

Sum-Product Laws and Efficient Algorithms for Imprecise Markov Chains

Jasper De Bock, Alexander Erreygers, Thomas Krak

UAI 2021, July 27- 30

Online





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Foundations Lab
for imprecise probabilities





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Uncertainty in
Artificial Intelligence
Research Group

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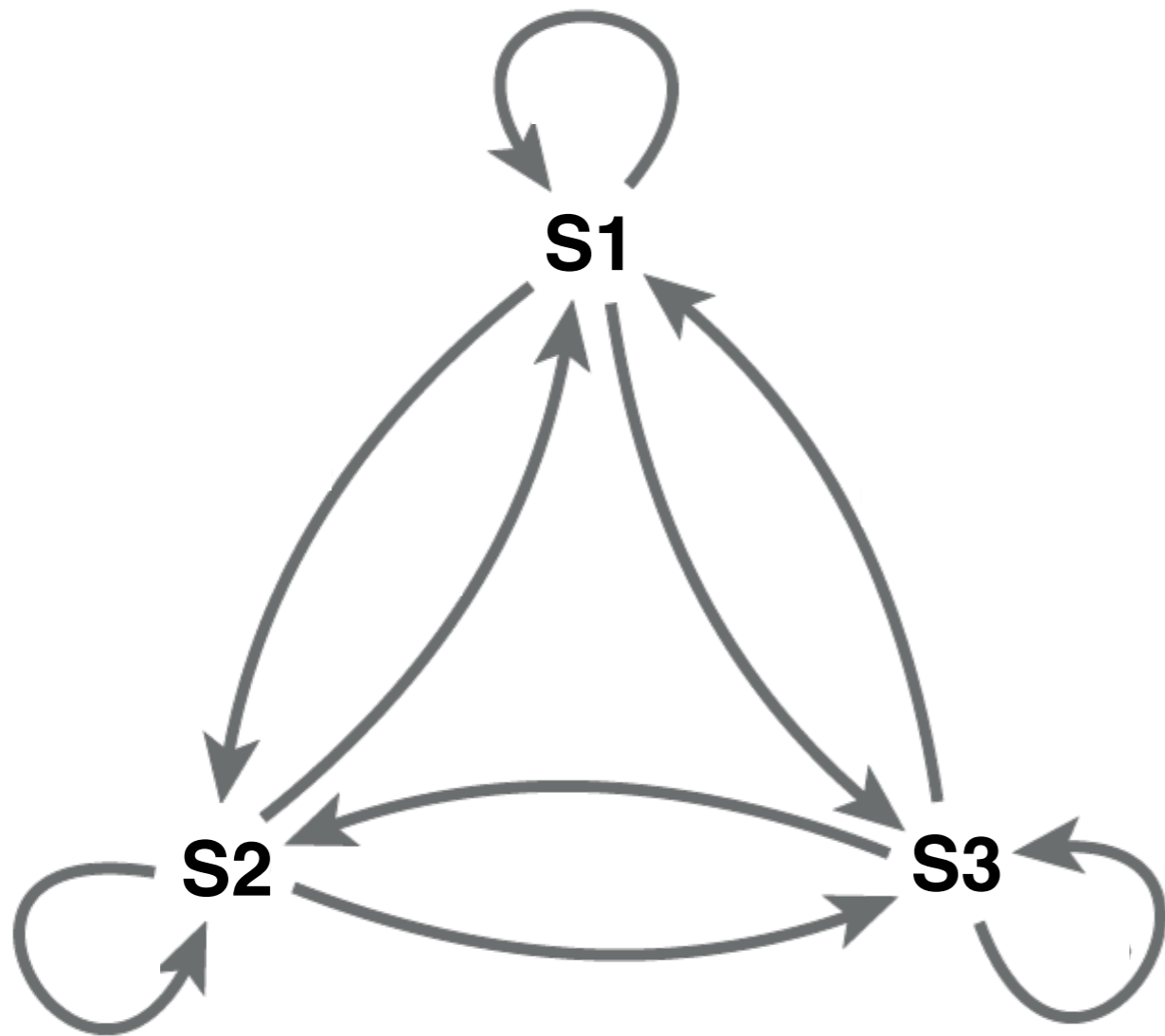
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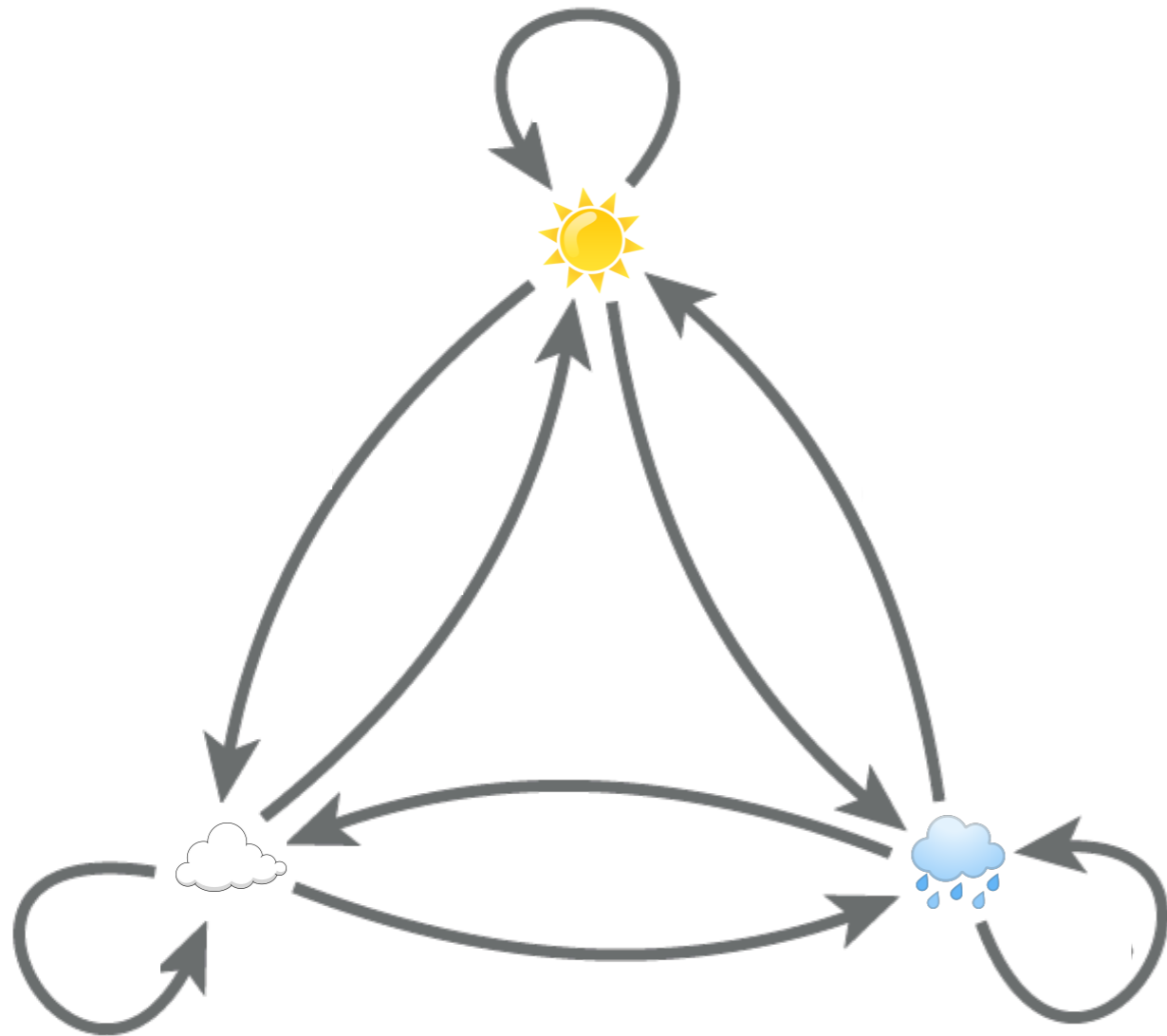
