UUvUG
workshop

Credal networks

an overview of different approaches

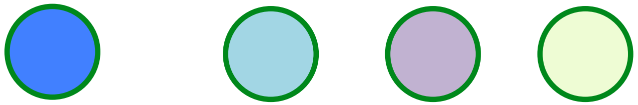
Jasper De Bock

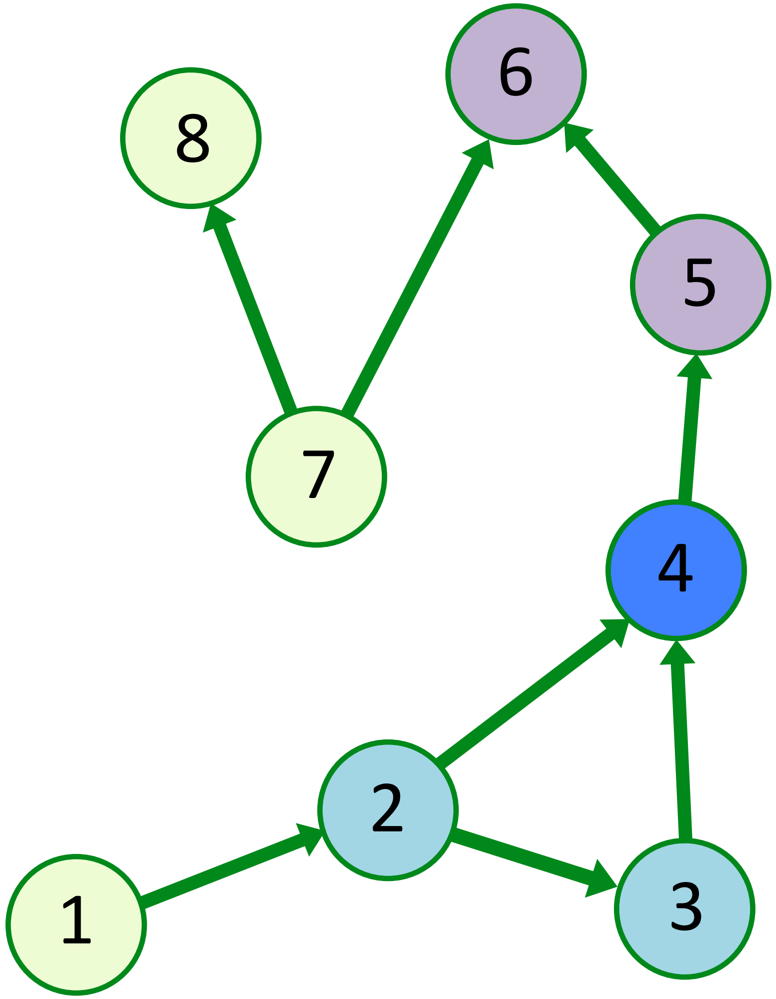
11 June 2013

Bayesian networks: basic setup

- Graphical structure: DAG

⇨ $\forall s \in G$: P(s), D(s), N(s)





Bayesian networks: basic setup

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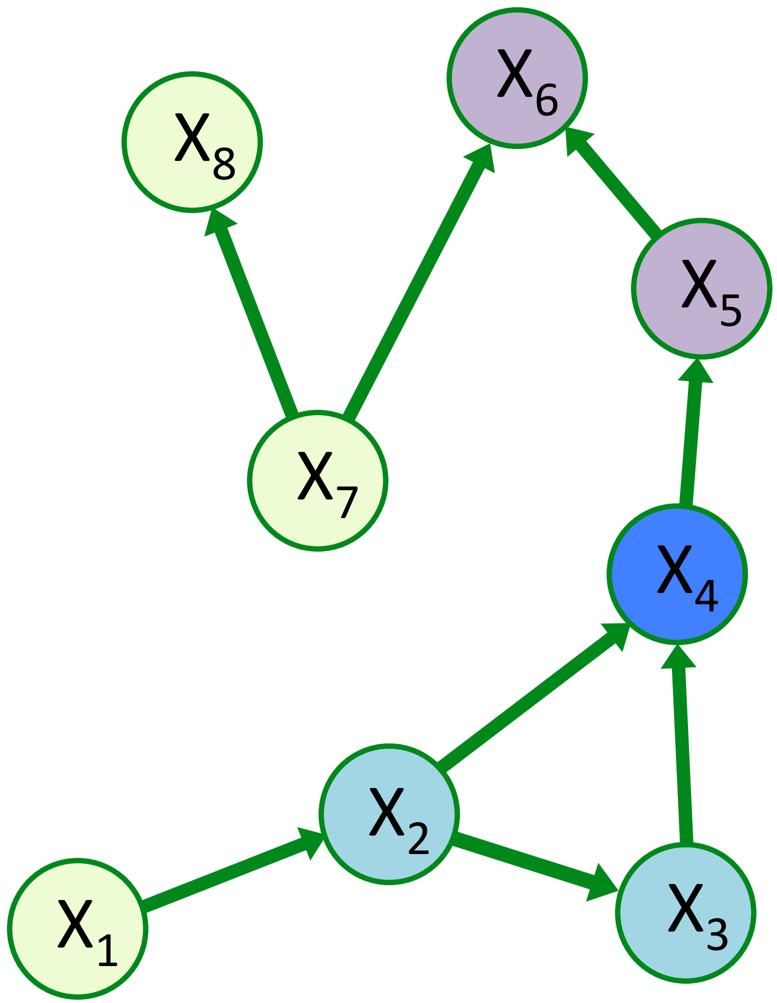
  $\Longrightarrow$ $\forall s \in G$: P(s), D(s), N(s)
- Variables X_s take values x_s in a finite non-empty set $\mathcal{X}_s$



Bayesian networks: basic setup

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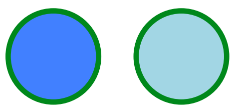
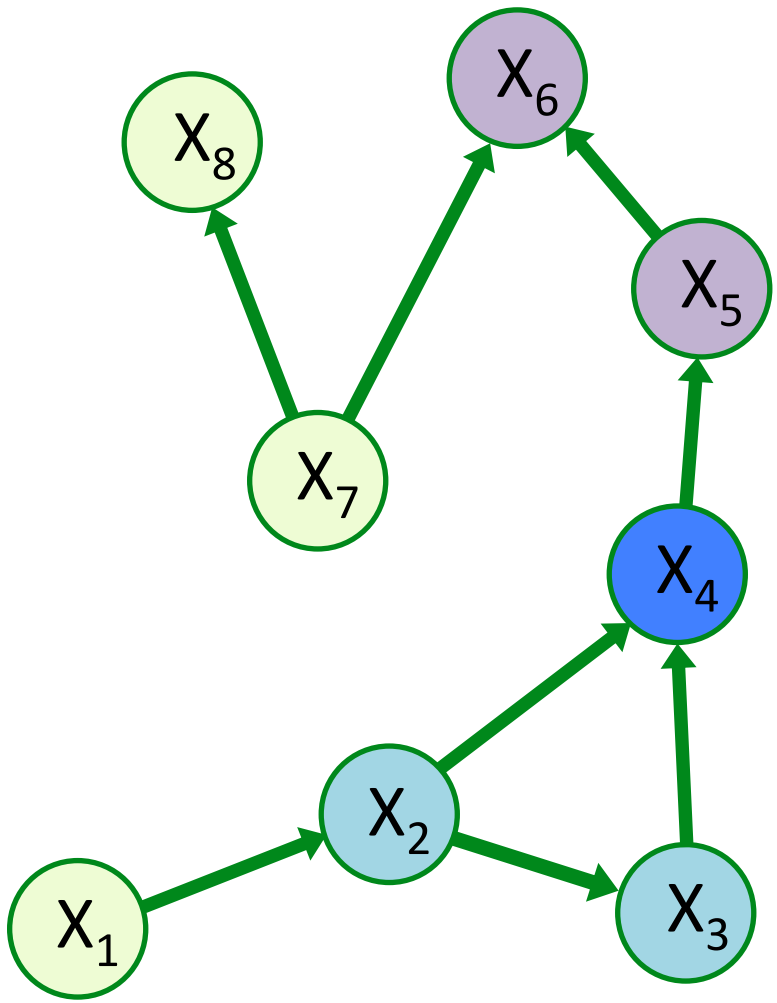
  Example: $q(X_4 | x_{\{2,3\}})$





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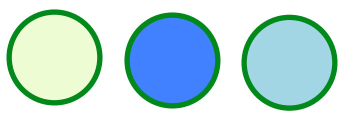
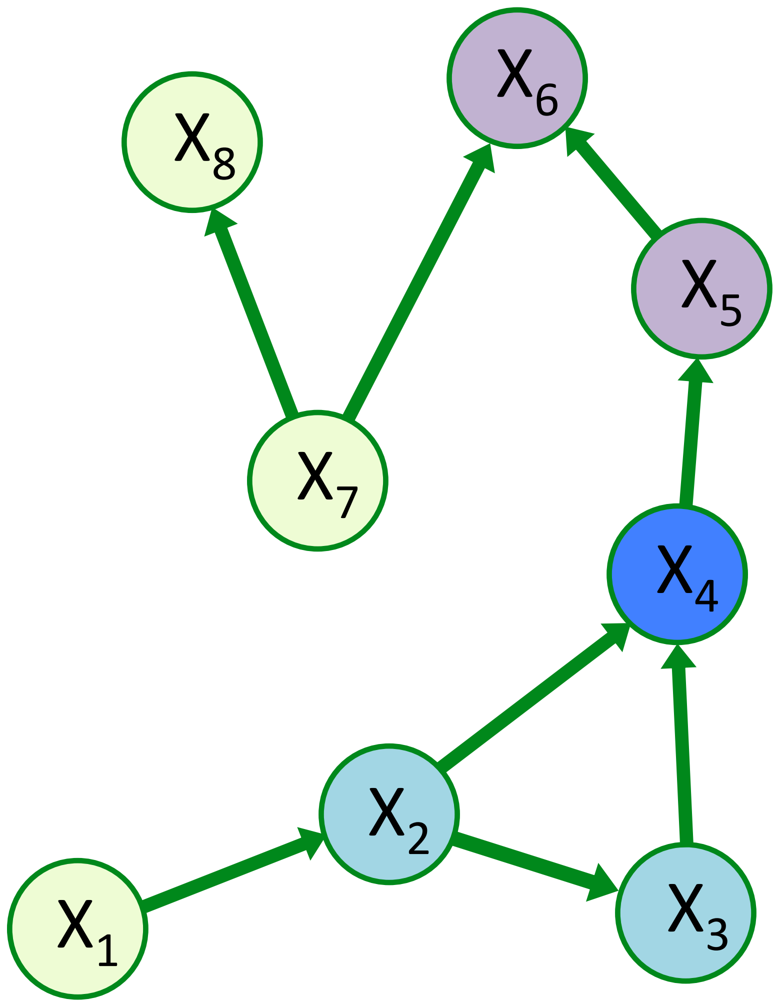
- Independence assumptions:
  $\forall s \in G$: I(N(s), s|P(s))





Bayesian networks: joint model $p(X_G)$?

