This study presents a data-driven analysis of non-stationary temperature extremes using a structural time series approach combined with Extreme Value Theory (EVT). We extend the Dynamic Generalized Extreme Value (DGEV) framework by integrating an Expectation-Maximization (EM) algorithm for robust parameter estimation. Using over a century of daily temperature records from Uccle, Belgium, we model seasonal extremes and averages within a state-space framework, where the location parameter evolves stochastically. The structural approach allows us to explicitly separate trend and seasonality, providing deeper insights into long-term changes in extreme temperatures. Our results reveal a significant warming trend, with extreme minimum temperatures increasing more rapidly than maximum temperatures. Additionally, we show a quick emergence of previously impossible events following a power-law growth, emphasizing the increasing likelihood of record-breaking extremes. We compare our structural dynamic models to traditional non-dynamic models, demonstrating that allowing time-varying parameters improves the representation of climate trends. The model is validated through sequential Monte Carlo estimation and goodness-of-fit diagnostics, highlighting the effectiveness of our approach in capturing the evolving distribution of extremes. These findings underscore the importance of structural modeling in extreme value analysis and provide a flexible, data-driven framework for studying climate-driven changes in extremes.