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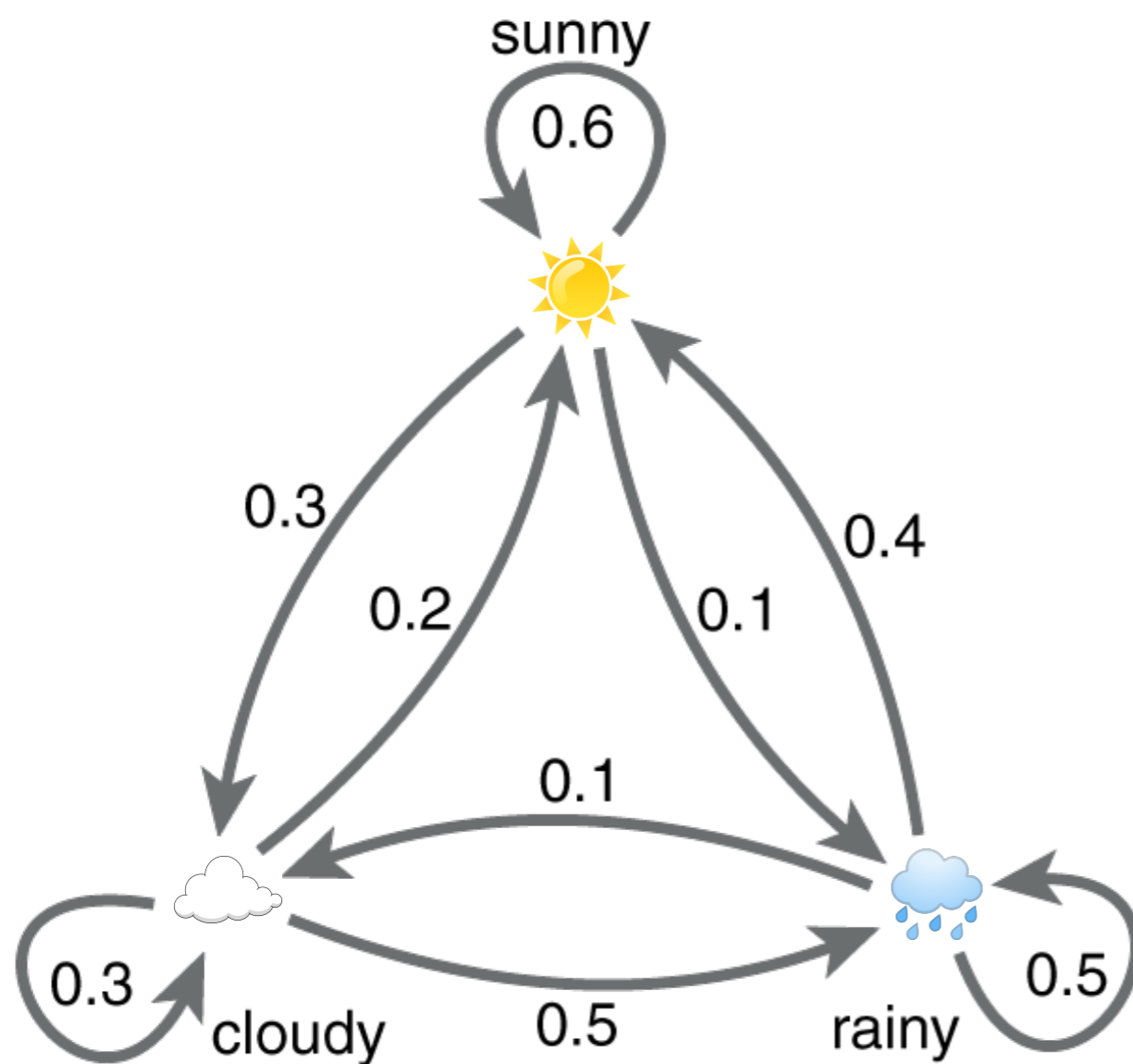
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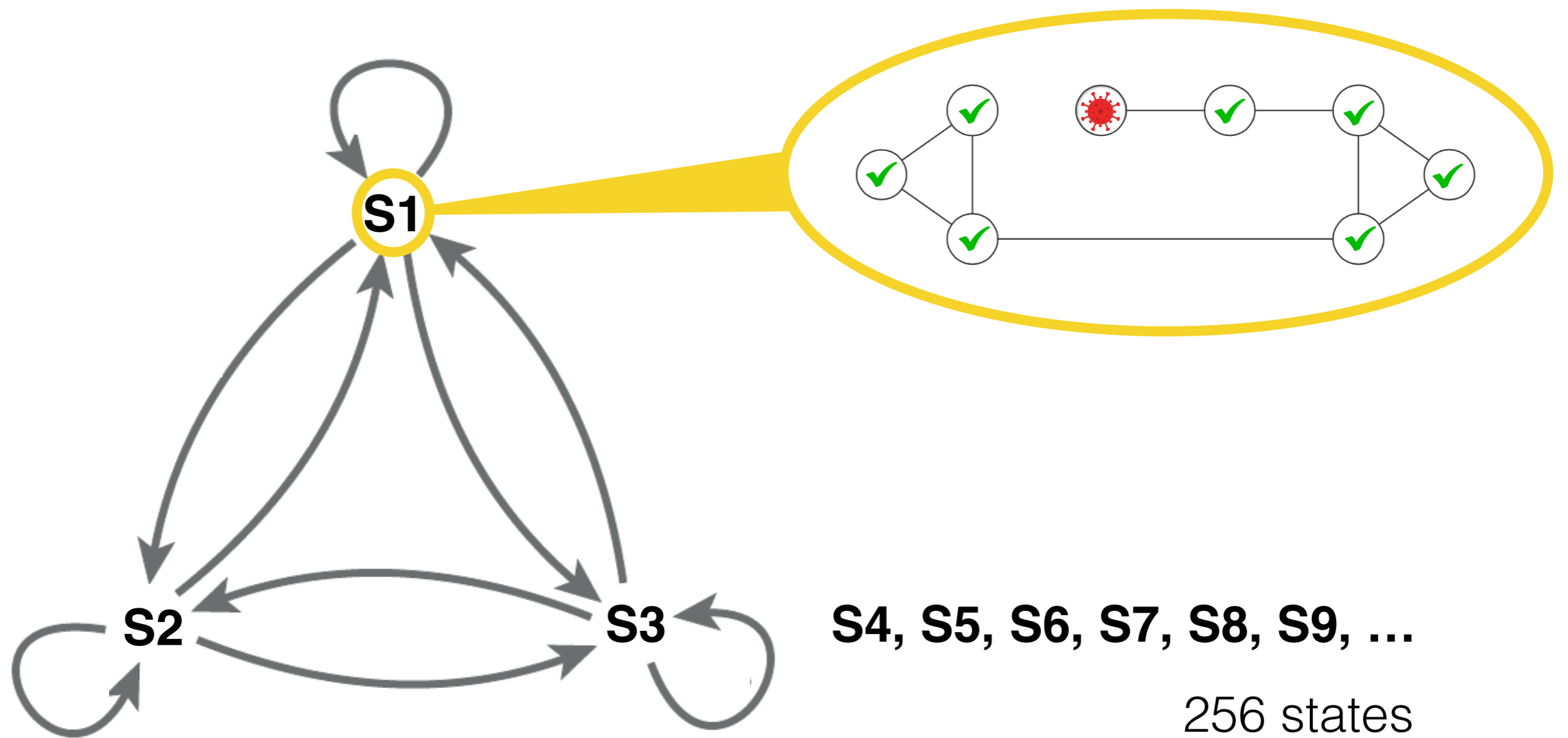
Sum-Product Laws and Efficient Algorithms for Imprecise Markov Chains



