WaFT workshop

on imprecise and game-theoretic probabilities

Imprecise Bernoulli processes

Jasper De Bock & Gert de Cooman

5 April 2012

Classical Bernoulli processes

Classical Bernoulli processes

An infinite sequence of binary random variables

$$X_1 , X_2 , \dots , X_n , \dots$$

assuming values in the set

$$\mathscr{X} = \{ a , b \}$$

Classical Bernoulli processes

An infinite sequence of binary random variables

$$X_1 , X_2 , \dots , X_n , \dots$$

defining properties

IID

- **I**ndependent
- **I**dentically **D**istributed

Classical Bernoulli processes

An infinite sequence of binary random variables

$$X_1 , X_2 , \dots , X_n , \dots$$

! IMPLICIT ASSUMPTION !

a single **Bernoulli experiment X_i** has a **precise** and **precisely known** **probability mass function**

Classical Bernoulli processes

$$\boxed{P(X_i = a) = \theta \qquad P(X_i = b) = 1 - \theta}$$

with a fixed $\theta \in [0, 1]$

a single **Bernoulli experiment X_i** has a **precise** and **precisely known** **probability mass function**

Classical Bernoulli processes

$$\boxed{P(X_i = a) = \theta \qquad P\,(X_i = b) = 1 - \theta}$$

with a fixed $\theta \in [0, 1]$

IID

Independent

Identically Distributed

Classical Bernoulli processes

$X_1, X_2, \ldots, X_n$

BINOMIAL DISTRIBUTION
with parameters θ and n

Classical Bernoulli processes

$X_1, X_2, \dots, X_n$

BINOMIAL DISTRIBUTION

with parameters θ and n

For every x = $(x_1, \dots, x_n)$ in $\mathscr{X}^n$:

Probability of occurence p(x) = $\theta^{n(a)}(1- \theta)^{n(b)}$

Classical Bernoulli processes

$X_1 , X_2 , \ldots , X_n$

BINOMIAL DISTRIBUTION
with parameters θ and n

For every x = $(x_1, \ldots, x_n)$ in $\mathscr{X}^n$:

Probability of occurence p(x) = $\theta^{n(a)}(1- \theta)^{n(b)}$

For every gamble (real valued map) f on $\mathscr{X}^n$:

Expected value

$$E(f) = Mn_n(f \mid \theta) = \sum_{x \in \mathcal{X}^n} f(x)p(x)$$

Sensitivity analysis in Bernoulli processes

Sensitivity analysis in Bernoulli processes

An infinite sequence of binary random variables

$$X_1 , X_2 , \ldots , X_n , \ldots$$

defining properties

IID

Independent

Identically **D**istributed

Sensitivity analysis in Bernoulli processes

An infinite sequence of binary random variables

$$X_1 , X_2 , \dots , X_n , \dots$$

! Introducing imprecision !

a single **Bernoulli experiment X_i** has a

precise and **~~precisely~~ known**

probability mass function

Sensitivity analysis in Bernoulli processes

$P(X_i = a) = \theta$ $P(X_i = b) = 1 - \theta$

θ varies over an interval $[\underline{\theta}, \overline{\theta}]$

a single **Bernoulli experiment X_i** has a **precise** and ~~**precisely known**~~ **probability mass function**

Sensitivity analysis in Bernoulli processes

$$P(X_i = a) = \theta \qquad P(X_i = b) = 1 - \theta$$

θ varies over an interval $[\underline{\theta}, \overline{\theta}]$

IID

- **I**ndependent
- **I**dentically **D**istributed

Sensitivity analysis in Bernoulli processes

For a fixed $\theta \in [0, 1]$:

For every gamble f on $\mathscr{X}^n$:

Expected value: $E(f) = Mn_n(f \mid \theta) = \sum_{x \in \mathcal{X}^n} f(x)p(x)$

Sensitivity analysis in Bernoulli processes

For a fixed $\theta \in [0,1]$:

For every gamble f on $\mathscr{X}^n$:

Expected value: $E(f) = Mn_n(f \,|\, \theta\ ) = \sum_{x \in \mathcal{X}^n} f(x)p(x)$

If θ varies over an interval $[\underline{\theta}, \overline{\theta}]$:

Sensitivity analysis in Bernoulli processes

For a fixed $\theta \in [0,1]$:

For every gamble f on $\mathscr{X}^n$:

Expected value: $E(f) = Mn_n(f \mid \theta) = \sum_{x \in \mathcal{X}^n} f(x)p(x)$

If θ varies over an interval $[\underline{\theta}, \overline{\theta}]$:

Lower and upper expected value:

$$\overline{E}(f) = \max\{\mathrm{Mn}_n(f|\theta) : \theta \in [\underline{\theta}, \overline{\theta}]\}$$

$$\underline{E}(f) = \min\{\mathrm{Mn}_n(f|\theta) : \theta \in [\underline{\theta}, \overline{\theta}]\}$$

Imprecise Bernoulli processes

Imprecise Bernoulli processes

An infinite sequence of binary random variables

$$X_1 , X_2 , \dots , X_n , \dots$$

! dropping both assumptions !

a single **Bernoulli experiment X_i** has a
~~precise~~ and **~~precisely~~ known**
probability mass function

Imprecise Bernoulli processes

An infinite sequence of binary random variables

$$X_1 , X_2 , \dots , X_n , \dots$$

a single **Bernoulli experiment X_i** is regarded as **inherently imprecise**

We do not assume the existence of an underlying precise probability distribution

Imprecise Bernoulli processes

No underlying precise probability distribution!

A set $\mathcal{D}$ of desirable gambles

We model a **subject's beliefs** regarding the possible **outcomes Ω of an experiment** by looking at the **gambles he is willing to accept**

Imprecise Bernoulli processes

No underlying precise probability distribution!

A set $\mathcal{D}$ of desirable gambles

Rationality criteria:

COHERENT

C1. *if* $f = 0$ *then* $f \notin \mathcal{D}$;
C2. *if* $f > 0$ *then* $f \in \mathcal{D}$;
C3. *if* $f \in \mathcal{D}$ *then* $\lambda f \in \mathcal{D}$ *[scaling]*;
C4. *if* $f_1, f_2 \in \mathcal{D}$ *then* $f_1 + f_2 \in \mathcal{D}$ *[combination]*.

($f > 0$ iff $f \geq 0$ and $f \neq 0$)

Imprecise Bernoulli processes

No underlying precise probability distribution!

A **coherent** set $\mathcal{D}$ of desirable gambles

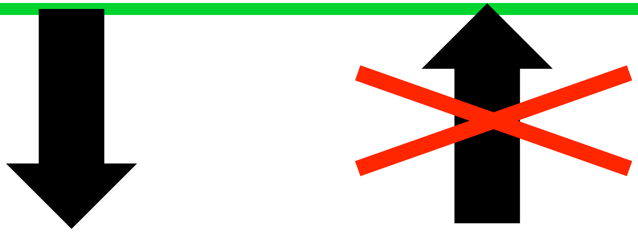
↓

Set of probability mass functions

Imprecise Bernoulli processes

No underlying precise probability distribution!

