak order result of the aggregation.

Let $\pi_{xy}(p) = \sum_{v \in V} S_{xy}(p,v)$, $N_x^+(p) = \sum_{y \in X} \pi_{xy}(p)$, $N_x^-(p) = \sum_{y \in X} \pi_{yx}(p)$,

$\beta_x(p) = N_x^+(p) - N_x^-(p)$.

Let *p* be a profile. We shall write $\bar{p}$ a profile such that $\forall x, y \in X, v \in V, S_{xy}(p,v) = S_{yx}(\bar{p}, v)$. Thus, $\bar{p}$ denotes a profile where each voter has reversed his preferences. Let σ be a permutation of the alternative set *X*. σ(*p*) will denote the profile obtained from *p* by relabelling the elements of *X* according to σ.

III. Characterization of Borda rule.

The Borda rule is, as we told in the introduction, a well known method in social choice theory. Its purpose is to aggregate the information contained in a set of crisp relations in order to obtain a choice set containing the most preferred alternatives or a ranking (weak order). In this paper, we are interested in the aggregation of valued relations and we propose the following generalization of the Borda method :

a) for each alternative x compute the generalized Borda score $\beta_x(p)$.

b) build a relation $\gtrsim(p)$ by means of the rule:

$$\beta_x(p) \geq \beta_y(p) \Leftrightarrow x \gtrsim (p) y .$$

Let us now define the axioms that we will use to characterize the Borda method.

A1. *Neutrality* : Let σ be a permutation on the alternative set X. $\sigma(\gtrsim(p))$ will denote the weak order obtained from $\gtrsim(p)$ by relabelling the elements of X according to σ. The mapping $\gtrsim$ is neutral if $\gtrsim(\sigma(p)) = \sigma(\gtrsim(p))$.

A2. *Consistency* : Let S_1 and S_2 be a partition of V. To subset S_1 (resp. S_2) corresponds the profile p_1 (resp. p_2). To V corresponds the profile p also noted p_1+p_2. When $p_1 = p_2 = p$, we will write : $p_1+p_2 = 2p$. Consistency imposes that:

- $x \gtrsim (p_1) y$ and $x \gtrsim (p_2) y \Rightarrow x \gtrsim (p_1+p_2) y$ and
- $x > (p_1) y$ and $x \gtrsim (p_2) y \Rightarrow x > (p_1+p_2) y$.

In a less formal way, if two groups agree to consider x better than y, then the group consisting of the two groups considers x better than y.

A3. *Faithfulness* : if $|V| = 1$ and the relation S used by the only voter is a crisp weak order then $\gtrsim(p)$ is the same weak order.

A4. *Cancellation* : $\forall\, x, y \in X$ $\pi_{xy}(p) = \pi_{yx}(p) \Rightarrow \gtrsim(p) = X^2$. Cancellation is much less satisfactory here than in the crisp case. In the crisp case, $\pi_{xy}(p)$ is the number of voters in favour of x rather than y. In the valued case, it is a sum of valuations. Hence, it is stronger in the valued case than in the crisp case. And it raises the following remark : $\pi_{xy}(p)$ is a sum of valuations; but is it meaningful to add valuations ? There is no general answer as this depends upon how the valuations have been obtained and what they represent.

Now we can propose a characterisation of the Borda method, whose proof is very similar to the proof given by Debord.

Theorem 1. The Borda method is the only neutral (A1), consistent (A2) and faithful (A3) ordering function that satisfies cancellation (A4).

Before proving this theorem, we shall go through five lemmas.

Let $\mathcal{P}_x$ be the set of all permutations σ on X such that $\sigma(x) = x$.

Let's define $p_x = \sum_{\sigma \in \mathcal{P}_x} \sigma(p)$. It is a profile consisting of the juxtaposition of the profiles obtained by all the permutations σ on X leaving x unchanged. By construction, we have :

$\forall\, y, z \neq x$, $\pi_{yz}(p_x) - \pi_{zy}(p_x) = 0$ (because z and y have symmetrical positions in p_x) and

$\forall\, z \neq x$, $\pi_{xz}(p_x) - \pi_{zx}(p_x) = (n-1)!\, \beta_x(p)$.

