Limit behaviour of imprecise Markov chains

Ergodicity versus weak ergodicity

Jasper De Bock

ImPRooF 2022, Cartagena

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We are SIPTA, the Society for Imprecise Probabilities: Theories and Applications, and we are
convinced that there is more to uncertainty than probabilities. There is much more, in fact.
Would you like to use probabilities but don't know their
ues? Or would you like to
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Limit behaviour of imprecise Markov chains

Ergodicity versus weak ergodicity

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$P($  $) = {}^{1}/_{2}$

$$X_1 \quad X_2 \quad X_3 \quad X_4 \quad \cdots \quad X_n \quad X_{n+1} \quad \cdots$$

$\lim_{n\to\infty}$ #  / (#  + # ) $= 1/2$

$P($  $) = 1/2$

Strong law of large numbers

$$X_1 \quad X_2 \quad X_3 \quad X_4 \quad \cdots \quad X_n \quad X_{n+1} \quad \cdots$$

identically distributed with $P($$) = {}^1\!/_2$

+ independence assumption

$\Rightarrow \lim_{n \to \infty} \dfrac{\#}{\# + \#} = {}^1\!/_2$, where the numerator is #  and the denominator is #  + # 

almost surely **!**

Strong law of large numbers

$\in \mathcal{X}$ finite

$$X_1 \; X_2 \; X_3 \; X_4 \; \cdots \; X_n \; X_{n+1} \; \cdots$$

identically distributed with mass function P

+ independence assumption

$$\Rightarrow \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} f(X_i) = E(f)$$

almost surely

Strong law of large numbers

$$X_1 \; X_2 \; X_3 \; X_4 \; \cdots \; X_n \; X_{n+1} \; \cdots$$

identically distributed with mass function P

~~+ independence assumption~~

$$\Rightarrow \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} f(X_i) = E(f) \quad ?$$

almost surely

Markov chains

$$\begin{matrix} & & & & & x & & \\ & & & & & \| & & \\ X_1 & X_2 & X_3 & X_4 & \cdots & X_n & X_{n+1} & \cdots \end{matrix}$$

identically distributed

+ independence

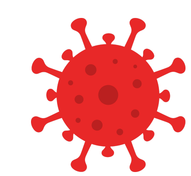
Markov assumption

$$P_x(X_{n+1})$$

Markov chains

- ✓ **Reliability engineering** (failure probabilities, …)
- ✓ **Queuing theory** (waiting in line …)
  - dimensioning of call centers
  - router queues on the internet

- ✓ **Chemical reactions** (time-evolution …)



- ✓ **Epidemiology** (time until threshold…)
- ✓ **Pagerank**
- ✓ …

Strong law of large numbers ?

$$X_1 \; X_2 \; X_3 \; X_4 \; \cdots \; X_n \; X_{n+1} \; \cdots$$

transition probabilities P_x for each x in $\mathcal{X}$

+ Markov assumption

$$\Rightarrow \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} f(X_i) = \boxed{E(f)} \; ?$$

almost surely

Markov chains

$$\begin{array}{c} x \\ \Vert \end{array}$$

$$X_1 \quad X_2 \quad X_3 \quad X_4 \quad \cdots \quad X_n \quad X_{n+1} \quad \cdots$$

identically distributed

\+ independence

$$P_x(X_{n+1})$$

Markov assumption

limit expectation

ergodicity

$$E(f) \Rightarrow E_\infty(f) := \lim_{n\to\infty} E(f(X_n)|X_1 = x)$$

Pointwise ergodic theorem

$$X_1 \;\; X_2 \;\; X_3 \;\; X_4 \;\; \cdots \;\; X_n \;\; X_{n+1} \;\; \cdots$$

transition probabilities P_x for each x in $\mathcal{X}$

+ Markov assumption + ergodicity

$$\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} f(X_i) = E_\infty(f)$$

almost surely

How to check ergodicity?