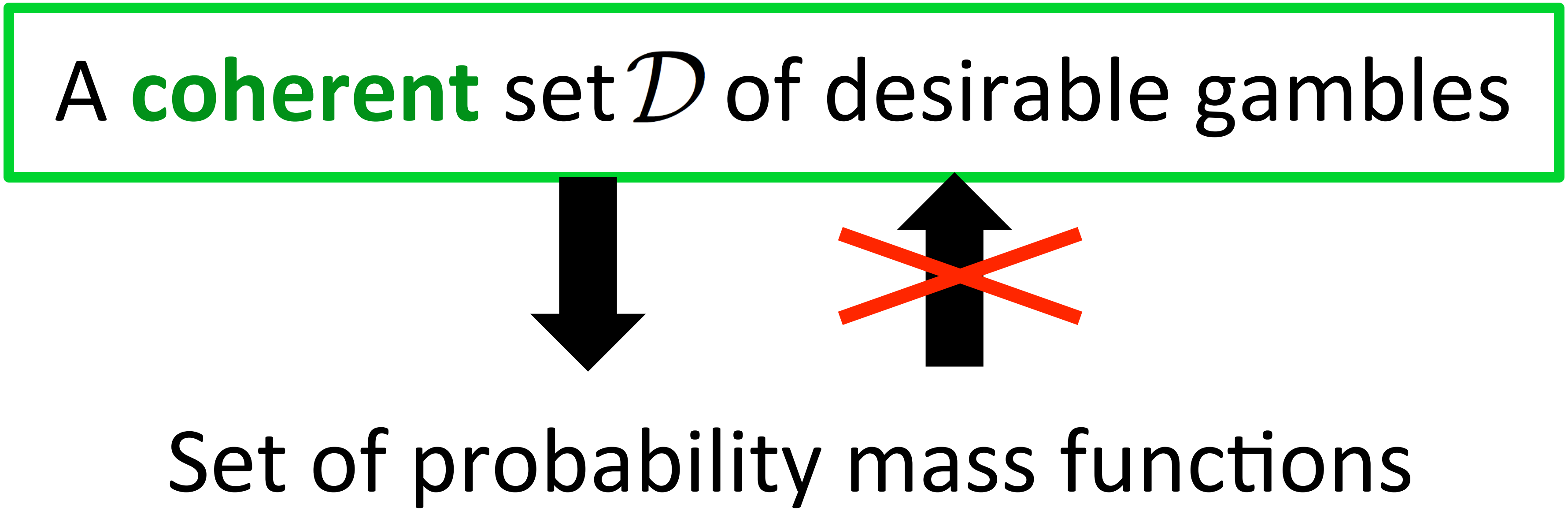
A **coherent** set $\mathcal{D}$ of desirable gambles



Set of probability mass functions

Imprecise Bernoulli processes

No underlying precise probability distribution!



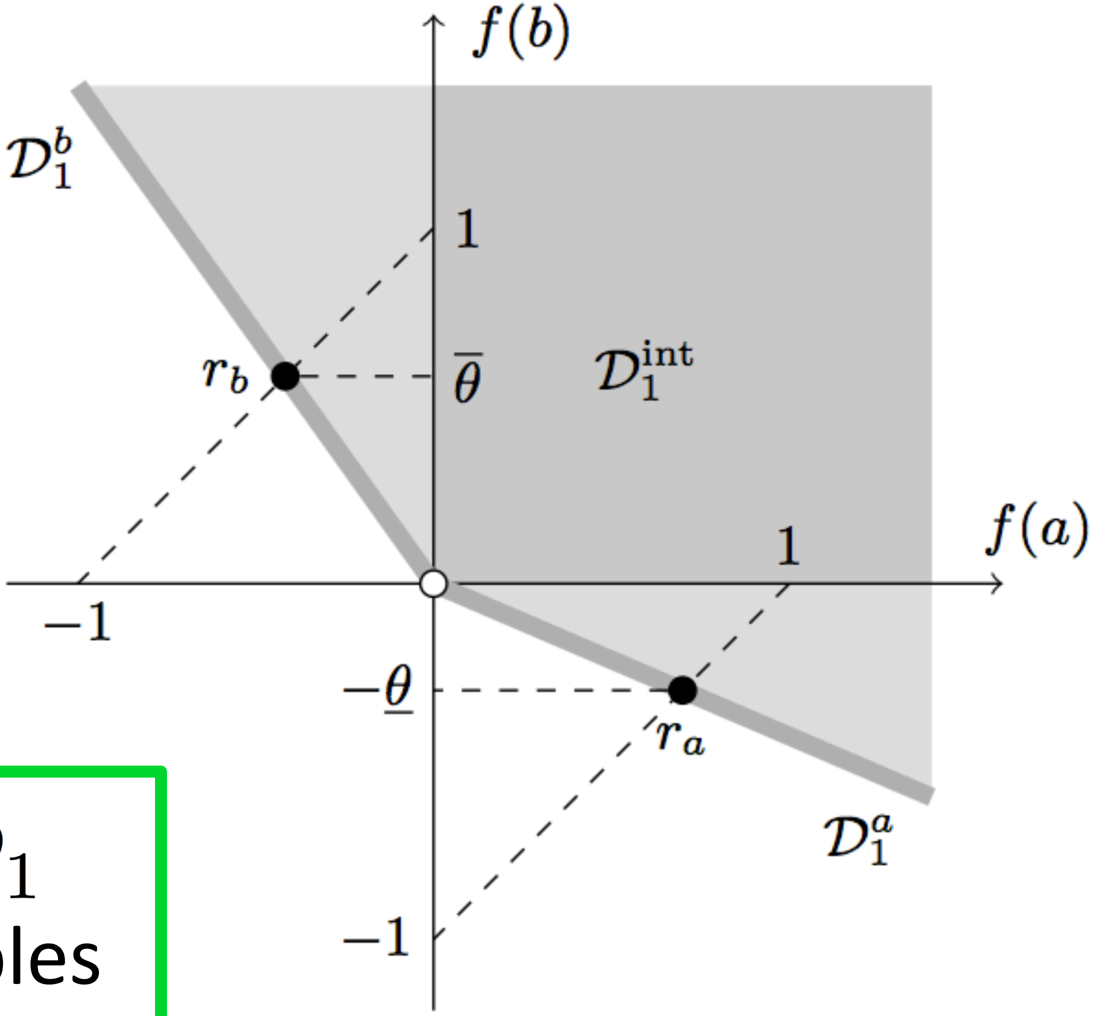
A **coherent** set $\mathcal{D}$ of desirable gambles

Set of probability mass functions

Just a mathematical connection!

