
**A holistic model to infer mathematics performance: the interrelated impact of student, family and school context variables**

**Abstract**

The present study aims at exploring predictors influencing mathematics performance. More in particular, the study focuses on internal students’ characteristics (gender, age, metacognitive experience, mathematics self-efficacy) and external contextual factors (GDP of school location, parents’ educational level, teachers’ educational level, and teacher beliefs). A sample of 1 749 students and 91 teachers from Chinese primary schools were involved in the study. Path analysis was used to test the direct and indirect relations between the predictors and mathematics performance. Results reveal that a large proportion of mathematics performance can directly be predicted from students’ metacognitive experiences. In addition, other student characteristics and contextual variables influence mathematics performance in direct or indirect ways.

Key words: mathematics performance, metacognition, mathematics learning, path analysis
Introduction

Students’ mathematics literacy is essential for their further schooling and their success in the future workplace. Therefore, exploring and understanding the factors that influence mathematics learning is an important topic. Available researches present a variety of views concerning the factors influencing mathematics performance. Those factors can be clustered into two groups: (1) internal student characteristics, such as gender (Hyde, Feenema, & Lamon, 1990), metacognition (Desoete & Roeyers, 2001), and math self-efficacy (Pajares & Graham, 1999), and (2) external or contextual variables, such as GDP (Gross Domestic Product) of the geographical school location (Young, 1998), parents’ educational level (Sirin, 2005), teachers’ educational level (Goldhaber & Brewer, 2000) and teacher beliefs (Mandeville & Liu, 1997).

A large body of the available studies focus only on the impact of internal variables. Such studies ignore the specific and interaction effect of external variables, such as family and school context. Nevertheless, contextual variables are also considered as important educational factors that are related to math performance (see e.g., Reusser, 2000). As Vygotsky (1978) mentioned in title of his famous book that the development of the higher psychological processes are the result of the interaction between the human being and his culture in the society. Based on the Vygotsky (1978)’s cultural-historical activity theory, Engestrom (1987) developed his famous activity theory which emphasized the situation and the person’s activity. Later, studies of expansive learning conducted in the Helsinki Center followed these ideas. Although, there is a long theoretical history for the contextual variables and the interaction effects between the internal variables and contextual variables, the studies from the holistic perspective are limited. From Year 2000, more and more studies have been set up – in many cases in an international setting - to construct holistic models about mathematics performance and to make explicit implications for practitioners (such as Howie & Plomp, 2006).

In the present study, we also adopt such a holistic approach to study mathematics performance by bringing together both internal and external student variables that influence mathematics learning and performance.

Conceptual model

Internal variables

Demographic variables: Age and Gender

Studies show that age is a highly significant predictor of mathematics performance (Kyriakides & Luyten, 2009). Gender - as proposed in the meta-analysis research of Hyde, Feenema and Lamon (1990) and Else-Quest, Hyde and Linn (2010) – clearly predicts learning performance. However, only a limited amount of studies have explored gender differences in mathematics performance at primary school level (Fennema, 1974; Hyde, et al., 1990). In addition, the available empirical evidences show that gender difference tends to decrease (e.g., Eisenberg, Martin, & Fabes, 1996; Hyde & Mertz, 2009) or even disappear with age (e.g., Frost, Hyde, & Fennema, 1994; Pajares & Graham, 1999).
Internal variables and mathematics performance

Metacognition and mathematics performance

Recent studies suggest that metacognition is a significant predictor of learning performance in general and mathematical performance in particular (e.g., Ozsoy, 2010; Veenman, Van Hout-Wolters, & Afflerbach, 2006). Flavell (1979) defined the concept of “metacognition” as “thinking about thinking”. Furthermore, metacognition can be defined in terms of metacognitive knowledge and metacognitive experiences (Efklides, 2001, 2008; Flavell, 1981).

In the one hand, metacognitive knowledge includes information about tasks, strategies, and goals (Flavell, 1979). Research points out that when learners are sufficiently aware of their metacognitive knowledge and therefore the way their own mind works, the lower mathematics achiever can learn better after intervention by the metacognitive knowledge (Maqsud, 1998).

In another hand, metacognitive experience is “what person is aware of and what she or he feels when coming across a task and processing the information related to it” (Efklides, 2008, p.279). They take the form of metacognitive feelings, and metacognitive judgments/estimates. Metacognitive experiences make the learner aware of his/her cognition and trigger control processes that serve the pursued goal of the self-regulation process (Efklides, 2006; Efklides, 2008). When students have metacognitive experiences and know how to capitalize on these experiences, they have more chance to be a successful mathematics problem solving endeavour (Foong, 1993).