- Graphical structure: DAG
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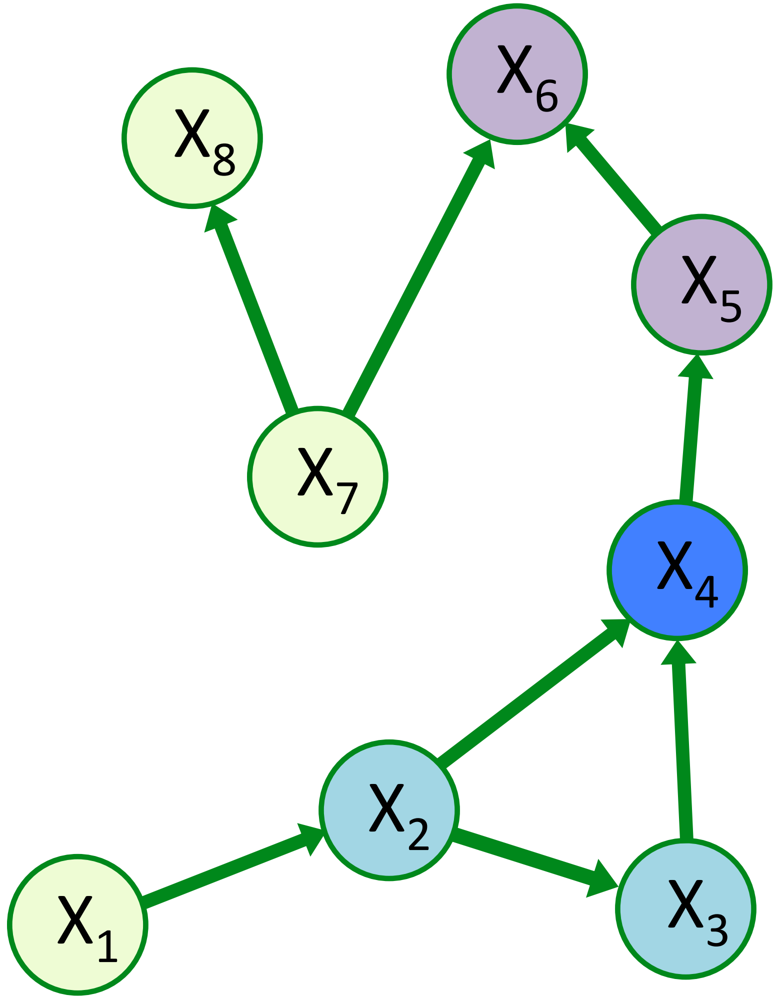
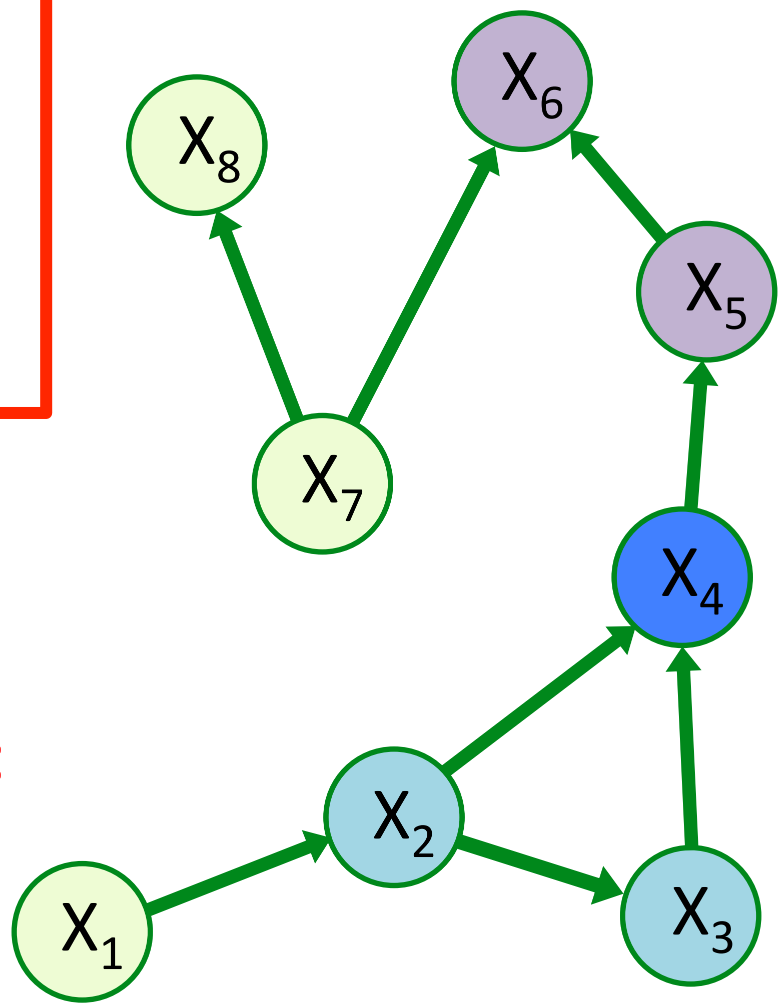


Bayesian networks: joint model $p(X_G)$ **?**

$$p(x_s \mid x_{P(s)}, x_{N(s)}) \;=\; p(x_s \mid x_{P(s)})$$

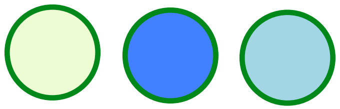
EQUIVALENT ⬍

$$p(x_{N(s)} \mid x_{P(s)}, x_s) \;=\; p(x_{N(s)} \mid x_{P(s)})$$



- Independence assumptions:

  $\forall s \in G$: $I(N(s), s \mid P(s))$



Bayesian networks: joint model $p(X_G)$ **?**

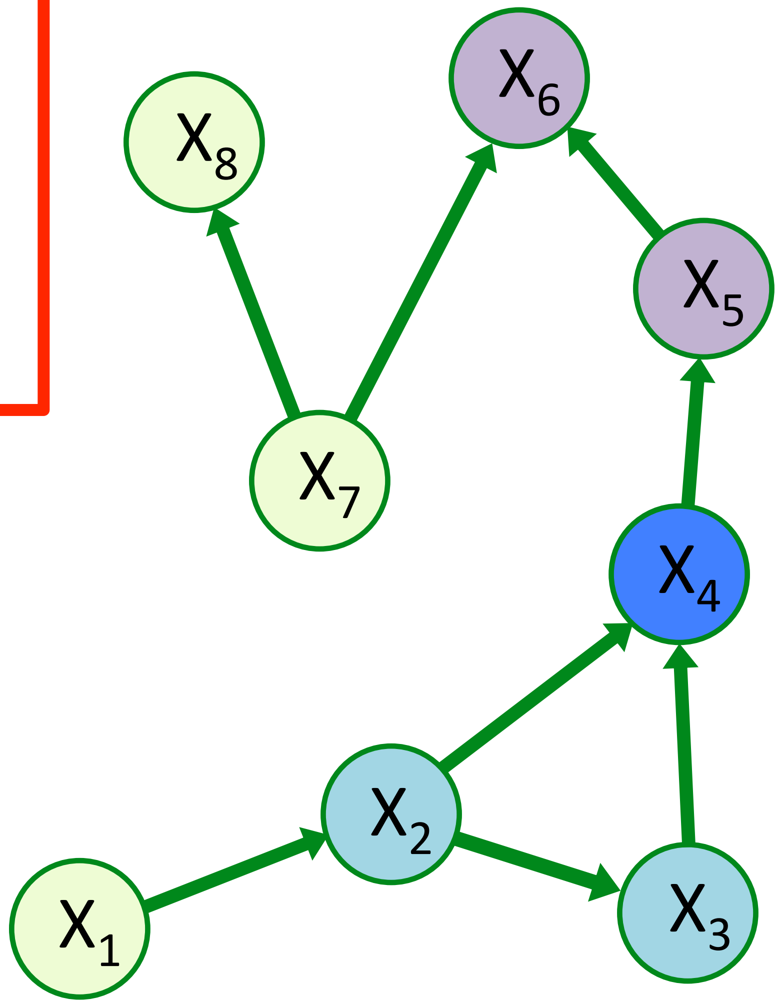
$$p(x_s | x_{P(s)}, x_{N(s)}) = \boxed{p(x_s | x_{P(s)})}$$

