Induced modulation instability and recurrence in nematic liquid crystals

J. Beeckman,1,2∗ X. Hutsebaut,1 M. Haelterman,1 and K. Neyts2

1Service d'optique et acoustique, Université libre de Bruxelles CP 194/5
50 avenue F.D. Roosevelt, 1050 Bruxelles, Belgium

2Liquid Crystals & Photonics Group, Department of Electronics and Information Systems,
Ghent University, Sint-Pietersnieuwstraat 41, 9000 Gent, Belgium

*Corresponding author: jeroen.beeckman@elis.ugent.be

Abstract: We study induced modulation instability in a nematic liquid crystal cell. Two broad elliptical beams along one direction are launched into the cell. The two beams have slightly different angle in order to create a sinusoidally varying intensity at the entrance of the cell. In this way, the gain of perturbations with different spatial frequency is investigated. The evolution of the optical pattern, for certain conditions, shows a recurrence of the signal. We believe that this is the manifestation of the Fermi-Pasta-Ulam recurrence and to the best of our knowledge, the first experimental observation of this phenomenon in the spatial optical domain. Numerical simulations show a good agreement with the experimental findings.

© 2007 Optical Society of America

OCIS codes: (160.3710) Materials: Liquid crystals; (190.0190) Nonlinear optics: Nonlinear optics; (260.5950) Self-focusing.

References and links
1. Introduction

Plane waves, propagating in a nonlinear self-focusing medium are unstable and small perturbations of the plane wave are amplified along the propagation. This phenomenon is referred to as modulation instability and it is present in a number of subfields of physics. The explanation of this phenomenon is the energy transfer between spectral modes, namely between the zero order mode (the plane wave) and higher-order Fourier components in the signal. The gain of the signal is not spectrally flat, which means that some frequencies have a higher gain than others. In this work, we are not only interested in the initial gain of the perturbation, but also in the long term evolution of the beam propagation. In the early 1950s, Fermi, Pasta and Ulam numerically studied the long term evolution of the energy transfer between spectral modes in the context of nonlinear discrete systems [1]. It was commonly admitted at that time that in the long term, the system would exhibit ‘thermalization’, which is an equipartition of energy between the modes. Instead they found that the system exhibits a complicated quasi-periodic behavior, with only a few modes involved. This discovery had a great impact on nonlinear science and is referred to as Fermi-Pasta-Ulam (FPU) recurrence. After that, a lot of theoretical and numerical work has been carried out, showing that FPU recurrence is possible for many nonlinear wave equations, for example for the Korteweg-de Vries equation [2], the Nonlinear Schrödinger equation [3] or for saturable nonlinearities [4].

In contrast with the theoretical and numerical work available, experimental observations of the FPU recurrence are rather uncommon. The main experimental problem is to keep the theoretical models valid over long propagation distances. Experiments have been demonstrated in electrical networks [5], in hydrodynamics [6] and recently in magnetic feedback rings [7]. In the optical domain, the FPU recurrence has been observed in 2001 by Van Simaeys et al. [8] in optical fibers. The interest of this latter work has been underlined by N.N. Akhmediev in [9], describing the phenomenon as a Décou Vu in Optics. In the optical domain, the FPU recurrence is thus only demonstrated in the temporal domain. In this work we present for the first time, a demonstration of FPU recurrence in the spatial optical domain.

Nematic liquid crystals have proved to be excellent materials for investigation of various optical nonlinear phenomena. Generation of spatial solitons has been demonstrated for different configurations [10–12] with only a few mW of light power. The nonlinearity is highly nonlocal [13] and the nonlocality can be tuned by using either different cell thickness [14] or by using the time dependence of the nonlinearity [15,16]. In most of the reported work, the nonlinearity used is optical director reorientation [17]. Due to a torque on the molecules when an electric field
is present, the director (which is the average molecular orientation) tends to reorient parallel to the electric field. This in turn leads to an increase of refractive index, which gives rise to the self-focusing effect, necessary for the generation of bright solitons. In 2003, Peccianti et al. experimentally demonstrated modulation instability of a broad elliptical beam [18, 19].