transition matrix

$$T = \begin{matrix} & \text{sunny} & \text{cloudy} & \text{rainy} \\ \text{sunny} & \begin{matrix} \text{sunny} \\ 0.6 \end{matrix} & \begin{matrix} \text{cloudy} \\ 0.3 \end{matrix} & \begin{matrix} \text{rainy} \\ 0.1 \end{matrix} \\ \text{cloudy} & \begin{matrix} \text{sunny} \\ 0.2 \end{matrix} & \begin{matrix} \text{cloudy} \\ 0.3 \end{matrix} & \begin{matrix} \text{rainy} \\ 0.5 \end{matrix} \\ \text{rainy} & \begin{matrix} \text{sunny} \\ 0.4 \end{matrix} & \begin{matrix} \text{cloudy} \\ 0.1 \end{matrix} & \begin{matrix} \text{rainy} \\ 0.5 \end{matrix} \end{matrix}$$

Sum-Product Laws and Efficient Algorithms for Imprecise Markov Chains



Sum-Product Laws and Efficient Algorithms for Imprecise Markov Chains

✓ **Reliability engineering** (failure probabilities, ...)

✓ **Queuing theory** (waiting in line ...)

- dimensioning of call centers

- router queues on the internet



✓ **Chemical reactions** (time-evolution ...)



✓ **Epidemiology** (time until threshold...)

