Parametric Macromodeling for Sensitivity Responses From Tabulated Data

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***Abstract*—This letter presents a parametric macromodeling technique which accurately describes the parameterized frequency behavior of electromagnetic systems and their corresponding parameterized sensitivity responses with respect to design parameters. The technique is based on the interpolation of a set of state-space matrices with a proper choice of the interpolation scheme, so that parametric sensitivity macromodels can be computed. Pertinent numerical results validate the proposed parametric macromodeling approach.**

***Index Terms*—Interpolation, parametric macromodeling, parametric sensitivity.**

I. Introduction

EFFICIENT and accurate design space exploration, design optimization, and sensitivity analysis call for the development of parameterized macromodels which describe the parameterized frequency behavior of the original model and the parametric sensitivity responses over the entire design space of interest.

One of the most common approaches in calculating local sensitivities is the adjoint variable method [1], [2], by which the sensitivity information can be obtained from at most two systems analyses regardless of the number of design parameters. However, these methods involve the calculation of system matrix derivatives, which are most frequently estimated by means of finite difference approximations.

Recently, some interpolation-based parametric macromodeling techniques have been presented in [3]–[6], which interpolate an initial set of univariate macromodels, called *root macromodels*. In [3], [4], the interpolation is performed at an input-output level, while in [5], [6] it is applied to the internal state-space matrices of the *root macromodels*. Both poles and residues are parameterized in [5], [6], which enhances the modeling capability as compared to [3], [4], where only residues are parameterized.

This letter proposes a parametric macromodeling technique, which is able to build parametric sensitivity responses over the entire design space of interest. As in [5], [6], an interpolation process of the internal state-space matrices of the *root macromodels* is performed. However, in [5], [6], the focus is on parametric macromodeling which ensures stability and passivity over the design space of interest. This is not necessary for the calculation of parametric sensitivities, which allows the use of more powerful interpolation schemes. The suitable choice of interpolation schemes at least continuously differentiable allows to obtain parametric sensitivity macromodels, which are analytical models and no finite difference approximation is used. Also, [5], [6] solve computationally expensive linear matrix inequalities to guarantee a passivity-preserving interpolation, which can be avoided in the present work. Pertinent numerical results validate the proposed parametric macromodeling approach.

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II. Creation of Root Macromodels

Starting from a set of data samples $\{(s,\vec{g})_k, \mathbf{H}(s,\vec{g})_k\}_{k=1}^{K_{tot}}$, a set of frequency-dependent rational macromodels is built for a set of design space points by means of the Vector Fitting (VF) technique [7]. Each *root macromodel* has the form

$$\mathbf{R}_{\vec{g}_k}(s) = \sum_{n=1}^{N_P} \frac{\mathbf{c}_n^{\vec{g}_k}}{s - a_n^{\vec{g}_k}} + \mathbf{d}^{\vec{g}_k} \tag{1}$$

where $a_n^{\vec{g}_k}$, $\mathbf{c}_n^{\vec{g}_k}$ and $\mathbf{d}^{\vec{g}_k}$ represent poles, residues and feed forward terms, respectively at the design point $\vec{g}_k = (g_{k_1}^{(1)}, \ldots, g_{k_N}^{(N)})$. The idea of VF is to recast the nonlinear problem (1) into a linear problem by introducing a set of starting poles b_n, which are chosen by a rule presented in [7] and an unknown function $\sigma(s)$ such that

$$\begin{bmatrix} \sigma(s)\mathbf{R}_{\vec{g}_k}(s) \\ \sigma(s) \end{bmatrix} = \begin{bmatrix} \sum_{n=1}^{N_P} \frac{\mathbf{c}_n^{\vec{g}_k}}{s-b_n} + \mathbf{d}^{\vec{g}_k} \\ \sum_{n=1}^{N_P} \frac{\tilde{\mathbf{c}}_n^{\vec{g}_k}}{s-b_n} + 1 \end{bmatrix}. \tag{2}$$

From (2) we have

$$\sum_{n=1}^{N_P} \frac{\mathbf{c}_n^{\vec{g}_k}}{s - b_n} + \mathbf{d}^{\vec{g}_k} = \left[\sum_{n=1}^{N_P} \frac{\tilde{\mathbf{c}}_n^{\vec{g}_k}}{s - b_n} + 1\right] \mathbf{R}_{\vec{g}_k}(s). \tag{3}$$

The unknowns $\mathbf{c}_n^{\vec{g}_k}$, $\tilde{\mathbf{c}}_n^{\vec{g}_k}$ and $\mathbf{d}^{\vec{g}_k}$ in (3) are found by solving an overdetermined linear problem over several frequency samples iteratively [7]. If unstable poles are generated during an iteration, a pole-flipping scheme is used to enforce stability [7]. Passivity can be assessed using a half-size singularity test matrix based on the admittance state-space model of $\mathbf{R}_{\vec{g}_k}(s)$ [8], while the enforcement can be achieved by perturbing the terms $\mathbf{c}_n^{\vec{g}_k}$ and $\mathbf{d}^{\vec{g}_k}$ in (1) using the technique presented in [9].

