Mapping the possible occurrence of archaeological finds by Bayesian inference

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- Archaeological prospection & predictive modeling
- Objectives
- Mapping: Bayes: deductive, inductive, mixed model
  Regression indicator kriging
- Precision+accuracy: MBF evidence filter + ROC validation
- Results and conclusions
Archaeological prospection

- EU Valetta Treaty: protecting the cultural heritage
  
  Increased need for info on presence, quantity and quality of archaeological heritage → “find databases” with data of various origin:

1. **Reconnaissance walking**: field loc ±; amateur work; only positives
2. **Line walking**: scanning line fragments; field loc; common; pos+neg
3. **Grid walking**: exhaustive; GPS field loc; less common; pos+neg
Predictive modeling

- Two “schools of ecological determinism”:
  - **Deductive**: knowledge-driven
    Most common; based on (reconstructed) soilscape
    Criticism: personal bias, unknown portability
  - **Inductive**: data-driven
    Still few examples; Discr. Analysis, logistic regression, CART, kriging
    Criticism: dependency (unfavorable) data configuration
- Problems:
  - Data configuration often suboptimal;
  - Information “non-find” often not used.
Research Q’s + Objectives

1. What mapping method works in difficult data configurations and with + - data?
   • Test potential of Bayesian methods
   • Compare with IRK
   • Compare deductive and inductive methods

2. Political sensitivity: display what (when) on map?
   • Evidence strength filter
Research area, sampling

- 19x14 km² area in N Flanders
- Pleistocene coversand, alluvial plains, tertiary outcrops
- Mostly line walks in threatened (plowed agricultural) fields
- Fields randomly selected (10% sampled area = validation)
- Focus on final-Paleolithic and Mesolithic artifacts
- Finds & non-finds stored in 10x10 m grid (protocol)
Research area: auxiliary variables

- Relevancy for settlement, hunting, gathering

- DEM
- Wetness
- N-wind exposure
- Distance to open water
- Drainage class
- Soil texture class
Bayesian mapping

At a location,
1. Prob(occur.|attributes)
2. CP(attributes)
3. P(attributes)
Combination:
(e.g. Aspinall, 1992):

\[ P(o \mid f) = \frac{P(f \mid o) \cdot P(o)}{P(f)} \]  (Bayes’ rule)

\[ P(f \mid o) = \prod_{i=1}^{n} co_i \]  and  \[ P(f \mid a) = \prod_{i=1}^{n} ca_i \]

\[ P(f) = P(f \mid o) \cdot P(o) + P(f \mid a) \cdot P(a) \]

\[ P(o \mid f) = \frac{P(o) \prod_{i=1}^{n} co_i}{P(o) \prod_{i=1}^{n} co_i + P(a) \prod_{i=1}^{n} ca_i} \]

Prior occurrence counted as fraction (occur.+abs.)
CP’s individual attributes counted and combined
Bayesian mapping (*BayesPMap*)

\[
P(o \mid f) = \frac{P(o) \prod_{i=1}^{n} c_{oi}}{P(o) \prod_{i=1}^{n} c_{oi} + P(a) \prod_{i=1}^{n} c_{ai}}
\]

1. **Deductive method**  *eqv Aspinall, 1992*
   - \( n=1 \) (deductive map with \( m \) classes)

2. **Inductive method**  *eqv Gorsevski et al., 2003*
   1. Fuzzy \( k \)-means clustering of auxiliary info (*Matlab-FuzMe*)
   2. \( n=\)no. clusters, with \( \Sigma m \) membership classes

3. **Mixed method**  *eqv Aspinall, 1992*
   1. Selection of auxiliary information
   2. Supervised classification in classes (distribution + - histogram, \( \chi^2 \) test) (*interactive software*)
   3. \( n=\)no. of auxiliary variables; \( m \) classes
Comparing mapping approaches

Auxiliary info maps
- **Mixed** knowledge+infer
  - X²-analysis
  - Reclassified aux maps

Inductive data driven
- Fuzzy k-means clustering
  - Membership -class maps

Deductive knowledge driven
- Expectation class map

Knowledge

GIS-counts:
- Priors+ conditionals
- Bayesian inference & mapping
- Probability map archaeol. find of Type X

Findings

- Indicator Regression Kriging
- Bayesian inference & mapping
Map precision: Evidence filter

Minimum Bayes Factor MBF
(Goodman, 1999): Gaussianity $\rightarrow$

Low MBF $\Rightarrow$ strong evidence against $H_0$: $\mu=$prior

At pixel: $\mu=$prior, $x= P(o|f)$, $\sigma=?$

Bootstrap resampling pos/neg dataset
  "distribution of CP’s $\rightarrow$ SD$_{coi}$

1. Error propagation rules
   - SD $P(o|f)$
2. Calc MBF per pixel
3. Apply filter for display

$$\frac{Pr(x | \mu = \mu, \sigma)}{Pr(x | \mu = x, \sigma)} \rightarrow e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2} = e^{-\frac{1}{2}Z^2}$$
Map accuracy: ROC and AUC

1. Receiver Operator Characteristic

Call these pixels “negative”
Call these pixels “positive”

with non-finds
with finds

True Positives
False Positives

ROC curve

2. Area Under Curve
(eqv. Mann-Whitney $U$)
Results: Quality of Bayesian maps

**ROC**

- **Predictive**
  - pos+neg
  - pos

- **Inductive**
  - pos+neg
  - pos

- **Mixed**
  - pos+neg
  - pos

**Off** very weak → evidence filter → very strong
Results: comparison with IRK

- IRK performance depends on data configuration
  - Random validation fields: poor performance
  - Random validation pixels: excellent performance

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<th>Method</th>
<th>Specs</th>
<th>AUC</th>
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<tr>
<td>Predictive Bayes</td>
<td>+ and -</td>
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<td>Mixed Bayes</td>
<td>+</td>
<td>0.83</td>
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Best map

Probability map for mixed model
Conclusions

1. Bayesian predictive mapping may outperform IRK (depending on data configuration).

2. Bayesian models:
   - Imported deductive models and inductive models based on fuzzy $k$-means clustering performed comparably (AUC);
   - Inductive model passed evidence filter easily, deductive model didn’t;
   - Mixed model performed best: Interaction archaeologist/pedometrician!

3. Usage of both + and – data improves performance (AUC+evidence filter).

4. MBF-based evidence filter useful for politically sensitive maps.