Lemma 1. If $\gtrsim$ satisfies cancellation (A4) and $\beta_x(p) = 0$,then $\gtrsim(p_x) = X^2$.

Proof. If $\beta_x(p) = 0$, then $\forall\ y, z \in X$, $\pi_{yz}(p_x) - \pi_{zy}(p_x) = 0$, and by cancellation, $\gtrsim(p_x) = X^2$. ■

Lemma 2. If $\gtrsim$ satisfies neutrality (A1) and consistency (A2), then $[\forall y \in X\ \ x \gtrsim (p) y$ and $\gtrsim(p) \neq X^2\ ] \Rightarrow x > (p_x) y \quad \forall y \in X$.

Proof. By neutrality, $\forall y \in X, \forall \sigma \in \mathcal{P}_x\ \ x \gtrsim (\sigma(p)) y$.

$\gtrsim(p) \neq X^2 \Rightarrow \forall y \in X, \exists\ \sigma \in \mathcal{P}_x : x > (\sigma(p)) y$. By consistency, $\forall y \in X\ x > (p_x) y$. ■

Lemma 3. If $\gtrsim$ satisfies neutrality (A1), consistency (A2), cancellation (A4) and $\forall x \in X$, $\beta_x(p) = 0$, then $\gtrsim(p) = X^2$.

Proof. Suppose that $\gtrsim(p) \neq X^2$. Then, by lemma 2, as $\gtrsim(p)$ is a weak order, $\forall y \in X, \exists\ x : x > (p_x) y$. But by lemma 1 we know that $\forall x \in X \gtrsim(p_x) = X^2$. Contradiction. ■

Lemma 4. If $\gtrsim$ satisfies cancellation (A4) and consistency (A2), then $x \gtrsim (p) y \Leftrightarrow y \gtrsim (\bar{p}\ ) x$.

Proof. Suppose that $x \gtrsim (p) y$ and $\neg\ y \gtrsim (\bar{p}\ ) x$. By consistency, $x > (p+\bar{p}\ ) y$. But $\forall\ y, z \in X\ \pi_{yz}(p+\bar{p}\ ) = \pi_{zy}(p+\bar{p}\ )$. Thus, by cancellation, $x \sim (p+\bar{p}\ ) y$. Contradiction. ■

Lemma 5. if $\gtrsim$ satisfies neutrality (A1), consistency (A2) and cancellation (A4), then $\forall x \in X$ $\beta_x(p) = \beta_x(q) \Rightarrow \gtrsim(p) = \gtrsim(q)$.

Proof. $\forall x \in X\ \beta_x(p + \bar{q}\ ) = 0$. By lemma 3, $\gtrsim(p + \bar{q}) = X^2$.

Suppose that $x \gtrsim (p) y$ and $\neg\ x \gtrsim (q) y$. By lemma 4, $\neg\ y \gtrsim (\bar{q}) x$ and by consistency, $x > (p+\bar{q}\ ) y$. Contradiction. ■

We are now ready to give the proof of theorem 1. Here is shortly how we will do: For any profile p, we construct a profile p^* consisting of well known profiles p_i^*. We show that $\gtrsim(p)$ must be equal to $\gtrsim(p^*)$. And, using consistency, we find $\gtrsim(p^*)$ from $\gtrsim(p_i^*)$.

Proof of theorem 1. Let us sort the elements of X according to their generalized Borda score and rename them as follows: x_1 for the element with the highest generalized Borda score, x_2 for the next one, ... and x_n for the element with the lowest generalized Borda score. This is licit since $\gtrsim$ is neutral. Let $\delta_i(p) = \beta_{x_i}(p) - \beta_{x_{i+1}}(p)$.

