An Iron Loss Model for Electrical Machines using the Preisach theory

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Abstract—In this paper we deal with a mathematical model for the evaluation of the electromagnetic iron losses in rotating electrical machines under no load conditions. The presented problems of electromagnetic field computations are coupled with refined material models based on the Preisach theory. The model is validated by the comparison of numerical results and experimental values from measurements.

I. INTRODUCTION

Iron losses can account for a significant part of the total losses of an electrical machine. At the other hand, nowadays the efficient use of electricity is strongly emphasized. Electrical drive systems offer considerable opportunity to obtain major improvements in this respect. Consequently, it is important to increase the accuracy and reliability of the modelling and simulation of the iron losses. In [1] and [2] the calculation method of the iron losses in electrical machines is based on the local flux distribution and on the basic concept of loss separation.

In this paper we present the inclusion of the vector Preisach model, as described in [3], in the magnetic field calculations for a two dimensional domain D, shown in Fig.1. This domain D represents one tooth region of the stator of an asynchronous machine. The magnetic behaviour of the material can be described in terms of the macroscopic fields, taking into account the hysteresis phenomena. The three enforced flux patterns through the boundary parts \( \partial D_1, \partial D_2 \) and \( \partial D_3 \), see Fig.1, are obtained by local measurements in the electrical machine. On the basis of the computed field patterns in the domain D, the local excitation conditions for the magnetic material will be derived. The magnetodynamic models of one lamination, described in [4] and [5], will be used to investigate the local material response. This will lead to a detailed knowledge of the local iron losses. Finally, the global machine losses, evaluated from this new method, are compared with the measured machine losses.

II. MATERIAL MODELS

A. Scalar hysteresis model

If \( \vec{H} \) and \( \vec{B} \) are unidirectional, the BH-relation can be described by a scalar Preisach model in which the material is assumed to consist of small dipoles, each being characterized by a rectangular hysteresis loop with parameters \( \alpha \) and \( \beta \), see [6]. The characteristic parameters \( \alpha \) and \( \beta \) are distributed statistically according to a Preisach function \( P_\alpha(\alpha, \beta) \) which can be identified in a straightforward way. The BH-relation reads

\[
B(H, H_{\text{past}}) = \int_{-H_m}^{H_m} d\alpha \int_{-H_m}^{H_m} d\beta \eta_\alpha(\alpha, \beta, t) P_\alpha(\alpha, \beta). \tag{1}
\]

Here \( \eta_\alpha(\alpha, \beta, t) \) gives the value of the magnetisation \( M_\alpha \) for the dipole with parameters \( \alpha \) and \( \beta \) at time \( t \).

B. Vector hysteresis model in the x-y plane

In the vector model, as described in [3], the magnetic field vector \( \vec{H} \) and the magnetic induction vector \( \vec{B} \) are no longer unidirectional. The vector \( \vec{H} \) is projected on an axis \( \vec{d} \) (unit vector \( \vec{d}_d \)), which encloses an angle \( \theta \) with the fixed x-axis. The BH-relation is now given by

\[
B(H, H_{\text{past}}, \theta) = \frac{1}{\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta B_\theta(H_\theta, H_{\text{past}}, \theta) \vec{d}_\theta, \tag{2}
\]

with

\[
B_\theta(H_\theta, H_{\text{past}}, \theta) = \int_{-H_m}^{H_m} d\alpha \int_{-H_m}^{H_m} d\beta \eta_\theta(\theta, \alpha, \beta, t) P_\theta(\alpha, \beta), \tag{3}
\]

where \( \eta_\theta(\theta, \alpha, \beta, t) \) is obtained from the component \( H_\theta \) of the field along \( \vec{d}_\theta \). In (1), (2) and (3), \( H_{\text{past}} \) expresses
the dependency of the induction $B$ on the history of $H$, reflected in the functions $\eta_i$ and $\eta_2$, see e.g. [3], [6].

C. Extension to a rate dependent hysteresis model

The frequency dependence of the hysteresis effects may have a large influence on the magnetic behaviour of the material as pointed out in [1]. In the paper [4], the rate-dependent scalar Preisach model has been evaluated experimentally. Unfortunately, at present no experimental validated rate-dependent vector hysteresis model is available for non-oriented electrical steels.

III. TWO LEVEL MACHINE MODEL

A. First level: tooth region model

We consider the single tooth region of Fig.1, where the electrical conductivity $\sigma$ is assumed to be zero. The relevant Maxwell equations for the magnetic field $H = H_x \hat{1}_x + H_y \hat{1}_y$ and the magnetic induction $B = B_x \hat{1}_x + B_y \hat{1}_y$, in the domain $D$ are

$$\nabla \times H = 0, \; \nabla \times B = 0,$$

where the relation between $H$ and $B$ is defined by the material characteristics obtained from the vector Preisach hysteresis model, mentioned above. Enforcing a total flux $\phi_s(t)$ through the parts $\partial D_s, s=1,2,3$, of $\partial D$, we arrive at the boundary conditions (BCs)

$$\phi_s(t) = \int_{\partial D_s} B \cdot \hat{n} \, dl, \; t > 0, \; s = 1,2,3,$$

An assumed zero flux leakage through $\partial D_4, \partial D_5$ and $\partial D_6$ (shown in Fig.1) results in the additional BCs:

$$B \cdot \hat{n} = 0 \text{ on } \partial D_s, \; t > 0, \; s = 4, 5, 6.$$

As the enforced fluxes $\phi_s(t), s = 1,2,3$, are periodic in time, we may use a complex Fourier decomposition for the local vector field $B(x,y,t)$, viz

$$B(x,y,t) = \sum_{k=-\infty}^{+\infty} B_k(x,y) \cdot e^{(kw+\omega_k)}.$$  

Here, $\omega$ is $2\pi$ times the basic frequency; $\omega_k$ and $B_k$ are the phase angle and the amplitude of the $k$-th harmonic of $B$. Using the local field patterns obtained from the tooth region model, see (7), we may investigate the local material behaviour from a lamination model.