EQUIVALENT $\Updownarrow$

$$\cancel{p(x_{N(s)} | x_{P(s)}, x_s) = p(x_{N(s)} | x_{P(s)})}$$

- Local uncertainty models:
  mass functions $q(X_s | x_{P(s)})$

- Independence assumptions:
  $\forall s \in G: I(N(s), s | P(s))$



Bayesian networks: joint model $p(X_G)$ **?**

$$\boxed{p(x_s \mid x_{P(s)}, x_{N(s)}) = p(x_s \mid x_{P(s)})}$$

$$= q(x_s \mid x_{P(s)})$$

- Local uncertainty models:
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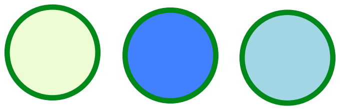
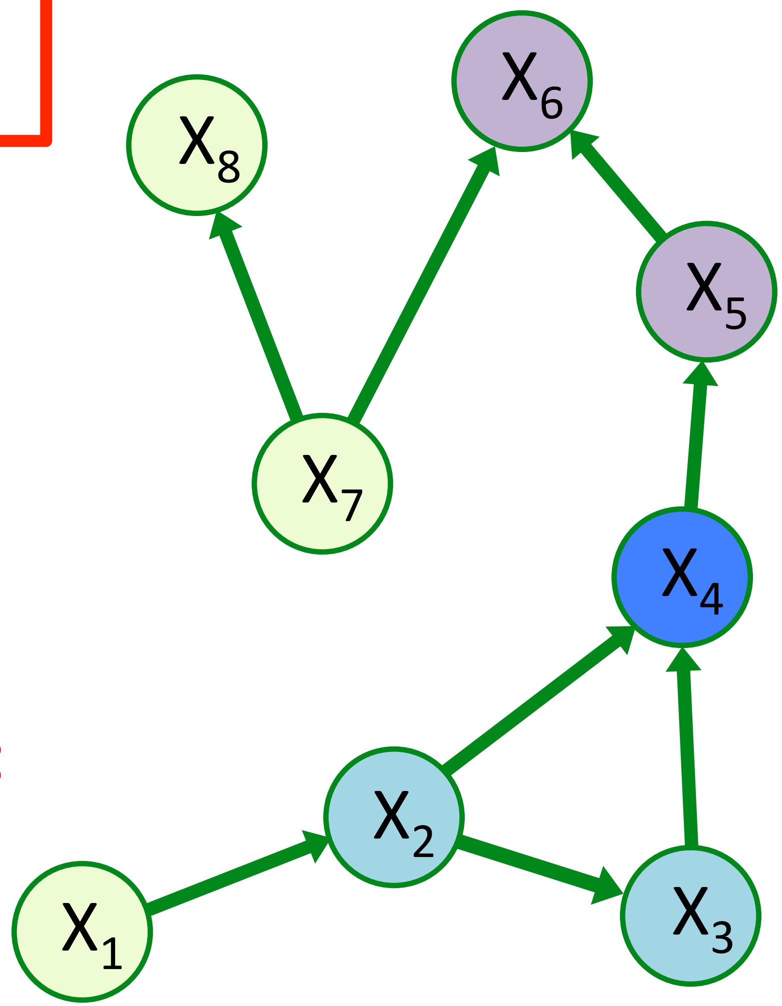
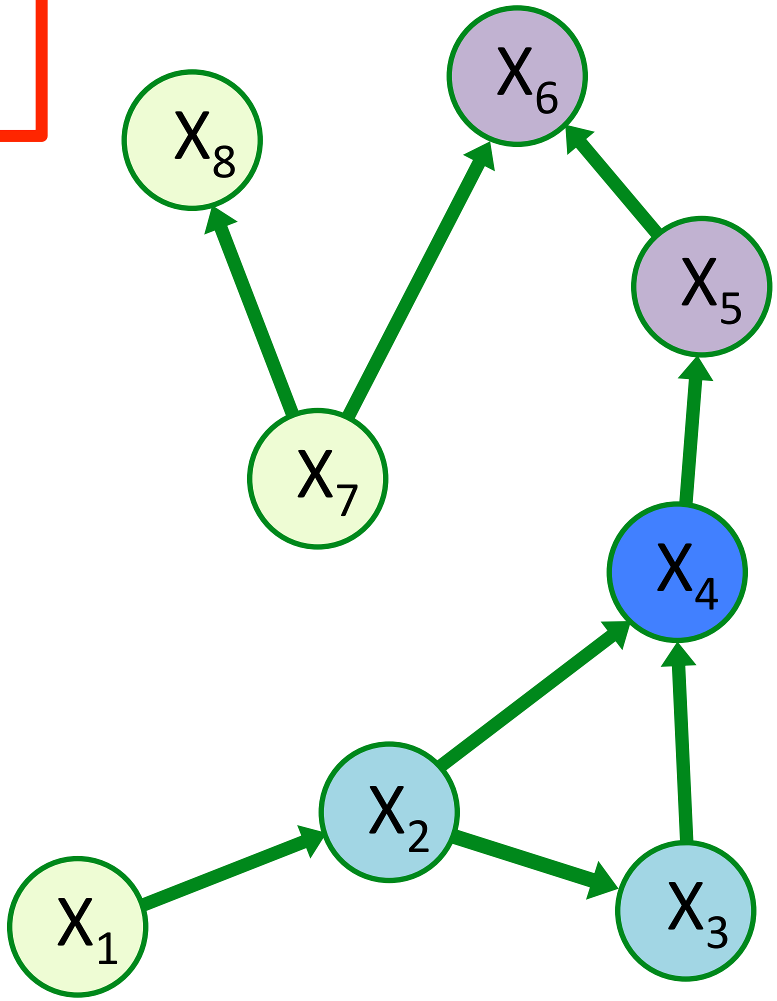




Bayesian networks: joint model $p(X_G)$

$$\boxed{p(x_s \mid x_{P(s)}, x_{N(s)}) = \boxed{\begin{array}{c} p(x_s \mid x_{P(s)}) \\ = \\ q(x_s \mid x_{P(s)}) \end{array}}}$$

$$\boxed{p(x_G) = \prod_{s \in G} q(x_s \mid x_{P(s)})}$$



Credal networks: basic setup

- Graphical structure: DAG

  $\Longrightarrow \quad \forall s \in G$: P(s), D(s), N(s)

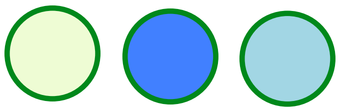
- Variables X_s take values x_s in a finite non-empty set $\mathcal{X}_s$

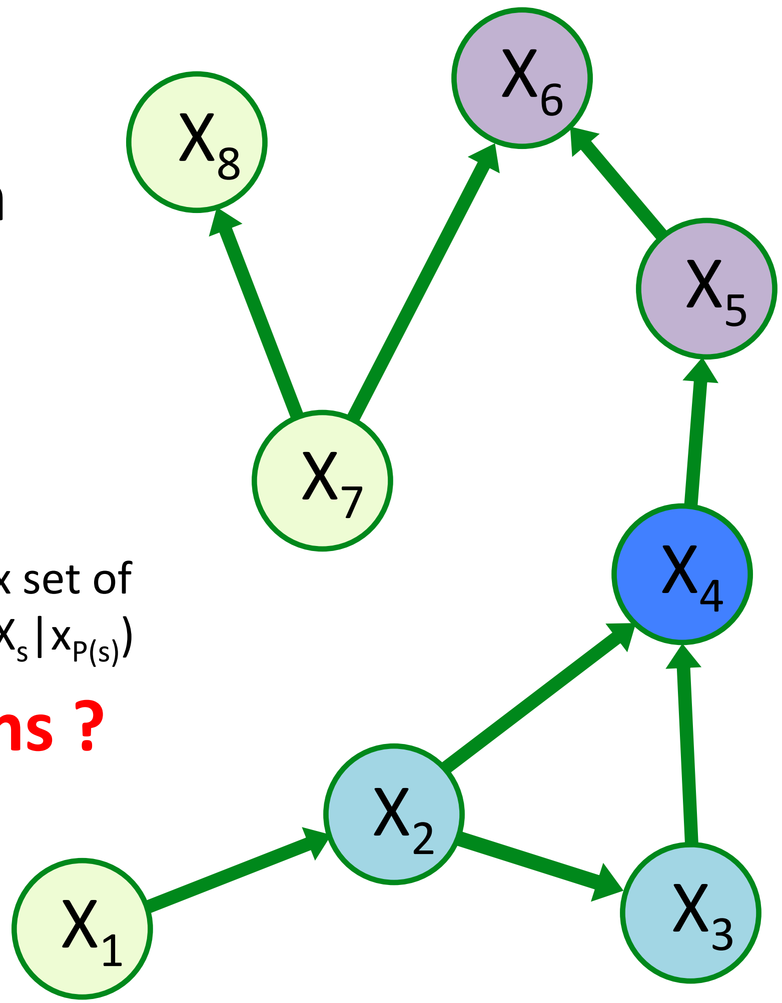
- Local uncertainty models: credal sets $\mathcal{M}(X_s|x_{P(s)})$ **?**

  Closed and convex set of mass functions $q(X_s|x_{P(s)})$

- **? Independence assumptions ?**

  $\forall s \in G$: **?I?**(N(s), s|P(s))





Credal networks: joint model $\mathcal{M}(X_G)$?

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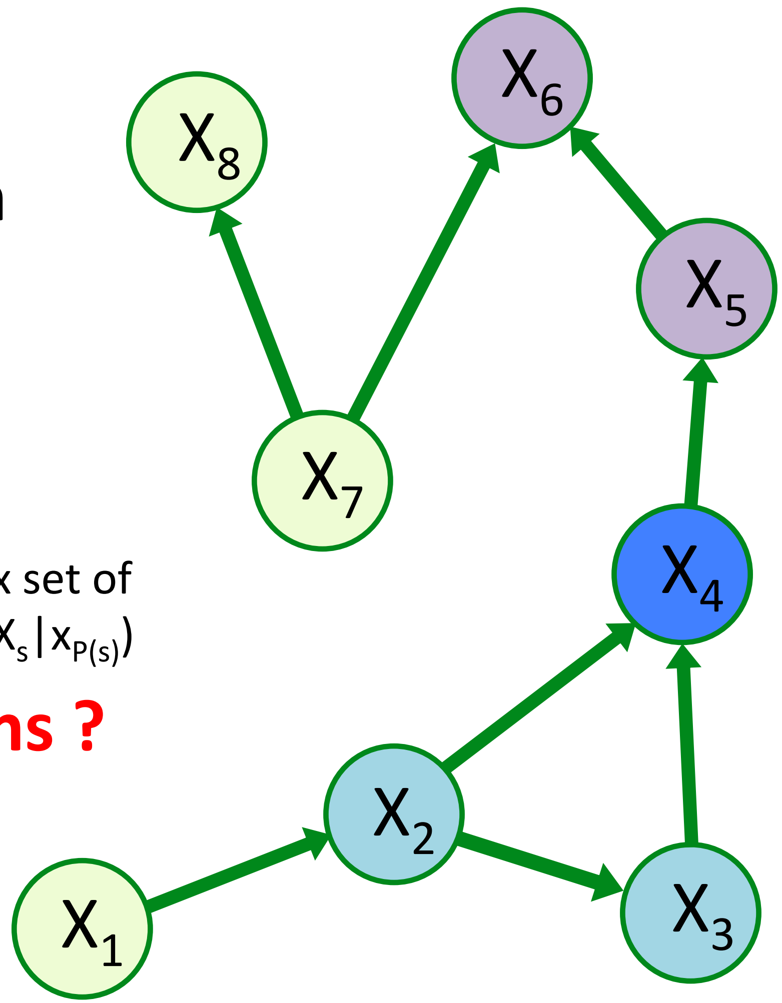


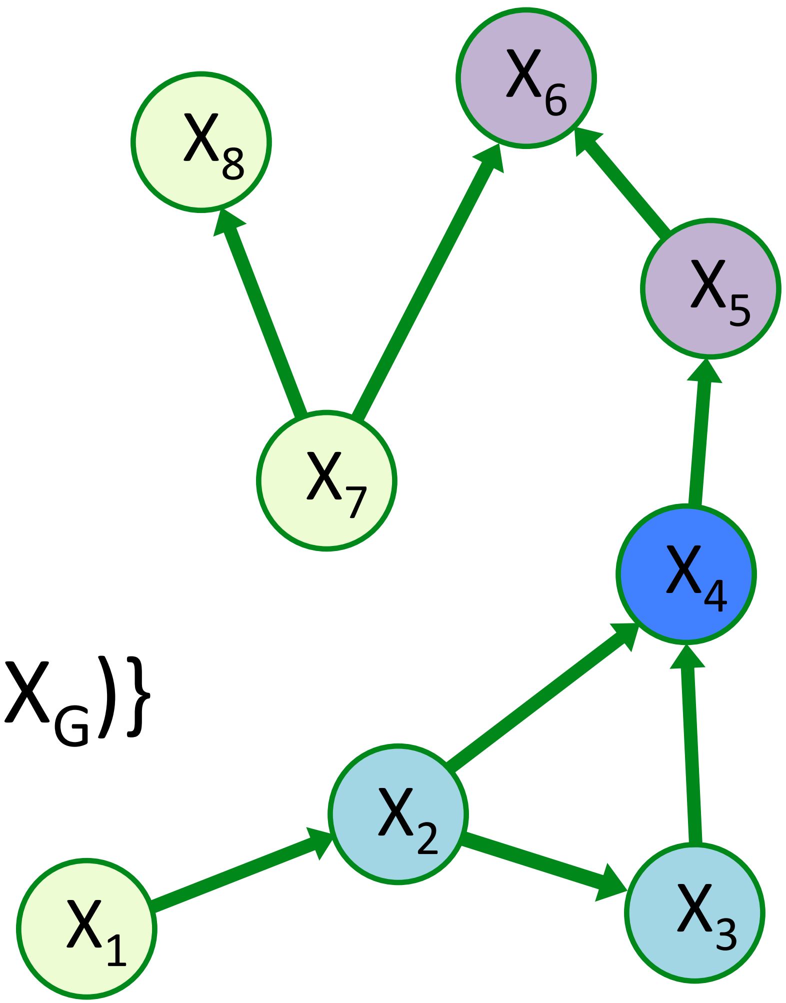
Credal networks: joint model $\mathcal{M}(X_G)$?

