Interpreting, axiomatising and representing

coherent choice functions

in terms of desirability

Jasper De Bock & Gert de Cooman

ISIPTA 2019

Interpreting, axiomatising and representing

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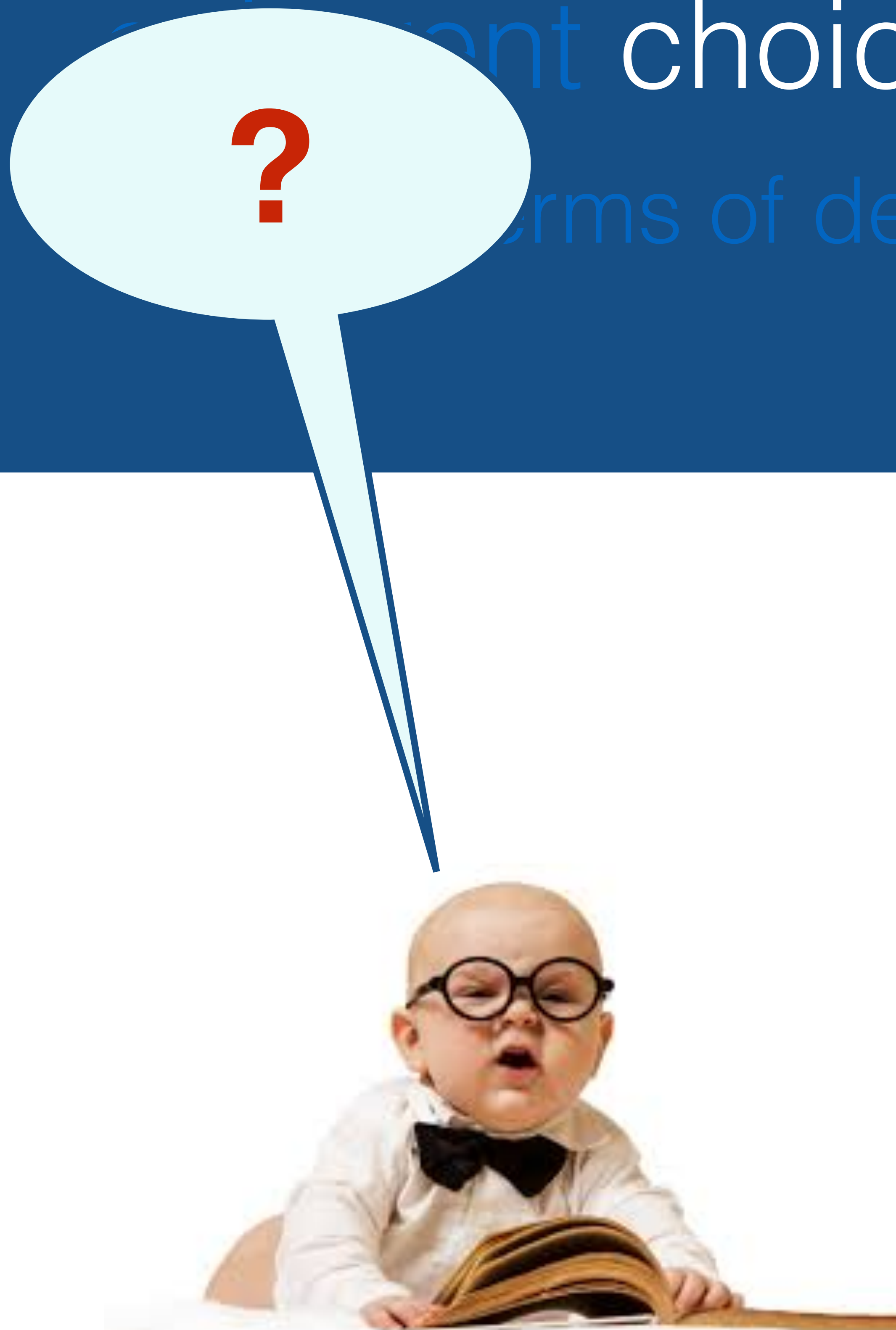

Interpreting, axiomatising and representing

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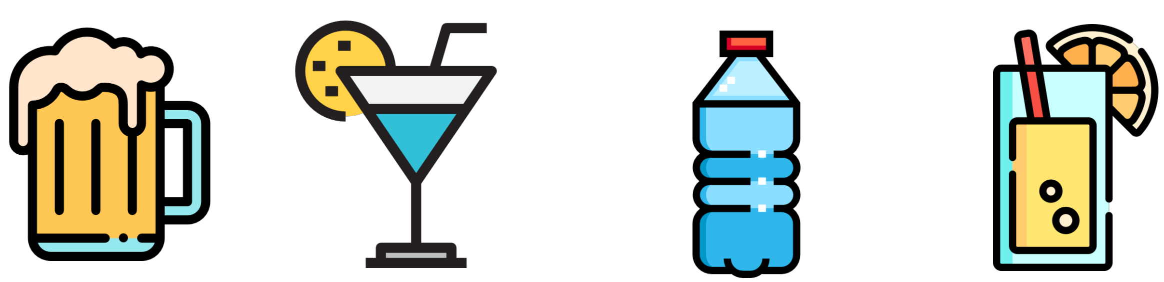
$$C(1\ 2\ 3\ 4) = ?$$

Interpreting, axiomatising and representing coherent choice functions in terms of desirability

$C($  $) = $?