Having a top class

+

Being top class regular

ergodicity

limit expectation

$$E_\infty(f) := \lim_{n\to\infty} E(f(X_n)|X_1 = x)$$

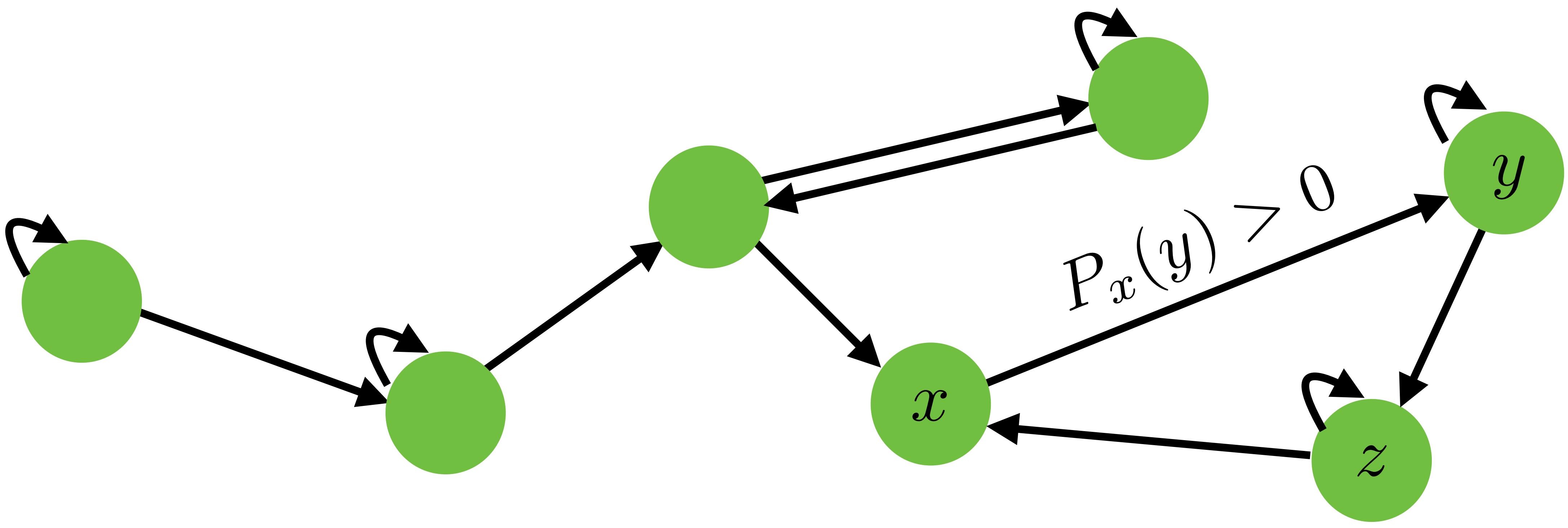
How to check ergodicity?

Accessibility relation

$$x \xrightarrow{1} y \Leftrightarrow P_x(y) > 0$$

$$x \xrightarrow{n} y \Leftrightarrow x = z_1 \xrightarrow{1} z_2 \xrightarrow{1} z_3 \xrightarrow{1} \cdots \xrightarrow{1} z_{n-1} \xrightarrow{1} z_n = y$$

$$x \rightarrow y \Leftrightarrow (\exists n \in \mathbb{Z}_{\geq 0}) x \xrightarrow{n} y$$



