Decision making in credal networks

Estimating state sequences in strong iHMMs

Jasper De Bock

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$\hat{x}_{1:n}$ is maximal

$$\Updownarrow$$

$$(\forall x_{1:n} \in \mathscr{X}^n)(\exists p \in \mathscr{M})\, p(x_{1:n}, o_{1:n}) \leq p(\hat{x}_{1:n}, o_{1:n})$$

$$\Updownarrow$$

$$(\forall x_{1:n} \in \mathscr{X}^n)(\exists p \in \mathscr{M})\, \frac{p(x_{1:n}, o_{1:n})}{p(\hat{x}_{1:n}, o_{1:n})} \leq 1$$

$$\Updownarrow$$

$$(\forall x_{1:n} \in \mathscr{X}^n) \min_{p \in \mathscr{M}} \frac{p(x_{1:n}, o_{1:n})}{p(\hat{x}_{1:n}, o_{1:n})} \leq 1$$

$$\Updownarrow$$

$$\max_{x_{1:n}} \min_{p} \frac{p(x_{1:n}, o_{1:n})}{p(\hat{x}_{1:n}, o_{1:n})} \leq 1$$

$\hat{x}_{1:n}$ is maximal

$$\Updownarrow$$

$$\max_{x_{1:n}} \min_{p} \frac{p(x_{1:n}, o_{1:n})}{p(\hat{x}_{1:n}, o_{1:n})} \leq 1$$

$$\Updownarrow$$

$$\max_{x_{1:n}} \min_{p} \frac{p(x_1)}{p(\hat{x}_1)} \frac{p(o_1|x_1)}{p(o_1|\hat{x}_1)} \frac{p(x_2|x_1)}{p(\hat{x}_2|\hat{x}_1)} \frac{p(o_2|x_2)}{p(o_2|\hat{x}_2)} \cdots \frac{p(x_n|x_{n-1})}{p(\hat{x}_n|\hat{x}_{n-1})} \frac{p(o_n|x_n)}{p(o_n|\hat{x}_n)} \leq 1$$

$$\Updownarrow$$

$$\max_{x_1} \min_{p} \frac{p(x_1)}{p(\hat{x}_1)} \frac{p(o_1|x_1)}{p(o_1|\hat{x}_1)} \max_{x_2} \min_{p} \quad \cdots \quad \max_{x_n} \min_{p} \frac{p(x_n|x_{n-1})}{p(\hat{x}_n|\hat{x}_{n-1})} \frac{p(o_n|x_n)}{p(o_n|\hat{x}_n)} \leq 1$$

There exists a maximal sequence that starts with $\hat{x}_1$

$$\Updownarrow$$

$$(\exists \hat{x}_{2:n}) \max_{x_{1:n}} \min_{p} \frac{p(x_{1:n}, o_{1:n})}{p(\hat{x}_{1:n}, o_{1:n})} \leq 1$$

$$\Updownarrow$$

$$\min_{\hat{x}_{2:n}} \max_{x_{1:n}} \min_{p} \frac{p(x_1)}{p(\hat{x}_1)} \frac{p(o_1|x_1)}{p(o_1|\hat{x}_1)} \frac{p(x_2|x_1)}{p(\hat{x}_2|\hat{x}_1)} \frac{p(o_2|x_2)}{p(o_2|\hat{x}_2)} \cdots \frac{p(x_n|x_{n-1})}{p(\hat{x}_n|\hat{x}_{n-1})} \frac{p(o_n|x_n)}{p(o_n|\hat{x}_n)} \leq 1$$

$$\Updownarrow$$

$$\max_{x_1} \min_{p} \frac{p(x_1)}{p(\hat{x}_1)} \frac{p(o_1|x_1)}{p(o_1|\hat{x}_1)} \min_{\hat{x}_{2:n}} \max_{x_{2:n}} \min_{p} \frac{p(x_2|x_1)}{p(\hat{x}_2|\hat{x}_1)} \cdots \frac{p(x_n|x_{n-1})}{p(\hat{x}_n|\hat{x}_{n-1})} \frac{p(o_n|x_n)}{p(o_n|\hat{x}_n)} \leq 1$$

Final comments / questions / discussion suggestions

- An equally efficient algorithm exists for the epistemic irrelevance approach (De Bock & de Cooman 2011).
- Can be extended to general networks as long as there is no missing data.
- Similar (but approximate) things can be done for E-admissability.
- Which decision criterium performs best?
- Which decision criterium is the most tractable?
- Is (conditional) maximin tractable in the strong independence approach?
- Is E-admissability tractable for large solution spaces?
- Useful applications of imprecise Viterbi?