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$C\big($  $\big) =$ 

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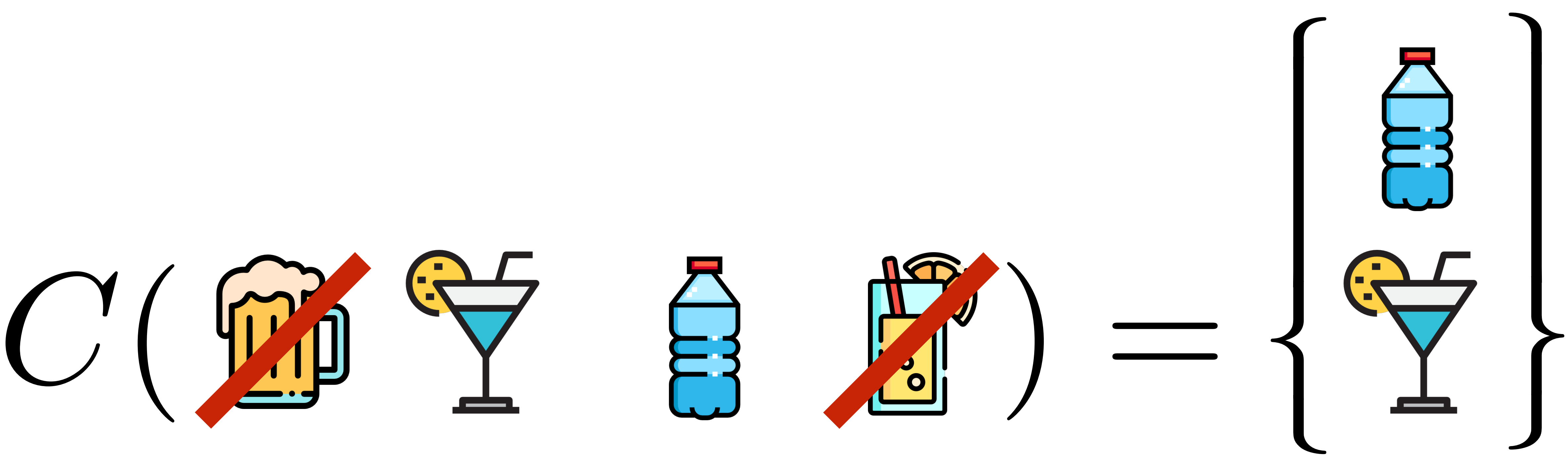

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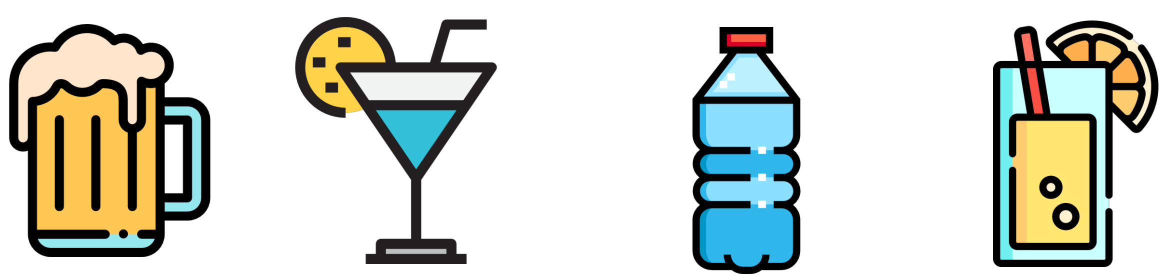
$$C\left(\; v_1 \quad v_2 \quad v_3 \quad v_4 \; \right) = \text{?}$$



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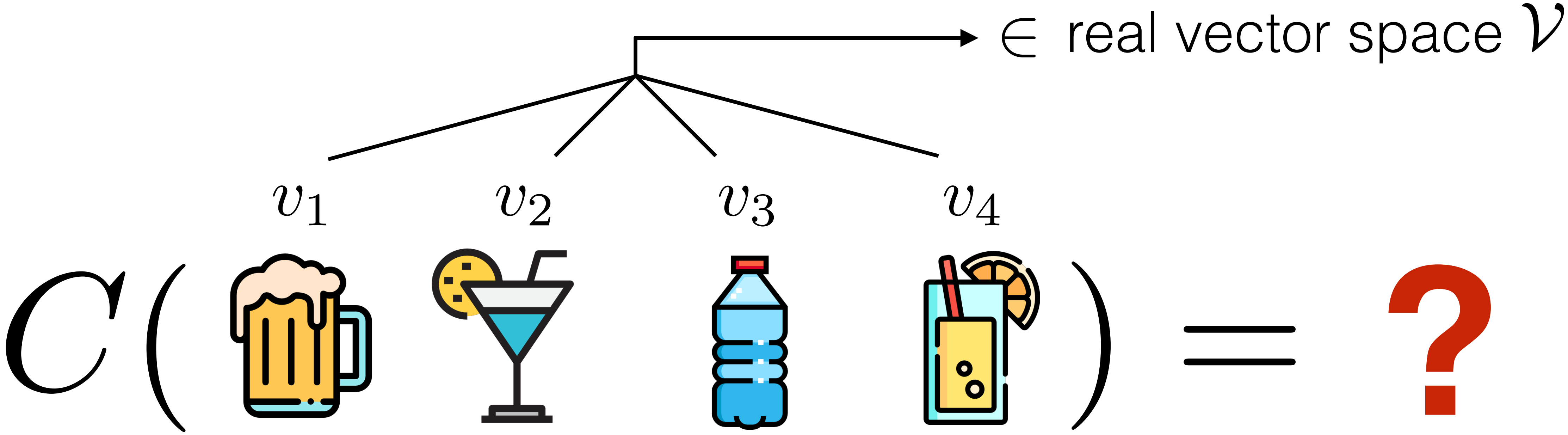