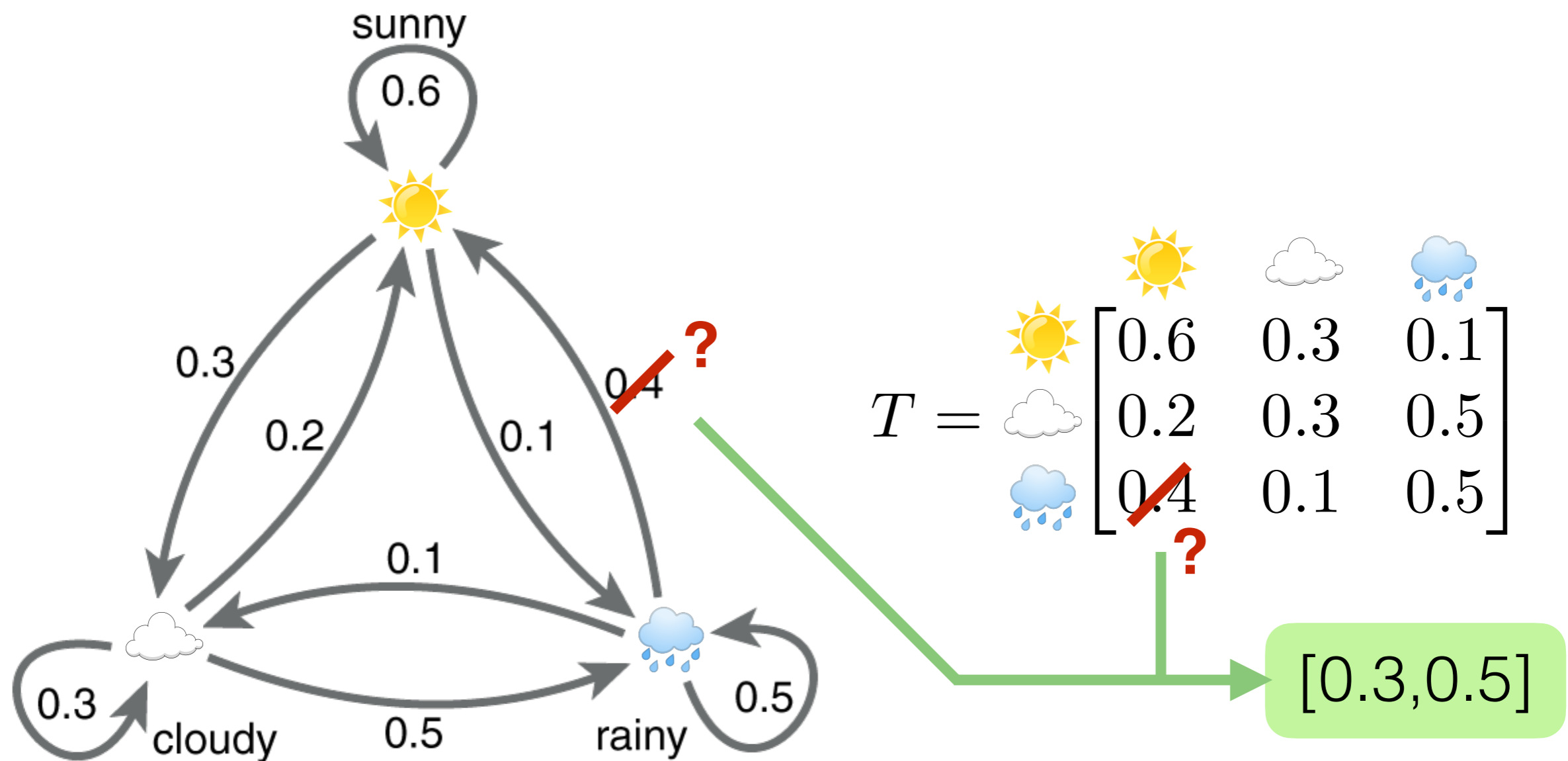


✓ **Pagerank**

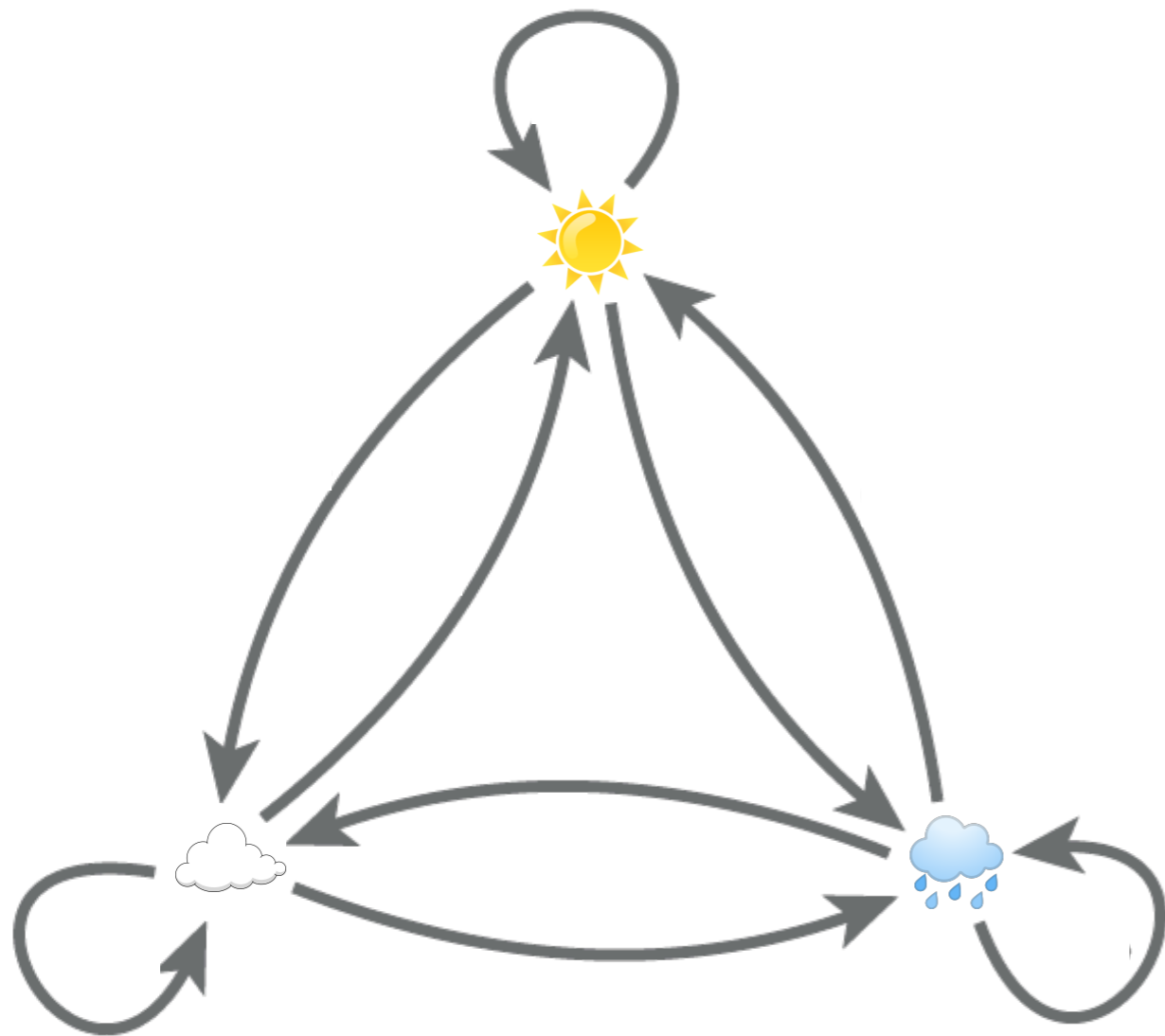


✓ ...

Sum-Product Laws and Efficient Algorithms for Imprecise Markov Chains



Sum-Product Laws and Efficient Algorithms for Imprecise Markov Chains



set of
transition matrices

$$\mathcal{T} \in T = \begin{matrix} \text{Sun} \\ \text{Cloud} \\ \text{Rain} \end{matrix} \begin{bmatrix} \text{Sun} & \text{Cloud} & \text{Rain} \\ & ? & \\ & & \end{bmatrix}$$