In the present work the spectral dependence of the modulation instability is investigated experimentally and numerically by seeding the instability by means of a second broad elliptical beam co-propagating with a slightly oblique angle with the fundamental beam. In this way, the beam intensity exhibits an initial sinusoidal perturbation along the transverse direction and the spatial frequency is controlled by changing the angle between the two beams. Not only the initial gain of the signal is observed, but also the long-term behavior of the beam propagation.

2. Experimental set-up

The cell used in the experiment is very similar to the ones used in previous publications [11, 13]. It consists of two glass plates with a transparent electrode and an alignment layer (not shown in Fig. 1). These glass plates are glued together with spacers in between to ensure a homogeneous thickness, which in this work is 53 \( \mu \)m. A third glass plate is glued perpendicularly to these two glass plates to ensure a good incoupling of the beams (also not shown in Fig. 1). The rubbing of the alignment layers is such that the molecules are planar and oriented along the \( z \) axis. The nematic liquid crystal used is E7 from Merck. A voltage can be applied over the LC layer so that the director can tilt. The director remains in the \( xz \) plane so that the director orientation can be described by the angle \( \theta \), which is the angle with respect to the \( z \) axis.

![Fig. 1. Schematic representation of the cell containing the liquid crystal. Two beams with a linear polarization are launched into the liquid crystal layer: one beam along the \( z \) axis (\( k_0 \)) and the other beam (\( k_1 \)) in the \( yz \) plane with a small angle \( \beta \) with respect to the \( z \) axis.](image)

Two Gaussian elliptical beams are launched into the cell by means of a cylindrical lens. One beam is launched along the \( z \) direction, while the other lower power beam is launched in the \( yz \) plane with a small angle \( \beta \) with respect to the \( z \) axis. Both beam are broad along the \( y \) direction and the waist of both beams is estimated from measurements to be 2 \( \mu \)m and 0.4 mm respectively along the \( x \) and the \( y \) direction.

Launching two broad elliptical beams with an adjustable angle inside the cell requires a careful design of the set-up. The set-up that is used is schematically depicted in Fig. 2. A laser beam with a wavelength of 780 nm passes respectively through a rotatable half-wave plate, a linear polarizer, a lens and a Wollaston prism. In this way, two beams are generated with a controllable intensity. The high power beam, which is launched straight into the cell (along the \( z \) direction), passes through a system of mirrors that acts as a delay line, to avoid problems with spatial coherence. The other beam passes through another half-wave plate at 45\(^\circ\), so both beams have the same polarization. Two rotatable mirrors are used to adjust the angle at which the beam enters the cell. The beam propagation in the cell can be observed due to scattering of the light.
in the nematic liquid crystal. Via a lens system and a CCD-camera, the beam propagation can be easily observed. The set-up is placed on a vibration-controlled optical table, because the interferometric set-up is very sensitive to vibrations.

The beam amplitude at the entrance of the cell (z = 0) can then be written as:

\[ E(x, y, z = 0) = [E_0 + E_1 \exp(-jk_0 \sin \beta y)] \exp\left(-\frac{x^2}{\Delta x^2} - \frac{y^2}{\Delta y^2}\right) \]

and the intensity as (with \( I_0 = E_0^2 \) and \( I_1 = E_1^2 \)):

\[ I(x, y, z = 0) = \left[I_0 + I_1 + 2 \sqrt{I_0 I_1} \cos(k_y y)\right] \exp\left(-\frac{2x^2}{\Delta x^2} - \frac{2y^2}{\Delta y^2}\right) \]

The sinusoidal modulation of the beam intensity has a spatial frequency \( k_y \), which relates to the spatial period \( T \) as \( k_y = \frac{2\pi}{T} \). Since the beam is broad along the \( y \) direction, one can neglect the Gaussian profile in good approximation, when looking only at a limited window near the maximum of the profile along the \( y \) direction.