As Flavell(1987) reveals, the young children have more trouble than older children in metacognitive experience, such as comprehending their own feelings of incomprehension. Students with good metacognitive experience will achieve higher on mathematics than their peers (Desoete, & Roeyers, 2001). Based on Efklides’s study on mathematics problem-solving (2006), the metacognitive experiences is interrelated with the feeling of familiarity (FOF), feeling of difficulty (FOD), feeling of confidence (FOC) and feeling of satisfaction (FOS). Feeling of confidence and feeling of satisfaction are retrospective rather than prospective. If the feeling of confidence is required retrospectively, it is mainly related to estimated solution correctness and feeling of difficulty experienced during of task solving process. However, the feeling of confidence can be required prospective, and it is related to processing fluency and knowledge accessibility. It is assumed that the future metacognitive experience can be revised according to this performance information and the metacognitive experience can help persons to solve similar problems in the future (Akama, & Yamauch, 2004). Then in this paper, the focus will be on the metacognitive experience of young students in primary school.

Mathematics self-efficacy and mathematics performance

Self-efficacy can be defined as the belief in one’s capacity to organize and execute actions required to attain a level of performance (Bandura, 1993; 1997). Previous studies reveal that the mathematics performance is correlated to the math self-efficacy (Hackett & Betz, 1989). Students with a higher level of self-efficacy adopt a wider variety of cognitive strategies and reflect a higher level of cognitive engagement (Pintrich & DeGroot, 1990). It has been shown that self-efficacy mediates between other variables and mathematics
performance (Pajares & Miller, 1994). Mathematics problem solving is directly and indirectly affected by the math self-efficacy (Pajares & Graham, 1999). Bandura (1997) claims – for instance - that self-efficacy helps to develop interest in a task. When students feel up to the task, they are more likely to be interested in and be persistent (Bandura, 1997). Previous studies reveal that the students from different cultures can hold different self-efficacy beliefs (Klassen, 2004). Thus, it is also worthwhile to examine the effect of self-efficacy in different cultural settings.

There are also some interrelations between the internal variables themselves. Gender is assumed to affect the self-efficacy (Betz & Hackett, 1981; Hackett & Betz, 1989). For example, gifted boys have been found to be biased as to their expertise. This bias resulted in being overconfident (Pajares, 1996). Girls reported a lower level of mathematics self-efficacy than did boys (Seegers, & Boekaerts, 1996). But the relationship between gender and mathematics self-efficacy is still unclear (Post-Kammer & Smith, 1985; Skaalvik & Rankin, 1994). Self-efficacy mediates the effect of gender and prior experience on math problem-solving performance (Pajares & Graham, 1999). Also, studies show that metacognitive experiences control the impact of self-efficacy on performance (Akama, 2006; Panaoura, 2007).

External contextual factors

Family related variables and mathematics performance

SES is a complex variable that comprises – depending on the author or study – a different set of variables. parental educational level, parental occupation and home resources or wealth (Sirin, 2005). The link between the socioeconomic status (SES) of parents and mathematics performance has been subject of numerous studies; see e.g., the TIMSS and PISA research (Marks, 2006; Ming & Zeng, 2008; Webster & Fisher, 2000). Students with highly educated fathers and mothers perform considerably better than the other student which parents hold a medium schooling degree (Fertig, 2003). Of the predictors of mathematics performance at age 10, the effect size of mother education level is higher than the father education level (Melhuish, Sylva, Sammons, Siraj-Blatchford, Taggart, Phan, & Malin, 2008).

Besides the mathematics performance, parents’ SES also influence the development of students’ internal variables. For example, Vygotsky (1978) and Wertsch (1985) state that metacognition is affected by family social interactions. Previous research reveals explicitly how metacognition is related to environmental factors, such as the socioeconomic status (Pappas, Ginsburg, & Jiang, 2003), collaboration styles of mothers with their preschoolers during problem solving (Moss, 1990) and family culture (Eills, 1997). In this context, Schommer (1990) shows that higher educated parents expect to a larger extent that their children take up responsibilities at home and expect their children to think more independently. In addition, a supportive parenting style has proven to lead to higher levels of self-efficacy and subsequent school achievement (Whitbeck, Simons, Conger, Wickrama, Ackley, & Elder, 1997).