Imprecise Bernoulli processes

a single
Bernoulli experiment

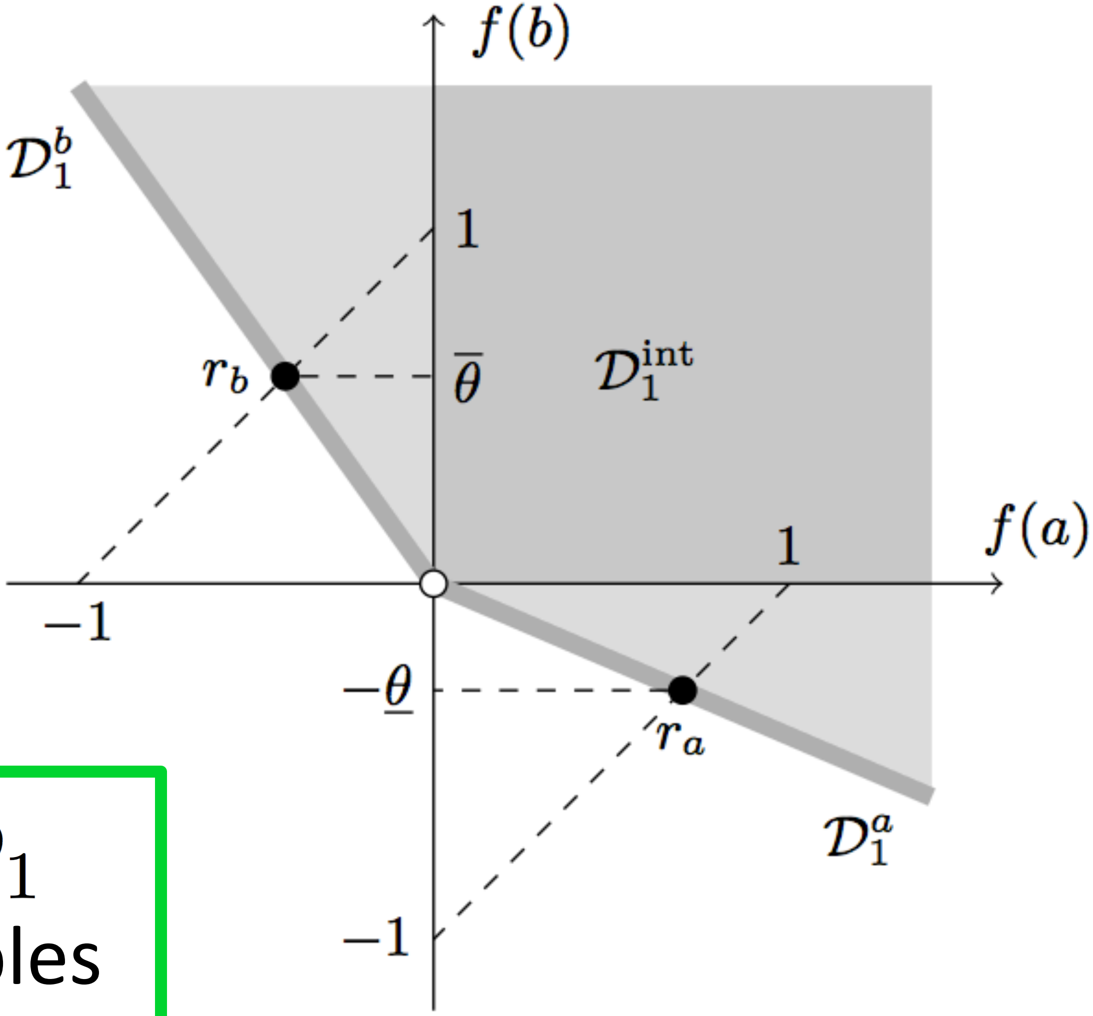


A coherent set $\mathcal{D}_1$ of desirable gambles

Imprecise Bernoulli processes

Due to coherence:

$$0 \leq \underline{\theta} \leq \overline{\theta} \leq 1$$



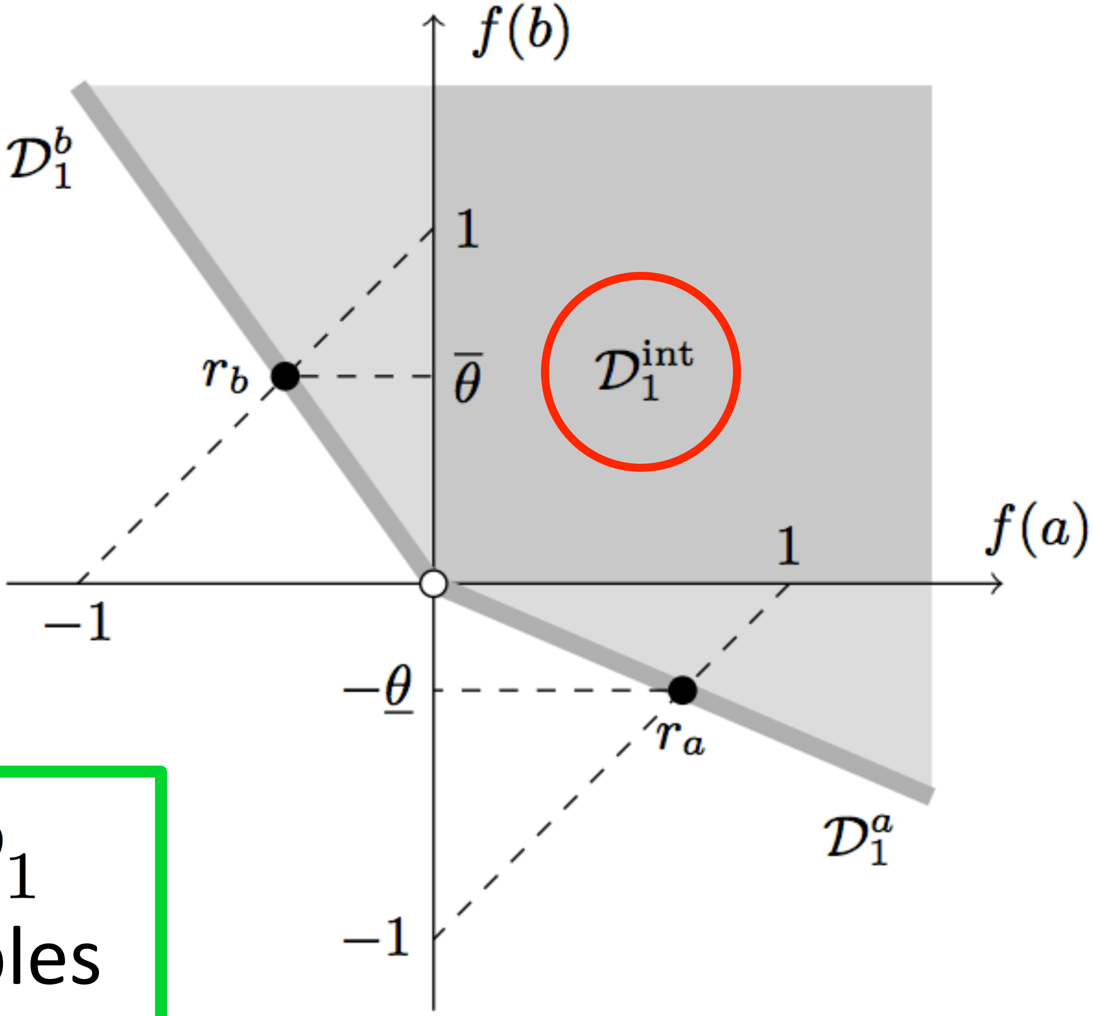
A coherent set $\mathcal{D}_1$ of desirable gambles

Imprecise Bernoulli processes

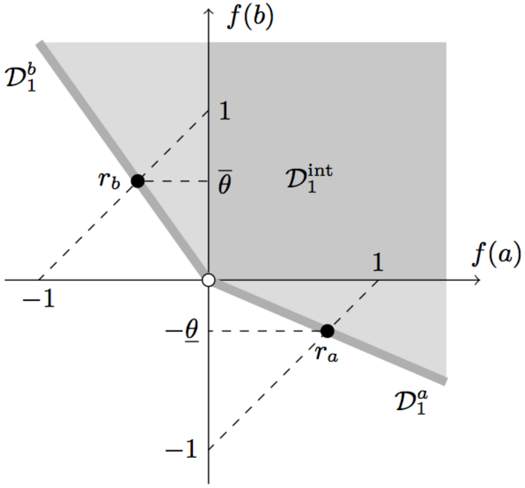
Due to coherence:

$$0 \underset{<}{\cancel{\leq}} \underline{\theta} \underset{<}{\cancel{\leq}} \overline{\theta} \underset{<}{\cancel{\leq}} 1$$

A coherent set $\mathcal{D}_1$ of desirable gambles



Imprecise Bernoulli processes



Due to coherence:

$$0 \underset{<}{\cancel{\leq}} \underline{\theta} \underset{<}{\cancel{\leq}} \overline{\theta} \underset{<}{\cancel{\leq}} 1$$

A coherent set $\mathcal{D}_1$ of desirable gambles

Associated set of mass functions:

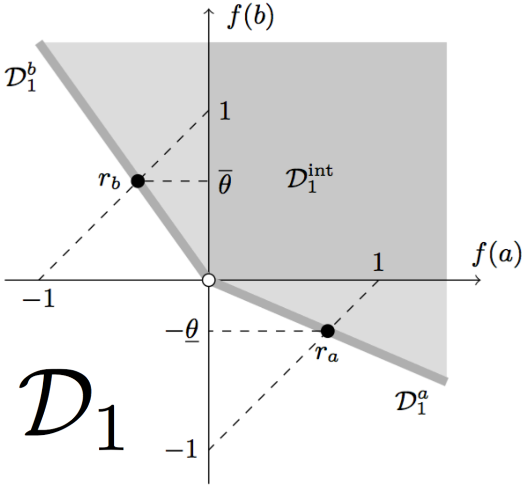
$P(X_i = a) = \theta$

$P(X_i = b) = 1 - \theta$

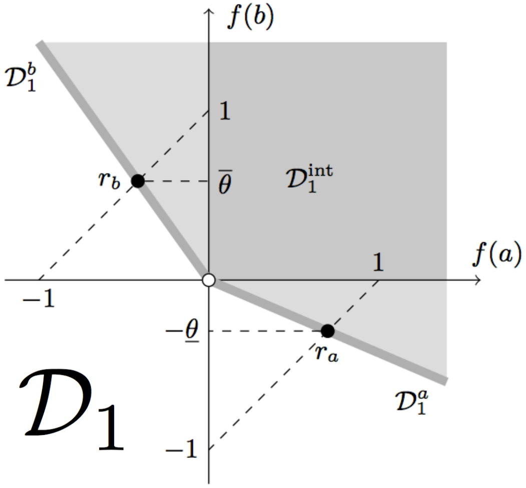
θ varies over $[\underline{\theta}, \overline{\theta}]$