### 3. Experimental results

Figure 3 shows the evolution of the beam propagation for a spatial frequency of 0.124 \( \mu \text{m}^{-1} \) and different optical powers. The entrance of the cell is at the left side and the light is propagating towards the right. At the entrance, a lot of light scattering is visible, which is mainly caused by the glue between the glass interfaces and by inhomogeneities in the director orientation near the entrance. For the high power situation of Fig. 3(d) the high power beam has a total optical power of 335 mW, while the power of the low power beam is 8 mW. The figures with lower power are obtained with both beams equally attenuated, so the relative modulation depth at the left side is the same for every figure. Note that different images are patched together and some discontinuities can be observed. This is caused by fluctuations in the beam propagation, mainly caused by fluctuations in the liquid crystal and mechanical vibrations.

For the lowest power situation (Fig. 3(a)), one can observe some change from the initial sinusoidal modulation along the propagation. It should be noted however that there is no clear amplification of the initial modulation. The signal gets more or less scrambled, probably due to inhomogeneities and fluctuations in the liquid crystal layer. For higher power (Fig. 3(b)), clearly some focusing of the initial modulation can be seen. The maximum intensity of the peaks increases and the peaks become narrower. The maximum intensity is obtained after about 1 mm of propagation. For higher powers (Fig. 3(c) and (d)), the amplification of the initial modulation peaks is larger. This is deducted from the fact that the maximum peak intensity becomes higher.
and that the maximum shifts toward the entrance of cell \((i.e.\) to the left). For the highest power situation, this maximum is obtained after about 0.5 mm of propagation distance.

Fig. 3. Evolution of the beam propagation for a spatial frequency of 0.124 \(\mu\text{m}^{-1}\) (period 50.7 \(\mu\text{m}\)), for different optical power: 0.04 (a), 0.18 (b), 0.27 (c) and 0.41 mW/\(\mu\text{m}\) (d). The scale is the same along the \(y\) and \(z\) direction.

It should be noted that the maximum peak intensity is not reached exactly at the same propagation distance for all peaks. In Fig. 3(c) and (d) it is clearly visible that for the middle region the maximum peak intensity is closer to the entrance than for the higher and lower regions. This can be explained by the fact that the two incident beams have a Gaussian profile along the \(y\) direction, with lower intensity for the outer regions. More important is the fact that after the initial focusing of the beam, the peaks are spreading again. This is similar to the 'breathing' of soliton-like beams with a power excess, as for example described in [20]. Here, the spreading however does not lead to a second focusing of the peaks. Instead, one gets a situation where the defocusing regime results in a rather chaotic creation of beam filaments. This is somewhat similar to the multisoliton generation that was observed for a wide focused beam [19].

Figure 4 shows the beam propagation for different spatial frequencies. One can see that the gain of the signal is dependent on the spatial frequency. The position of the maximum peak intensity appears closer to the entrance of the cell for higher spatial frequencies. The dependence of the gain on the spatial frequency will be discussed further in this article.

For every spatial frequency, the peaks first focus and then defocus. However, a return to the
initial sinusoidal state cannot be observed for any spatial frequency in Fig. 4. It is only for large spatial frequencies and lower optical powers that the beam returns more or less to its initial sinusoidally modulated state.

Figure 5 shows the Fourier transform of the intensity pattern. The value at zero frequency represents the total intensity evolution of the beam, which allows us to estimate the loss coefficient to be 2 cm$^{-1}$. This includes both absorption and scattering losses. For the maximum propagation distance considered (1.5 mm) about 26% of the light is lost, which means that losses are acceptably low. We can see clearly in the graph the build-up of multiple orders (up to 7 orders can be distinguished). The noise at $z = 0$ is due to scattering of light at the entrance as mentioned earlier, so we cannot draw any conclusions from the first few 100 µm of propagation. However we can clearly see that the height of the different orders first increases and then decreases again, although from the intensity patterns one can see that it is not a clear return to the initial state.