Interpreting, axiomatising and representing

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in terms of desirability

elementary concept

desirable option v

$\in \mathcal{V}$

strictly preferred over 0

Interpreting, axiomatising and representing

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elementary concept

desirable option v

$\in \mathcal{V}$

strictly preferred

over 0

derived notion 1

set of desirable options D

$\subseteq \mathcal{V}$

coherence axioms!

ELSEVIER International Journal of Approximate Reasoning 24 (2000) 125–148
INTERNATIONAL JOURNAL OF APPROXIMATE REASONING
www.elsevier.com/locate/ijar

Towards a unified theory of imprecise probability

Peter Walley

Received 1 September 1999; accepted 1 December 1999

Abstract

Coherent upper and lower probabilities, Choquet capacities of order 2, belief functions and possibility measures are amongst the most popular mathematical models for uncertainty and partial ignorance. Examples are given to show that these models are not sufficiently general to represent some common types of uncertainty. In particular, they are not sufficiently informative about expectations and conditional probabilities. Coherent lower previsions and sets of probability measures are considerably more general, but they may not be sufficiently informative for some purposes. Two other models for uncertainty, which involve partial preference orderings and sets of desirable gambles, are discussed. These are more informative and more general than the previous models, and they may provide a suitable mathematical foundation for a unified theory of imprecise probability. © 2000 Elsevier Science Inc. All rights reserved.

Keywords: Choquet capacity; Coherence; Comparative probability; Credal sets; Desirable gambles; Foundations of probability; Interval-valued probability; Lower prevision; Lower probability; Partial preference ordering; Uncertainty measures

1. Introduction

Can there be a unified theory of imprecise probability? At present there are numerous mathematical models, interpretations and applications of imprecise probabilities. The various articles in this volume and in [3,4] give some idea of