III. Parametric Sensitivity Macromodeling

Each *root macromodel* $\mathbf{R}_{\vec{g}_k}(s)$, corresponding to a specific design space point $\vec{g}_k = (g_{k_1}^{(1)}, \ldots, g_{k_N}^{(N)})$, is converted from a pole-residue form (1) into a barycentric form

$$\mathbf{R}_{\vec{g}_k}(s) = \frac{\sum_{n=1}^{N_P} \frac{\mathbf{F}_n^{\vec{g}_k}}{s-b_n} + \mathbf{F}_0^{\vec{g}_k}}{\sum_{n=1}^{N_P} \frac{\mathbf{f}_n^{\vec{g}_k}}{s-b_n} + \mathbf{f}_0^{\vec{g}_k}} \quad (4)$$

with basis poles b_n, which make the conversion well conditioned [5]. The barycentric form (4) is then converted into a state-space form

$$\mathbf{R}_{\vec{g}_k}(s) = \mathbf{C}_{\vec{g}_k}\left(s\mathbf{I} - \mathbf{A}_{\vec{g}_k}\right)^{-1}\mathbf{B}_{\vec{g}_k} + \mathbf{D}_{\vec{g}_k}. \quad (5)$$

The state-space realization (5) of (4) can provide a smooth parameterization of the state-space matrices [5], which is important for the following interpolation process. Next, this set of state-space matrices is interpolated to build a parametric macromodel $\mathbf{R}(s,\vec{g})$ [5], [6]. A simple and computationally cheap interpolation scheme is the piecewise linear interpolation method. Since it is not continuously differentiable, it cannot be used to generate parametric sensitivity macromodels. A proper choice of an interpolation scheme which is at least continuously differentiable is necessary. In this letter, two interpolation methods are investigated, namely the cubic spline (CS) interpolation and the piecewise cubic Hermite interpolation (PCHIP), which are briefly described in what follows.

A. Cubic Spline (CS) Interpolation

Given some data samples $(x_i, y_i)_{i=1}^n$, the CS interpolation method builds a cubic polynomial $s^i(x)$ for each interval of the dataset $x_i \le x \le x_{i+1}$, $i = 1,\ldots,n$. The coefficients of the cubic polynomials are obtained by imposing the first and second order derivative continuity at each data point along with a *not-a-knot* end condition [10]. Once these coefficients are computed, the derivatives of the overall spline interpolation function can be analytically calculated in terms of its coefficients. If the data under interpolation is in matrix form, each entry of the matrices is independently interpolated.

The univariate CS interpolation can be extended to higher dimensions by means of a tensor product implementation [10].

B. Piecewise Cubic Hermite Interpolation (PCHIP)

The PCHIP method is a monotonic shape preserving interpolation scheme. As in the CS interpolation, each data interval is modeled by a cubic polynomial with additional constraints to preserve the monotonicity locally [11]. An extension to higher dimension can be performed by a tensor product implementation [10]. The calculation of derivatives is done in the same way as in the CS interpolation case. This interpolation scheme works better for non-smooth datasets, wherein CS could result in overshoots or oscillatory behavior. However, PCHIP is only continuous in first derivatives, which affects the smoothness of the derivatives [11].

IV. Parametric Sensitivity Macromodels

The set of *root macromodel* state-space matrices $\mathbf{A}_{\vec{g}k}, \mathbf{B}_{\vec{g}k}, \mathbf{C}_{\vec{g}k}, \mathbf{D}_{\vec{g}k}$ is interpolated entry-wise and the multivariate models $\mathbf{A}(\vec{g}), \mathbf{B}(\vec{g}), \mathbf{C}(\vec{g}), \mathbf{D}(\vec{g})$ are built, yielding a parametric macromodel over the entire design space

$$\mathbf{R}(s,\vec{g}) = \mathbf{C}(\vec{g})\left(s\mathbf{I} - \mathbf{A}(\vec{g})\right)^{-1}\mathbf{B}(\vec{g}) + \mathbf{D}(\vec{g}). \quad (6)$$

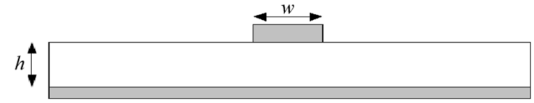

Fig. 1. Cross section of the microstrip.