Imprecise Bernoulli processes

Single **Bernoulli experiment** $\mathcal{D}_1$



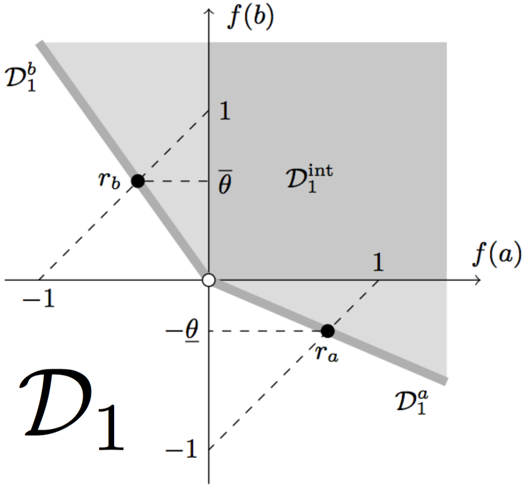
Imprecise Bernoulli processes



Single **Bernoulli experiment** $\mathcal{D}_1$

Imprecise Bernoulli process

Imprecise Bernoulli processes

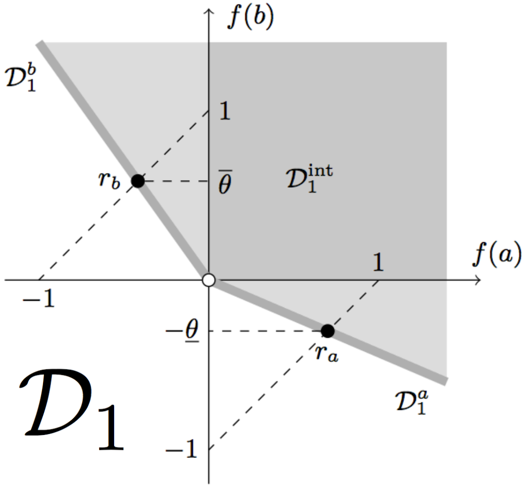


Single **Bernoulli experiment** $\mathcal{D}_1$

- **Exchangeability**
- **Epistemic independence**

Imprecise Bernoulli process

Imprecise Bernoulli processes



Single **Bernoulli experiment** $\mathcal{D}_1$

- **Exchangeability**
- **Epistemic independence**

Imprecise Bernoulli process

Imprecise Bernoulli processes

Exchangeability

Consider any permutation **π** of the set of indices **$\{1, 2, \dots, n\}$**

For any **$x = (x_1, x_2, \dots, x_n)$** in $\mathscr{X}^n$ we let **$\pi x := (x_{\pi(1)}, x_{\pi(2)}, \dots, x_{\pi(n)})$**

For any **gamble f** on $\mathscr{X}^n$ we let **$\pi^t f := f \circ \pi$**, so **$(\pi^t f)(x) = f(\pi x)$**

Imprecise Bernoulli processes

Exchangeability

Consider any permutation π of the set of indices **{1, 2, … , n}**

For any $\mathbf{x} = (x_1, x_2, \dots, x_n)$ in $\mathscr{X}^n$ we let $\pi\mathbf{x} := (x_{\pi(1)}, x_{\pi(2)}, \dots, x_{\pi(n)})$

For any **gamble f** on $\mathscr{X}^n$ we let $\pi^t f := f \circ \pi$, so $(\pi^t f)(\mathbf{x}) = f(\pi\mathbf{x})$

$X_1, X_2, \dots, X_n$ is assessed to be exchangeable

⟺ You are willing to exchange **f** for $\pi^t f$

Exchangeability

Consider any permutation π of the set of indices $\{1, 2, \dots, n\}$

For any $x = (x_1, x_2, \dots, x_n)$ in $\mathscr{X}^n$ we let $\pi x := (x_{\pi(1)}, x_{\pi(2)}, \dots, x_{\pi(n)})$

For any **gamble f** on $\mathscr{X}^n$ we let $\pi^t f := f \circ \pi$, so $(\pi^t f)(x) = f(\pi x)$

$X_1, X_2, \dots, X_n$ is assessed to be exchangeable

⟺ You are willing to exchange **f** for $\pi^t f$

$\mathcal{D}_n$ is exchangeable

⟺ $f - \pi^t f$ is (weakly) desirable $\approx$ $f - \pi^t f \in \mathcal{D}_n$

Imprecise Bernoulli processes

Exchangeability

Infinite exchangeable sequence $X_1 , X_2 , \dots , X_n , \dots$

Family of sets $\mathcal{D}_n$ of desirable gambles on $\mathscr{X}^n$

(for all $n \in \mathbb{N}_0$)

- Time consistent
- Each $\mathcal{D}_n$ is coherent
- Each $\mathcal{D}_n$ is exchangeable

Imprecise Bernoulli processes

Exchangeability

Infinite exchangeable sequence $X_1, X_2, \ldots, X_n, \ldots$

Family of sets $\mathcal{D}_n$ of desirable gambles on $\mathscr{X}^n$

(for all $n \in \mathbb{N}_0$)

- Time consistent
- Each $\mathcal{D}_n$ is coherent
- Each $\mathcal{D}_n$ is exchangeable

How to impose this property?

Imprecise Bernoulli processes

Exchangeability

!

BINOMIAL DISTRIBUTION (θ and n)

For every gamble f on $\mathscr{X}^n$:

$$E(f) = Mn_n(f \mid \theta) = \sum_{x \in \mathcal{X}^n} f(x)p(x)$$

↳ $\theta^{n(a)}(1- \theta)^{n(b)}$

Imprecise Bernoulli processes

Exchangeability

BINOMIAL DISTRIBUTION (θ and n)

For every gamble f on $\mathscr{X}^n$:

$$E(f) = \boxed{Mn_n(f \mid \theta)} = \sum_{x \in \mathcal{X}^n} f(x)p(x)$$

↳ $\theta^{n(a)}(1-\theta)^{n(b)}$

Polynomial function of θ

$p(\theta) := Mn_n(f \mid \theta)$ $(\deg(p) \leq n)$

Imprecise Bernoulli processes

Exchangeability

Infinite exchangeable sequence $X_1, X_2, \dots, X_n, \dots$

Family of sets $\boxed{\mathcal{D}_n}$ of desirable gambles on $\mathscr{X}^n$

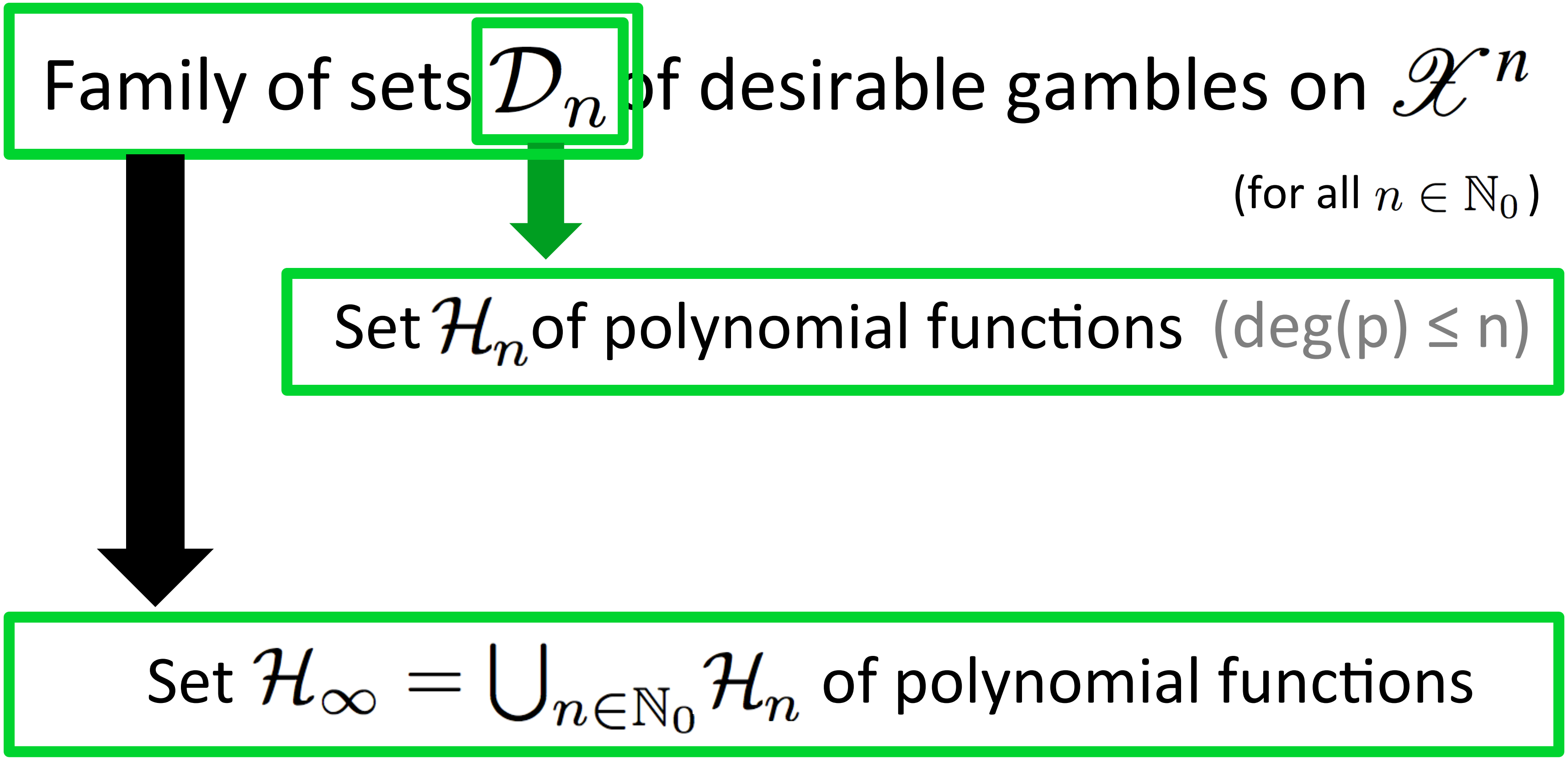
(for all $n \in \mathbb{N}_0$)

Set $\mathcal{H}_n$ of polynomial functions (deg(p) ≤ n)

Imprecise Bernoulli processes

Exchangeability

Infinite exchangeable sequence **$X_1, X_2, \ldots, X_n, \ldots$**



Family of sets $\mathcal{D}_n$ of desirable gambles on $\mathscr{X}^n$

(for all $n \in \mathbb{N}_0$)

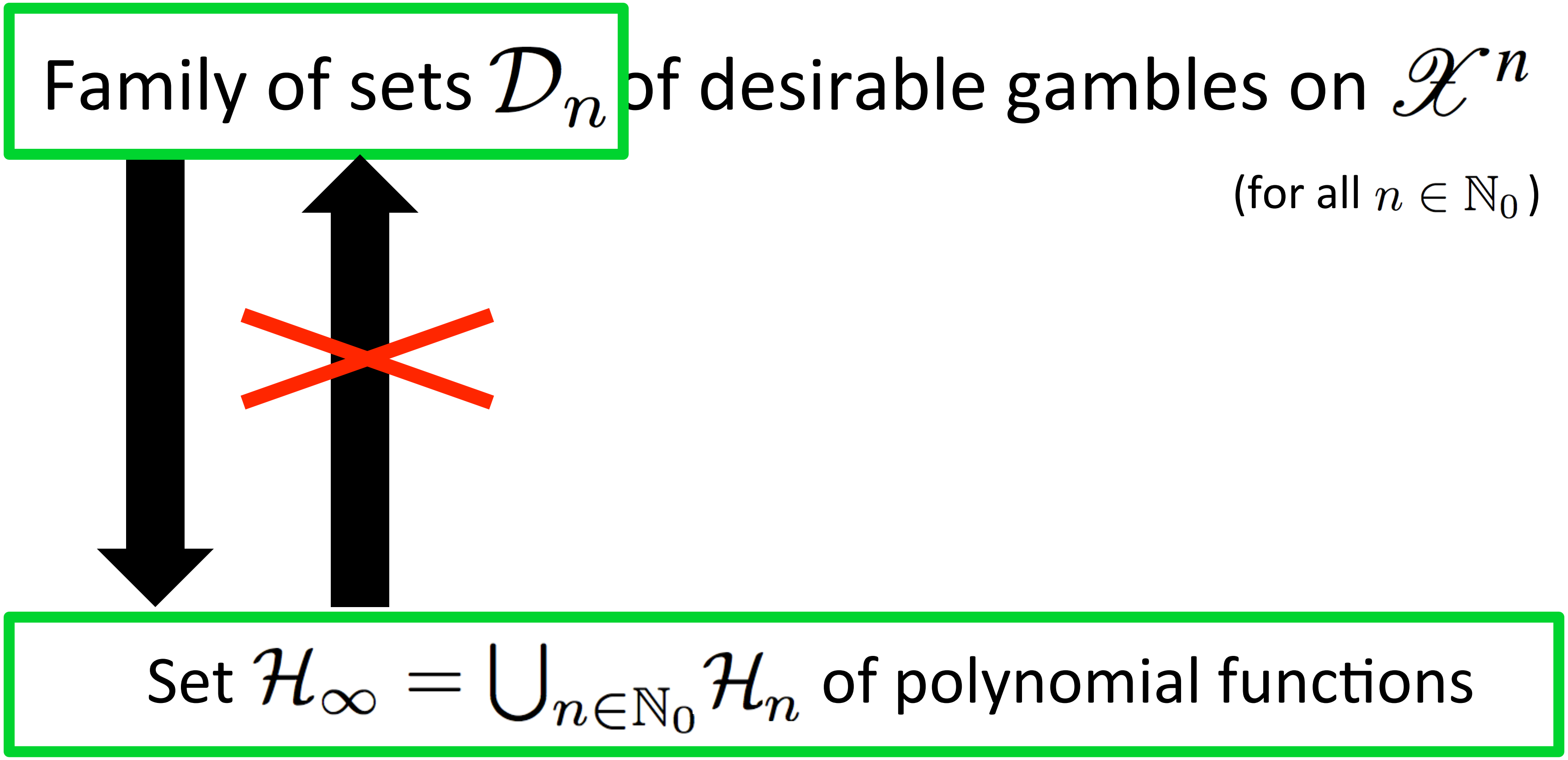
Set $\mathcal{H}_n$ of polynomial functions (deg(p) ≤ n)

Set $\mathcal{H}_\infty = \bigcup_{n\in\mathbb{N}_0}\mathcal{H}_n$ of polynomial functions