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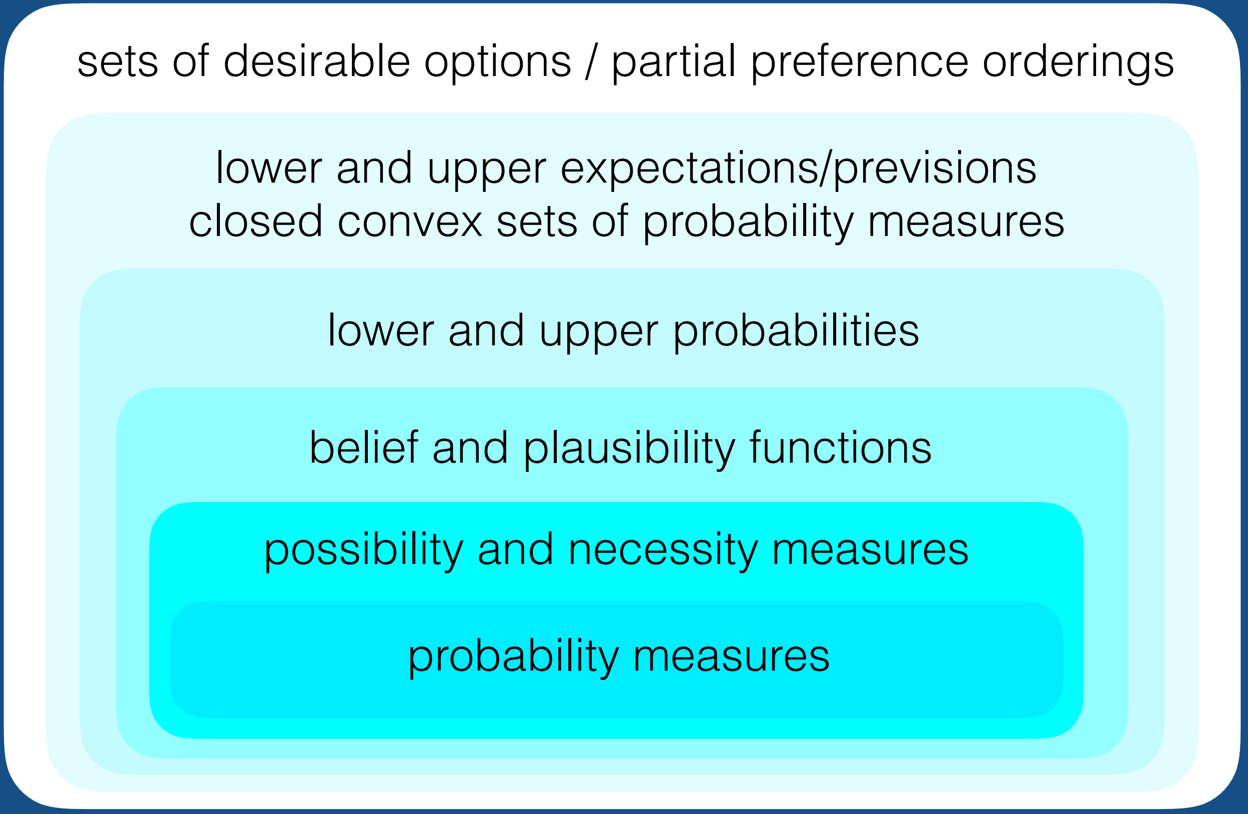
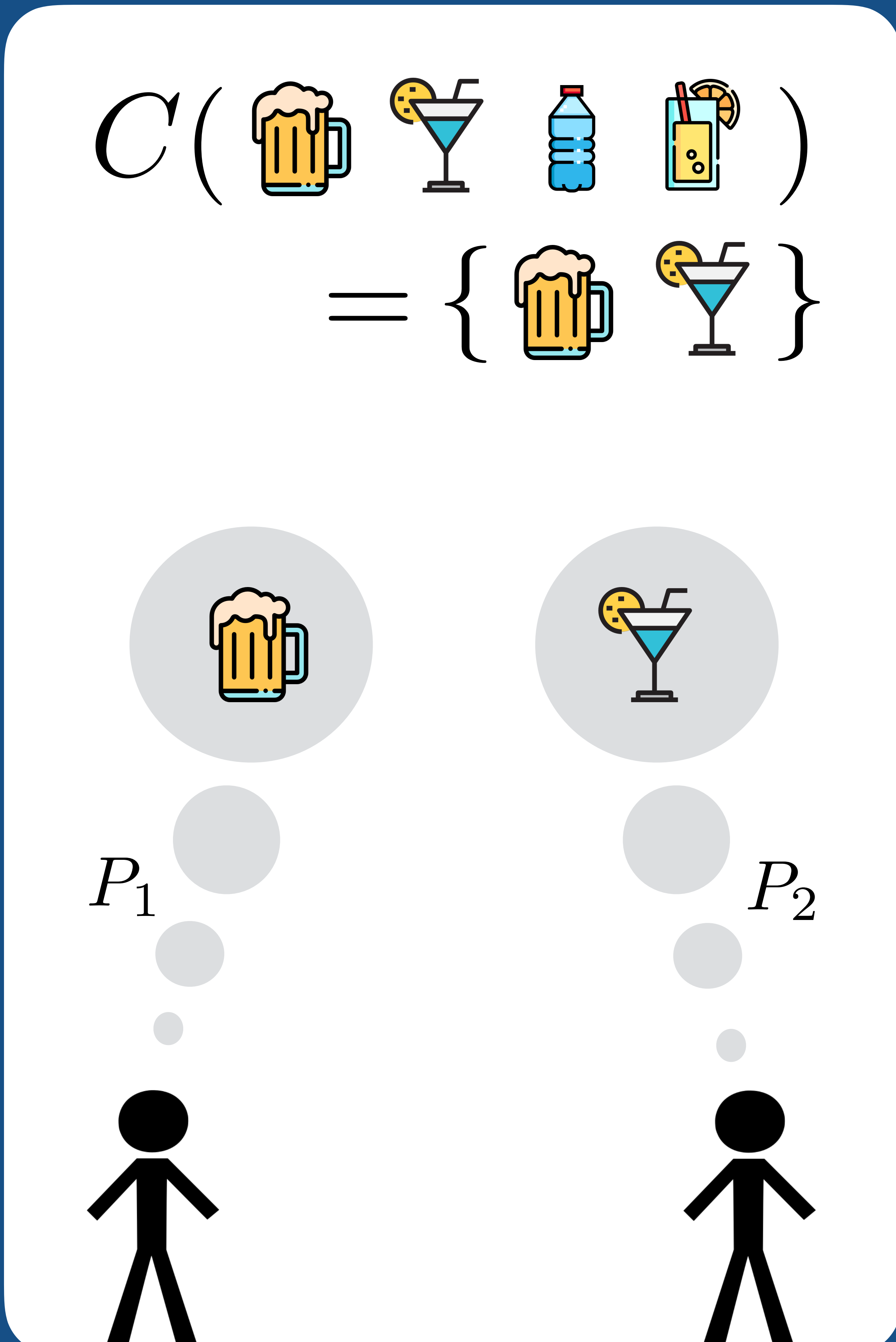


derived notion 1

set of desirable options D

$\subseteq \mathcal{V}$

coherence axioms!