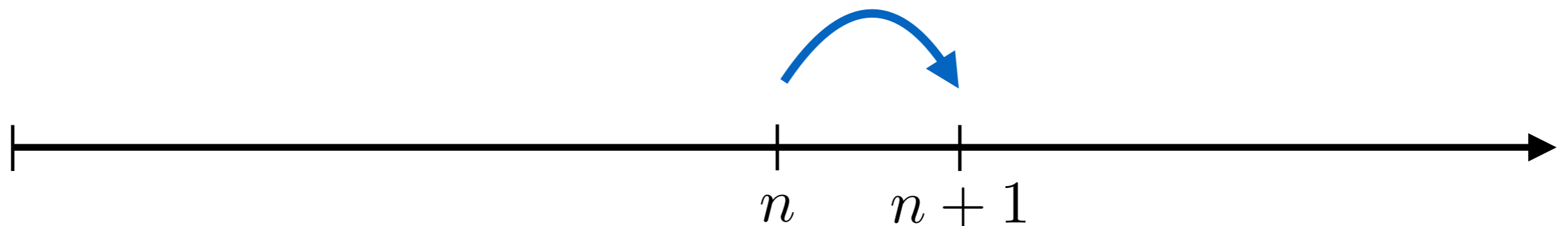
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TYPE 1

Imprecise Markov chain under complete independence

set of transition matrices

$\mathcal{T}_n \leftarrow$
 \cup
 $\mathcal{T}_n \leftarrow$



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TYPE 2

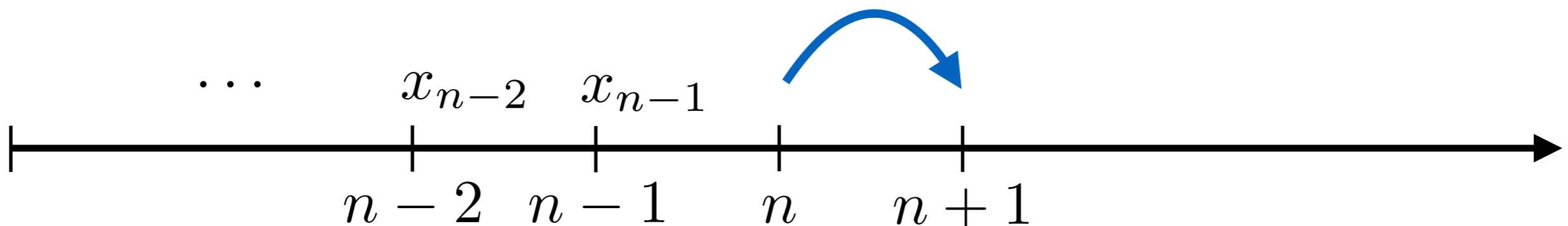
Imprecise Markov chain under epistemic irrelevance