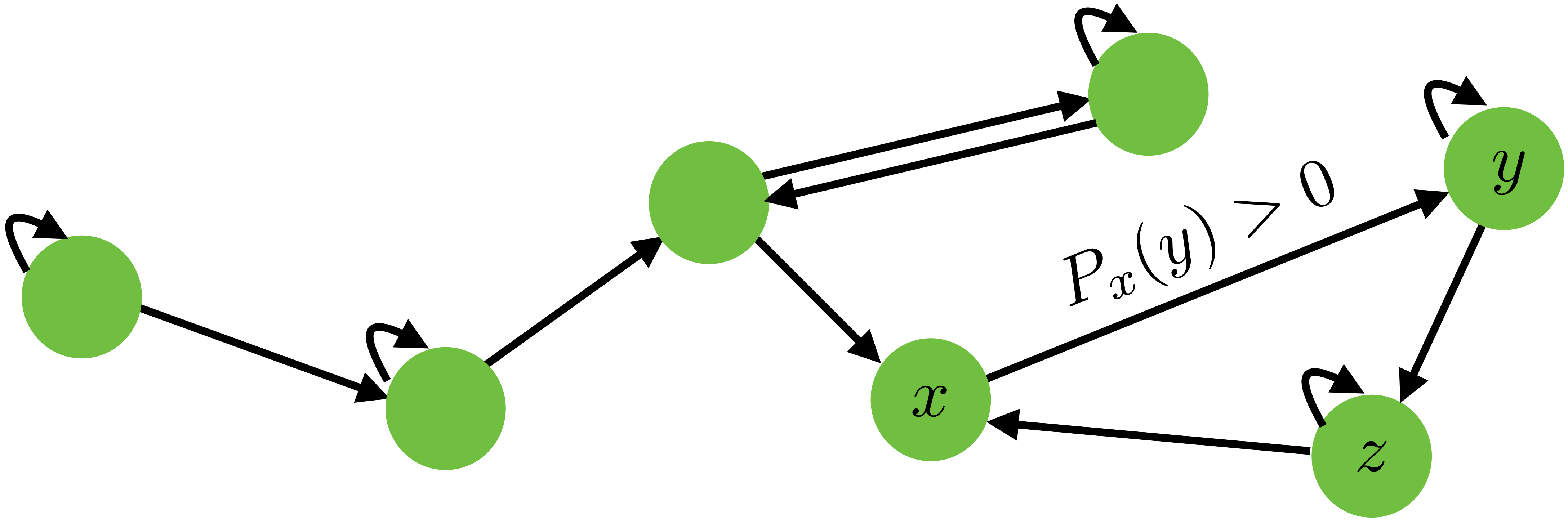
Having a top class

$$\mathcal{R} := \{y \in \mathcal{X} : (\forall x \in \mathcal{X}) x \to y\} \neq \emptyset$$

+

Being top class regular

$$(\exists n \in \mathbb{Z}_{\geq 0})(\forall k \geq n)(\forall x, y \in \mathcal{R}) x \xrightarrow{k} y$$



Having a top class

$$\mathcal{R} \coloneqq \{y \in \mathcal{X} \colon (\forall x \in \mathcal{X}) x \to y\} \neq \emptyset$$

+

Being top class regular

$$(\exists n \in \mathbb{Z}_{\geq 0})(\forall k \geq n)(\forall x, y \in \mathcal{R}) x \xrightarrow{k} y$$

ergodicity

limit expectation

$$E_\infty(f) \coloneqq \lim_{n \to \infty} E(f(X_n) | X_1 = x)$$

How to find the limit expectation?

limit distribution π_∞

$$(\forall x \in \mathcal{X})\pi_\infty(x) = \sum_{y\in\mathcal{X}} \pi_\infty(y) P_y(x)$$

$$\sum_{x\in\mathcal{X}} \pi_\infty(x) f(x)$$

$$||$$

$$E_\infty(f) := \lim_{n\to\infty} E(f(X_n)|X_1 = x)$$

How to find the limit expectation?

transition operator T

$$Tg(x) := \sum_{y \in \mathcal{X}} P_x(y) g(y)$$

$$\lim_{n \to \infty} T^n f(x)$$

$$||$$

$$E_\infty(f) := \lim_{n \to \infty} E(f(X_n) | X_1 = x)$$

Pointwise ergodic theorem

$$X_1 \;\; X_2 \;\; X_3 \;\; X_4 \;\; \cdots \;\; X_n \;\; X_{n+1} \;\; \cdots$$

transition probabilities P_x for each x in $\mathcal{X}$

+ Markov assumption + ergodicity

$$\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} f(X_i) = E_\infty(f)$$

almost surely

Pointwise ergodic theorem

$$X_1 \; X_2 \; X_3 \; X_4 \; \cdots \; X_n \; X_{n+1} \; \cdots$$

transition ~~probabilities~~ P_x for each x in $\mathcal{X}$

+ Markov assumption + ergodicity

→ $$\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} f(X_i) = E_\infty(f)$$?

almost surely

Imprecise Markov chains

$$X_1 \quad X_2 \quad X_3 \quad X_4 \quad \cdots \quad X_n = x \quad X_{n+1} \quad \cdots$$

$\mathcal{P}_x$ set of transition probabilities

~~Markov assumption~~

Imprecise Markov assumption

Imprecise Markov chains

$$X_1 \quad X_2 \quad X_3 \quad X_4 \quad \cdots \quad X_n \quad X_{n+1} \quad \cdots$$

$X_n = x$

$\mathcal{P}_x$ set of transition probabilities

~~Markov assumption~~

Imprecise Markov assumption

Type 1 (epistemic irrelevance)

Set of all stochastic processes such that

$$P(X_{n+1} | X_{1:n} = x_{1:n}) \in \mathcal{P}_{x_n}$$

Imprecise Markov chains

$$X_1 \quad X_2 \quad X_3 \quad X_4 \quad \cdots \quad \underset{}{\overset{x}{\overset{\|}{X_n}}} \quad X_{n+1} \quad \cdots$$

$\mathcal{P}_x$ set of transition probabilities

~~Markov assumption~~

Imprecise Markov assumption

Type 2 (complete independence)

Set of all stochastic processes such that

$$P(X_{n+1}|X_{1:n} = x_{1:n}) = P(X_{n+1})|X_n = x_n)$$

$$\in \mathcal{P}_{x_n}$$

Imprecise Markov chains

$$X_1 \quad X_2 \quad X_3 \quad X_4 \quad \cdots \quad X_n \quad X_{n+1} \quad \cdots$$