sets of desirable options / partial preference orderings

lower and upper expectations/previsions
closed convex sets of probability measures

lower and upper probabilities

belief and plausibility functions

possibility and necessity measures

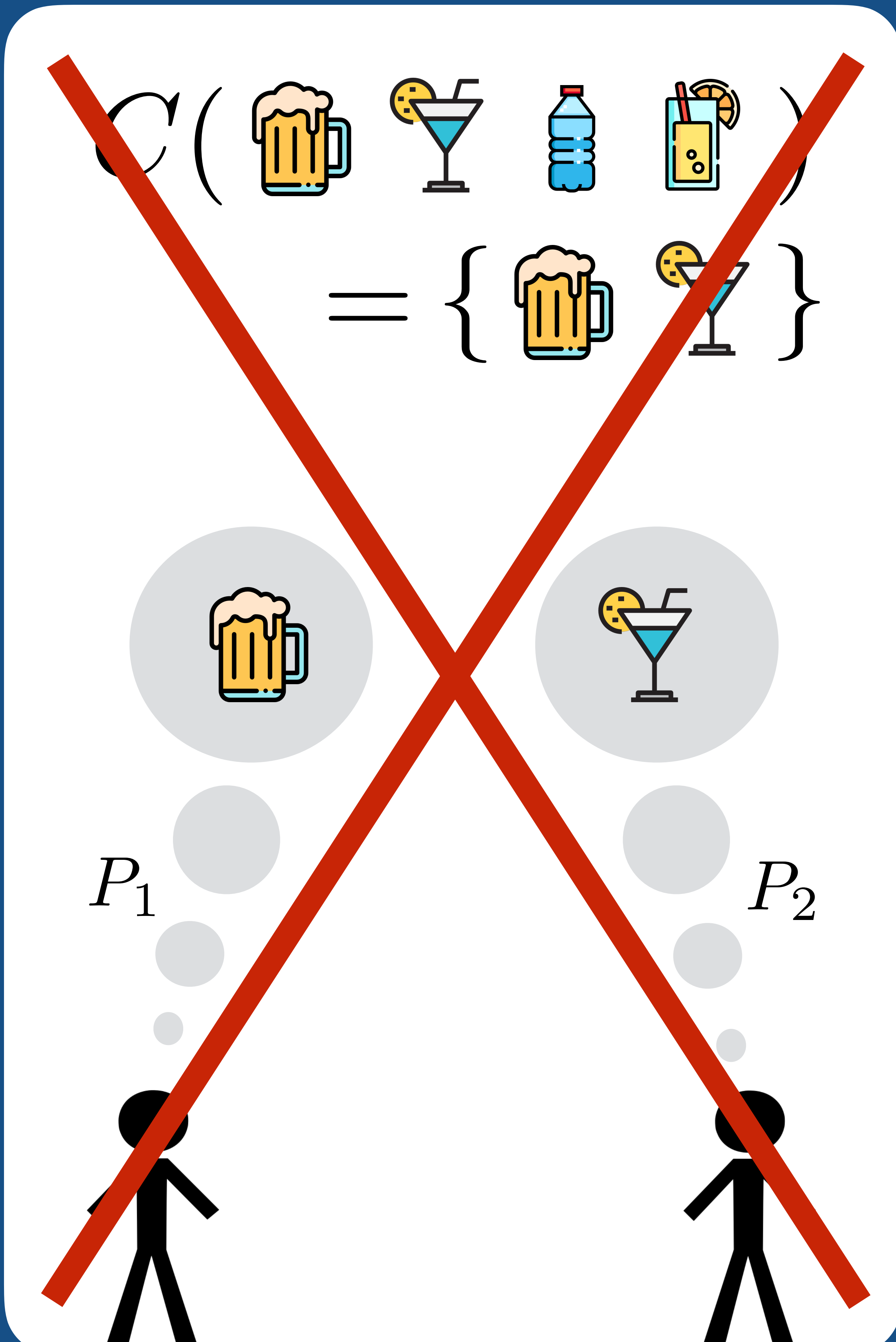
probability measures

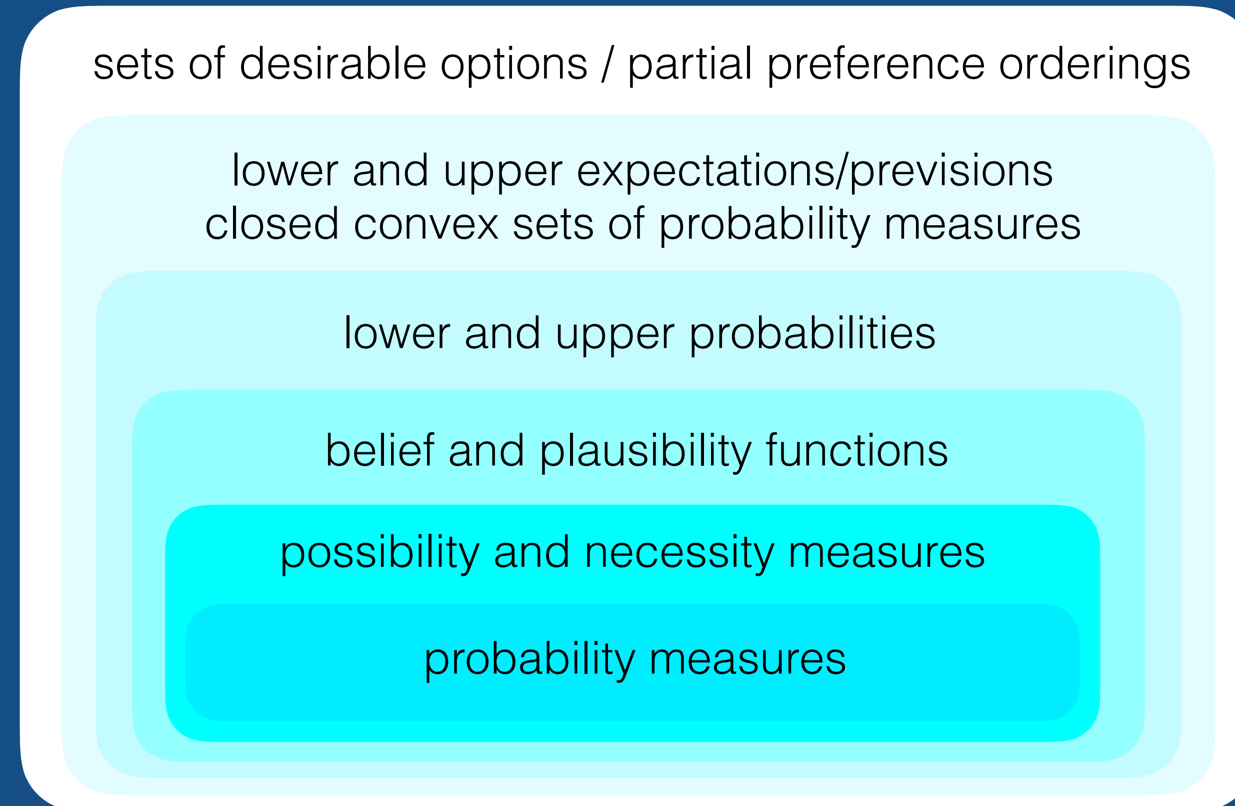
derived notion 1

set of desirable options D

$\subseteq \mathcal{V}$

coherence axioms!





derived notion 1

set of desirable options D

$\subseteq \mathcal{V}$

coherence axioms!

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elementary concept

