Higher spin operators

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Abstract

In this talk, I will introduce higher spin operators. These must be seen as generalizations of the well known Dirac operator and the Rarita-Schwinger operator, describing the behaviour of some elementary particles in quantum theory.

We, however, will take a look at these operators from a function theoretical point of view. The operators should be seen as objects acting on functions $f(\underline{x})$ on $\mathbb{R}^m$, the m-dimensional real space. They will take values in more complicated irreducible representations for the spin group. This brings us to an representation theoretical part of this theory. It is important to note that each irreducible representation of $Spin(m)$ can be represented by its highest weight. This is a vector $\lambda = (l_1, l_2, ..., l_k)$, where all l_i are integer valued, or all l_i are half integer valued. This vector will determine the representation entirely. Therefor, we will denote the corresponding representation by means of its highest weight vector from now on.

The next step is to make a connection to function theory. It can be shown that each of the irreducible representations stated above, can be expressed as a specific space of polynomials. In the case of integer valued highest weights:

$$\lambda = \mathcal{H}_\lambda,$$

the space of simplicial harmonic polynomials in k vector variables $\underline{u}_i$, homogeneous of degree l_i in $\underline{u}_i$, and which are $\mathbb{C}$-valued. In the case of half integer valued highest weights, we have:

$$\lambda = \mathcal{S}_\lambda,$$

the space of simplicial monogenic polynomials in k vector variables $\underline{u}_i$, homogeneous of degree l_i in $\underline{u}_i$, and which are $\mathbb{S}$-valued (the spinor space).

Considering the operators again, there are 2 major classes we will discuss, one being the operators of the form

$$\mathcal{Q}_\lambda : \mathcal{C}^\infty(\mathbb{R}^m, \mathcal{S}_\lambda) \to \mathcal{C}^\infty(\mathbb{R}^m, \mathcal{S}_\lambda).$$

These are so called higher spin Dirac operators. The second class will contain the operators of the form

$$\mathcal{T}_\lambda^{\lambda-\delta_i} : \mathcal{C}^\infty(\mathbb{R}^m, \mathcal{S}_\lambda) \to \mathcal{C}^\infty(\mathbb{R}^m, \mathcal{S}_{\lambda-\delta_i}).$$

Here, $\lambda - \delta_i$, is the vector λ, where 1 is substracted from the i-th component.

The final aim of this talk will be to discuss an explicit, inductive construction of these operators, using the notion of so called twisted operators.

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