Master thesis topic (Masterproefonderwerp)	Finsler geometry approach to Beltrami framework for image processing
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Location – (Locatie)	Technicum, De Sterre, thuis

References (Referenties):

1. R. Kimmel, *Numerical Geometry of Images*, Springer-Verlag, 2004.
2. M. Dahl, *A Brief Introduction to Finsler Geometry*, Lecture Notes, 2006. (https://math.aalto.fi/~fdahl/finsler/finsler.pdf)
3. J. Stojanov, *Anisotropic frameworks for dynamical systems and image processing*, PhD Thesis, University of Novi Sad, 2015.

Description of the problem (Probleemstelling)	Scientists from different fields of science and engineering are interested in image processing tasks due to their importance in medical sciences, geology and geography as well as in art investigation for restauration of old paintings. Quite often we have to deal with complicated scenarios and suitable representation and visualization of images is extremely important in order to be able to understand visual processing and to interpret image content properly. Geometric methods represent classical but at the same time modern tools for image processing and image analysis. The partial differential equations approaches such as scale-space methods and gradient descent methods represent powerful tools in image processing. Although differential geometry found its application in pattern recognition, shape reconstruction, edge detection, color-image enhancement and segmentation, new developments are yet to come. That this is indeed the case we can see in [1], which gives extremely nice survey of existing and widely applicable differential geometry methods and explains geometric framework in image processing in the combination with already mentioned PDE tools. Well known Beltrami flow, that represents one of the most important geometric frameworks in image processing, is also presented in [1] with all its potentials. In general, the geometric framework views images as manifolds (generalization of surfaces with locally Euclidean structure) embedded in a higher dimensional space-feature manifold. In Figure 1. we have a grayscale image represented as a surface (2D manifold), but also color and multispectral images as well as 3D medical images can be seen in this framework. In this framework many image processing problems can be formulated as the computation of minimal surfaces i.e. surfaces that locally minimize their area. A special type of a manifold known as a Riemannian manifold is well suited for the representation of digital images due to the possibility to measure distances between the elements of a manifold. A very recent trend in geometrical image processing builds a generalized model of the Beltrami framework by the means of Finslerian geometry, which is a generalization of Riemannian geometry in the sense that objects in Finsler geometry typically depend not only on the position on the manifold, but also on the approaching direction between them (see [2]). In the generalized approach the images are still treated in the same manner as mentioned before, i.e. as manifolds embedded in a higher dimensional space and the goal is to find the minimal surface. For example, noise is represented as points of high curvature so the goal is to smooth the image surface by reducing the number of points of high curvature. The Beltrami flow is obtained by minimizing the area of the image manifold with respect to the intensity components: the surface moves according to the intensity component in the direction of the normal of the surface, which is the only direction which changes the shape of the image. In this way we are indeed changing the intensity components and removing the noise. Other image processing problems can be seen in this framework.

Goal of the thesis (Doelstelling)	This thesis should combine already developed theoretical techniques in image processing and computer vision (introduced in [1,3]) with a solid mathematical theory of curves and surfaces in order to conduct experimental results and practical validation. The main goal of the thesis is to do the practical validation of the theoretical results presented in [3]. Since Beltrami flow already showed remarkable results, the new generalized Beltrami framework, so far only from the theoretical point of view, is also extremely promising method. As mentioned, practical validation is yet to be conducted and this should be the main task in this master thesis. The results should be compared with the traditional as well as with the state-of-the-art methods for image enhancing. All the existing code for the traditional Beltrami framework will be available to the student. The developed method will be applied in concrete image processing applications, which will be chosen in the agreement with the student and depending on his/her affinities.

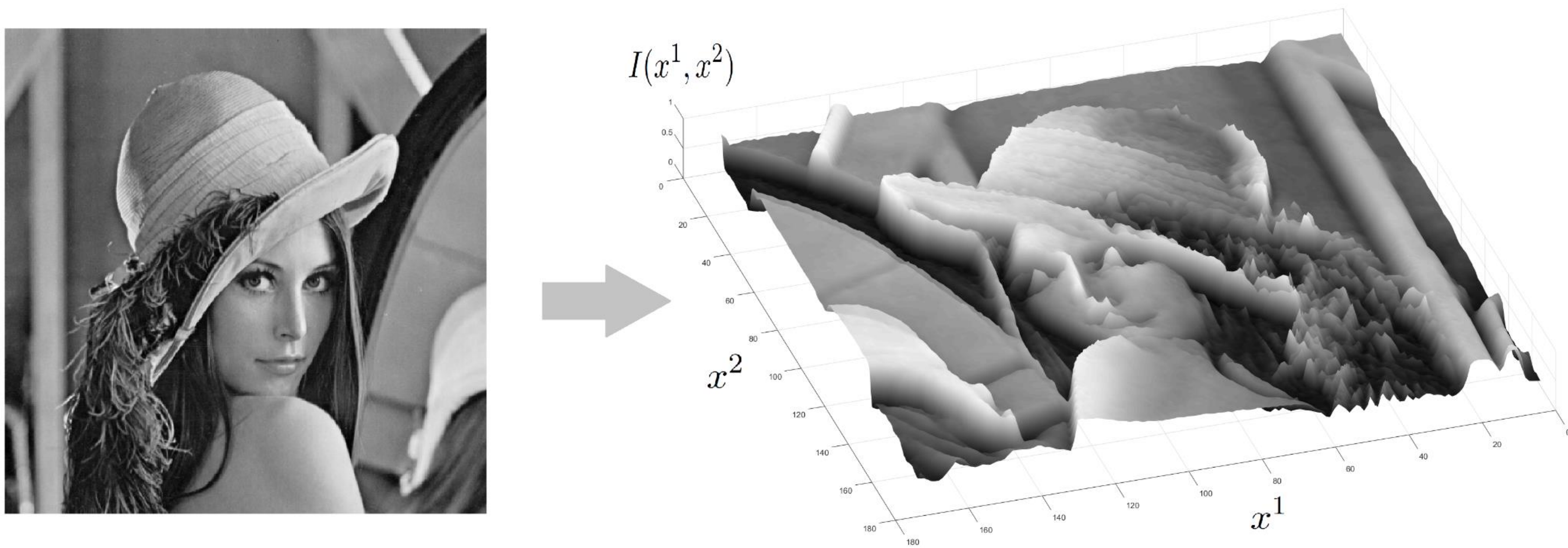

Figure 1: Grayscale "Lena" image (left) can be considered as a 2D surface $S = (x^1, x^2, I(x^1, x^2))$ embedded in a 3D space (right); the surface point $(x^1, x^2, I(x^1, x^2))$ has the gray level $I(x^1, x^2)$.