desirable option v

$\in \mathcal{V}$

strictly preferred over 0

derived notion 2

desirable option set A

$\subseteq \mathcal{V}$

contains at least one desirable option!

Interpreting, axiomatising and representing

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derived notion 3

! set of desirable option sets !

derived notion 2

desirable option set A

$\subseteq \mathcal{V}$

contains at least one desirable option!

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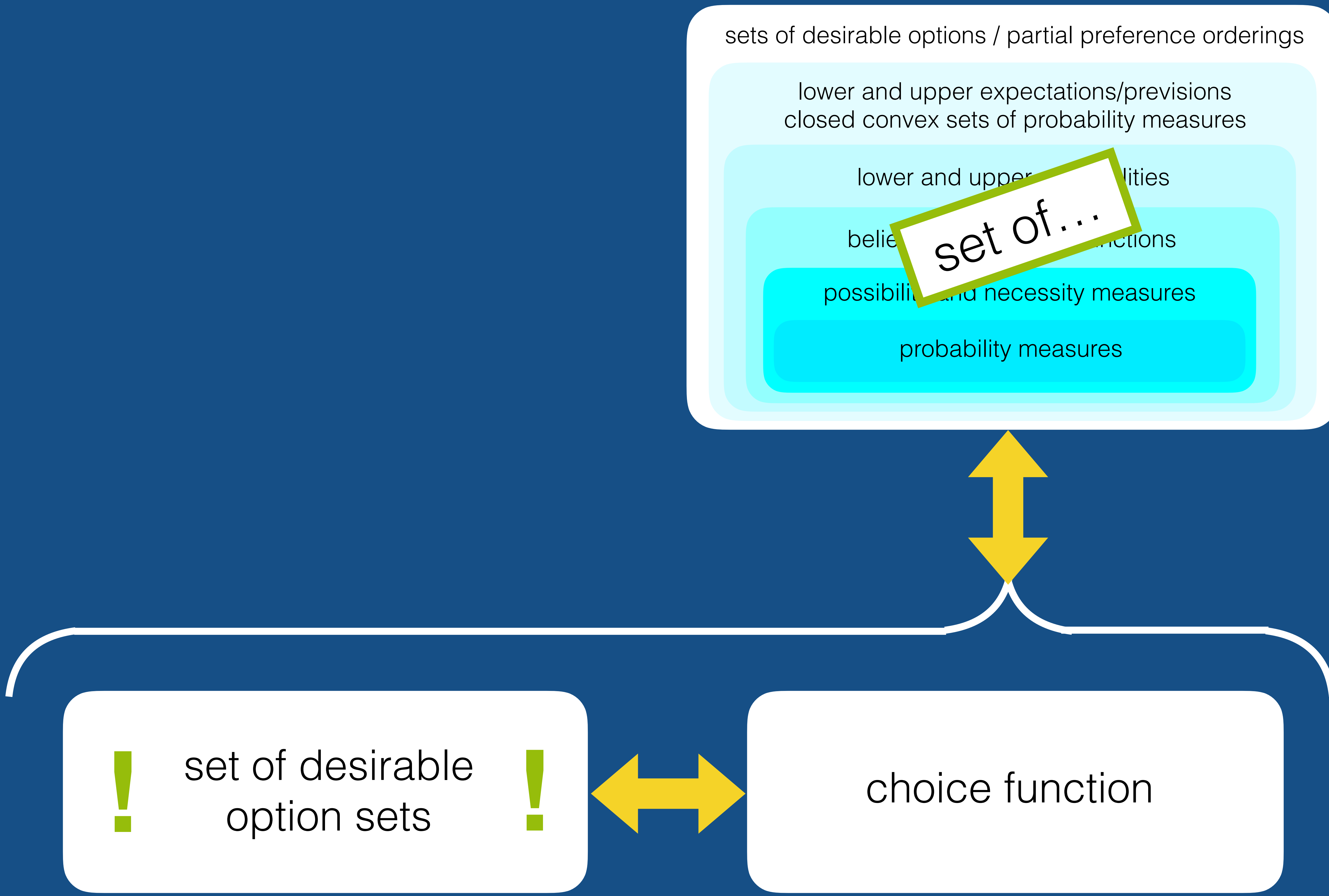
! set of desirable option sets ! $\longleftrightarrow$ choice function