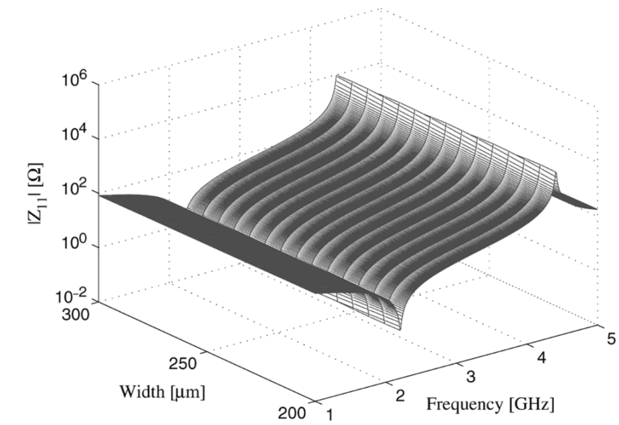

Fig. 2. Magnitude of Z_{11} for $h = 450\ \mu\mathrm{m}$.

A parametric macromodel of sensitivity responses is obtained by differentiating (6) with respect to the design parameters $\vec{g}$

$$\begin{aligned}\frac{\partial}{\partial\vec{g}}\mathbf{R}(s,\vec{g}) = {} & \frac{\partial\mathbf{C}(\vec{g})}{\partial\vec{g}}\left(s\mathbf{I} - \mathbf{A}(\vec{g})\right)^{-1}\mathbf{B}(\vec{g}) \\ & + \mathbf{C}(\vec{g})\left(s\mathbf{I} - \mathbf{A}(\vec{g})\right)^{-1}\frac{\partial\mathbf{A}(\vec{g})}{\partial\vec{g}} \\ & \times \left(s\mathbf{I} - \mathbf{A}(\vec{g})\right)^{-1}\mathbf{B}(\vec{g}) \\ & + \mathbf{C}(\vec{g})\left(s\mathbf{I} - \mathbf{A}(\vec{g})\right)^{-1}\frac{\partial\mathbf{B}(\vec{g})}{\partial\vec{g}} + \frac{\partial\mathbf{D}(\vec{g})}{\partial\vec{g}}. \quad (7)\end{aligned}$$

The derivatives of $\mathbf{A}(\vec{g}), \mathbf{B}(\vec{g}), \mathbf{C}(\vec{g}), \mathbf{D}(\vec{g})$ are computed efficiently and analytically using the CS and PCHIP schemes.

V. Numerical Results

In this example, a microstrip with a length of 2 cm is modeled. Its cross section is shown in Fig. 1. The relative permittivity of the substrate is equal to $\epsilon_r = 4.1$. A trivariate macromodel is built as a function of the width $W \in [200-300]\ \mu\mathrm{m}$ of the strip and the height $h \in [400-500]\ \mu\mathrm{m}$ of the substrate in addition to frequency $freq \in [1-5]$ GHz.

The two-port open-circuit impedance parameter matrix $\mathbf{Z}(s,W,h)$ has been computed by means of the analytical quasi-TEM model presented in [12] on a grid of $150 \times 15 \times 15$ samples $(freq, W, h)$. The accuracy of the model $\mathbf{R}(s,\vec{g})$ and its derivatives with respect to the original analytical quasi-TEM model $\mathbf{Z}(s,\vec{g})$ for the two interpolation methods is measured in terms of the relative error defined as

$$\mathrm{E}_{rel} = \left|\frac{\mathbf{R}(s,\vec{g}) - \mathbf{Z}(s,\vec{g})}{\mathbf{Z}(s,\vec{g})}\right|; \quad \vec{g} = (W,h) \in \text{validation grid}. \quad (8)$$

A set of stable and passive *root macromodels* has been built for 8 values of W and 8 values of h using VF, and 14 poles were selected for the *root macromodels* using an error-based bottom up approach. The remaining data are used for validation. Each *root macromodel* has been converted to a state-space form (5) and the state-space matrices have been interpolated using the CS and PCHIP interpolation methods. The maximum relative error

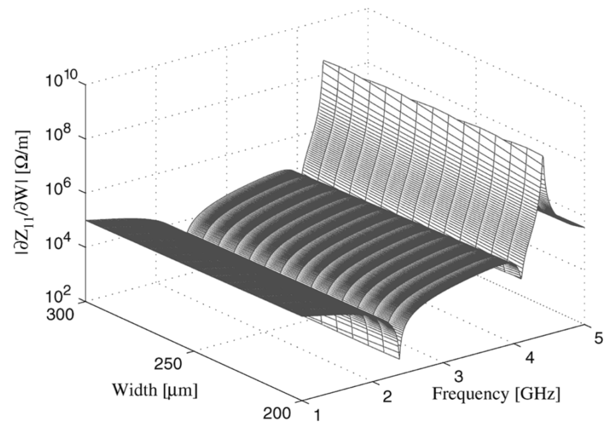

Fig. 3. Magnitude of $\partial Z_{11}/\partial W$ for $h = 450\ \mu\mathrm{m}$.