Imprecise Bernoulli processes

Exchangeability

Infinite exchangeable sequence **$X_1, X_2, \ldots, X_n, \ldots$**



Family of sets $\mathcal{D}_n$ of desirable gambles on $\mathscr{X}^n$

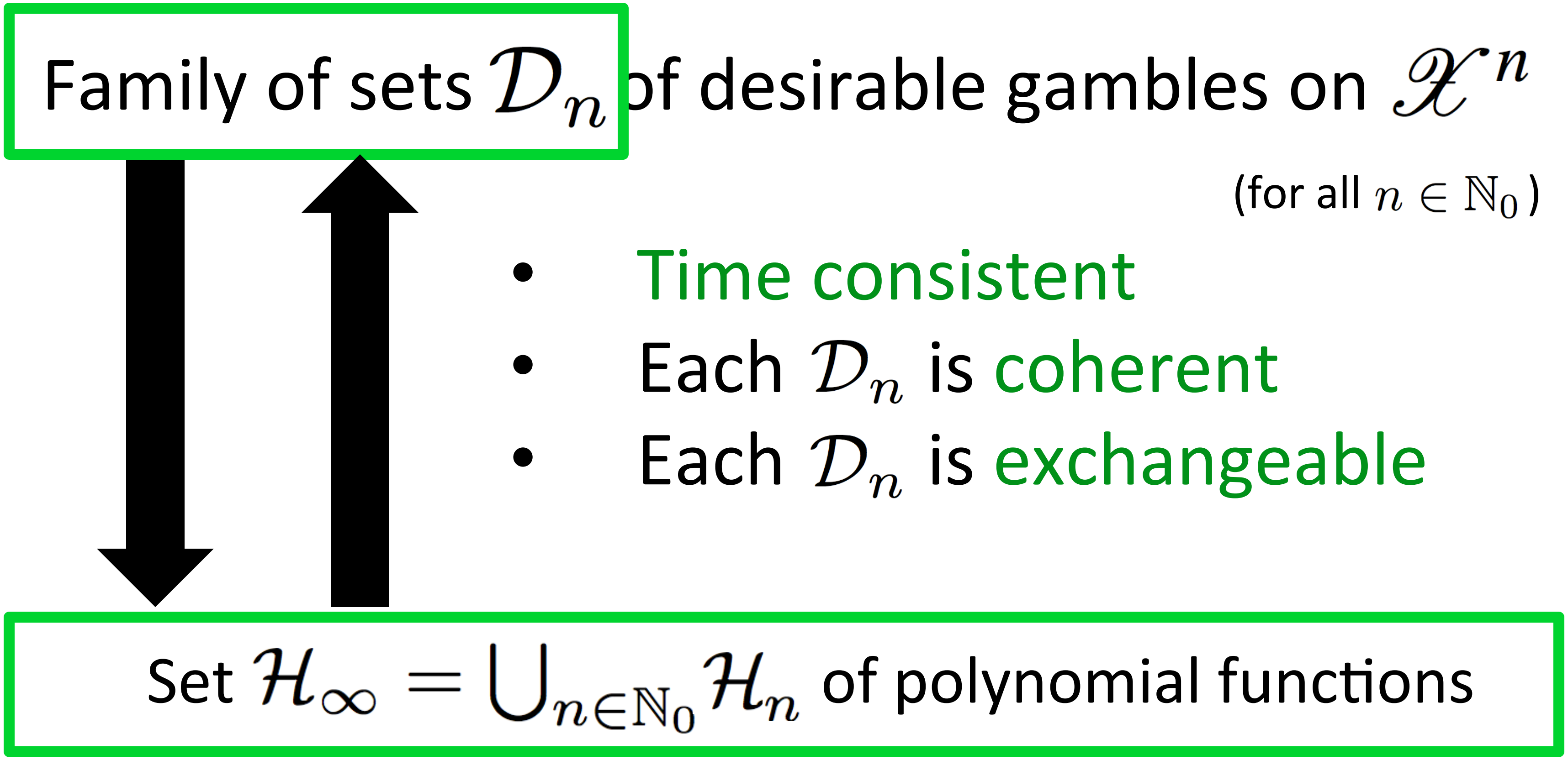
(for all $n \in \mathbb{N}_0$)

Set $\mathcal{H}_\infty = \bigcup_{n\in\mathbb{N}_0} \mathcal{H}_n$ of polynomial functions

Imprecise Bernoulli processes

Exchangeability

Infinite exchangeable sequence **X_1, X_2, … , X_n, …**



Family of sets $\mathcal{D}_n$ of desirable gambles on $\mathscr{X}^n$

(for all $n \in \mathbb{N}_0$)

- Time consistent
- Each $\mathcal{D}_n$ is coherent
- Each $\mathcal{D}_n$ is exchangeable

Set $\mathcal{H}_\infty = \bigcup_{n\in\mathbb{N}_0} \mathcal{H}_n$ of polynomial functions

Imprecise Bernoulli processes

Exchangeability

Infinite exchangeable sequence $X_1 , X_2 , \dots , X_n , \dots$

Bernstein coherent:

B1. *if* $p = 0$ *then* $p \notin \mathcal{H}$;

B2. *if* $p \in \mathcal{V}^+$, *then* $p \in \mathcal{H}$;

B3. *if* $p \in \mathcal{H}$ *then* $\lambda p \in \mathcal{H}$;

B4. *if* $p_1, p_2 \in \mathcal{H}$ *then* $p_1 + p_2 \in \mathcal{H}$.

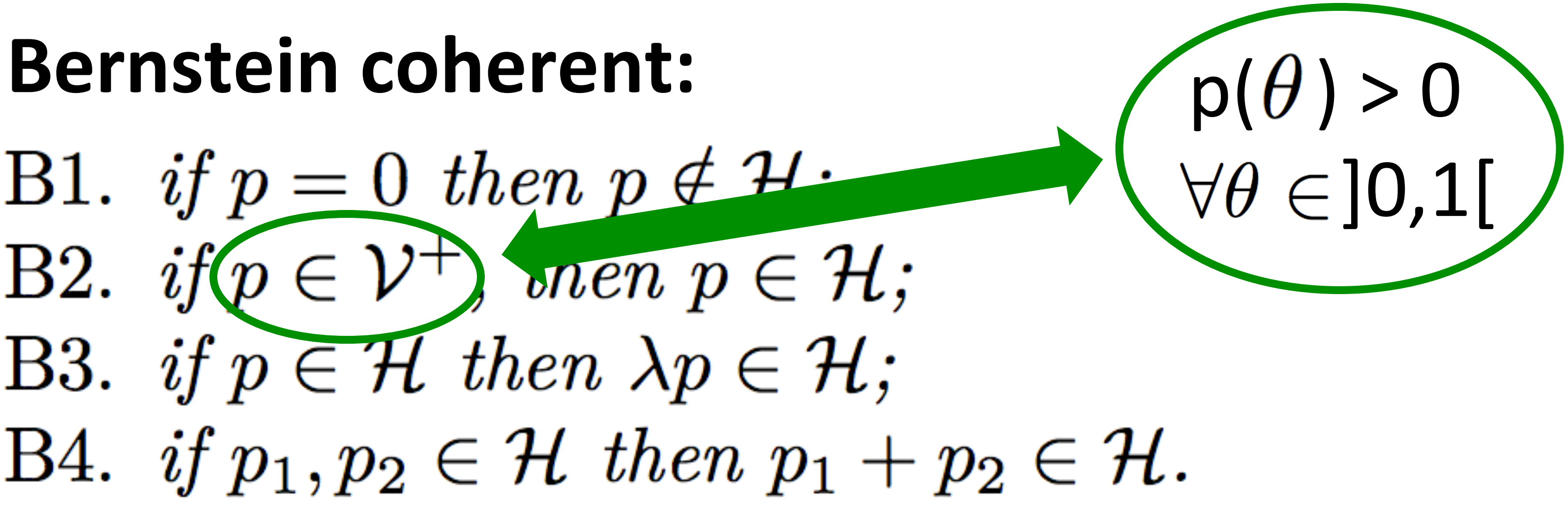
Set $\mathcal{H}_\infty = \bigcup_{n \in \mathbb{N}_0} \mathcal{H}_n$ of polynomial functions