Teacher quality and mathematics performance


It is widely accepted that learning is influenced by a variety of academic contextual elements (e.g., Salomon & Perkins, 1997), such as teacher quality defined by their educational level, (Mandeville & Liu, 1997; Smith, Desimone, & Ueno, 2005). Teachers who have a standard certification have a statistically significant positive impact on student math test scores while teachers hold other certification or are not certified do not have the impact (Goldhaber & Brewer, 2000).

Teachers’ beliefs about mathematics teaching have been revealed to influence mathematics performance in general, and mental calculation in particular (Stigler, 1984). Previous studies show that teachers with cognitive constructivist orientation were associated with their students’ larger achievement gains in mathematical word problems (Staub & Stern, 2002).

Also, teachers have a direct and indirect impact on math score and on mediating internal variables. The teacher impact on internal variables is found in studies about metacognition. It has been shown that different teaching methods might hinder or improve metacognitive processing (Nist, Holschuh, & Sharman, 1995; Van Keer & Verhaeghe, 2005). When it comes to the impact on self-efficacy, Siegle and McCoach (2007) reveals that teaching methods improve the students’ mathematics self-efficacy.

**Contextual variables and mathematics performance**

The Chinese educational context is different from other countries. In addition, also within China, regions differ widely as to their economical activity. This is reflected in a large difference in the regional Gross Domestic Product (GDP). The regional GDP will affect mediating variable that impact mathematics performance, such as the investments in schools, instructional media, teacher professional development, etc. (Perry & McConney, 2010).

**Towards a holistic conceptual model**

Given the fact that most previous studies focus either on the relationship between internal or external variables that affect mathematics learning and performance, the present study adopts a holistic model approach that includes all these variables in a model to study mathematics performance. From a theoretical point of view, this is meant to be an important addition to the existing literature about mathematics education.

Figure 1 represents our conceptual model in a graphical way. Elementary mathematics can be seen as a broad domain, comprising various subdomains such as arithmetics and numerical facility skills (Desoete, Stock, Schepens, Baeyens, & Roeyers, 2009; Dowker, 2005). In our model, mathematics performance (MP) and mental calculations (MC) are regarded as dependent variables. MP represents complex mathematics performance while MC represents basic number retrieval processes.

We further distinguish internal variables such as grade, gender, mathematics self-efficacy (MSS), and metacognitive experiences measured by metacognition calibration score (MCS). As external variables, the Gross Domestic Product (GDP) of the region are positioned as contextual variables. The father’s (FEL) and mother’s educational level (MEL) are representing variables in the family context. Further, a teacher’s educational level (TEL) and teacher beliefs are positioned in the school context. The
following beliefs are included in the study: Teacher beliefs about Student Learning (SL), Teacher belief about Stage of Learning (L), and Teacher beliefs about Teaching Practices (TP).

<Insert Figure 1 about here>

Considering the proposed conceptual model, three research questions are put forward:

(1) Which internal variables contribute to mathematics performance in elementary schools in the Chinese context?

(2) Which external variables influence mathematics performance in elementary schools in the Chinese context?

(3) What is the interaction between internal factors and external factors? How do internal factors mediate the relationship between external factors and mathematics performance?

Research Design

Sample

A sample of 1749 pupils (female = 49%) was involved in the study. In addition, the teachers of these pupils and information about their school were included in the study. The sampling was based on the following stratification variables: pupils are enrolled in grade two to grade six in 18 different schools, from five provinces in China, reflecting different levels in gross domestic product (see Table 1). The GDP distribution shows that 58.66% of pupils originate from a high GDP province and 41.34% from a low GDP province. Within each GDP level, there were equal numbers of boys and girls. Research data also includes information from 91 teachers, of which 3.30% got a senior school degree, 36.26% obtained a pre-Bachelor degree, and 60.44% of the teachers got a Bachelor degree.

<Insert Table I about here>

Research instruments

Questions about background variables such as gender and grade were included in the students’ questionnaire. Information about parents’ educational level was obtained from the teacher. The gross domestic product index (GDP) was derived from the 2005 report of the Chinese Economic Ministry. The teachers were asked to fill out the teacher questionnaire that comprises beliefs related research instruments and questions about other background variables; e.g., their educational training level (TEL).

Mathematics performance

A mathematics test was designed for this study, with different forms for each grade. The test covers the three general elementary mathematics domains: number and algebra, shape and space, statistics and probability (MOE, 2001). In each test form, anchor items were defined in each form in order to be able to calibrate all the different test forms. This comprehensive mathematics performance test was analyzed by Item Response Theory (IRT). Mathematics Performance (MP) was calibrated with the BiLog-MG3 programme.
The internal consistency (Cronbach’s α) of each grade level test ranges from 0.93 to 0.96.

