Quaternion sparse representation model for color image processing
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Image processing, Convolutional neural networks, Dictionary learning, Sparse coding
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1. W. Samek, T. Wiegand, K.R. Müller, *Explainable Artificial Intelligence: Understanding, Visualizing and Interpreting Deep Learning Models*, 2018.

2. P. Vardan, Y. Romano, M. Elad. Convolutional neural networks analyzed via convolutional sparse coding, 2017.

3. V. Papyan, Y. Romano, J. Sulam, M. Elad, *Theoretical Foundations of Deep Learning via Sparse Representations*, 2018.

	Many problems like image and video restoration, compression and coding, digital image inpainting and content analysis benefit from the sparse representation model. As these techniques are powerful and widely applicable, sparse representations of signals (including images and higher-dimensional data), attract the interest of researchers from different fields. The goal of such a sparse representation is to approximate well a signal using only few elements from a (typically redundant) dictionary (see Figures 1 and 2). One of the best known and widely used approaches for dictionary learning is the so-called K-SVD method. K-SVD is an extension of the K- means clustering method that allows efficient learning of the dictionary using the singular value decomposition (SVD). The common dictionary learning techniques, including the recent K-SVD methods, treat signals in a unified way irrespective of their dimensionality and the nature of different channels in the case of multicomponent data (such as color, multispectral or hyperspectral images). All the data within a 2-D window (in the case of a greyscale image) or a 3-D window (in the case of a multicomponent image) are simply stacked in an array, using a given scanning order and as such treated as a single 1-D vector.
Description of the problem (Probleemstelling)	A very recent trend in signal processing and machine learning attempts to build an improved sparse representation model of color images by introducing quaternions into dictionary construction. Quaternions are four-dimensional generalization of complex numbers (with three imaginary units instead of one). Due to their property to describe efficiently rotations in 3D, quaternions have many applications in theoretical and applied mathematics but also in different fields of engineering such as computer graphics and computer vision as well as in various applications including biomedical image processing, remote sensing, hyperspectral image processing and many others. The quaternionic representation with three imaginary units is also ideally suited for representing three color channels, and therefore quaternions have already been used extensively in color image processing. A very recent method, so-called K-QSVD, which is a generalization of the K-SVD algorithm in the quaternionic framework, already showed remarkable results (see Figure 3). The potentials of quaternions in improving sparse representations of multicomponent images are yet to be explored, starting from the first encouraging results. The motivation is that the coefficient matrix preserves not only the correlation among the channels but also the orthogonal property. According to recent studies, this proves to be important in terms of computation complexity but also in terms of color fidelity in the reconstruction. The experiments of reconstruction, denoising, inpainting, and super-resolution on natural color images prove its advantages in characterizing color structures and preserving the interrelationship among color channels compared to other classical methods. However, many aspects of this approach are yet to be explored, both theoretically and in terms of the practical design. In this master thesis, the student will be guided by supervisors from the Clifford research group and the Image Processing and Interpretation research group.

	This thesis should combine emerging and hugely popular technologies in image processing and computer vision with a solid mathematical theory to build a sound framework that will be validated in some concrete
Goal of the thesis (Doelstelling)	applications but even more widely applicable. There are many aspects of this approach yet to be investigated, so the main goal of the thesis is to further explore the possibilities of already developed K-QSVD algorithm. The developed method will be applied in the concrete image processing application of image demosaicing and color video processing. Demosaicing is the process of reconstructing a full-resolution color image from the color samples output from an image sensor overlaid with a color filter array (a color filter array is a mosaic
	of color filters in front of the image sensor). Next, in the sparse coding step of the K-QSVD algorithm different algorithms can be used, most notable ones being QOMP and quaternion basis pursuit. The efficiency of QOMP algorithm should be compared with the efficiency of the recently introduced quaternion basis pursuit algorithm (solved by ADMM technique), in terms of the approximation power. All the results obtained so far are obtained by using QOMP algorithm, so a lot of space remains for improvement and future work. In these applications, the use of quaternionic dictionaries will be practically evaluated and compared to some of the current state- of-the-art methods. All the material and starting code will be made available to the student.

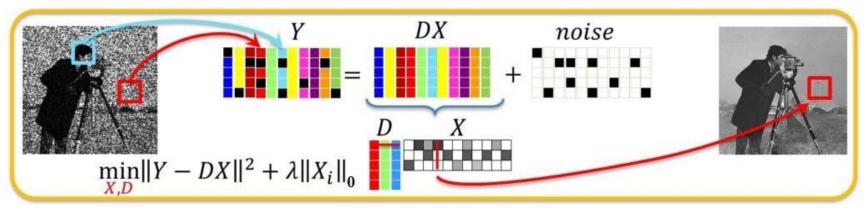


Figure 1: For input data Y, dictionary learning method aims to find dictionary D and a representation matrix X such that its columns Xi are sparse enough (<u>https://research.csiro.au/data61/</u>).



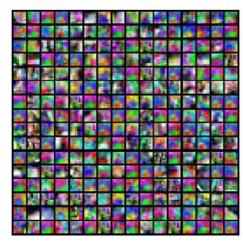


Figure 2: An illustration of the color image dictionaries learned by K-SVD (on the left) and K-QSVD algorithm (on the right).



**Figure 3**: An example showing the application of K-QSVD in image inpainting. Left: damaged image (70% missing); Right: an image reconstructed using K-QSVD.