Imprecise Bernoulli processes

Exchangeability

Infinite exchangeable sequence **$X_1, X_2, \dots, X_n, \dots$**

Bernstein coherent:



$p(\theta) > 0$
$\forall\theta \in]0,1[$

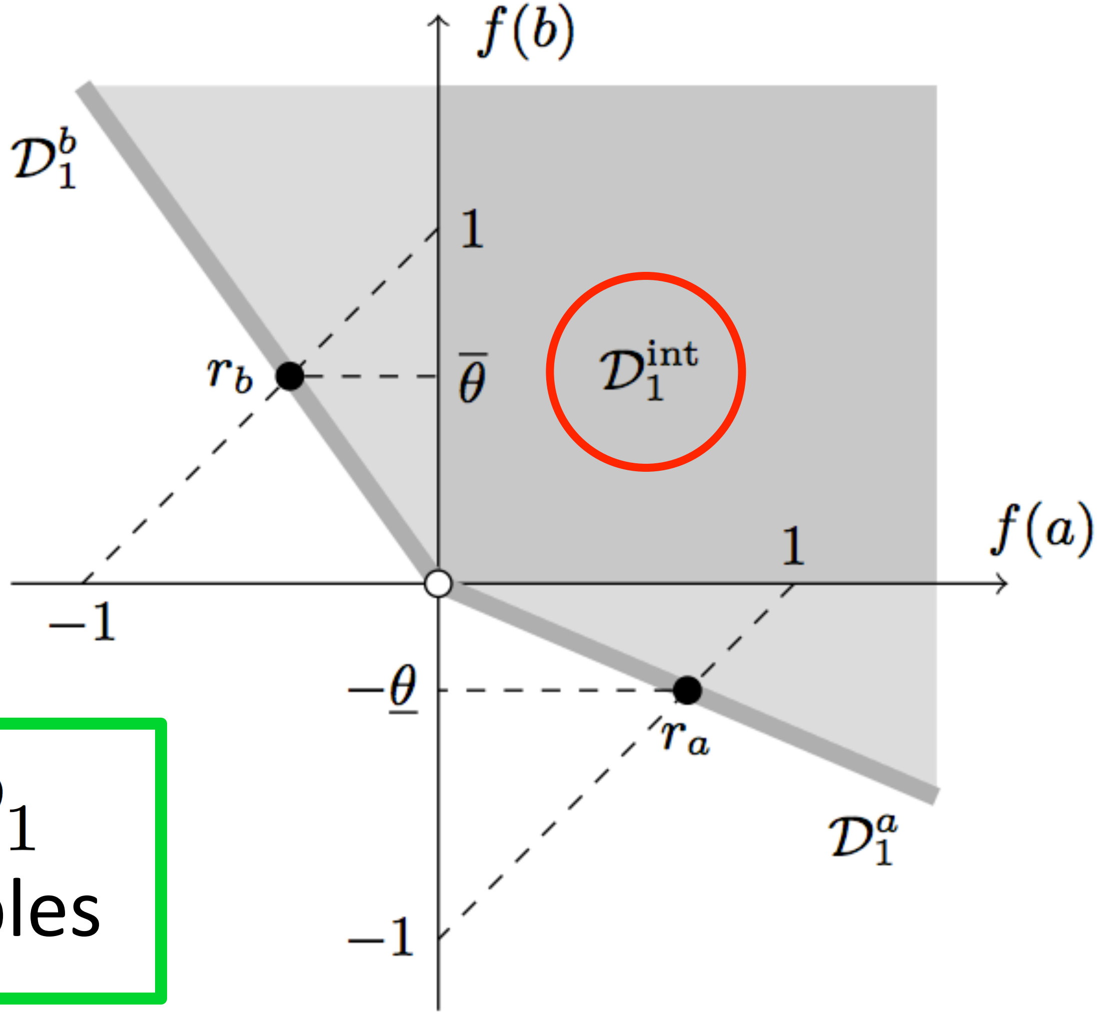
B1. *if* $p = 0$ *then* $p \notin \mathcal{H}$;
B2. *if* $p \in \mathcal{V}^+$, *then* $p \in \mathcal{H}$;
B3. *if* $p \in \mathcal{H}$ *then* $\lambda p \in \mathcal{H}$;
B4. *if* $p_1, p_2 \in \mathcal{H}$ *then* $p_1 + p_2 \in \mathcal{H}$.

Set $\mathcal{H}_\infty = \bigcup_{n\in\mathbb{N}_0} \mathcal{H}_n$ of polynomial functions

Imprecise Bernoulli processes

Exchangeability

$$0 \underset{<}{\cancel{\leq}} \underline{\theta} \underset{<}{\cancel{\leq}} \overline{\theta} \underset{<}{\cancel{\leq}} 1$$