$X_n = x$

$\mathcal{P}_x$ set of transition probabilities

~~Markov assumption~~

Imprecise Markov assumption

Type 3 (repetition independence)

Set of all stochastic processes such that

$$P(X_{n+1}|X_{1:n} = x_{1:n}) = P(X_{n+1})|X_n = x_n)$$
$$= P(X_1|X_0 = x_n) \in \mathcal{P}_{x_n}$$

Pointwise ergodic theorem ?

$$X_1 \;\; X_2 \;\; X_3 \;\; X_4 \;\; \cdots \;\; X_n \;\; X_{n+1} \;\; \cdots$$

set of transition probabilities $\mathcal{P}_x$ for each x in $\mathcal{X}$

+ imprecise Markov assumption

$$\Rightarrow \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} f(X_i) = \boxed{E_\infty(f)} \; ?$$

almost surely

Imprecise Markov chains

$$X_1 \quad X_2 \quad X_3 \quad X_4 \quad \cdots \quad X_n = x \quad X_{n+1} \quad \cdots$$

$\mathcal{P}_x$ set of transition probabilities

~~Markov assumption~~

Imprecise Markov assumption

limit lower/upper expectation — ergodicity

$$\cancel{E_\infty(f)} \Rightarrow \underline{E}_\infty(f) := \lim_{n\to\infty} \underline{E}(f(X_n)|X_1 = \cancel{x})$$

$$-\underline{E}_\infty(-f) = \overline{E}_\infty(f) := \lim_{n\to\infty} \overline{E}(f(X_n)|X_1 = \cancel{x})$$

Imprecise pointwise ergodic theorem

$$X_1 \quad X_2 \quad X_3 \quad X_4 \quad \cdots \quad X_n \quad X_{n+1} \quad \cdots$$

set of transition probabilities $\mathcal{P}_x$ for each x in $\mathcal{X}$

+ imprecise Markov assumption + ergodicity

Type 1
Type 2

$$\underline{E}_\infty(f) \leq \liminf_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} f(X_i)$$

$$\leq \limsup_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} f(X_i) \leq \overline{E}_\infty(f)$$

almost surely **!**

(with **lower** probability 1)

De Cooman et al. (2016)

How to check ergodicity?

Hermans & De Cooman (2012)

Having a top class

+ Being top class regular

+ Being top class absorbing

Type 1
Type 2

ergodicity

limit lower expectation

$$\underline{E}_\infty(f) := \lim_{n \to \infty} \underline{E}(f(X_n)|X_1 = \cancel{x})$$

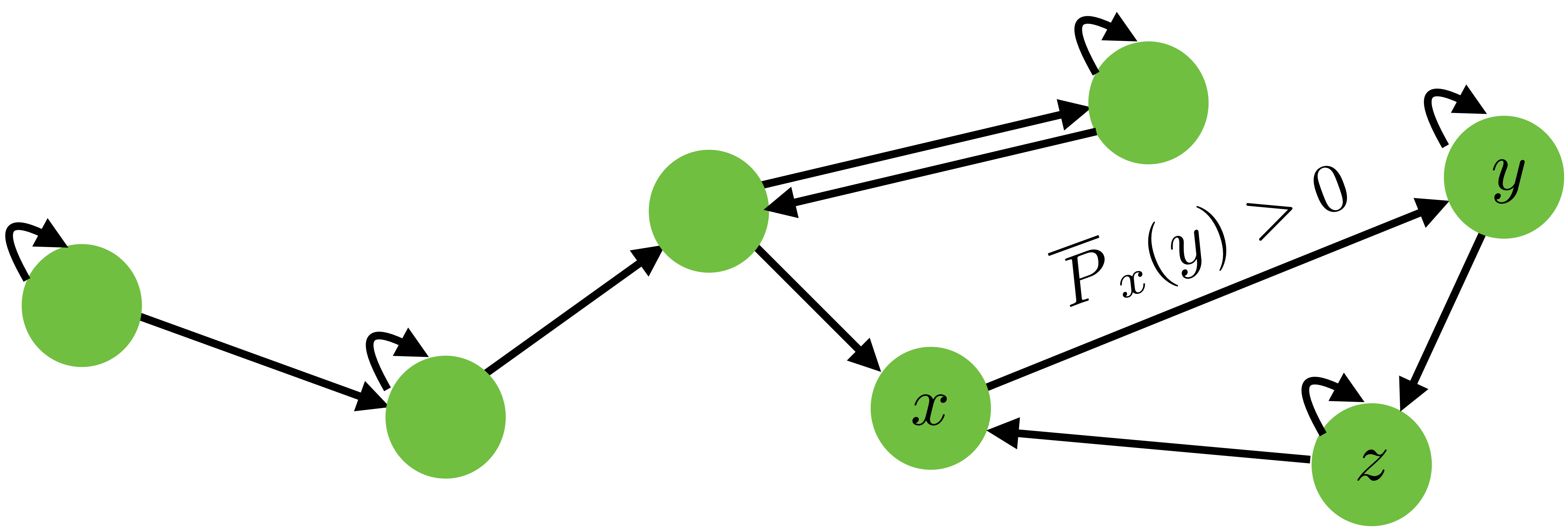
How to check ergodicity?

