A Relative Shape Comparison Technique to Compare Shapes of Polygons

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1. Introduction

Traditionally mainly quantitative methods are used for shape representation. Specific domains of interest for shape representation include computer vision, pattern recognition, and image analysis. Recently a subfield of artificial intelligence, namely qualitative spatial reasoning, has been developed in order to model commonsense knowledge of space (Cohn, 1995). This approach is useful to deal with geographic space and, therefore, to build new generation Geographic Information Systems (GIS) (Egenhofer, and Mark, 1995). In other words, shape comparison techniques based on qualitative spatial reasoning techniques could form a solid basis for comparing shapes between entities in geographic space, like lakes, (vague) boundaries of cities, geomorphological features, … In this paper we show how a technique based on a qualitative calculus can be used to compare the shape of polygons. We illustrate the potentials of this so-called Relative Shape Comparison Technique (RSC-Technique) by the use of an illustrative example with relevance for all kinds of researchers working with spatially referenced entities, more specifically working in Geographic Information Science.

In (Van de Weghe, Cohn, and De Maeyer, 2004) a qualitative spatio-temporal representation for moving objects based on describing their relative trajectories, the so-called Relative Trajectory Calculus (RTC), was developed. Important in this calculus is that an object moving during a period P (starting at time point T1 and ending at time point T2) is represented by means of a vector starting at T1 and ending at T2. If multiple objects are moving between these two time points, the entire configuration can be represented by a set of vectors, each one representing a movement of a specific object. On the other hand, when these vectors are not considered as movements between two time points, but as arcs at a single moment in time, a configuration composed out of multiple arcs can be analyzed by use of the transformed RTC, which we will call the Relative Shape Comparison Technique (RSC-Technique).

2. Relative Shape Comparison Technique – an Informal Account

2.1. Relative Position of two Arcs

Two arcs A (a1,a2) and B (b1,b2) can be compared relatively, based on the following construction rules (see Figure 1):

- draw a line L through the origin of the vectors (a1 and b1)
- project the destination points (a2 and b2) on this line L ? a2’ and b2’

Based on the subsequent functions, a label consisting of three characters can be generated:

- character 1: orientation constraint (orientation of A towards b1)
  - if vector a1a2’ is directed towards b1 then –
  - directed away from b1 then +
  - orthogonal with L then 0
- character 2: orientation constraint (orientation of B towards a1)
  
  if vector b1b2' is directed towards a1 then –
  directed away from a1 then +
  orthogonal with L then 0

- character 3: length constraint (relative length of both arcs)
  
  if |A| < |B| then –
  if |A| > |B| then +
  if |A| = |B| then 0

Using this technique we can characterize a couple of arcs by a label consisting of three characters representing the relative position of these two arcs. It can be proved that there are 27 different label possibilities. In Figure 2 an example of each label is visualized.

2.2. Relative Position of multiple Arcs

Of course things will become more interesting though much more complex when considering a configuration of multiple arcs. As stated before, every couple of arcs can be represented by a label of three characters. A configuration of n arcs will result in n² labels which can be reduced to (n² - n)/2 labels: n pairs of arcs have no meaning because they are composed of identical arcs and (n² - n)/2 pairs of arcs have reversible labels.

3. Comparison of Shapes of Polygons by Use of an Illustrative Example

As a direct result it is possible to search for smaller specific arc-configurations in a huge set of arcs as typically used in the field of pattern recognition. Each arc of a polygon can be represented via a vector. This way a polygon can be oriented in a unique way and as a direct result the RSC-Technique can be applied to polygons, in order to describe similarity between different shapes. This is explained by means of an example: The evolution of a lake during nine time stamps (see Figure. 3). Consider a lake at a certain moment in time T1 (the lake does not change in number of vertices during the evolution; each vector is labeled at the beginning). Due to some conditions (e.g. global warming) the lake shrinks year after year. After nine years, the surface of the lake has decreased significantly, but what has happened to its shape during the process? Which form is the most and which one is the least similar to the initial form of the lake at T1?

Shape-space analysis, a statistical tool for measuring the geometry of random objects where location, rotation and scale information can be ignored, can be used to compare the shape of the lake at different moments in time. (Dryden, and Mardia, 1998). Unfortunately, what is meant by the term shape similarity is application dependent, which has resulted in many shape similarity measures (see: Latecki, and Lakmper, 2000). In this work we use the introduced RSC-Technique to analyze shape similarities. To be able to answer this, one has to define what is meant by 'shape similarity'. One could for example state that polygons are equivalent if they have the same topology. In this way the nine forms represented in the example are equivalent. On the other hand one could state that two polygons are equivalent if both polygons perfectly match. This way there is no equivalence between any of the couples of polygons in the example.

A comparison is worked out between the shape of the lake at T1 and the altered shapes at later time stamps. In order to enhance the calculation of the matrices (see Table 1), we created some code in AutoCAD MAP®.
Firstly, we define:

Equivalence in Shape: Two shapes are equivalent in shape if they are represented by the same matrix constructed via the RSC-Technique.

Similarity between different Shapes (S):

\[ 100 \times \frac{\text{Number of non-equal cells in the Reduced Matrix}}{\text{total number of cells in Reduced Matrix}} \]

(the Reduced Matrix contains \((n^2-n)/2\) cells; see section: 2.2.)

Solution of the comparison (based on Table 1):

1. The lake is equivalent in shape at T1 and at T9.
2. The Similarity Measures are:
   - S12 (similarity measure between shape at T1 and at T2): 93%
   - S13: 75%
   - S14: 68%
   - S15: 64%
   - S16: 64%
   - S17: 71%
   - S18: 93%
   - S19: 100%

4. Future Work

In reality different polygons will often count a different number of nodes. This problem can be overcome by some simple line simplification techniques of which the line simplification algorithm reported by Douglas and Peucker (Douglas, and Peucker, 1973) is the most widely used.

A well known problem for geographers working with maps dating from different periods is the diversity in accuracy. Due to the acknowledgement of techniques and tools to measure and map reality, one could generally say that recent maps are more accurate then old maps. But what is the difference in accuracy? We believe that the RTC-technique can be used to generate a solid technique to compare accuracies of different maps representing the same study area at a different moment in time. In the near future this difficulty will be studied in depth.

Finally the RTC-technique has to be compared with other qualitative shape representations and alternative quantitative shape measures.

References


Douglas, D.H., and Peucker, T.K. (1973), Algorithms for the reduction of the number of points required to represent a digitized line or its caricature, Canadian Cartographer, 10, pp. 112-122

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Biography

Nico Van de Weghe is Research Assistant of the Fund for Scientific Research. He is a geographer working on a PhD concerning the temporal aspect in geographic information systems.
Figure 1.

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Figure 2.

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Figure 3.
Table 1.