Research points at a positive relation between mathematics performance and mental calculations (Adams & Hitch, 1997; Ashcraft & Kirk, 2001; Hitch, 1978). Therefore, next to a mathematics performance test, we also administer the Arithmetic Number Fact Test (Tempo Test Rekenen, TTR; De Vos, 1992). The TTR is a mental calculation test, presenting pupils with 200 arithmetic number-fact problems (e.g. 5 x 9 =...). Subjects have to solve as many number-fact problems as possible in 5 minutes time. The subjects were presented with a Chinese version of the test. The test helped to determine a mental calculation scores (MC) that build on an effective and efficient basic number fact retrieval.

Metacognitive experiences

There are different methods of assessing metacognition (Desoete, 2008; Veenman et al., 2006). Self-ratings are usual measures to determine metacognitive experiences. Calibration studies - in the context of primary education - have been proven to result in a reliable measurement of metacognitive experiences (Desoete & Roeyers, 2006). The studies where a comparison is made between the predicted success or failure in carrying out a task and the actual performance quality after the task have been carried out (Desoete & Roeyers, 2006; Grimes, 2002). In higher education, Grimes (2002) revised the calibration approach introduced by Lichtenstein and Fischhoff (1977), again resulting in a reliable measurement of metacognitive experiences.

After administration of the mathematics performance test - following the post diction paradigm’ - subjects were invited to predict the level of their test performance after the test (e.g., ‘I think I will obtain 70/100 on this test’). In line with Lichtenstein and Fischhoff (1977) and Grimes (2002), a metacognition calibration score (MCS) was calculated in the following way:

\[
\text{Metacognition Calibration Score} = \frac{(\text{Actual score} - \text{Expected score})^2}{\text{Expected Score}}. (1)
\]

A Mathematics self-efficacy

A mathematics self-efficacy scale (MSS) was developed on the base of the instrument of Marat (2005). The students were asked to answer these questions one day after the test. The original scale is based on twelve items and has 85 items. For example, the first item asks “How well do you believe you can calculate accurately numerical problems mentally?” Respondents have to indicate their reaction to each item on a Likert scale, ranging from Not well at all (coded 1) to very well (coded 5). The instrument was presented to Chinese primary school learners in a pilot study prior to the present study. Items were deleted with a item-total correlation <.30. Cronbach’s alpha reliability of the final version was .97. A one-factor model was confirmed on 77 items by Exploratory Factor Analysis (EFA, principle component analysis with orthogonal-varimax-rotation). This single component accounted for 34.41% of the item variance. The eigenvalue of this single factor was 22.92.

Teacher beliefs
The teachers completed the Mathematics Beliefs Scales (MBS) developed by Fennema, Carpenter, and Loef (1990). The Chinese version of the scale consists of 16 of the original 18 items. It is structured into three subscales: (1) teacher beliefs about how children learn, labelled as the student learning factor (6 items); (2) beliefs about the teacher role to teach computational and application skills, labelled as the stages of learning factor (4 items); (3) teacher beliefs about teacher practices (6 items). Item Likert scale categories ranged from Not agree at all (= 1) to agree very well (= 5). The survey was completed by 83.33% of the teachers; some teachers could not attend the administration session due to unforeseen timing problems. The reliability of the whole scale is .81 (Cronbach’s alpha). The reliability of the subscale are .68, .65 and .62, respectively.

Data analysis

Both multiple regression and path analysis are useful approaches to test educational models (Anderson, & Evans, 1974; Goldberger, & Duncan, 1973). In this context, path analysis presents a number of advantages. Firstly, path analysis is helpful to understand the direct and indirect relationships between variables, while multiple regression does mainly center on direct effects. Secondly, the maximum likelihood method in path analysis (using Amos) builds on a less restrictive set of assumptions, while ordinary least squares (OLS) estimation for multiple regression (using SPSS) is more restricted; such as multicollinearity. Thirdly, for a complex model, the probability of type I errors α for the significance test should be adjusted for each stage in a regression analysis (Opp & Schmidt, 1976, pp.155 in Tacq, 1997). But adjusted significance tests help to meet the difficulties in regression analysis. In recent years, a variety of indices has been developed to measure the goodness of fit of a path model (See Hu, & Bentler, 1998; Marsh, Hau & Grayson, 2005), while multiple regression is limited to straightforward test equations. But, in path analysis we have to cope with limitations as to the directionality in the relationships. The directions of “arrows” in path analysis represent specific hypotheses; they are mostly grounded in previous studies. In the literature, a discussion can be found about the causal order in the variables (see e.g., Opp and Schmidit,1976). Some authors state that this can result in the development of premature models (Tacq, 1997, p.176).