sets of desirable options / partial preference orderings
lower and upper expectations/previsions
closed convex sets of probability measures
lower and upper
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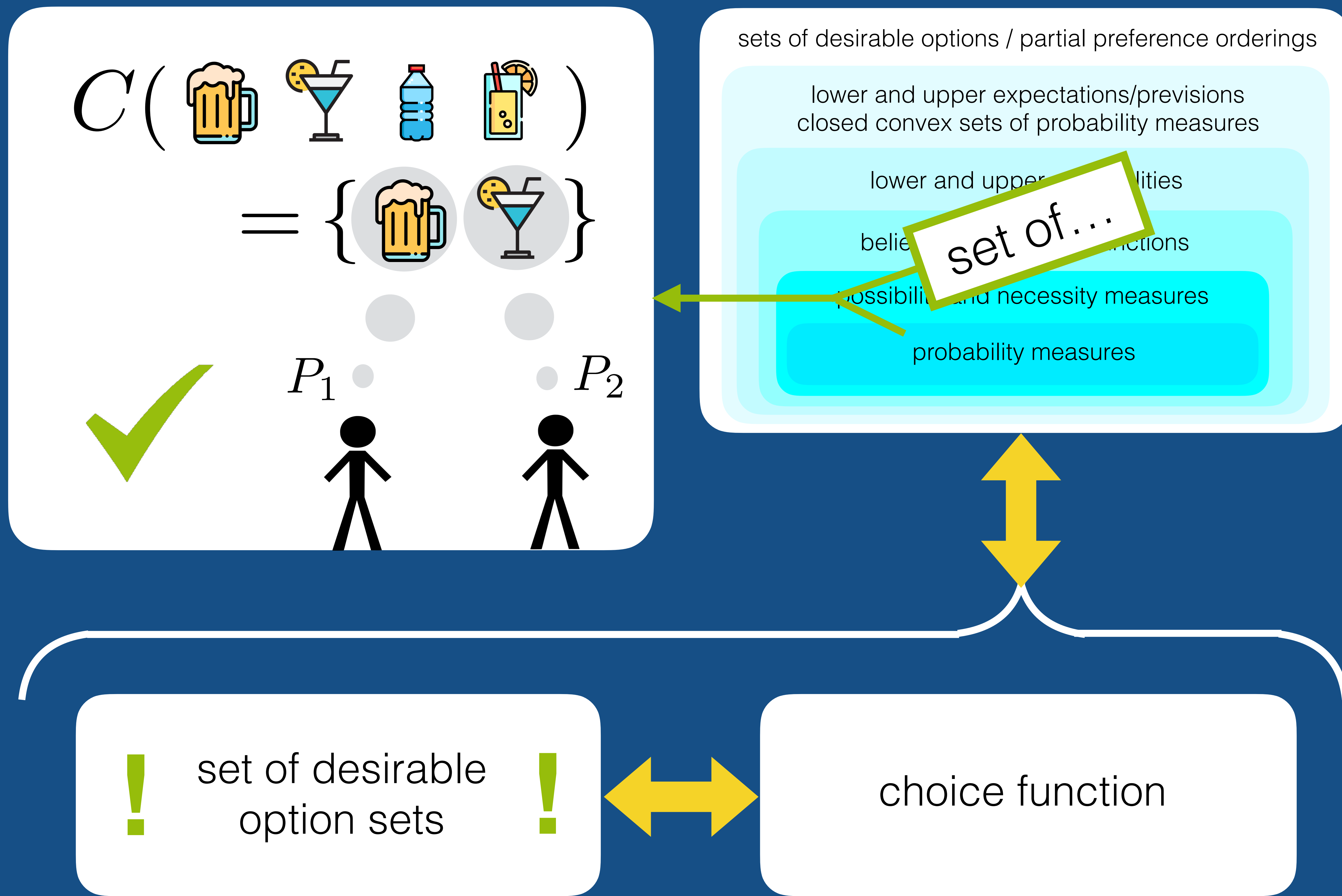
necessity measures
probability measures
set of…
!
set of desirable option sets
!
choice function