Let p_i^* be a profile consisting of only one crisp weak order h_i : $(x_1\ ...\ x_i)(x_{i+1} ...\ x_n)$ with two equivalence classes. Let p^* be a profile defined by $p^* = \delta_1(p)\, p_1^* + \delta_2(p)\, p_2^* + ... + \delta_{n-1}(p)\, p_{n-1}^*$. It is easy to verify that $\forall i\ \beta_{x_i}(p^*) = n\ \beta_{x_i}(p)$. Hence, by lemma 5 and consistency, $\gtrsim(p) = \gtrsim(p^*)$. We show it just for x_1. $\beta_{x_1}(p_1^*) = n\text{-}1$, $\beta_{x_1}(p_2^*) = n\text{-}2$, ..., $\beta_{x_1}(p_{n-1}^*) = 1$. $\beta_{x_1}(p^*) = \delta_1(p)\, \beta_{x_1}(p_1^*) + \delta_2(p)\, \beta_{x_1}(p_2^*) + ... + \delta_{n-1}(p)\, \beta_{x_1}(p_{n-1}^*) = (\beta_{x_1}(p) - \beta_{x_2}(p))\ (n\text{-}1) + (\beta_{x_2}(p) - \beta_{x_3}(p))\ (n\text{-}2) + ... + (\beta_{x_{n-1}}(p) - \beta_{x_n}(p))\ (1) = \beta_{x_1}(p)\ (n\text{-}1) - \beta_{x_2}(p) - \beta_{x_3}(p) - ... - \beta_{x_{n-1}}(p) - \beta_{x_n}(p) = \beta_{x_1}(p)\ (n\text{-}1) + \beta_{x_1}(p) = n\ \beta_{x_1}(p)$. Thus $\gtrsim(p) = \gtrsim(p^*)$. By faithfulness, $\gtrsim(p_i^*)$ is the weak order h_i given by the only voter. By consistency, $\gtrsim(\delta_i(p)\, p_i^*) = \gtrsim(p_i^*)$. And $\gtrsim(p^*)$ is equal to the weak order defined by :

$x_i \gtrsim (p^*) x_j \Leftrightarrow \beta_{x_i}(p^*) \geq \beta_{x_j}(p^*)$. ■

IV. Discussion.

The result presented in theorem 1 characterizes the Borda method when applied to a profile of valued relations. The valued relations must belong to a set D containing the weak orders and stable by permutation and transposition. The restrictions on the set D are very weak. D can be for example the set of all weak orders, the set of all semi-orders, the set of all interval orders or ... (It was already so in Debord, 1987, for the Borda method as a choice method). It can be also the set of all valued relations or the set of all valued relations equivalent to a homogeneous family of semi-orders (for definition, see Roubens and Vincke, 1985), a homogeneous family of interval orders or ... Hence our result places very few restrictions on D and can be applied in many different cases.

If we want to aggregate asymmetric relations, e.g. the asymmetric part of all asymmetric valued relations, our result is no more valid because the set D would not contain all weak orders. Let us change axiom 3 as follows :

A3*. Faithfulness : $|V|= 1$ and the relation S used by the only voter is a crisp strict weak order, i.e. an asymmetric and negatively transitive relation (Roberts, 1979), then $\gtrsim(p)$ is the weak order whose asymmetric part is the strict weak order of the only voter.

Moreover if we define a profile as a mapping p from V to a set D stable by transposition and permutation and containing the set of all *strict* weak orders, then we can apply our result to the aggregation of asymmetric relations, e.g. to the asymmetric part of semi-orders or the asymmetric part of a homogeneous family of interval orders or .... In particular, it can be applied to the valued relations used in PROMETHEE as shown in the next section.

A last remark : our result can easily be transposed to the choice method problem. The axioms need very few reshaping. Faithfulness becomes : if there is only one voter and his relation is a crisp weak order then the choice set is the first equivalence class of his weak order. Consistency : if the intersection between the choice sets of two groups is not empty, then the choice set of the group consisting of the two groups is the intersection.