set of transition matrices

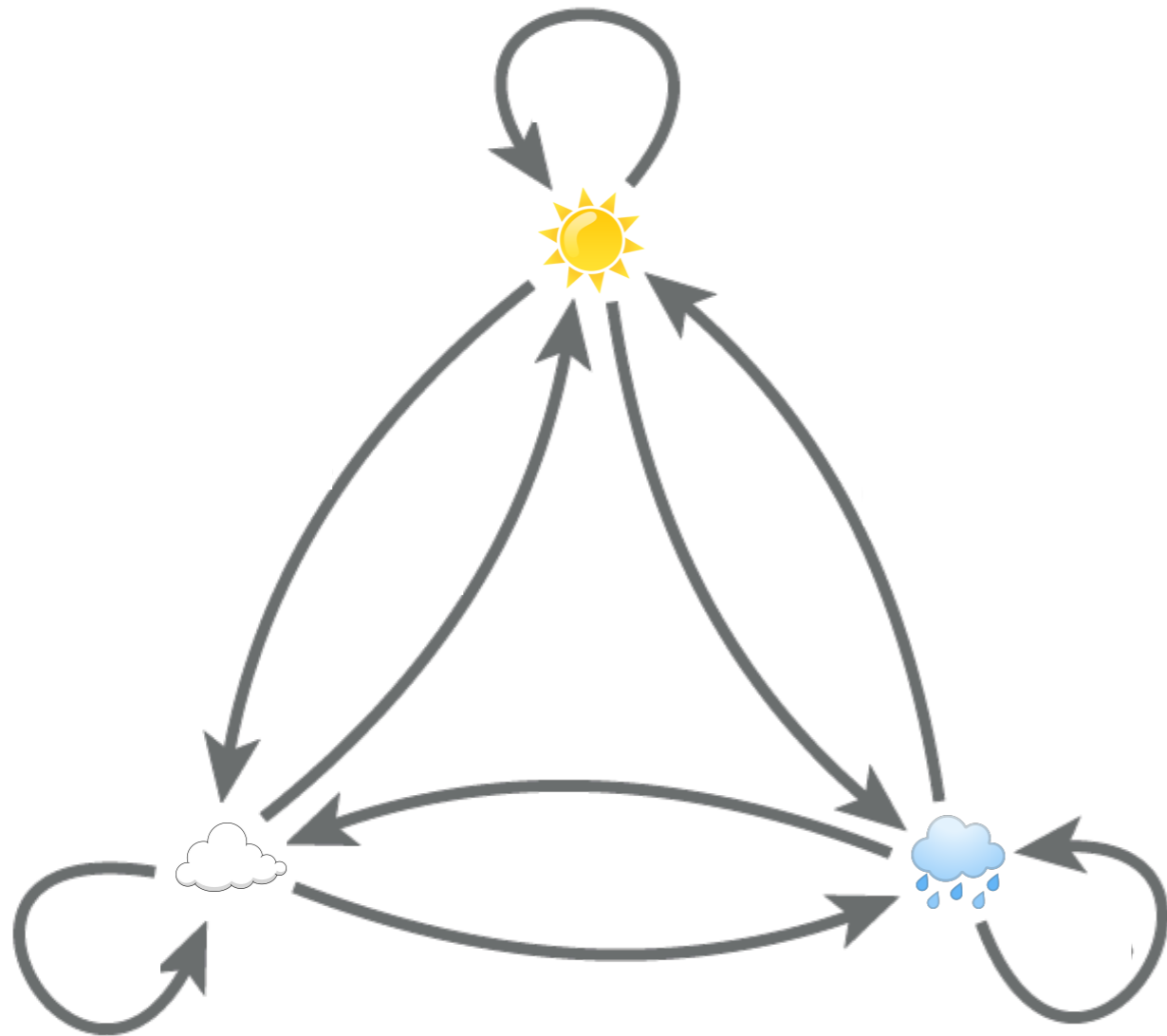
\mathcal{T}_n

\cup

$T_{n, x_{n-1}, x_{n-2}, \dots}$



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set of
transition matrices

$$\mathcal{T} \in T = \begin{matrix} \text{Sun} & \text{Cloud} & \text{Rain} \\ \begin{matrix} \text{Sun} \\ \text{Cloud} \\ \text{Rain} \end{matrix} & \begin{bmatrix} \text{Sun} & \text{Cloud} & \text{Rain} \\ \text{?} & & \\ & & \end{bmatrix} \end{matrix}$$

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probabilities and expectations at a single time point

the probability of reaching a set of goal states while only visiting safe states

expected transition count between given states

expected time until a given event occurs

expected time averages and frequencies of events

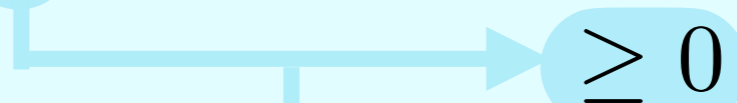
X_n

n



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inferences in sum-product form

$$f(X_{1:n}) = \sum_{k=1}^n g_k(X_k) \prod_{\ell=1}^{k-1} h_{\ell}(X_{\ell})$$

$$f(X_{1:n}) = \sum_{k=2}^n g_k(X_{k-1}, X_k) \prod_{\ell=1}^{k-1} h_{\ell}(X_{\ell})$$