In the previous studies, it was found that when adding the variables of the students variables of metacognitive experience and mathematics self-efficacy etc., the effect of teachers' variables disappeared (See Authors, 2011). The results implied that there are some indirect effects from the teachers' variables on the mathematics learning performance. In the present study, the path analysis by Amos 6.0 will be used to analyze the data. In the present study, we carry out both path analysis (Amos 6.0), and multiple regression analysis (SPSS). The latter is applied to check whether the results of the path analysis can be confirmed. A variety of statistical procedures was applied in line with the research questions. Firstly, correlation analysis was applied to test associations between the variables in the model. Secondly, in order to test the complete model, structural equation modelling (AMOS 6.0) was applied to test direct and/or indirect relationships (Arbuckle, 2005).
Results

Description and correlation analysis

Table 2 summarizes the description of the variables in our study according to the endogenous and exogenous student, family and teacher characteristics. At the general level, the means of the MCS is 10.64 (SD=11.88) and the means of MP is .83 (SD=.96).

<Insert Table 2 about here>

Table 3 gives an overview of the bivariate correlation between the research variables in our model. The results reflect significant interrelationships between all variables. Higher levels of mathematics performance was correlated with higher metacognitive experience (=smaller difference between expected score and real score) on MCS (r=-.66, p<.00). This result is in line with the result of previous study that there is significant correlation between metacognitive experiences and mathematics performance from grade 3 through grade 5 (Sperling et al., 2002). The mathematics performance decreased from lower deviance between actual score and predicted score to higher deviance of metacognitive experiences.

<Insert Table 3 about here>

Path analysis models

Three consecutive models were tested in this analysis approach. In a first model (Model A), internal characteristics were included and linked to the dependent variables. In a second model (Model B), the effects of the external family contextual variables were added. In the third final model, the additional effect of external school variables was explored. Also, the structural integrity of the model was tested. For reasons of parsimony, variables with non significant regression weights are not reported in Table 4. In view of decisions about the number, type and cut-off values for Goodness-of-fit criteria, we built on the work of a variety of authors (e.g., Schermelleh-Engel, Moosbrugger & Müller, 2003; Shulruf, Hattie & Dixon, 2007). The following “goodness-of-fit” indices were adopted: relative chi-square ($\chi^2$/df) index, Goodness-of-fit index (GFI); adjusted GFI (AGFI) and Normed Fit Index (NFI) that makes the calculations independent of degrees of freedom (cut-off value ≥ 0.95), the Root Mean Square Error of Approximation (RMSEA, cut-off value 0.08).

In the first model 58% of the variance in mathematics performance (MP) can be attributed to the ability of mental calibration of basic number retrieval (MC), metacognitive experience (MCS), Mathematics self-efficacy (MSS) and grade. The Grade ($\beta = .23$), MCS ($\beta = -.65$), MSS ($\beta = .19$), MC($\beta = .06$) are found to be predictors of mathematics performance. In the second model – after adding the family variables – the coefficient of direct effect from the mathematics performance changes. GDP of the school level affects directly mathematics performance ($\beta = -.12$). Father educational level do not play an important role on the mathematics performance while mother educational level have an indirectly influence through mathematics self-efficacy. And the effect of influences of
Grade on mathematics performance disappeared. In the third model, when academic variables are added to the model, 56% of the variance in mathematics performance can be attributed to the complex interplay of the variables. Both direct and indirect effects on MP can be observed.

The final path model is presented in Fig.2, reporting the standardized path coefficients.

In the third model, the impact of MC on MP is not that important ($\beta = .08$). Here, arithmetical mental calculation can be assessed to control for the children with deficits in semantic memory (Ashcraft, 1992; Dehaene, 1992; Logie, Gilhooly, & Wynn, 1994). Students with learning difficulties often have problems with basic mental calculation tasks, especially due to deficits in semantic memory (Wilson & Swanson, 2001). This suggests that once children reach a baseline level, complex mathematics problem solving does no longer depend largely on the basic fact number retrieval system.