V. Application to the PROMETHEE II method.

PROMETHEE is a multicriteria decision aid method that appeared in 1982 and that has been further developed by Brans and Mareschal(1994) and which is now widely used. It is based on the outranking approach and, like most methods based on this approach, it lacks sound theoretical and axiomatic foundations. The PROMETHEE method can be briefly described as follows :

PROMETHEE considers multicriteria decision problems of the following type :

"Max" $\{ f_1(x_j), f_2(x_j), ..., f_i(x_j), ..., f_k(x_j) \}$ s.t. $x_j \in X = \{ x_1, x_2, ..., x_j, ..., x_n \}$ or $\{x, y, ... \}$. The elements of X are interpreted as possible decisions or alternatives which are evaluated on k criteria $f_1, ..., f_k$.

In order to take into account the deviations and the scales of the criteria, a preference function $P(x,y)$ giving the degree of preference of x over y for criterion f is defined. $P(x,y)$ is a function of the difference $d= f(x) - f(y)$ such that $0 \le P(x,y) \le 1$ and :

$$\begin{cases} P(x,y) = 0 \text{ if } d \le 0 : \text{no preference or indifference} \\ P(x,y) \approx 0 \text{ if } d > 0 : \text{weak preference} \\ P(x,y) \approx 1 \text{ if } d >> 0 : \text{strong preference} \\ P(x,y) = 1 \text{ if } d >>> 0 : \text{strict preference .} \end{cases}$$

It is clear that P must be a non decreasing function of d, with a shape similar to that of fig.1. In the PROMETHEE method, a preference function is associated to each criterion f_i, i=1, ..., k.

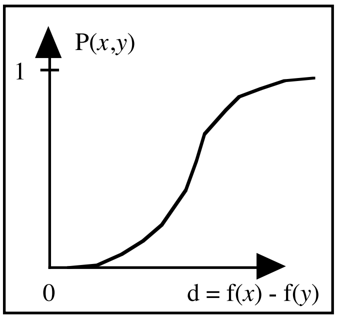

fig.1.: Preference function

A multicriteria preference index $\pi(x,y)$ of x over y can then be defined taking into account all the criteria :

$$\pi(x,y) = \sum_{i=1}^{k} w_i \; P_i(x,y) ,$$

where $w_i > 0$ (i = 1, ..., k) are weights associated to each criterion and sum up to unity. These weights are positive real numbers that do not depend on the scale of the criteria. The following properties obviously hold for the $\pi(x,y)$ values :

$$\begin{cases} \pi(x,y) \approx 0 \text{ implies a weak global preference of } x \text{ over } y \\ \pi(x,y) \approx 1 \text{ implies a strong global preference of } x \text{ over } y . \end{cases}$$

The net outranking flow is defined by :

$$\Phi(x) = \frac{1}{n-1} \sum_{y \pi x} \big(\pi(x,y) - \pi(y,x)\big) \quad .$$

The higher the net flow, the better the alternative. The PROMETHEE II ranking is defined by: $x \gtrsim y$ iff $\Phi(x) \geq \Phi(y)$.

Schematically the PROMETHEE method can be divided into three main steps :

- Construction of k valued relations P_i defined by the values $P_i(x,y)$ and corresponding to the k criteria. (step 1).
- Aggregation of the k valued relations into one valued relation π defined by the values $\pi(x,y)$. (step 2).
- Exploitation of the valued relation π and construction of the PROMETHEE II weak order. (step 3).

Bouyssou (1992) studied a problem very similar to step 3. Given a valued relation he used the so called *net flow method* to obtain a ranking (weak order). The net flow method is actually the same one as in step 3 of the PROMETHEE method. If the input of the net flow method happens to be a crisp relation then the net flow method is equivalent to the Copeland rule. Bouyssou (1992) showed that the net flow method is the only one satisfying the following axioms : *neutrality*, *strong monotonicity* (if the position of one alternative is improved in the valued relation, its position in the resulting ranking must also be improved) and *independence of circuits* (if the valued relation contains circuits and if all values of the arcs in the circuit are increased (decreased) by the same amount, the resulting ranking must not change). The difference between Bouyssou's problem and step 3 of the PROMETHEE method is that he considers in input any valued relation. But Bouyssou (1995) showed that the valued relations π obtained after the second step of the PROMETHEE method have some particular properties. Thus, one cannot obtain any valued relation π after step 2. In 1993, Bouyssou presented another characterization of the net flow method, in which he considered valued relations that are not necessarily without special properties. So that his result can be applied to the third step of the PROMETHEE method. He used the following axioms : *neutrality*, *cancellation*, *faithfulness*, *consistency*. Formally, his axioms are very different of ours because he considered one valued relation and not a profile. But the idea is the same.