Conditioning:

$\mathcal{M}(X_G)$

$\mathcal{M}(X_s | x_{P(s)})$

$= \{p(X_s | x_{P(s)}) : p(X_G) \in \mathcal{M}(X_G)\}$



Credal networks: joint model $\mathcal{M}(X_G)$?

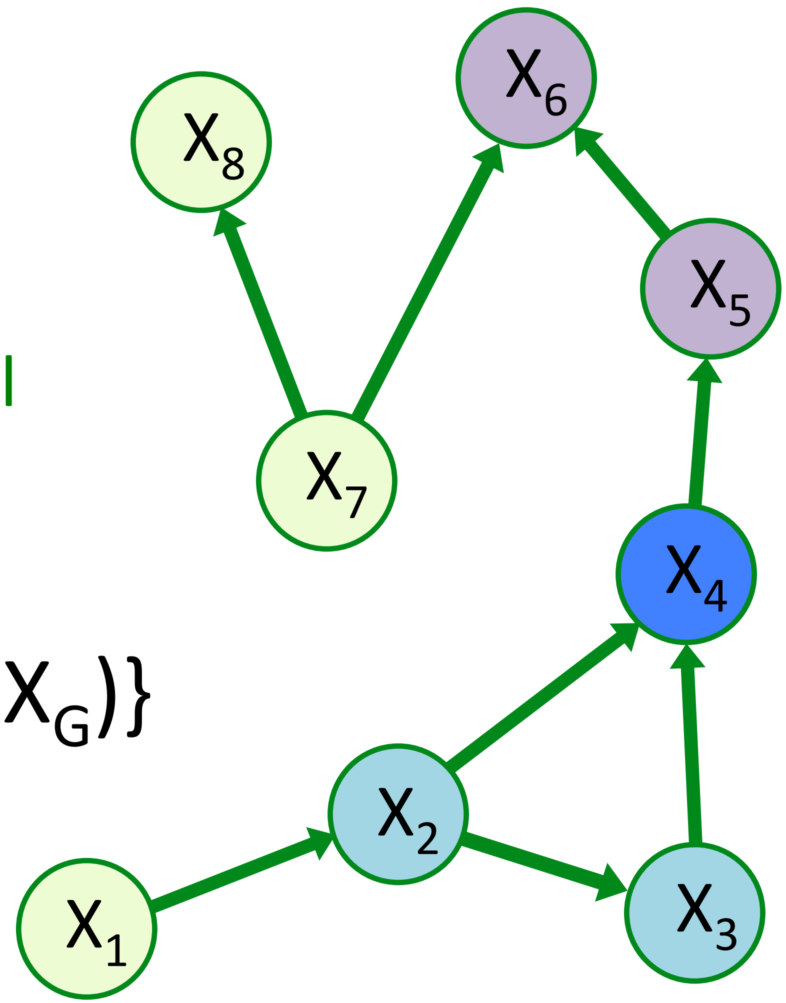
Conditioning:

$\mathcal{M}(X_G)$

$\downarrow$

$\mathcal{M}(X_s | x_{P(s)})$ = Given local credal set

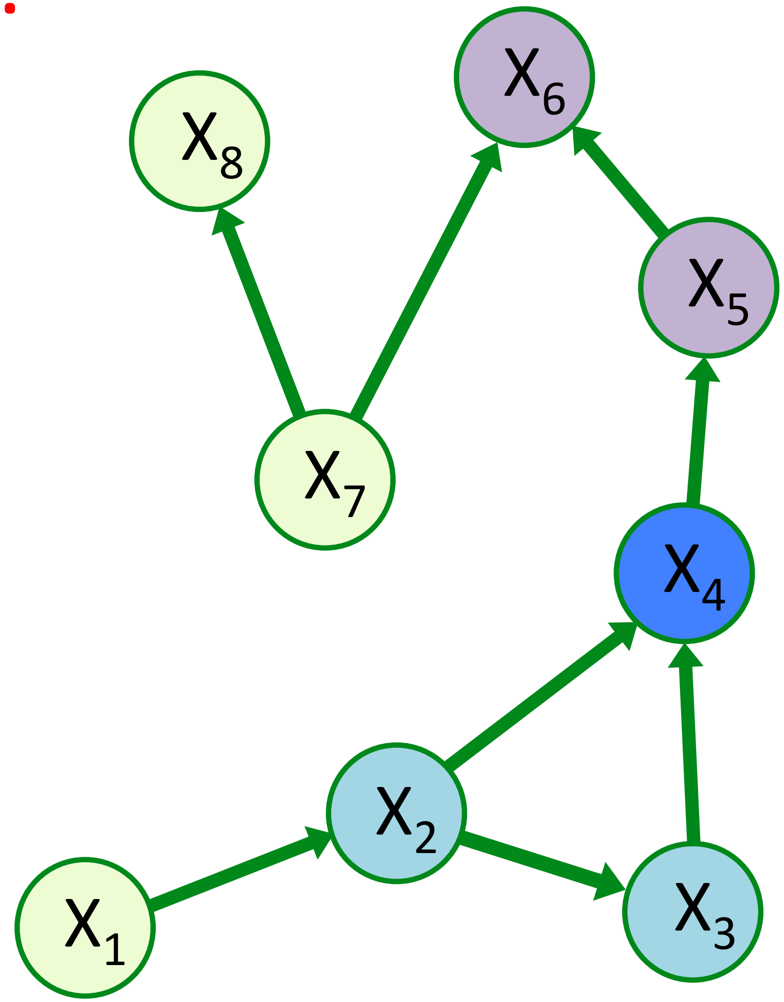
$= \{p(X_s | x_{P(s)}) : p(X_G) \in \mathcal{M}(X_G)\}$



Credal networks: joint model $\mathcal{M}(X_G)$?

? Independence assumptions ?

- Strong independence
- Epistemic irrelevance
- Epistemic independence



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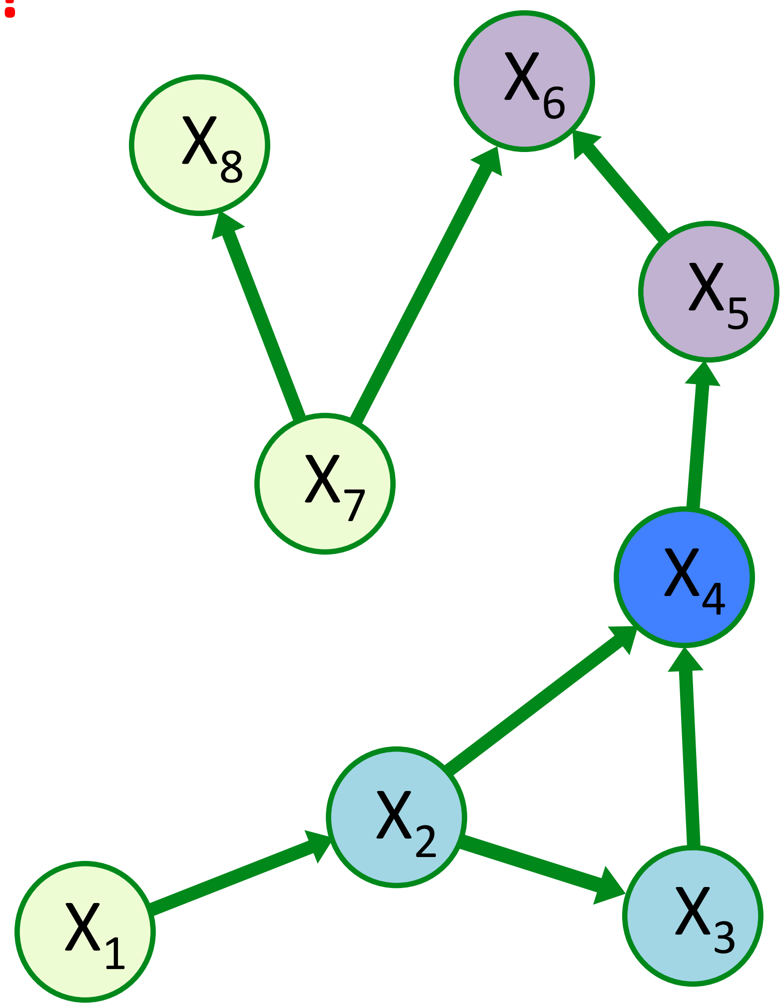
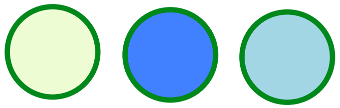


Credal networks: joint model $\mathcal{M}(X_G)$?