Accessibility relation

$$x \xrightarrow{1} y \Leftrightarrow \overline{P}_x(y) > 0$$

$$x \xrightarrow{n} y \Leftrightarrow x = z_1 \xrightarrow{1} z_2 \xrightarrow{1} z_3 \xrightarrow{1} \cdots \xrightarrow{1} z_{n-1} \xrightarrow{1} z_n = y$$

$$x \rightarrow y \Leftrightarrow (\exists n \in \mathbb{Z}_{\geq 0}) x \xrightarrow{n} y$$



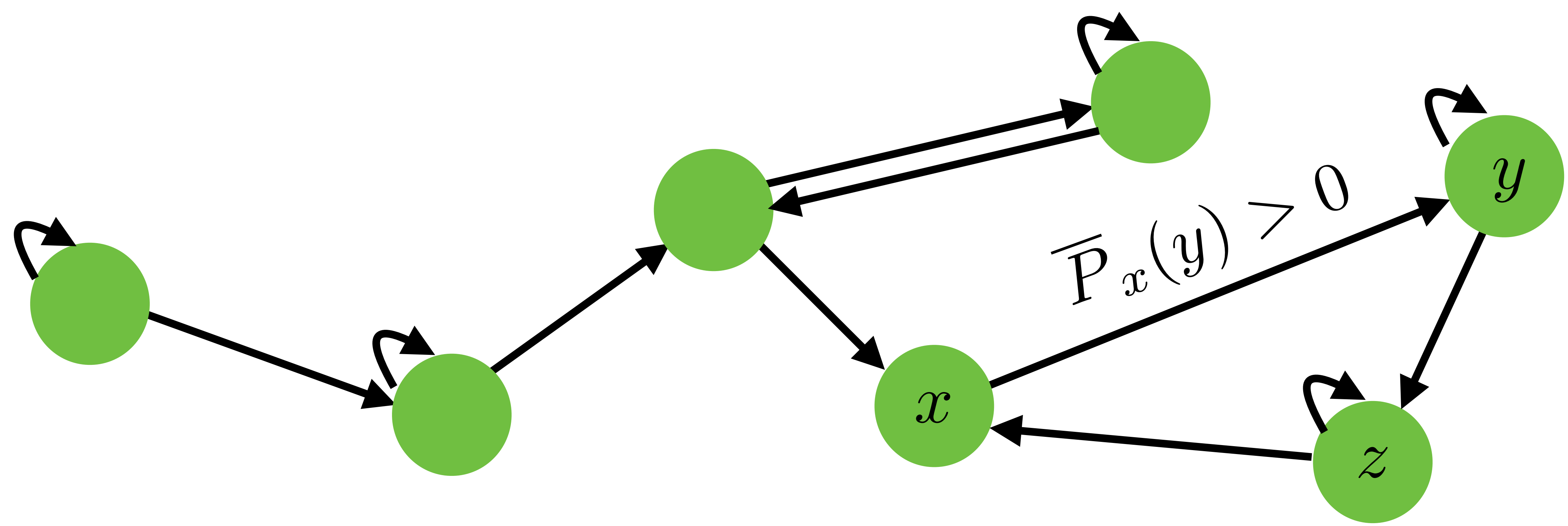
Having a top class

$$\mathcal{R} := \{y \in \mathcal{X} \colon (\forall x \in \mathcal{X}) x \to y\} \neq \emptyset$$

+

Being top class regular

$$(\exists n \in \mathbb{Z}_{\geq 0})(\forall k \geq n)(\forall x, y \in \mathcal{R}) x \xrightarrow{k} y$$



Having a top class

$$\mathcal{R} := \{y \in \mathcal{X} \colon (\forall x \in \mathcal{X}) x \to y\} \neq \emptyset$$

