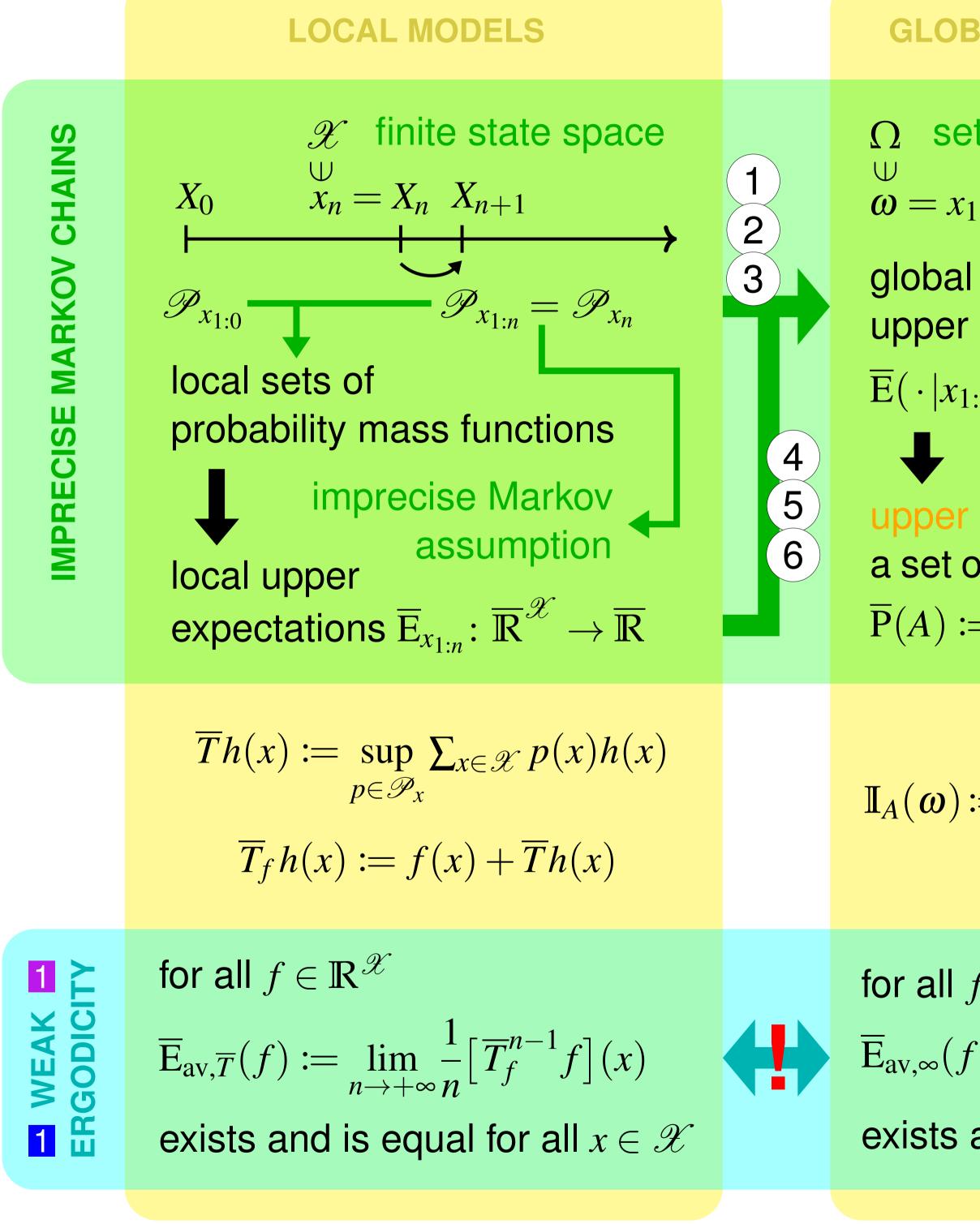
Average Behaviour of Imprecise Markov Chains: A Single Pointwise Ergodic Theorem Measure-theoretic models for Six Different Models 123456 23



Jasper.De Bock Natan.T'Joens Want more? 1 UGent.be

GLOBAL MODELS

 Ω set of paths

 $\omega = x_1 x_2 x_3 \dots x_n \dots$

upper expectations

 $\overline{\mathrm{E}}(\,\cdot\,|x_{1:n})\colon \overline{\mathbb{R}}^{\Omega}\to \overline{\mathbb{R}}$

upper probability of a set of paths $A \subseteq \Omega$ $\overline{\mathbf{P}}(A) \coloneqq \overline{\mathbf{E}}(\mathbb{I}_A)$

$$\boldsymbol{\omega}) \coloneqq \begin{cases} 1 \text{ if } \boldsymbol{\omega} \in A \\ 0 \text{ if } \boldsymbol{\omega} \notin A \end{cases}$$

for all $f \in \mathbb{R}^{\mathscr{X}}$ $\overline{\mathrm{E}}_{\mathrm{av},\infty}(f) \coloneqq \lim_{n \to +\infty} \overline{\mathrm{E}}\left(\frac{1}{n}\sum_{i=1}^{n} f(X_i) \, \big| \, x_1\right)$