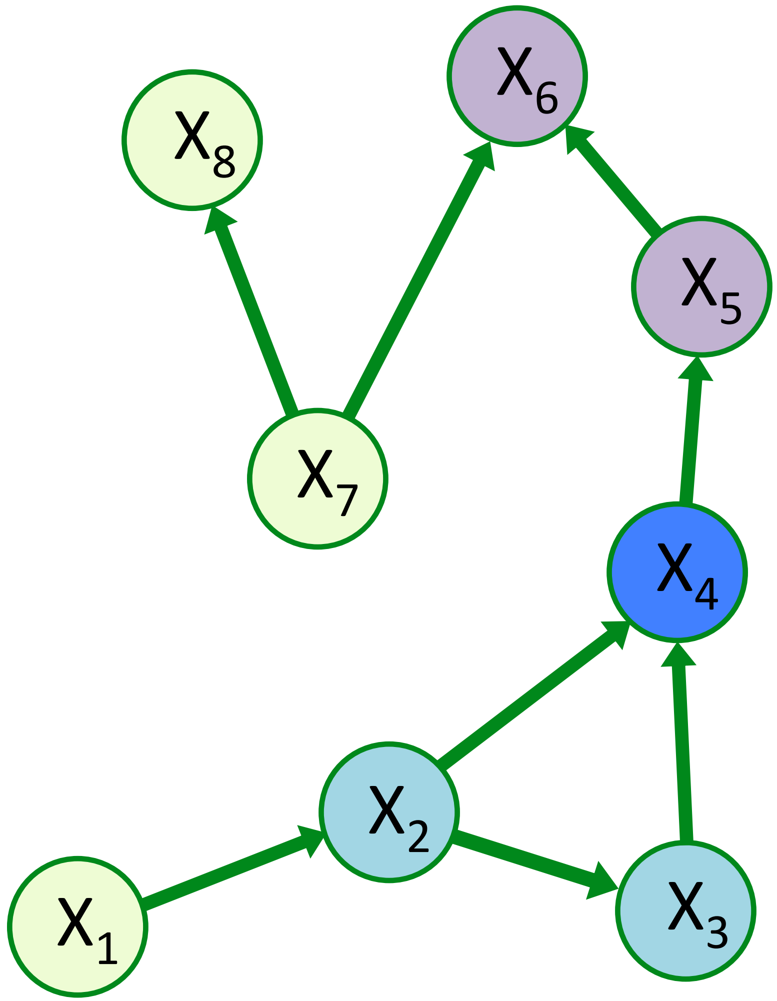
Strong independence

$\forall s \in G$: **SI**$(N(s), s | P(s))$



$\mathcal{M}(X_G)$ is the convex hull of a set of mass functions $p(X_G)$ that satisfy the usual independence assumption:

$\forall s \in G$: $I(N(s), s | P(s))$



Credal networks: joint model $\mathcal{M}(X_G)$?

Strong independence

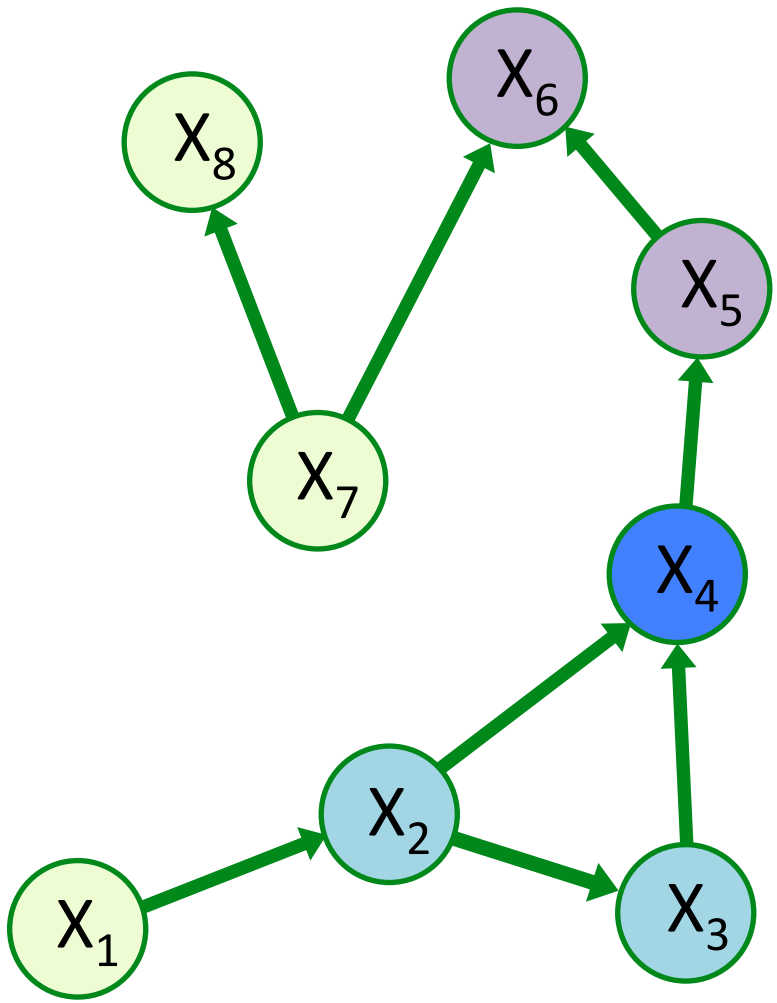
$\forall s \in G$: **SI**$(N(s), s \,|\, P(s))$

+

Local models $\mathcal{M}(X_s \,|\, x_{P(s)})$

not unique!

$\mathcal{M}(X_G)$



Credal networks: joint model $\mathcal{M}^{str}(X_G)$?

Strong independence

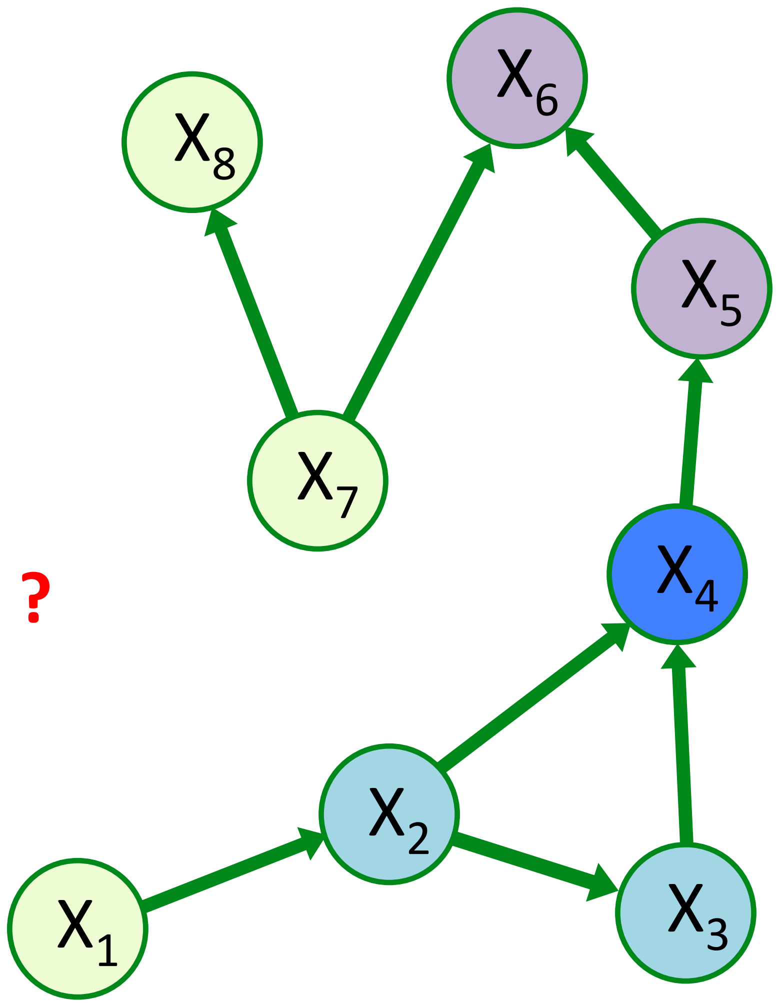
$\forall s \in G$: **SI**$(N(s), s \,|\, P(s))$

+

Local models $\mathcal{M}(X_s \,|\, x_{P(s)})$

⬇ largest solution!

The strong extension $\mathcal{M}^{str}(X_G)$ **?**



Credal networks: joint model $\mathcal{M}^{str}(X_G)$

Strong independence

$\forall s \in G$: **SI**$(N(s), s \mid P(s))$

+

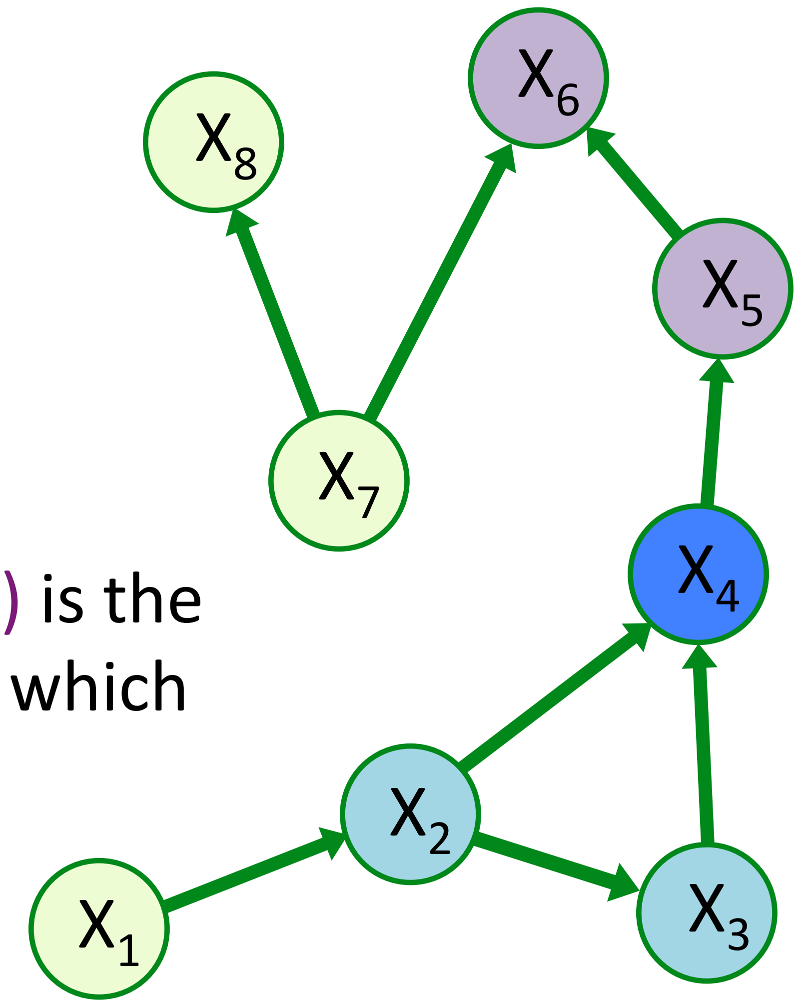
Local models $\mathcal{M}(X_s \mid x_{P(s)})$

⬇ largest solution!