The results reveal that the metacognitive experiences (MCS) clearly affect mathematics performance ($\beta = -.61$). Mathematics performance is negatively associated with the metacognition calibration score (MCS). Students with smaller difference between expected score and real score (=higher metacognitive experiences) reflect a higher mathematics performance. This means that the students, who are able to predict their score more accurately, end up with a higher mathematics performance. In addition, student with a higher mathematics self-efficacy (MSS) reflect a higher mathematics performance ($\beta = .17$). Considering the different aspects in mathematics performance, MC is clearly linked to MCS ($\beta = -.10$) and MSS ($\beta = .10$).

When considering the external variables, significant findings can be reported in the way that specific school and family context are related to mathematics performance. Comparing to the internal variables, the coefficients of external variables (GDP, $\beta = -.09$; TEL, $\beta = .13$; L, $\beta = .08$) are small but nevertheless statistically significant. Students with a higher educational level in a province with a higher GDP level obtain higher mathematics performance scores. The mother’s educational level (MEL) does not seem to have an indirect impact on mathematics performance by the mediating of students’ mathematics self-efficacy ($\beta = .11$).

Regarding the school variables, it is interesting to consider how they are intertwined with metacognitive experiences and mathematics self-efficacy as mediating variable. Teacher educational level (TEL, $\beta = -.08$) and teacher beliefs about the stage of learning (L, $\beta = -.05$) are related to the metacognitive experiences (MCS) and mathematics self-efficacy of pupils. Teacher educational level (TEL, $\beta = .06$) and teacher beliefs about stage of learning (L, $\beta = .11$) are positively related to mathematics self-efficacy (MSS). It is interesting – in this context – to see that teachers who have students with higher self-efficacy and higher metacognitive experiences, tend to reflect higher belief levels about the need to sequence the teaching of computational skills in the classroom. This
also implies that, although Chinese teachers strictly sequence the stage of teaching and learning for mathematics curriculum, does not restrict the development of students’ metacognitive experiences and mathematics self-efficacy. This seems to be in conflict with common conceptions about student-centred learning. But this has to be understood from the Chinese context. Also other authors referred in this context to the Paradox of Chinese Learner, which means that the seemingly unfavorable learning environment (focusing on rote learning and highly structured) yet produces students who outperform their counterparts in the West (Biggs & Watkins, 1996; Marton, Dall’Alba & Lai, 1993).

Focusing on the impact of family context, the GDP level of a school affects the level of metacognitive experiences (MCS, $\beta= .14$), and mathematics self-efficacy (MSS, $\beta=-.18$). Students enrolled in schools that are located in provinces with a higher GDP tend to predict the mathematics score more accurately and reflect a higher level of metacognitive experience. Mother’s educational level is related to MSS (MEL, $\beta=.11$) but not to MCS. We can assume that a higher mother’s educational level implies that she expects her children to take more responsibilities at home and in relation to their thinking and learning (Schommer, 1990). Also, it will be more likely that children will develop a higher level of self-efficacy.

In summary, the results of path analysis indicate that the external variables clearly affect internal variables and play as such an direct and indirect role in mathematics performance. A number of external variables add more explanatory power to the model. Compared to the other variables in the model, metacognitive experiences and mathematics self-efficacy are clearly dominant predictors for mathematics performance. This result is in line with previous studies.

Discussion, Limitations, and Conclusion

The aim of the present study was to re-examine the impact of students’ characteristics, family and school context on mathematics performance of primary students in China from holistic perspectives. It contributes to the existing holistic learning model (such as the cultural-historical learning theory) by the quantitative data analysis. And as a quantitative analysis, this study re-emphasizes the contextual variable and the interaction between internal and contextual variables by the real dataset for the activity learning (Engestrom,1987). It provides an important support for the activity learning in Finland from another research approaches.

Compared to earlier studies, additional variables were added to a conceptual model to study the direct and indirect impact of internal and external variables. In fact, this study is one of the expanded studies for the multilevel regression analysis. In the previous studies, the external variables had the effect on the mathematics performance until the internal variables were added. In the final model, most of the external variables cannot be included in the final model by multilevel analysis (Authors, 2011). This raised our questions about the interaction between the internal and external variables. The role of internal variables as mediators between external variables and mathematics performance
was studied in more detail in this study.