exists and is equal for all $x_1 \in \mathscr{X}$

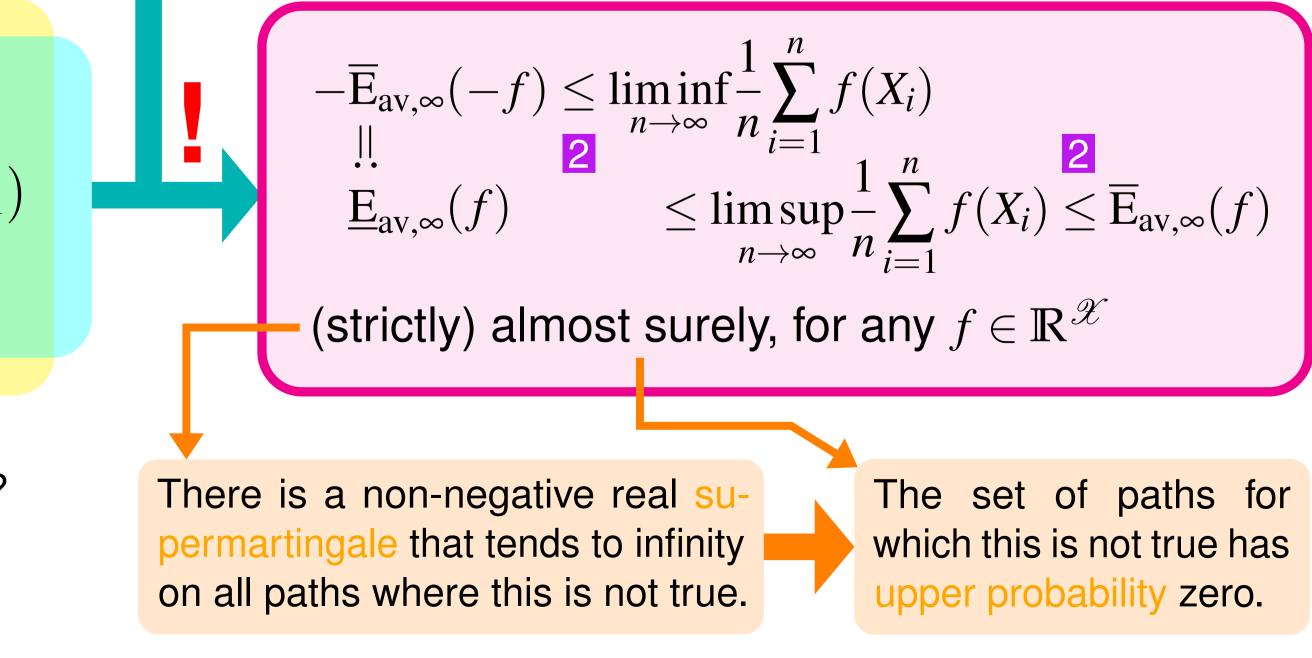
The local models $\mathscr{P}_{x_{1,n}}$ are used to define a set \mathbb{P} of compatible global probability measures. $\overline{\mathrm{E}}(h|x_{1:n})$ is the supremum over the expectations $E_P(h|x_{1:n})$ of the measures P in \mathbb{P} .

A probability measure *P* is compatible with $\mathscr{P}_{x_{1:n}}$ if $P(X_{n+1}|x_{1:n}) \in \mathscr{P}_{x_{1:n}}$

- the set of all com-İS patible probability measures that are time-homogeneous Markov chains
- \mathbb{P} is the set of all compatible 2 probability measures that are Markov chains
- \mathbb{P} is the set of all compatible 3 probability measures

3

 $\overline{\mathrm{E}}_{\mathrm{av},\overline{T}} = \overline{\mathrm{E}}_{\mathrm{av},\infty}$



- How do you check this in practice? 2 How different are they?
- 3 Don't you need measurability?

Game-theoretic models

The local models \overline{E}_{x_1, y_1} are used to define a set \mathbb{M} of compatible gambling strategies, called supermartingales. $\overline{\mathrm{E}}(h|x_{1:n})$ is the infimum capital needed at $x_{1:n}$ so as to hedge h with these gambling strategies.

A supermartingale *M* maps each state sequence $x_{1:n}$ to a capital $\mathcal{M}(x_{1:n})$, in such a way that the capital is expected to decrease: $\overline{\mathrm{E}}_{x_{1:n}}(\mathscr{M}(x_{1:n}X_{n+1})) \leq \mathscr{M}(x_{1:n})$



IM is the set of all extended real supermartingales that are bounded below (no unbounded borrowing)



IM is the set of all real supermartingales that are bounded below (no unbounded borrowing nor infinite spending)



IM is the set of all real supermartingales that are bounded (no unbounded borrowing or spending)

Differences with De Cooman et al. (2016):

They require ergodicity, which applies less often Our bounds are tighter They consider only one model (4)