B. Second level: lamination model

The magnetic behaviour of ferromagnetic laminations can be described in terms of the macroscopic fields, taking into account the interacting hysteresis and eddy current phenomena.

We consider a single lamination of thickness $2d$, see Fig.2. Throughout the sheet, the time dependent total flux vector $\vec{\phi}(t)$ flows parallel to the $(x,y)$-plane. This flux vector is constructed out of (7). The magnetic field and the magnetic induction in the lamination model take the form $\vec{H} = H_x \hat{1}_x + H_y \hat{1}_y$ and $\vec{B} = B_x \hat{1}_x + B_y \hat{1}_y$ respectively. As $d << w$ and $d << t$, eliminating the edge effects, we may assume $H_x, H_y$ and $B_x, B_y$ to vary in the $z$-direction only. The relevant Maxwell equations read

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \; \nabla \times \vec{H} = \vec{J}.$$  

Two different types of excitations can be considered.

1) Alternating excitation conditions: Here, we may assume $H_y, B_y$ and $\phi_y(t)$ to be identically zero. The equations above simplify to a parabolic differential equation (DE) for $H_x$

$$\frac{1}{\sigma} \frac{\partial^2 H_x}{\partial z^2} = \frac{\partial B_x}{\partial t}, 0 < z < d, t > 0,$$

along with the BCs

$$\frac{\partial H_x}{\partial z}(z = 0, t) = 0, \frac{\partial H_x}{\partial z}(z = d, t) = \frac{\sigma}{2} \frac{d\phi_x}{dt}, t > 0.$$  

Here, the magnetic induction $B_x(z,t)$ can be related to the magnetic field $H_x(z,t)$ by either the scalar rate independent or rate dependent Preisach hysteresis model, [4].

2) Rotational excitation conditions: Now, the governing DEs for the magnetic field $(H_x,H_y)$ are found to be:

$$\frac{1}{\sigma} \frac{\partial^2 H_x}{\partial z^2} = \frac{\partial B_x}{\partial t}, 0 < z < d, t > 0,$$

$$\frac{1}{\sigma} \frac{\partial^2 H_y}{\partial z^2} = \frac{\partial B_y}{\partial t}, 0 < z < d, t > 0,$$

while the BCs become

$$\frac{\partial H_x}{\partial z}(z = 0, t) = \frac{\partial H_x}{\partial z}(z = 0, t) = 0,$$

$$\frac{\partial H_x}{\partial z}(z = d, t) = \sigma \frac{d\phi_x}{dt}, \frac{\partial H_y}{\partial z}(z = d, t) = \sigma \frac{d\phi_y}{dt}, t > 0.$$  

![Figure 2: Geometry of magnetic model of one lamination](image-url)
\[ T_z - T_i \text{ is an integer multiple of the excitation period} \] are calculated by summing up the total hysteresis losses \( P_h \) and the eddy current losses \( P_e \), viz

\[ P_h = \frac{1}{2d} \int_{-d}^{d} \int_{T_i}^{T_f} (H_x \frac{\partial B_x}{\partial t} + H_y \frac{\partial B_y}{\partial t}) dt \]

\[ P_e = \frac{1}{2d} \int_{-d}^{d} \int_{T_i}^{T_f} \left( \left( \frac{\partial H_x}{\partial x} \right)^2 + \left( \frac{\partial H_y}{\partial z} \right)^2 \right) dt. \]

IV. NUMERICAL RESULTS

A. Tooth region model

We consider a three phase 3kW 4-pole induction motor, described in detail in [7]. Due to periodicity, only three neighbouring teeth in the stator will be considered. The enforced total fluxes \( \phi_s \) through the parts \( \partial D_s \), \( s = 1,2 \) in Fig.1 are obtained from local measurements in the electrical machine (notice that \( \phi_3 = -\phi_1 - \phi_2 \)). The flux through the gate \( \partial D_1 \) for each of the 3 neighbouring teeth is given in Table 1 by its Fourier decomposition

\[ \phi_1(t) = \sum A_k \cos(k \omega t + \gamma_k). \]

Table 2 shows the symmetry for each pair of positive and negative harmonics for point 1 in Fig.1 for tooth1. This corresponds to alternating field vectors. Notice that for point 2 this symmetry is lost, reflecting a rotational magnetic induction \( \vec{B} \). Similar remarks could be made for each point in tooth1, tooth2 and tooth3.

B. Lamination model

The local flux patterns obtained from the tooth model are used as input for the magnetodynamic model that invokes a \textit{vector rate independent} Preisach model to account

\[ \text{Figure 3: calc. local BH-loops for points 1 and 2 in tooth 1} \]

for rotational effects. The resulting BH-loops are shown in Fig.3 for the points 1 and 2 in tooth1. To evaluate the local losses, we add to the losses calculated from the model (11)-(13) extra dynamic electromagnetic losses to take into account the \textit{rate dependent} hysteresis effects. These extra dynamic losses represent the difference between the losses evaluated from the model (9)-(10), that is coupled first with the rate independent and next with the rate dependent Preisach model. Here, the alternating excitation used is the excitation which is obtained when we project the rotating excitation on that axis that gives rise to the maximum amplitude.

C. Machine losses

The global machine losses predicted by this combined tooth region-lamination model is equal to 65W, which is in reasonable agreement with measured machine losses of 72W, see [7].

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REFERENCES