The strong extension $\mathcal{M}^{str}(X_G)$ is the convex hull of those $p(X_G)$ for which

$$p(x_G) = \prod_{s \in G} q(x_s \mid x_{P(s)})$$

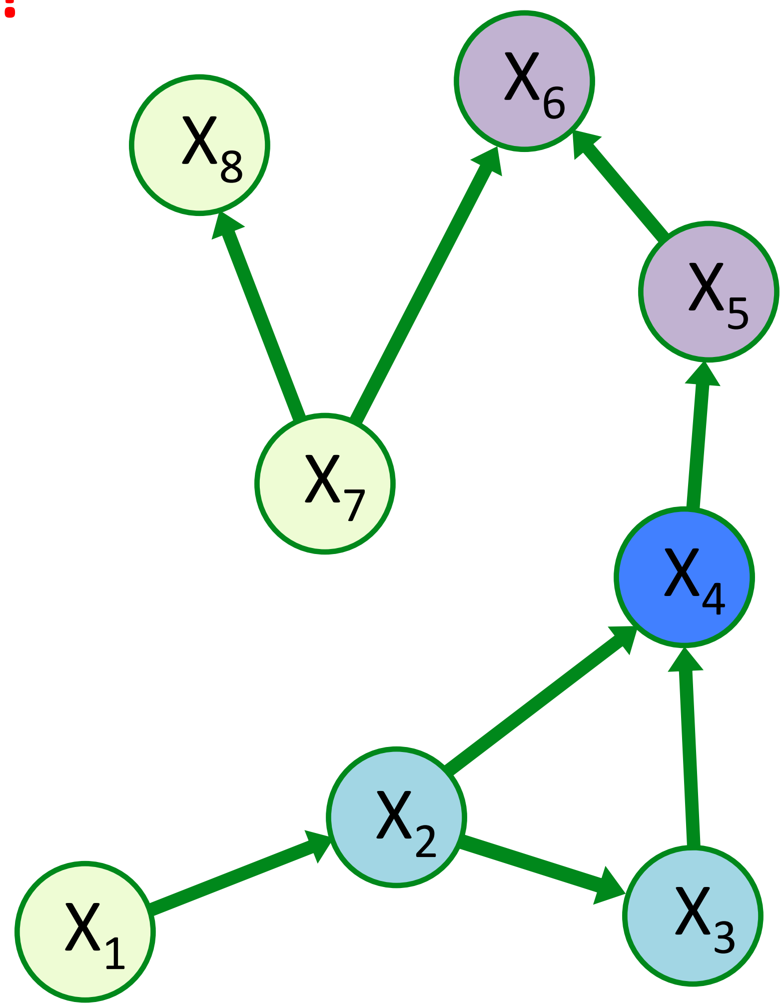
$$q(x_s \mid x_{P(s)}) \Subset \mathcal{M}(X_s \mid x_{P(s)})$$



Credal networks: joint model $\mathcal{M}(X_G)$?

? Independence assumptions ?

- Strong independence
- **Epistemic irrelevance**
- Epistemic independence

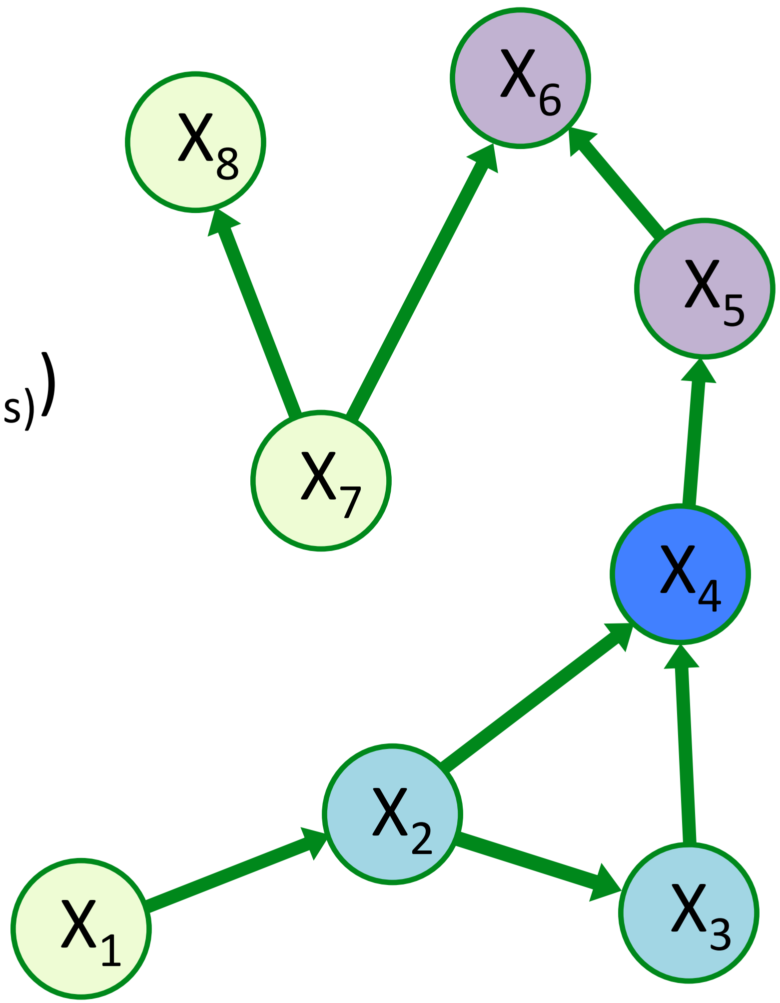


Credal networks: joint model $\mathcal{M}(X_G)$?

Epistemic irrelevance

$\forall s \in G$: **IR**$(N(s), s \mid P(s))$

$$\mathcal{M}(X_s \mid x_{P(s)}, x_{N(s)}) = \mathcal{M}(X_s \mid x_{P(s)})$$



Credal networks: joint model $\mathcal{M}(X_G)$?

Epistemic irrelevance

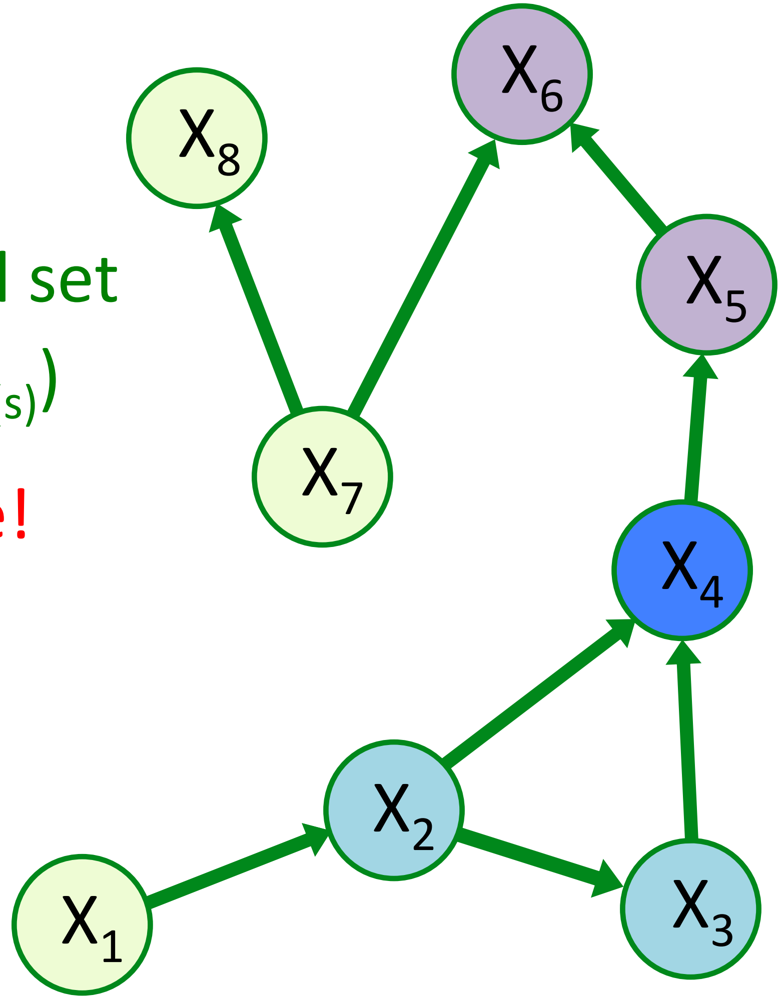
$\forall s \in G$: **IR**$(N(s), s \mid P(s))$

+ local credal set

$\mathcal{M}(X_s \mid x_{P(s)}, x_{N(s)}) = \mathcal{M}(X_s \mid x_{P(s)})$

⬇ not unique!

$\mathcal{M}(X_G)$



Credal networks: joint model $\mathcal{M}^{irr}(X_G)$?

Epistemic irrelevance

$\forall s \in G$: **IR**$(N(s), s | P(s))$

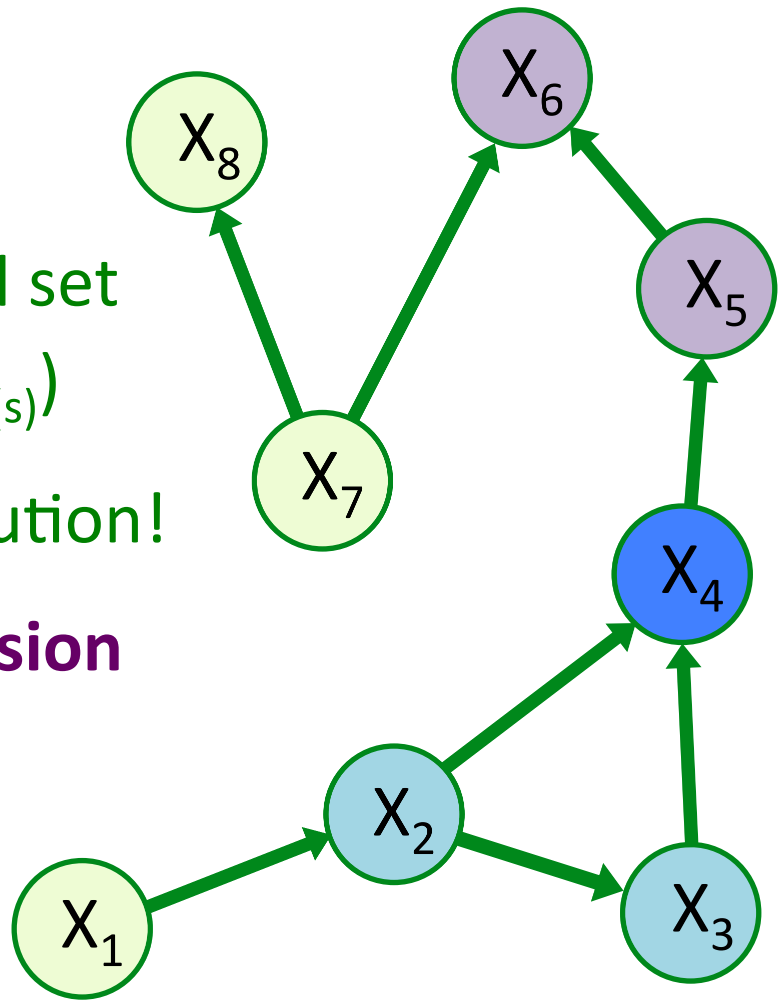
+ local credal set

$\mathcal{M}(X_s | x_{P(s)}, x_{N(s)}) = \mathcal{M}(X_s | x_{P(s)})$

largest solution!