Sum-Product Laws and Efficient Algorithms for Imprecise Markov Chains

inferences in sum-product form

sum-product laws for imprecise Markov chains

Sum-Product Laws and Efficient Algorithms for Imprecise Markov Chains

inferences in sum-product form



+



efficient algorithms for computing tight bounds on inferences

sum-product laws for imprecise Markov chains

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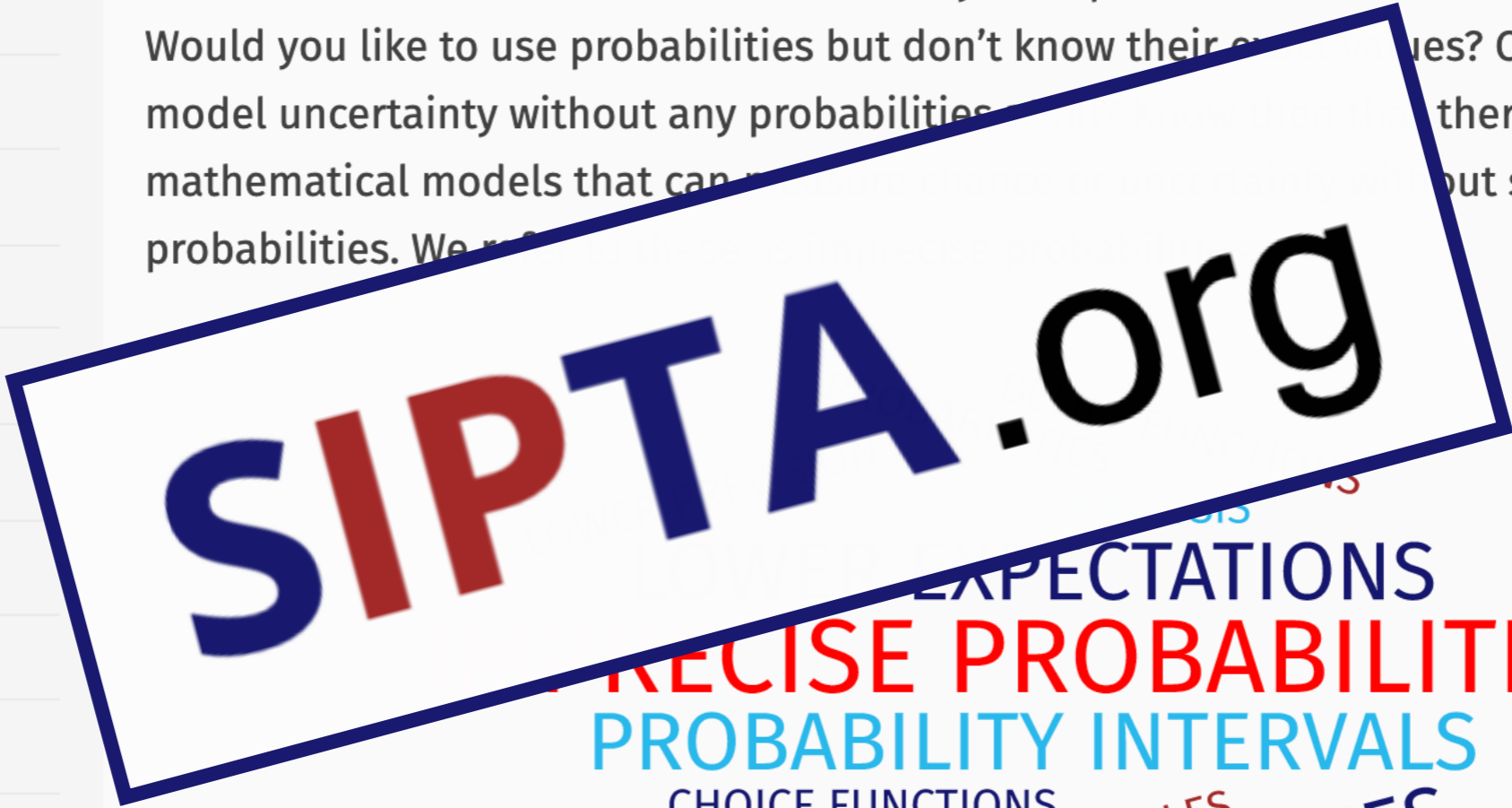
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Welcome

We are SIPTA, the *Society for Imprecise Probabilities: Theories and Applications*, and we are convinced that there is more to uncertainty than probabilities. There is much more, in fact. Would you like to use probabilities but don't know their exact values? Or would you like to model uncertainty without any probabilities? There are numerous mathematical models that can do this without sharp numerical probabilities. We...



EXPECTATIONS
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PROBABILITY INTERVALS
CHOICE FUNCTIONS
P-BOXES
CAPACITIES
SETS OF DESIRABLE GAMBLERS
SETS OF PROBABILITIES
PREFERENCE ORDERS
ROBUSTNESS