+

Being top class regular

$$(\exists n \in \mathbb{Z}_{\geq 0})(\forall k \geq n)(\forall x, y \in \mathcal{R}) x \xrightarrow{k} y$$

+

Being top class absorbing

$$R_0 := \mathcal{R}$$

$$R_{k+1} := R_k \cup \{x \in \mathcal{X} \setminus R_k \colon \underline{P}_x(R_k) > 0$$

$$\Rightarrow R_n = \mathcal{X} \text{ for } n = |\mathcal{R}^c|$$

How to check ergodicity?

ergodicity of $\mathcal{P}_\bullet$

Having a top class

\+ Being top class regular

\+ Being top class absorbing

Type 1
Type 2

ergodicity

limit lower expectation

$$\underline{E}_\infty(f) := \lim_{n\to\infty} \underline{E}(f(X_n)|X_1 = \cancel{x})$$

How to find the limit lower expectation?

Lower transition operator $\underline{T}$

$$\underline{T}g(x) := \inf_{P_x \in \mathcal{P}_x} \sum_{y \in \mathcal{X}} P_x(y) g(y)$$

$$\underline{E}_{\infty}^{\mathcal{P}_\bullet}(f) := \lim_{n \to \infty} \underline{T}^n f(x)$$

Type 1

$$||$$

Type 2

$$\underline{E}_{\infty}(f) := \lim_{n \to \infty} \underline{E}(f(X_n) | X_1 = x)$$

Imprecise pointwise ergodic theorem

$$X_1 \quad X_2 \quad X_3 \quad X_4 \quad \cdots \quad X_n \quad X_{n+1} \quad \cdots$$

set of transition probabilities $\mathcal{P}_x$ for each x in $\mathcal{X}$

+ imprecise Markov assumption + ergodicity of $\mathcal{P}_\bullet$

Type 1
Type 2
Type 3

$$\underline{E}_\infty^{\mathcal{P}_\bullet}(f) \leq \liminf_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} f(X_i)$$

$$\leq \limsup_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} f(X_i) \leq \overline{E}_\infty^{\mathcal{P}_\bullet}(f)$$

almost surely

(with lower probability 1)

Imprecise pointwise ergodic theorem

$$X_1 \;\; X_2 \;\; X_3 \;\; X_4 \;\; \cdots \;\; X_n \;\; X_{n+1} \;\; \cdots$$

set of transition probabilities $\mathcal{P}_x$ for each x in $\mathcal{X}$

+ imprecise Markov assumption + **weak** ergodicity

(of $\mathcal{P}_\bullet$)

Type 1

Type 2

Type 3

$$\underline{E}_{\mathrm{av},\infty}(f) \leq \liminf_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} f(X_i)$$

$$\leq \limsup_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} f(X_i) \leq \overline{E}_{\mathrm{av},\infty}(f)$$

almost surely

(with lower probability 1)

De Bock & T'Joens (2021)

Weak ergodicity

Having a top class

+ ~~Being top class regular~~

+ Being top class absorbing

weak ergodicity of $\mathcal{P}_\bullet$

Type 1

Type 2

Type 3

Less stringent than ergodicity!

weak ergodicity

limit lower expected average

$$\underline{E}_{\mathrm{av},\infty}(f) := \lim_{n\to\infty} \underline{E}\left({}^1/_n \sum_{i=1}^{n} f(X_i) \middle| X_1 = \cancel{x}\right)$$

Weak ergodicity

Having a top class

+ ~~Being top class regular~~

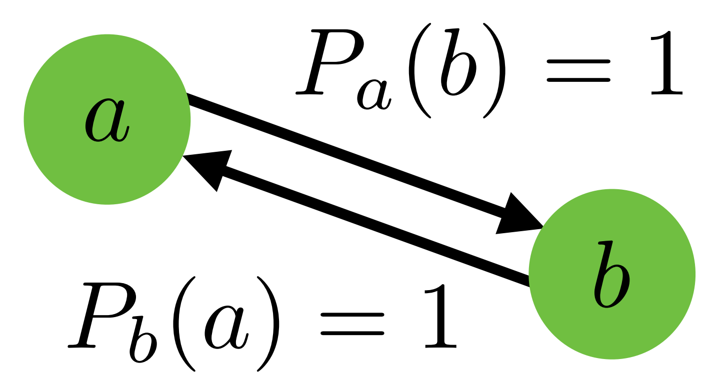
+ Being top class absorbing

weak ergodicity of $\mathcal{P}_\bullet$

Less stringent than ergodicity!