We now turn to the application of our result to steps 2 and 3. At the end of the first step, the set of the valued relations that can be obtained contains the set of all strict weak orders and is stable by transposition and permutation. This is true whether the shape of the k preference functions is fixed (the decision maker chooses any shape for his preference functions and keeps them) or not (the decision maker may choose and then change the shape of his preference functions). Moreover steps 2 and 3, considered together, are equivalent to the Borda method. So the characterisation given in §3 is valid for steps 2 and 3 of the PROMETHEE II method and it is thus a generalization of Bouyssou's result.

Up to now we didn't speak about weights. There are weights in the PROMETHEEE method and not in the Borda method. But the weights in the PROMETHEE method are such that giving a weight *w* to a criterion and 2*w* to another one is equivalent to consider a profile with the second criterion appearing two times more than the first one. And this raises no problem, as we considered that the size of *V* is not fixed.

Appendix .

In this appendix, we show that the four axioms (A1, A2, A3 and A4) we used in our characterization of the Borda rule are independent.

- Let $\beta^*_x(p) = \sum_{v \in V} N^+_x(p,v)$

and define $x \succsim (p) y$ iff $\beta^*_x(p) \geq \beta^*_y(p)$.

This rule satisfies neutrality, consistency and faithfulness but not cancellation.

- Let us define $x \succsim (p) y$ if $\beta_y(p) \geq \beta_x(p)$. This rule satisfies neutrality, consistency and cancellation but not faithfulness.

- Let ψ bijective : $X \rightarrow \{1,2,..,|X|\}$ and $\beta^{**}_x(p) = \sum_{y \neq x} \psi(y)\big(\pi_{xy}(p) - \pi_{yx}(p)\big)$ and define $x \succsim (p) y$ iff $\beta^{**}_x(p) \geq \beta^{**}_y(p)$.

This rule satisfies consistency, cancellation and faithfulness but not neutrality.

- Let $\phi : \mathbb{R} \rightarrow \{0,1\} : \phi(u) \rightarrow \begin{cases} 0 \text{ if } u \leq 0 \\ 1 \text{ if } u > 0. \end{cases}$

Define the following binary relation Q on X

$x\, Q(p)\, y$ if $\pi_{xy}(p) \,.\, \phi(\beta_x(p)) \geq \pi_{yx}(p) \,.\, \phi(\beta_y(p))$ and let

$x \succsim (p) y$ if $|\{z : x\, Q(p)\, z\}| > |\{z : y\, Q(p)\, z\}|$

or $|\{z : x\, Q(p)\, z\}| = |\{z : y\, Q(p)\, z\}|$ and $\beta_x(p) \geq \beta_y(p)$.

This rule satisfies neutrality, cancellation and faithfulness but not consistency.

To verify that it doesn't satisfy consistency, use the following profiles :

p_1 :

	a	b	c	d	e
a	0	1	0	1	1
b	0	0	0	1	1
c	0	1	0	1	1
d	0	0	0	0	0
e	0	0	0	0	0

	a	b	c	d	e
a	0	1	0	0	0
b	0	0	0	0	0
c	1	1	0	1	1
d	0	1	0	0	0
e	0	1	0	0	0

	a	b	c	d	e
a	0	0	0	0	0
b	1	0	1	1	1
c	0	0	0	0	0
d	1	0	1	0	0
e	1	0	1	0	0

p_2 :

	a	b	c	d	e
a	0	0	0	0	1
b	1	0	1	1	1
c	0	0	0	0	1
d	0	0	0	0	1
e	0	0	0	0	0

	a	b	c	d	e
a	0	1	1	1	1
b	0	0	0	1	1
c	0	0	0	1	1
d	0	0	0	0	0
e	0	0	0	0	0

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