The irrelevant (natural) extension

$\mathcal{M}^{irr}(X_G)$?



Credal networks: joint model $\mathcal{M}^{\mathrm{irr}}(X_G)$

Epistemic irrelevance

$\forall s \in G$: **IR**$(N(s), s \,|\, P(s))$

+ local credal set

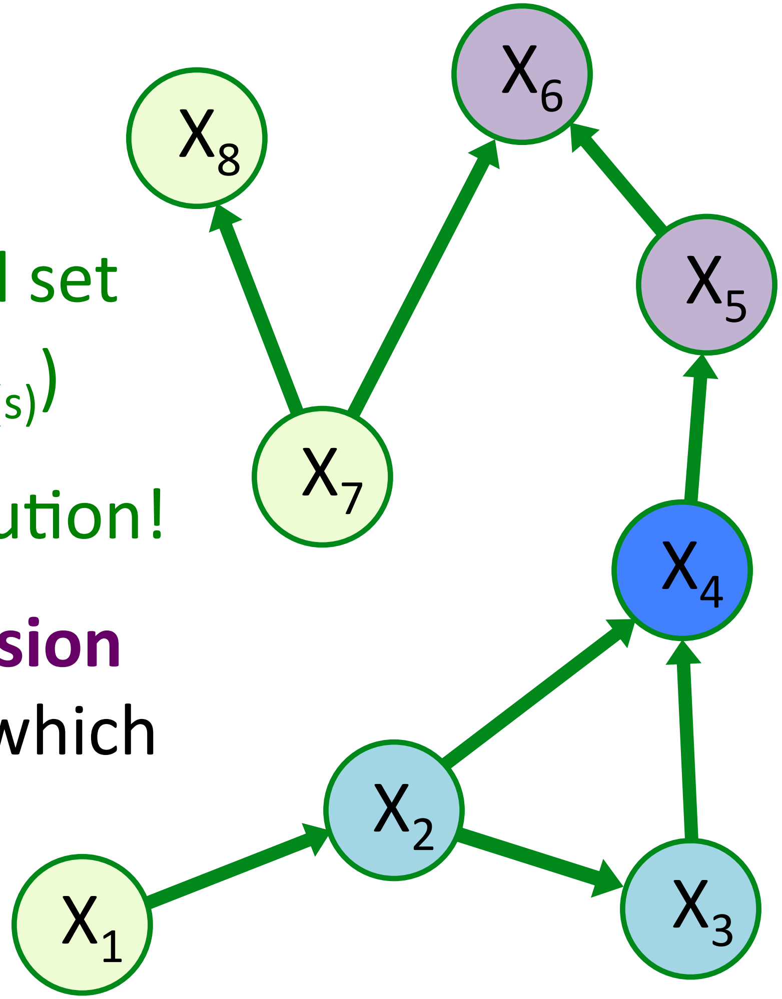
$\mathcal{M}(X_s \,|\, x_{P(s)}, x_{N(s)}) = \mathcal{M}(X_s \,|\, x_{P(s)})$

largest solution!

The irrelevant (natural) extension

$\mathcal{M}^{\mathrm{irr}}(X_G)$ is the set of $p(X_G)$ for which

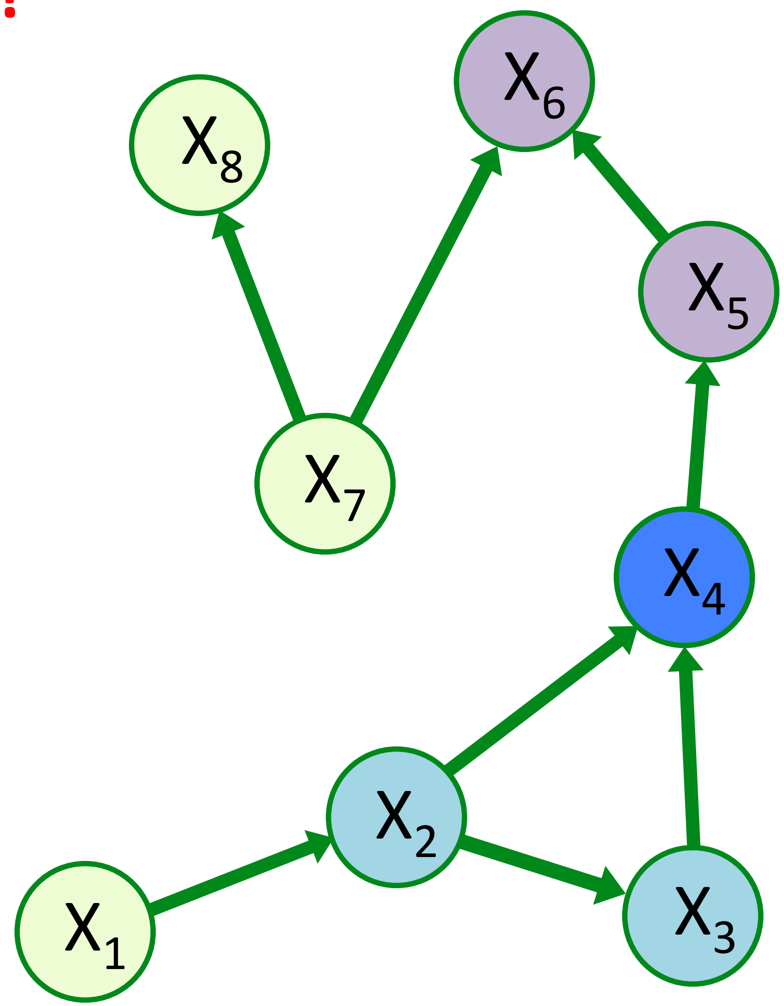
$P(X_s \,|\, x_{P(s)}, x_{N(s)}) \in \mathcal{M}(X_s \,|\, x_{P(s)})$



Credal networks: joint model $\mathcal{M}(X_G)$?

? Independence assumptions ?

- Strong independence
- Epistemic irrelevance
- **Epistemic independence**



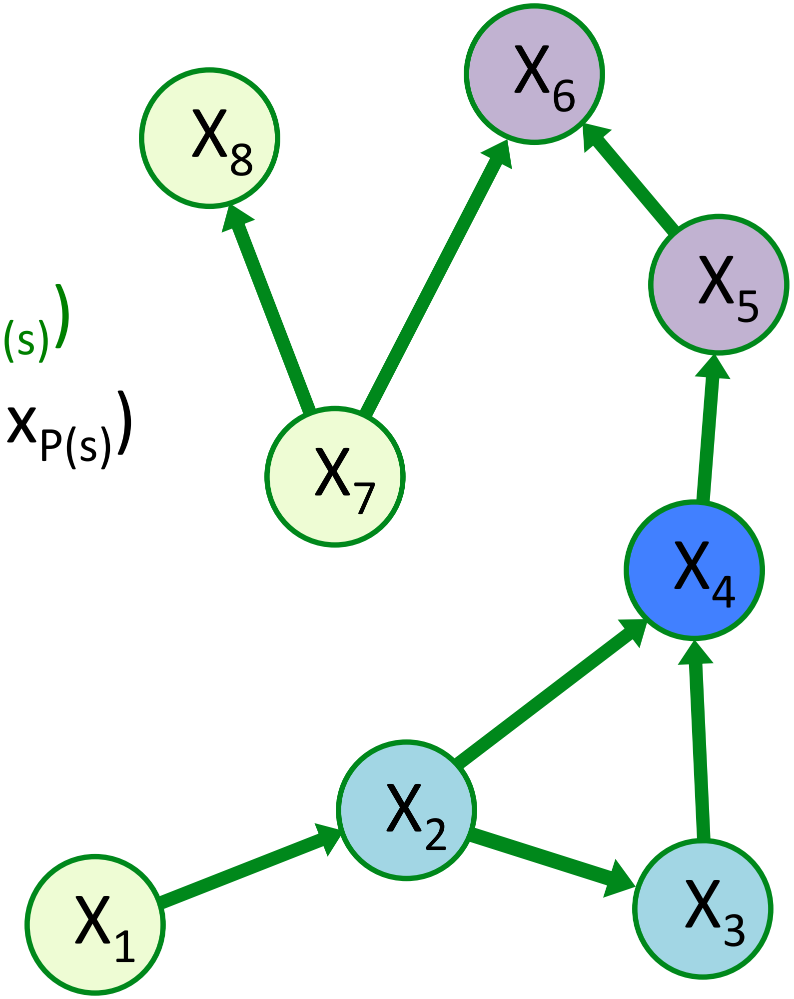
Credal networks: joint model $\mathcal{M}(X_G)$?

Epistemic independence

$\forall s \in G$: **EI**(N(s), s | P(s))

$$\mathcal{M}(X_s \mid x_{P(s)}, x_{N(s)}) = \mathcal{M}(X_s \mid x_{P(s)})$$

and $\mathcal{M}(X_{N(s)} \mid x_{P(s)}, x_s) = \mathcal{M}(X_{N(s)} \mid x_{P(s)})$



Credal networks: joint model $\mathcal{M}(X_G)$?

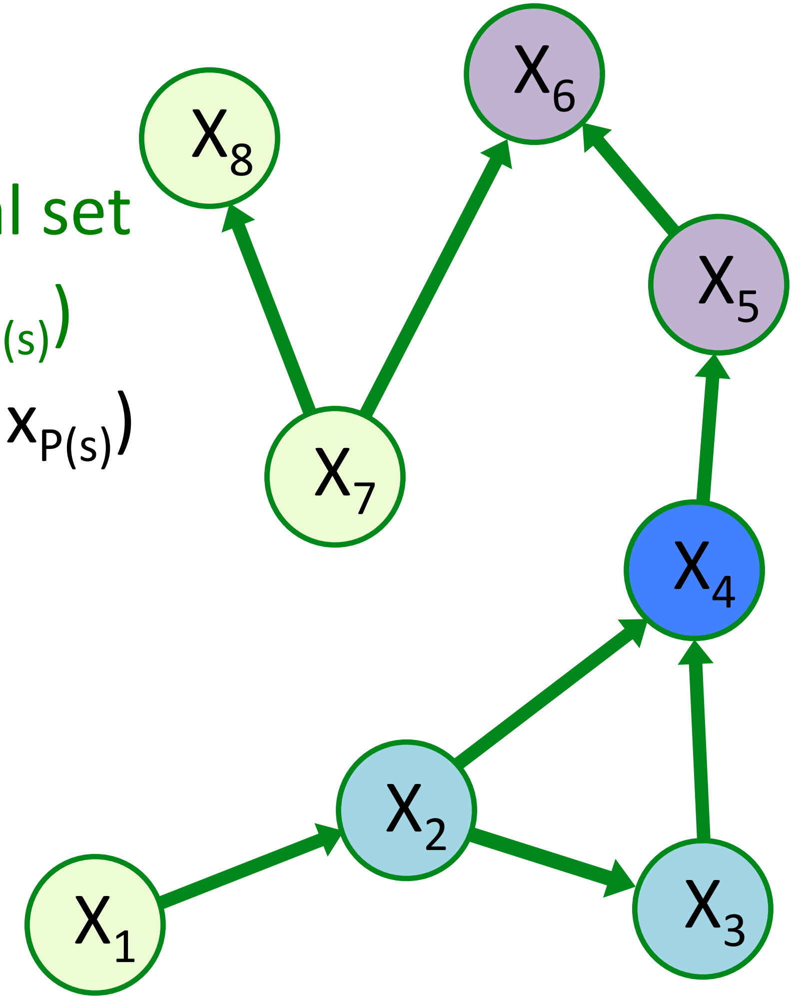
Epistemic independence

$\forall s \in G$: **EI**$(N(s), s \mid P(s))$

+ local credal set

$$\mathcal{M}(X_s \mid x_{P(s)}, x_{N(s)}) = \mathcal{M}(X_s \mid x_{P(s)})$$

and $\mathcal{M}(X_{N(s)} \mid x_{P(s)}, x_s) = \mathcal{M}(X_{N(s)} \mid x_{P(s)})$



Credal networks: joint model $\mathcal{M}(X_G)$?