$$\mathcal{X} = \{a, b\}$$

$$\mathcal{P}_x = \{P_x\}$$



$$\underline{E}_{\mathrm{av},\infty}(f) = \overline{E}_{\mathrm{av},\infty}(f) = \frac{1}{2}(f(a) + f(b))$$

Limit lower expected average

Operator $\underline{T}_f$

$$\underline{T}_f g(x) := f + \underline{T} g$$

$$\underline{E}^{\mathcal{P}\bullet}_{\mathrm{av},\infty}(f) := \lim_{n\to\infty} {}^1/_n \underline{T}_f^n 0(x)$$

$$||$$

$$\underline{E}_{\mathrm{av},\infty}(f) := \lim_{n\to\infty} \underline{E}({}^1/_n \textstyle\sum_{i=1}^n f(X_i)|X_1 = x)$$

Type 1
Type 2
Type 3

Imprecise pointwise ergodic theorem

$$X_1 \quad X_2 \quad X_3 \quad X_4 \quad \cdots \quad X_n \quad X_{n+1} \quad \cdots$$

set of transition probabilities $\mathcal{P}_x$ for each x in $\mathcal{X}$

+ imprecise Markov assumption + **weak** ergodicity

(of $\mathcal{P}_\bullet$)

⇒

$$\underline{E}^{\mathcal{P}_\bullet}_{\mathrm{av},\infty}(f) \leq \liminf_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} f(X_i)$$

Type 1
Type 2
Type 3

$$\leq \limsup_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} f(X_i) \leq \overline{E}^{\mathcal{P}_\bullet}_{\mathrm{av},\infty}(f)$$

almost surely
(with lower probability 1)

De Bock & T'Joens (2021)

Advantages of the new version

1. ~~ergodicity~~ $\Rightarrow$ weak ergodicity

2. the new bounds have an interpretation as limits of lower/upper expectations, also for Type 3

3. the new bounds are tighter !

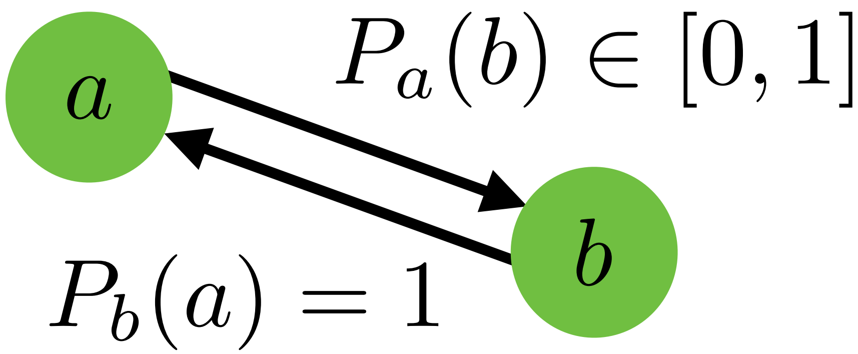
$$\underline{E}^{\mathcal{P}_\bullet}_{\infty}(f) \leq \underline{E}^{\mathcal{P}_\bullet}_{\mathrm{av},\infty}(f) \leq \overline{E}^{\mathcal{P}_\bullet}_{\mathrm{av},\infty}(f) \leq \overline{E}^{\mathcal{P}_\bullet}_{\infty}(f)$$

Advantages of the new version

$$\mathcal{X} = \{a, b\}$$

$$\mathcal{P}_a = \{P_a \colon P_a(b) = 1\}$$

$$\mathcal{P}_b = \{P_b \colon P_b(a) \in [0,1]\}$$



$$\underline{E}^{\mathcal{P}_\bullet}_{\infty}(f) = \min f \qquad \underline{E}^{\mathcal{P}_\bullet}_{\mathrm{av},\infty}(f) = \min\left\{ f(a), \frac{f(a)+f(b)}{2} \right\}$$

$$f = \mathbb{I}_a \Rightarrow \underline{E}^{\mathcal{P}_\bullet}_{\infty}(f) = 0 < \underline{E}^{\mathcal{P}_\bullet}_{\mathrm{av},\infty}(f) = \frac{1}{2}$$

3 the new bounds are tighter **!**

$$\underline{E}^{\mathcal{P}_\bullet}_{\infty}(f) \leq \underline{E}^{\mathcal{P}_\bullet}_{\mathrm{av},\infty}(f) \leq \overline{E}^{\mathcal{P}_\bullet}_{\mathrm{av},\infty}(f) \leq \overline{E}^{\mathcal{P}_\bullet}_{\infty}(f)$$