Firstly, the structural equation modelling (SEM) confirmed that internal variables such as students’ metacognitive experiences and self-efficacy play an important role on the mathematics performance. And also, students’ mother’s educational level, teachers’ educational level and teachers’ beliefs on the stage of learning were also related to mathematics performance of primary school children. Secondly, the study explores the interaction between the variables and provides an overview of the relationship between the variables and between the variables and math performance. This study seems to be more close to real educational context. Thirdly, the study provides the different cultural results to the existing studies. The sample of this study covers the students from the grade 2 to grade 6 in rural and urban areas.

In answer to research question 1, the results suggest that the largest proportion in mathematics performance variance could be explained by the internal variables; especially metacognitive experiences and self-efficacy. These results are in line with Kruger and Dunning (1999) and Kruger (2002). The knowledge that underlies mathematics ability is also the knowledge that underlies the ability to solve mathematical problems. Students with poor mathematics performance scores tend to overestimate their performance. As stated earlier, this shows how underachievers are presented with a dual burden: poor performance and poor metacognitive experiences.

As to research question 2, the path analysis results show that external factors such as teacher quality and teacher beliefs, mother educational level are important to be included in the model. Although mother education level does not have a direct impact on the mathematics performance, it has a influence on the mathematics self-efficacy and indirectly affects the mathematics performance. In school, teacher’s quality and beliefs affect the internal variables and mathematics performance.

For the research question 3, our data reveals that metacognitive experience control the impact of mathematics self-efficacy as we can see in a previous study (Akama, 2006). And the external factors have direct influence on mathematics performance and indirect influence through the internal factors. Teachers, who adopt the belief of stage of learning that strictly sequenced mathematics teaching is important, are linked to higher metacognitive experiences and mathematical self-efficacy in their students. These results are in line with the findings of An, Kulm and Wu (2004) who compared the mathematics teachers’ knowledge in U.S. and China and concluded that Chinese teachers especially emphasized the acquisition of both procedural and conceptual knowledge, which might explain the higher results of Chinese children on mathematical tasks.

Another interesting finding is that mother’s educational level (but not father’s educational level) is indirectly related to the mathematics performance of their children. Mothers seem to influence mathematical self-efficacy, but not the metacognitive experiences of their children. This is in line with prior research demonstrating that mothers are more involved in their children’s education than fathers (Epstein, 1986; Princiotta, Flanagan, & Germino Hausken, 2006 and with the results of Davis-Kean (2005) revealing that SES is indirectly related to children’s performance via parents’ beliefs and behaviour. Students with low-SES backgrounds were exposed to greater risks in mathematics performance (Borman & Overman, 2004; Coleman et al., 1966). The level of mother’s
education improves the mathematics self-efficacy of their children.

These results should be interpreted with care, since there are clearly limitations to the present study. Firstly, the findings of this study only refer to Chinese children and need to be replicated in other countries. Moreover, this study only included metacognitive experiences and did not take into accounts more complex metacognitive skills and knowledge. Although the metacognitive experiences take the form of metacognitive feelings, and metacognitive judgments/estimates, the measurement on the metacognitive experience here is only self-rating not including other measurements in this study. This measurement on the metacognitive experience should be expanded in the future.

Additional information in relation to classroom variables (teaching approach, textbooks used, homework ...) and family context (extra schooling activities at home, impact of brother or sisters ...) can be added to our model. Some external variables have been measured via the teacher (experience, educational level, gender). This can be criticized since our model wants to study the interplay between internal and external variables at the level of individual learners. In addition, metacognitive experiences with the expected score have some relation with student self-efficacy. More studies should be done in this area. In addition, in the present study, learners are approached as individuals. This can be criticized since the learners are nested within classes, within schools and within regions. This reflects a multilevel structure that should be respected when analyzing the impact of the variables on mathematics performance. Future studies should adopt a multilevel approach in the analysis of the data. Lastly, the SEM analysis approach was helpful to study the direct and indirect relationship between models, but it remains yet unclear whether all the relationships should be interpreted as causal relationships. More theoretical and empirical research is needed to underpin the nature of these relationships.

Despite these shortcomings, the study was helpful to illustrate the internal and external factors that are related to student mathematics performance at the primary school level. From a bio-ecological and transactional perspective (Kaiser, Hester, & Mc Duffie, 2001), a person’s cultural worldview constitutes a social and cultural difference and it causes differences in learning performance (Aleven, McLaren, Roll, & Koedinger, 2006). It might therefore be important to add these variables to the assessment approach and intervention strategies for students at risk in view of mathematical learning difficulties. In addition, our data suggests that in future research about internal variables and mathematics performance, other external variables should be incorporated, such as mothers’ educational level and family and teacher related variables.
References


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Authors (2011)
Fig. 1 Integrated model of the impact of various internal and external variables on mathematics performance.