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$= \{$ 🍺 🍸 $\}$
P_1
P_2
sets of desirable options / partial preference orderings
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set of…
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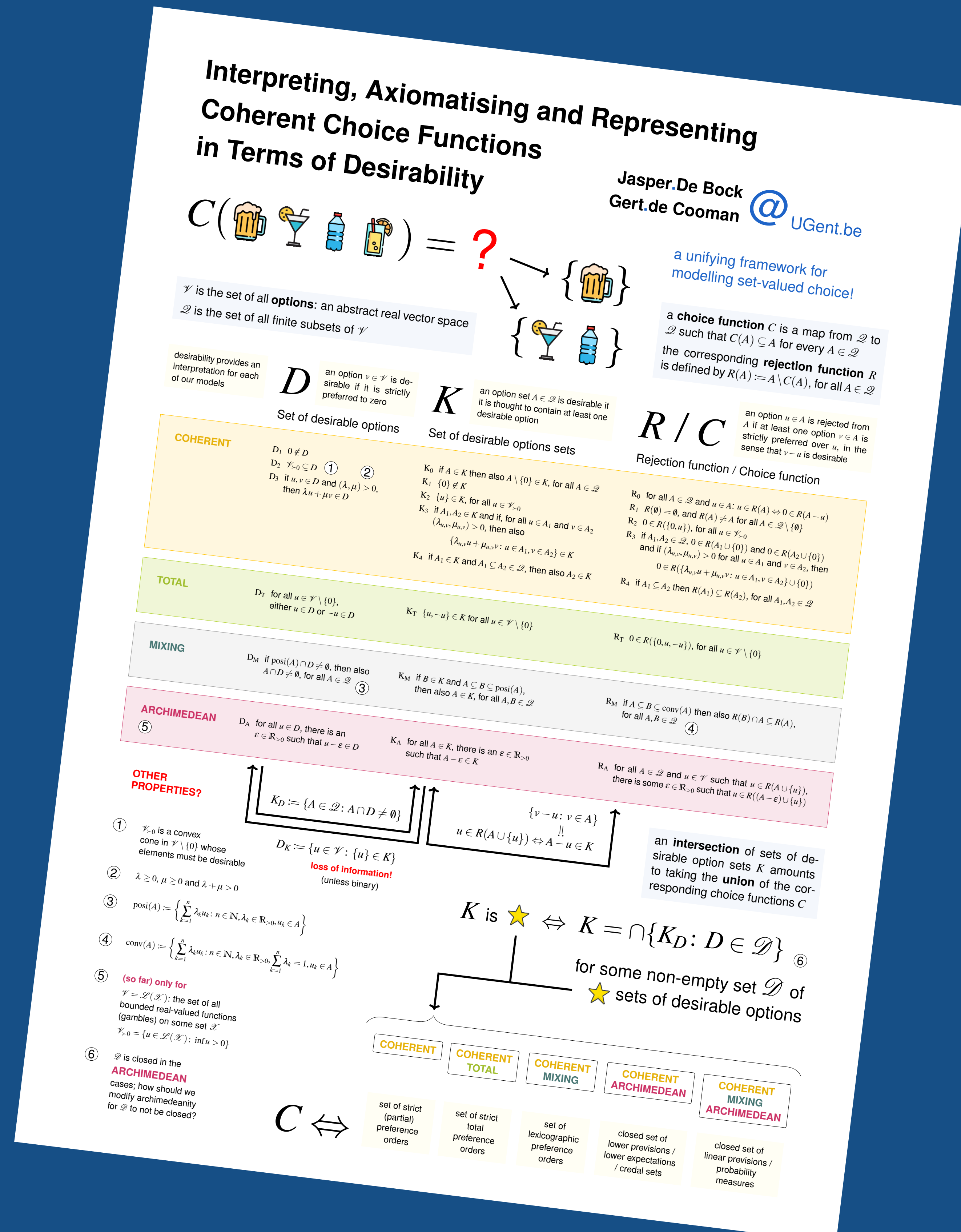
choice function


Interpreting, Axiomatising and Representing Coherent Choice Functions in Terms of Desirability
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Gert.de Cooman
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a unifying framework for modelling set-valued choice!
$\mathscr{V}$ is the set of all options: an abstract real vector space
$\mathscr{Q}$ is the set of all finite subsets of $\mathscr{V}$
a choice function C is a map from $\mathscr{Q}$ to $\mathscr{Q}$ such that $C(A) \subseteq A$ for every $A \in \mathscr{Q}$
the corresponding rejection function R is defined by $R(A) := A \setminus C(A)$, for all $A \in \mathscr{Q}$
desirability provides an interpretation for each of our models
D
an option $v \in \mathscr{V}$ is desirable if it is strictly preferred to zero
Set of desirable options
K
an option set $A \in \mathscr{Q}$ is desirable if it is thought to contain at least one desirable option
Set of desirable options sets
R / C
an option $u \in A$ is rejected from A if at least one option $v \in A$ is strictly preferred over u, in the sense that $v - u$ is desirable
Rejection function / Choice function
COHERENT
TOTAL
MIXING
ARCHIMEDEAN
OTHER PROPERTIES?
$K_D := \{A \in \mathscr{Q} : A \cap D \neq \emptyset\}$
$D_K := \{u \in \mathscr{V} : \{u\} \in K\}$
loss of information!
(unless binary)
an intersection of sets of desirable option sets K amounts to taking the union of the corresponding choice functions C
K is ☆ $\Leftrightarrow$ $K = \cap\{K_D : D \in \mathscr{D}\}$
for some non-empty set $\mathscr{D}$ of ☆ sets of desirable options
COHERENT
COHERENT TOTAL
COHERENT MIXING
COHERENT ARCHIMEDEAN
COHERENT MIXING ARCHIMEDEAN
$C \Leftrightarrow$
set of strict (partial) preference orders
set of strict total preference orders
set of lexicographic preference orders
closed set of lower previsions / lower expectations / credal sets
closed set of linear previsions / probability measures