Higher spin operators as generators of transvector algebras

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In higher spin Clifford analysis, any irreducible representation of the Spin(m)-group with a half-integer highest weight $\left(l_1 + \frac{1}{2}, \ldots, l_k + \frac{1}{2}, \frac{1}{2}, \ldots, \frac{1}{2}\right)$, where $l_1, \ldots, l_k$ are natural numbers, can be modelled by the space of simplicial monogenic polynomials in k vector variables $u_1, \ldots, u_k$, homogeneous of degree l_i in u_i for each $i \in \{1, \ldots, k\}$. This space is denoted by $\mathcal{S}_{l_1,\ldots,l_k}$, see [1].

The theory of generalised gradients (e.g. [2, 4]) tells us that the only conformally invariant first order differential operators acting on functions with values in $\mathcal{S}_{l_1,\ldots,l_k}$ (which can be identified with the space of polynomials $\mathcal{C}^\infty(\mathbb{R}^m, \mathcal{S}_{l_1,\ldots,l_k})$) are the higher spin Dirac operator $\mathcal{Q}_{l_1,\ldots,l_k}$, at most k twistor operators, and at most $k+1$ dual twistor operators.

In this talk, it will be shown that these differential operators can be seen as generators of a transvector algebra, hereby generalising the fact that the classical Dirac operator and its symbol generate the orthosymplectic Lie algebra $\mathfrak{osp}(1,2)$. To that end, we construct these operators using an extremal projector, an object that is naturally appearing in the theory of transvector algebras (see e.g. [3, 5, 6]).

References

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