Figure 6 shows the evolution of the first order of the Fourier intensity spectrum. This is the height of the peak appearing in the Fourier spectrum at the spatial frequency of the initial modulation. For every spatial frequency of the exciting beam, there is a build-up of the first order until it reaches a maximum and then it decreases again. First, one can observe that the position of the maximum appears closer to the cell entrance for higher spatial frequencies (as was also the case for the point of maximal focusing). Second, one can see that the height of the maximum decreases with increasing spatial frequency.

From the initial slope of the first order peak, one can calculate the gain of the initial perturbation. The result is shown in Fig. 7. One can see that, as expected, the gain increases with increasing optical power. The graph also shows that the gain exhibits a cut-off frequency, which means that above a certain spatial frequency, no gain is present. For small frequencies, the gain tends to go to zero too. In fact, the gain resembles very well the gain curves that were theoreti-
Fig. 5. Absolute value of the Fourier transform of the intensity $I(y,z)$ in Fig. 3(d). Artificial coloring is used for better visibility.

Fig. 6. Evolution of the fraction first order to zero order for the intensity evolution in Fig. 3(d), for 6 situations with different spatial frequencies of the exciting beam.
cally obtained, by Peccianti et al. for a nonlocal liquid crystal medium [18] and by Krolikowski et al. [21] for a nonlocal Kerr medium. The curves in Fig. 7 are obtained using the theoretical formula obtained in [21] for an exponential nonlocality. When using an exponential response function $R(y) = 1/(2\sigma) \exp(-|y|/\sigma)$ for the nonlocality of the nonlinearity:

$$\Delta n(I) = s \int_{-\infty}^{\infty} R(y'-y)I(y',z)dy'$$

the growth rate of the instability is given by:

$$|k_y| \sqrt{\frac{1}{\rho_0} + \frac{k_y^2}{4\rho_0}}$$

In these equations $\sigma$ represents the degree of nonlocality, $k_y$ is the spatial frequency and $\rho_0$ the intensity. The solid gray lines in Fig. 7 show the gain curves for a nonlocality of $\sigma = 8 \mu m$ and different intensities. Considering the simplicity of the model, the agreement with the experimental data is satisfactory.

![Fig. 7. Gain of the first order in function of spatial frequency of the exciting beam for different optical powers. The solid lines represent a fit with the gain for an exponential nonlocal Kerr nonlinearity.](image)

4. Numerical results

Numerical simulations were carried out based on the model that has been used already in some of our previous publications [15] and in publications of other groups [11, 22]. The model is based on the solution of the liquid crystal director orientation in two dimensions on one hand, ruled by the following partial differential equation:

$$\left( K_1 \cos^2 \theta + K_3 \sin^2 \theta \right) \frac{\partial^2 \theta}{\partial x^2} + \frac{1}{2} (K_3 - K_1) \sin^2 \theta \left( \frac{\partial \theta}{\partial x} \right)^2 + K_2 \frac{\partial^2 \theta}{\partial y^2} + \frac{1}{2} \varepsilon_0 \sin 2\theta \left( \Delta \varepsilon^s |E^s|^2 + \Delta \varepsilon^o |E^o|^2 \right) = 0.$$  

(5)

in which $K_1, K_2$ and $K_3$ are the Frank-Oseen elastic constants, $\theta$ is the angle of the director with respect to the $z$ axis, $E^s$ and $E^o$ are respectively the static and optical electric field along the $x$ direction and $\Delta \varepsilon^s$ and $\Delta \varepsilon^o$ are respectively the static and optical dielectric anisotropy of the...
liquid crystal. Together with this equation the static electric field distribution \(E^s(x, y)\) has to be solved.

The propagation of the optical field is calculated using a Beam Propagation Method using the following equation for the TM field:

\[
2j k_0 n_0 \frac{\partial E^o}{\partial z} + \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) E^o + k_0^2 (n^2 - n_0^2) E^o = 0
\]

In this equation, \(k_0\) is the wave number in vacuum, \(n_0\) is a background refractive index and \(n\) is the refractive index of the liquid crystal, depending on the angle \(\theta\). The numerical method is implemented for periodic boundaries in the \(y\) direction. In order to reduce the calculation time, only the propagation of one period \(T\) of the modulated beam is calculated. This means that the beam is infinite along the \(y\) direction and that the waist \(\Delta y\) in Eq. 1 is infinity.