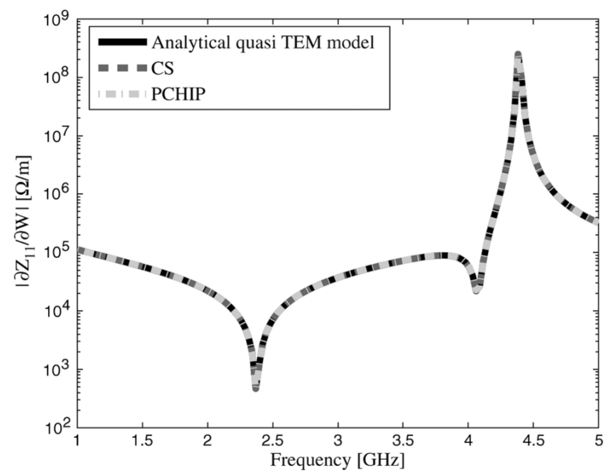

Fig. 4. Magnitude of $\partial Z_{11}/\partial W$ for $W = 250\ \mu\mathrm{m}$ and $h = 450\ \mu\mathrm{m}$.

(8) of the parametric macromodel of $\mathbf{Z}(s, W, h)$ is -62.23 dB and -57.78 dB, respectively, using the CS and PCHIP schemes. Then, the parametric sensitivities of $\mathbf{Z}(s, W, h)$ with respect to W and h has been computed by means of the derivatives (7) of the trivariate macromodels and the analytical quasi-TEM model. Figs. 2, 3 are plotted to show the parameterization of the $\mathbf{Z}$ and $\partial\mathbf{Z}/\partial W$ matrices. Fig. 2 shows the parametric behavior of the magnitude of the (1,1) entry of the $\mathbf{Z}(s, W, h)$ matrix (Z_{11}) as a function of frequency and W for $h = 450\ \mu\mathrm{m}$, while Fig. 3 shows the magnitude of the corresponding parametric sensitivity $\partial Z_{11}/\partial W$ obtained by the CS scheme. In order to visualize the modeling capability of the proposed method, the Fig. 4 shows sensitivity $\partial Z_{11}/\partial W$ obtained by the analytical quasi-TEM model, the CS and PCHIP methods as a function of frequency for the values $W = 250\ \mu\mathrm{m}$ and $h = 450\ \mu\mathrm{m}$, which have not been used for the generation of the *root macromodels*. A very good agreement between the methods can be observed.

Fig. 5 shows the relative error distribution (8) of the parametric sensitivity macromodel $\partial\mathbf{Z}/\partial W$ over the grid of $150 \times 15 \times 15$ samples $(freq, W, h)$. Similar results are obtained for $\partial\mathbf{Z}/\partial h$. We note that a good accuracy is achieved by both interpolation methods, but the CS scheme leads to a lower average error due to the continuity of the second derivative. However, in cases where the interpolation is performed on nonsmooth data sets, the CS scheme may result in oscillations of the derivatives. In those cases, PCHIP will result in a better accuracy.

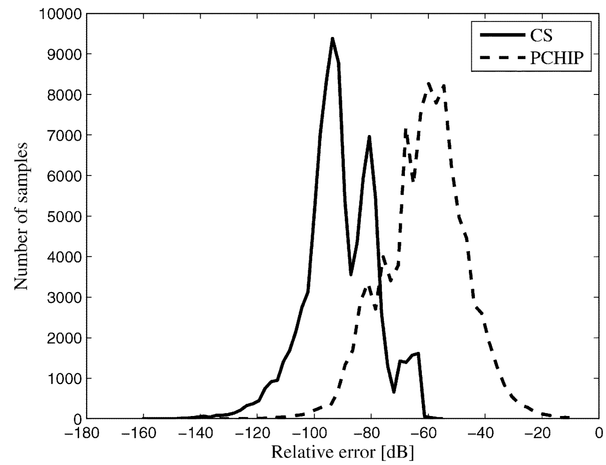

Fig. 5. Error distribution histogram for $\partial\mathbf{Z}/\partial W$.

VI. Conclusion

We have presented a new parametric macromodeling technique for building accurate parametric macromodels of system sensitivity responses with respect to design parameters. The technique is based on the interpolation of state-space matrices. A suitable choice of interpolation schemes allows to build accurate parametric sensitivity macromodels. Two different interpolation methods have been used and validated by numerical results, thereby demonstrating the accuracy and the modeling capability of the proposed method.

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