Note: GDP - Gross domestic product; MEL - Mother Educational Level; FEL - Father Educational Level; MP - Mathematics Performance; MSS - Mathematics Self-efficacy Score; MC - Veracity of Mental Calculation; Gender - Student’s Gender; TEL - Teachers’ Educational Level; SL - Teacher’s belief on Student Learning; L - Teacher’s belief on Stage of Learning; TP - Teacher’s belief on Teacher Practice; MCS - Metacognition Calibration Score.
Table 1 Stratification variables in the research sample.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Grade 2</th>
<th>Grade 3</th>
<th>Grade 4</th>
<th>Grade 5</th>
<th>Grade 6</th>
<th>Total</th>
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<td>273</td>
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Table 2 Description of the Characteristic of demographic variables

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<th>MP Means (SD)</th>
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<td>High school</td>
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<td>.83 (.96)</td>
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</table>

Note: GDP-Gross domestic product; TEL-Teachers’ Educational Level; FEL-Father Educational Level; MEL-Mother Educational Level.
Table 3 Bivariate correlation between research variables in the conceptual model (n=1,749).

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<thead>
<tr>
<th></th>
<th>MCS</th>
<th>MP</th>
<th>MC</th>
<th>MSS</th>
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<th>TEL</th>
<th>L</th>
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<tr>
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<td>.01</td>
<td>.19**</td>
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<td>-.15**</td>
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<td>.10**</td>
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<td>L</td>
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<td>.03</td>
<td>.15**</td>
<td>.09**</td>
<td>-.08**</td>
<td>-.15**</td>
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<td>GDP</td>
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<td>-.28**</td>
<td>-.09**</td>
<td>-.26**</td>
<td>.28**</td>
<td>-.17**</td>
<td>-.15**</td>
</tr>
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</table>

Note: GDP-Gross domestic product; L-Teacher’s belief on Stage of Learning; MCS-Metacognitive experiences as Calibration Score; MEL-Mother Educational Level; MP-Mathematics Performance; MSS-Mathematics Self-efficacy Score; TEL-Teachers’ Educational Level; MC- Mental Calculation.

*p<.05, **p<.01, ***p<.00
Table 4 Overview of the direct effects on MP: Standardised regression coefficients ($\beta$) and fit indices ($n=1,749$)

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<th>Model B</th>
<th>Model C</th>
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<td>n.s.</td>
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<td>-.62***</td>
<td>-.61***</td>
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<td>Mathematics Self-efficacy Score (MSS)</td>
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<td>.18***</td>
<td>.17***</td>
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</table>

<table>
<thead>
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<tr>
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<td>GDP</td>
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<td>.09***</td>
</tr>
<tr>
<td>Mother's educational Level (MEL)</td>
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<td>n.s.</td>
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</table>

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<thead>
<tr>
<th>Academic</th>
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</tr>
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<tbody>
<tr>
<td>Teacher Educational Level (TEL)</td>
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<td>-</td>
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</tr>
<tr>
<td>Belief on Stage of Learning (L)</td>
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<td>-</td>
<td>.08***</td>
</tr>
</tbody>
</table>

| Adjusted $R^2$           | .58       | .54       | .56       |
| Chi-square               | .06       | 5.47      | 8.43      |
| df (p-value)             | 1 (.80)   | 4 (.24)   | 7 (.30)   |
| GFI                      | 1.00      | 1.00      | 1.00      |
| AGFI                     | 1.00      | 1.00      | .99       |
| NFI                      | 1.00      | 1.00      | .99       |
| RMSEA                    | .00       | .01       | .01       |
| AIC                      | 28.016    | 39.466    | 66.43     |

Note. – not included in model, n.s. not significant; *p<.05, **p<.01, ***p<.005

Note: GDP-Gross domestic product; L-Teacher’s belief on Stage of Learning; MCS-Metacognition Calibration Score; MEL-Mother Educational Level; MP-Mathematics Ability; MSS-Mathematics Self-efficacy Score; TEL-Teachers’ Educational Level; MC-Mental Calculation.
Fig. 2 Result of the path analysis
Note: GDP-Gross domestic product; L-Teacher’s belief on Stage of Learning; MCS-Metacognition Calibration Score; MEL-Mother Educational Level; MP-Mathematics Ability; MSS-Mathematics Self-efficacy Score; TEL-Teachers’ Educational Level; MC- Mental Calculation.