Epistemic independence

$\forall s \in G$: **EI**$(N(s), s | P(s))$

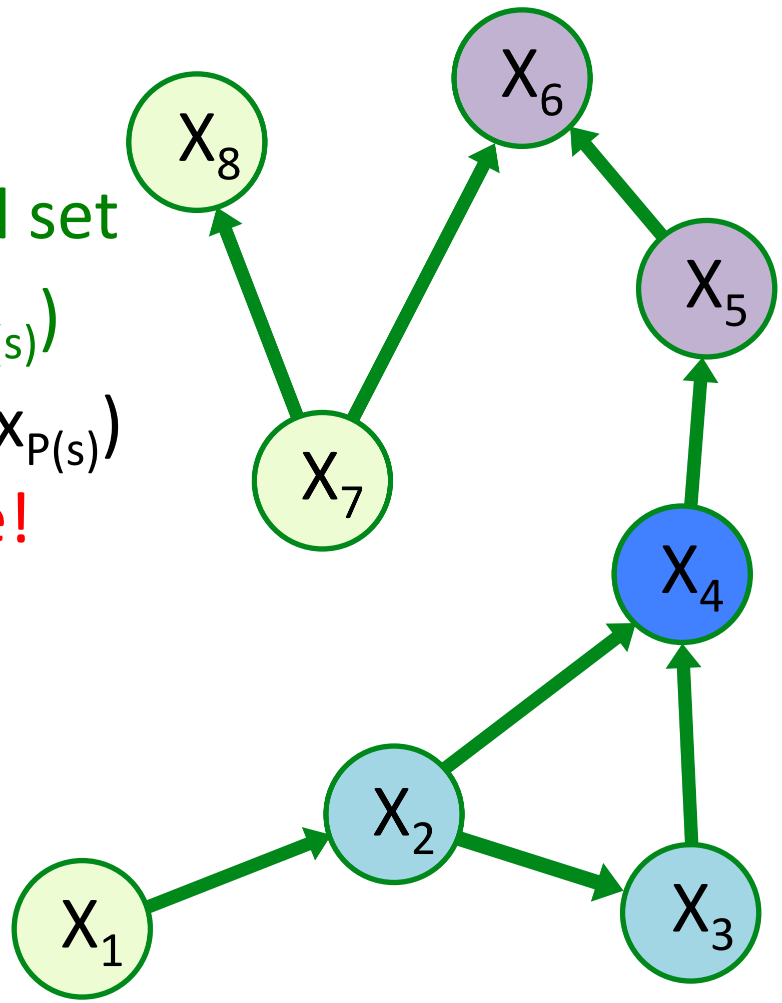
+ local credal set

$$\mathcal{M}(X_s | x_{P(s)}, x_{N(s)}) = \mathcal{M}(X_s | x_{P(s)})$$

and $\mathcal{M}(X_{N(s)} | x_{P(s)}, x_s) = \mathcal{M}(X_{N(s)} | x_{P(s)})$

not unique!

$$\mathcal{M}(X_G)$$



Credal networks: joint model $\mathcal{M}^{\text{ind}}(X_G)$?

Epistemic independence

$\forall s \in G$: **EI**$(N(s), s \,|\, P(s))$

+ local credal set

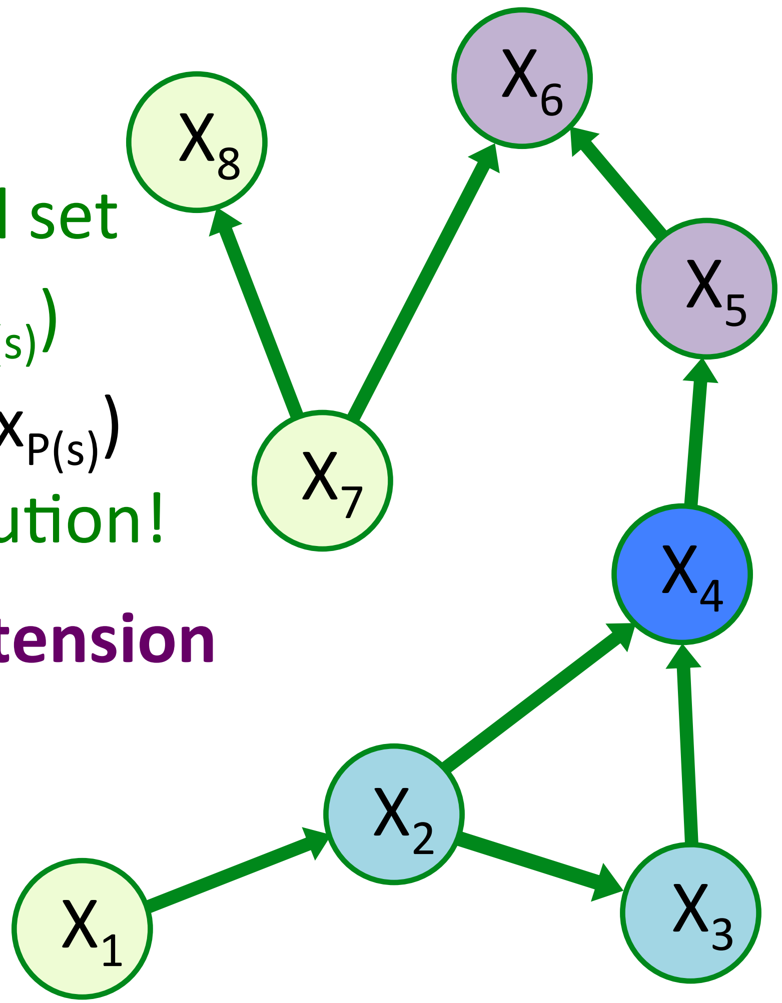
$$\mathcal{M}(X_s \,|\, x_{P(s)}, x_{N(s)}) = \mathcal{M}(X_s \,|\, x_{P(s)})$$

and $\mathcal{M}(X_{N(s)} \,|\, x_{P(s)}, x_s) = \mathcal{M}(X_{N(s)} \,|\, x_{P(s)})$

largest solution!

The independent (natural) extension

$\mathcal{M}^{\text{ind}}(X_G)$?



Credal networks: joint model $\mathcal{M}^{\text{ind}}(X_G)$?

Epistemic independence

$\forall s \in G$: **EI**$(N(s), s \mid P(s))$

+ local credal set

$$\mathcal{M}(X_s \mid x_{P(s)}, x_{N(s)}) = \mathcal{M}(X_s \mid x_{P(s)})$$

and $\mathcal{M}(X_{N(s)} \mid x_{P(s)}, x_s) = \mathcal{M}(X_{N(s)} \mid x_{P(s)})$

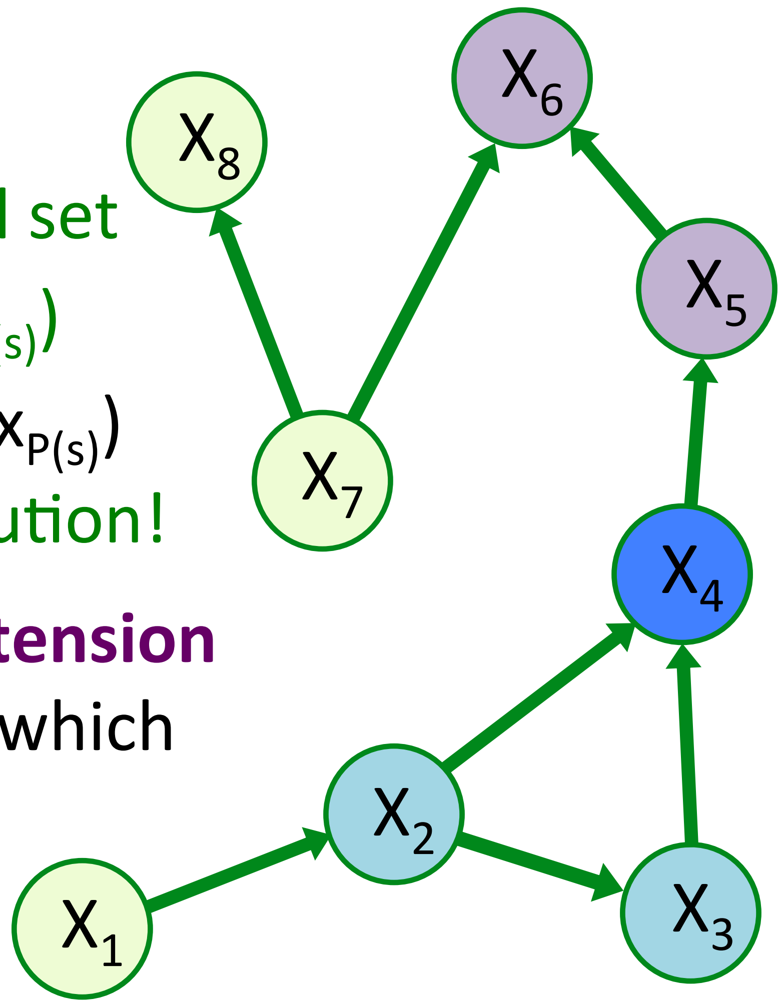
largest solution!

The independent (natural) extension

$\mathcal{M}^{\text{ind}}(X_G)$ is the set of $p(X_G)$ for which

$$p(X_s \mid x_{P(s)}, x_{N(s)}) \in \mathcal{M}(X_s \mid x_{P(s)})$$

and … (no easy description!)



Credal networks

Questions?