Figure 8 shows the beam propagation for different spatial frequencies. For the spatial frequencies considered, we can clearly see that the initial modulation is amplified. This results in the focusing of the different modulation peaks. After the maximum focusing the situation depends largely on the spatial frequency that is considered. For large spatial frequencies, we can observe a nearly periodical focusing and defocusing of the lobes. For smaller spatial frequencies, the beam propagation after the maximum focusing displays a rather chaotic behavior. The system does not return to its initial condition.

The simulations show a good agreement with the experimental results. This can be illustrated by comparing the propagation distance where the first order of the intensity spectrum reaches a maximum. The results from the numerical calculations are in good agreement with the experimental ones as shown in Fig. 9. In order to let both results agree, one should use a lower power in the simulations and the difference is a factor 2 to 3. In a previous publication we have noticed the same difference [15], so we are inclined to believe that the in-coupling losses are responsible for this discrepancy and that they are higher than expected.

5. Discussion

The fact that there is a rather chaotic behavior for small spatial frequencies is plausible when considering the evolution of the spectral orders. For large spatial frequencies (like \(0.4 \mu m^{-1}\) as in Fig. 8) only a limited number of orders come into play during the propagation. This is due to the fact that there is no gain of higher order modes, because their spatial frequency is located outside the gain window. For small spatial frequencies (like \(0.1 \mu m^{-1}\)), more orders come into play, because more higher orders are located inside the gain window. If there is only gain for the zero and first order, one can achieve a pseudo-periodic recurrence. When a large number of orders exhibit gain during propagation the energy transfer to the higher orders prevents the pseudorecurrence. A similar discussion can be found in [4], where the authors numerically investigated the pseudorecurrence in saturable self-focusing media.

In the experimental results, it was not possible to see a perfect return to the initial state. Nor was it possible to see a second recurrence for any of the spatial frequencies. This is in our opinion caused by the fluctuations in the liquid crystal. These fluctuations are caused by the thermal energy of the molecules. Due to elastic coupling of the molecules, the molecular fluctuations are a collective behavior, extending over periods of several molecular distances [17]. The effect of the fluctuations on the beam propagation can be seen in Fig. 4, because the figures are composed of different photographs which are taken with some time in between. One can see that the fluctuations cause small lateral shifts of the modulation peaks, which, in our opinion, prevents a second recurrence of the signal.
Fig. 8. Simulation of the beam propagation for an optical power of 0.14 mW/µm and for different spatial frequency (0.4, 0.2 and 0.1 µm⁻¹). The figures show the integration of the intensity along the x direction. The intensity is normalized to the average input intensity. The scales along the two directions are different.

Fig. 9. Position of the maximum of the first Fourier component along the propagation direction for the intensity profiles of Fig. 8.
6. Conclusions

In this work, induced modulation instability has been investigated in a nematic liquid crystal cell, both experimentally and numerically. We have performed measurements for different spatial frequencies of the input signal and in this way, we were able to produce the gain curve. The experiments showed a recurrence of the signal, namely that the modulation peaks are focused after which a defocusing occurs. It was not possible to see an exact recurrence to the initial modulation, nor it was possible to see a second recurrence of the signal. The numerical results reveal that a recurrence is not possible for small spatial frequencies, due to the fact that a lot of higher order modes come into play. For large frequencies, slightly below the maximal amplified frequency, simulations reveal that is should be possible to see multiple recurrences. The simulations do not take into account the fluctuations in the liquid crystal, which is, in our opinion, the effect preventing a second recurrence in the experiment.

Acknowledgements

Jeroen Beeckman is postdoctoral fellow of the Research Foundation - Flanders (FWO) and received a mobility grant from the same institution for working at the Université libre de Bruxelles. The project is a result of collaboration within the framework of the PHOTONNetwork and Photonics@be program of the Belgian Science Foundation.