Imprecise continuous time Markov chains

please…
not all over
again

Imprecise continuous time Markov chains

x

$X_0 \quad \cdots \quad X_t \quad X_{t+\Delta} \quad \cdots$

$\mathcal{Q}_x$ set of transition rates

Type 1 (epistemic irrelevance)
All compatible processes

Type 2 (complete independence)
All compatible Markov chains

Type 3 (repetition independence)
All compatible time-homogenous Markov chains

Ergodicity versus weak ergodicity

$$\underline{E}_\infty(f) := \lim_{t\to\infty} \underline{E}(f(X_t)|X_0 = \cancel{x})$$

ergodicity

limit lower expectation

De Bock (2016) $\Updownarrow$ **Type 1 & 2**

(weak) ergodicity of $\mathcal{Q}_\bullet$

Having a top class

\+ Being top class absorbing

pretty sure $\Updownarrow$ **Type 1, 2 & 3**

limit lower expected average

weak ergodicity

$$\underline{E}_{\mathrm{av},\infty}(f) := \lim_{t\to\infty} \underline{E}(1/t \int_0^t f(X_s)\mathrm{d}s|X_0 = \cancel{x})$$

Computing limit lower expectations

$$\underline{E}_\infty(f) := \lim_{t\to\infty} \underline{E}(f(X_t)|X_0 = x)$$

$$\|$$

$$\underline{E}^{\underline{Q}_\bullet}_\infty(f) = \lim_{t\to+\infty} e^{t\underline{Q}}(f)$$

Type 1 & 2

$$\underline{Q}_f g := f + \underline{Q}g$$

$$\underline{E}^{\underline{Q}_\bullet}_{\mathrm{av},\infty}(f) = \lim_{t\to+\infty} {}^1\!/_t e^{t\underline{Q}_f}(0)$$

Type 1, 2 & 3 pretty sure

$$\|$$

$$\underline{E}_{\mathrm{av},\infty}(f) := \lim_{t\to\infty} \underline{E}({}^1\!/_t \int_0^t f(X_s)\mathrm{d}s|X_0 = x)$$

Imprecise pointwise ergodic theorem

$$X_0 \qquad \cdots \qquad X_t \quad X_{t+\Delta} \cdots$$

set of transition rates $\mathcal{Q}_x$ for each x in $\mathcal{X}$

+ imprecise Markov assumption + ergodicity of $\mathcal{Q}_\bullet$

Type 1
Type 2
Type 3

$$\underline{E}_\infty^{\mathcal{Q}_\bullet}(f) \leq \liminf_{t\to\infty} \frac{1}{t}\int_0^t f(X_s)\mathrm{d}s$$
$$\leq \limsup_{t\to\infty} \frac{1}{t}\int_0^t f(X_s)\mathrm{d}s \leq \overline{E}_\infty^{\mathcal{Q}_\bullet}(f)$$

almost surely

(with lower probability 1)

Imprecise pointwise ergodic theorem

$$X_0 \quad \cdots \quad X_t \quad X_{t+\Delta} \cdots$$

set of transition rates $\mathcal{Q}_x$ for each x in $\mathcal{X}$

+ imprecise Markov assumption + weak ergodicity

(of $\mathcal{Q}_\bullet$)

Type 1
Type 2
Type 3

$$\underline{E}^{\mathcal{Q}_\bullet}_{\mathrm{av},\infty}(f) \leq \liminf_{t\to\infty} \frac{1}{t}\int_0^t f(X_s)\mathrm{d}s$$

$$\leq \limsup_{t\to\infty} \frac{1}{t}\int_0^t f(X_s)\mathrm{d}s \leq \overline{E}^{\mathcal{Q}_\bullet}_{\mathrm{av},\infty}(f)$$

almost surely

(with lower probability 1)


This is great, tell me more!

I didn't tell you…

…that this leads to a recursive method for solving particular kinds of nonlinear optimisation problems

…what happens if we allow dependence on the initial state

… that we're using a whole bunch of supermartingales under the hood

This is great,
tell me more!


This sounds good, but I work in Robust Finance. How is this useful to me?

Discussion

Our models are similar,
so how about our results?

Do you study (weak)
ergodicity in robust finance?

Are pointwise ergodic theorems
of interest in your community?

This sounds good, but I work in Robust Finance. How is this useful to me?

I couldn’t care less about imprecise Markov chains, let alone their limit behaviour…

We also work on…

More general imprecise stochastic processes

Imprecise randomness

Credal networks

Dealing with the curse of dimensionality in precise Markov chains

Choice functions

Quantum mechanics and IP

I couldn't care less about imprecise Markov chains, let alone their limit behaviour…

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