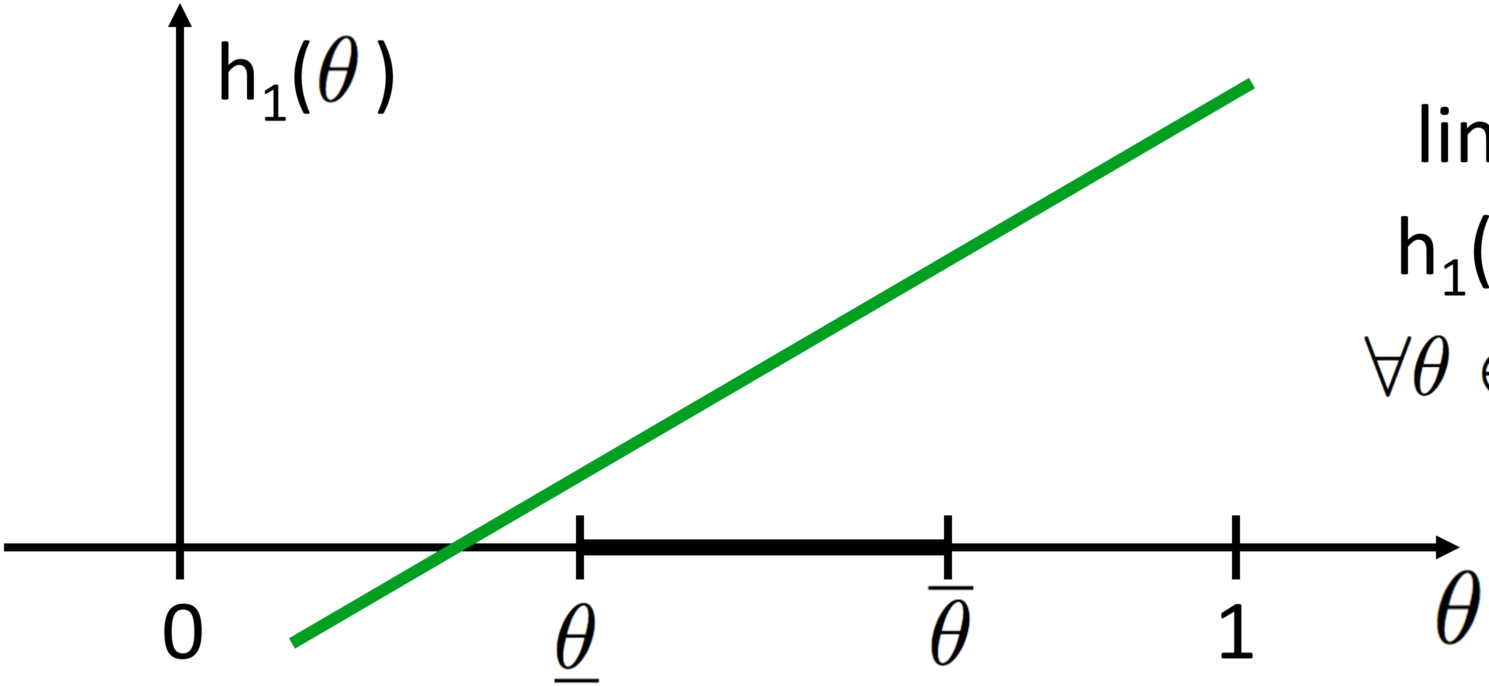


A coherent set $\mathcal{D}_1$ of desirable gambles

Imprecise Bernoulli processes

Exchangeability

$\mathcal{H}_1$



linear h_1

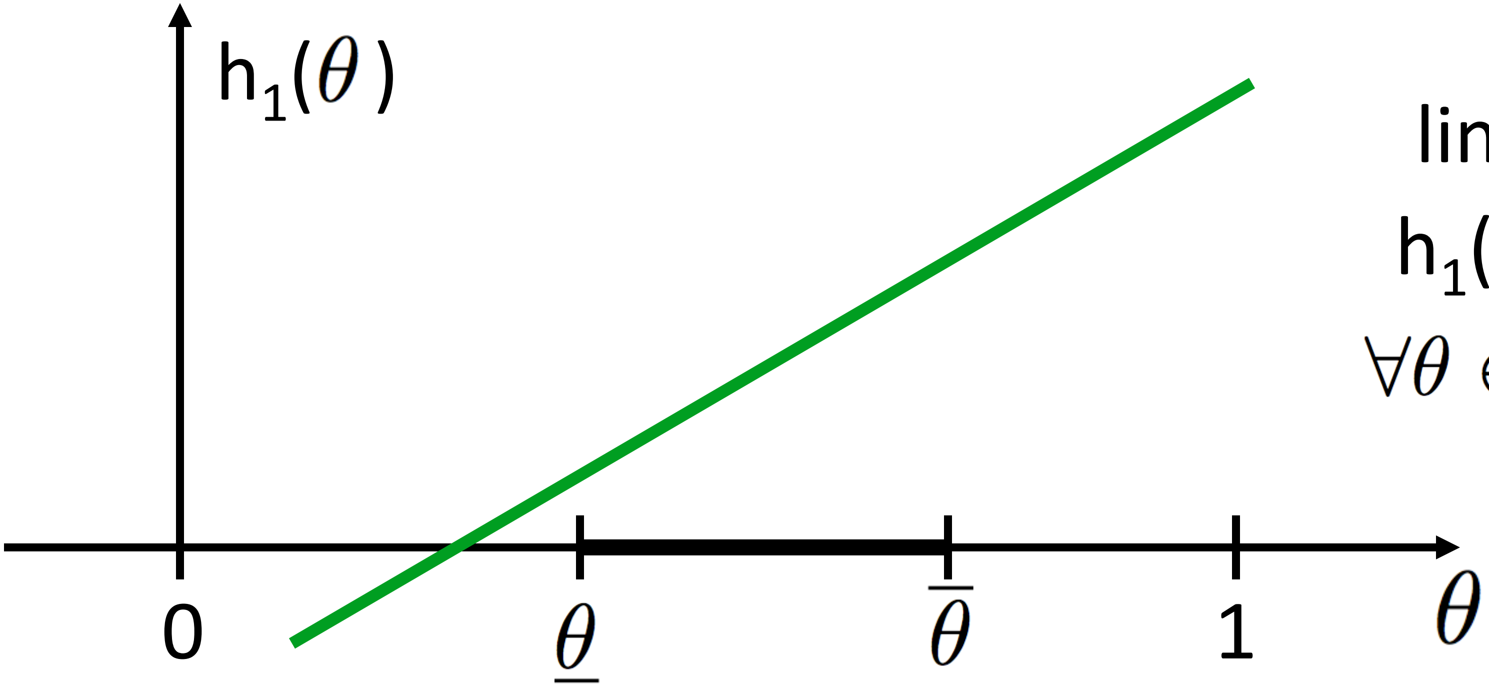
$h_1(\theta) > 0$

$\forall\theta \in [\underline{\theta}, \overline{\theta}]$

Imprecise Bernoulli processes

Exchangeability

$\mathcal{H}_1$



linear h_1

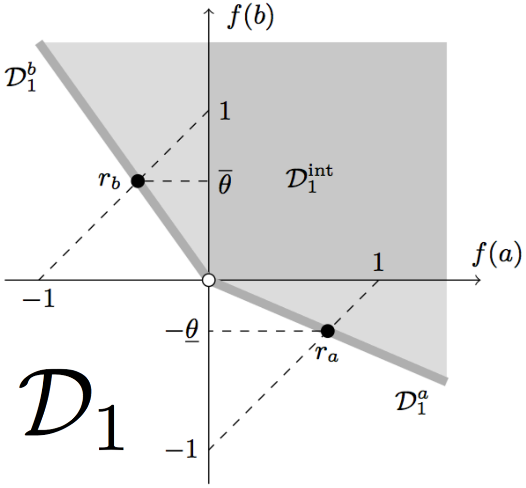
$h_1(\theta) > 0$

$\forall\theta \in [\underline{\theta}, \overline{\theta}]$

Set $\mathcal{H}_\infty = \bigcup_{n\in\mathbb{N}_0} \mathcal{H}_n$ of polynomial functions

?

Imprecise Bernoulli processes



Single **Bernoulli experiment** $\mathcal{D}_1$

- **Exchangeability**
- **Epistemic independence**

Imprecise Bernoulli process

Imprecise Bernoulli processes

Epistemic independence

Infinite sequence $X_1, X_2, \dots, X_n, \dots$

assessment of epistemic independence

Imprecise Bernoulli processes

Epistemic independence

Infinite sequence $X_1 , X_2 , \ldots , X_n , \ldots$

assessment of epistemic independence

⇕

Learning the value of any **finite number of variables does not change our beliefs about** any finite subset of the **remaining, unobserved ones**.

Imprecise Bernoulli processes

Epistemic independence

Infinite sequence $X_1, X_2, \dots, X_n, \dots$

assessment of epistemic independence

Set $\mathcal{H}_\infty = \bigcup_{n\in\mathbb{N}_0}\mathcal{H}_n$ of polynomial functions

?

Imprecise Bernoulli processes

Epistemic independence

Infinite sequence $X_1, X_2, \ldots, X_n, \ldots$

assessment of epistemic independence

$$h \in \mathcal{H}_\infty \begin{array}{l} \Leftrightarrow \theta h \in \mathcal{H}_\infty \\ \Leftrightarrow (1-\theta) h \in \mathcal{H}_\infty \end{array}$$

Set $\mathcal{H}_\infty = \bigcup_{n \in \mathbb{N}_0} \mathcal{H}_n$ of polynomial functions

?

Imprecise Bernoulli processes

Epistemic independence

$$h_1 \in \mathcal{H}_1 \in \mathcal{H}_\infty \Rightarrow \theta^k(1-\theta)^l h_1 \in \mathcal{H}_\infty$$

$$h \in \mathcal{H}_\infty \begin{array}{l} \Leftrightarrow \theta h \in \mathcal{H}_\infty \\ \Leftrightarrow (1-\theta) h \in \mathcal{H}_\infty \end{array}$$

Set $\mathcal{H}_\infty = \bigcup_{n \in \mathbb{N}_0} \mathcal{H}_n$ of polynomial functions

?

Imprecise Bernoulli processes

Epistemic independence

$$h_1 \in \mathcal{H}_1 \in \mathcal{H}_\infty \Rightarrow \theta^k (1- \theta)^l h_1 \in \mathcal{H}_\infty$$

B4. *if* $p_1, p_2 \in \mathcal{H}$ *then* $p_1 + p_2 \in \mathcal{H}$

$$\Rightarrow p\, h_1 \in \mathcal{H}_\infty \quad \boxed{p \in \mathcal{V}^+}$$

Imprecise Bernoulli processes

Epistemic independence

$$h_1 \in \mathcal{H}_1 \in \mathcal{H}_\infty \Rightarrow \theta^k (1-\theta)^l h_1 \in \mathcal{H}_\infty$$

B4. *if* $p_1, p_2 \in \mathcal{H}$ *then* $p_1 + p_2 \in \mathcal{H}$

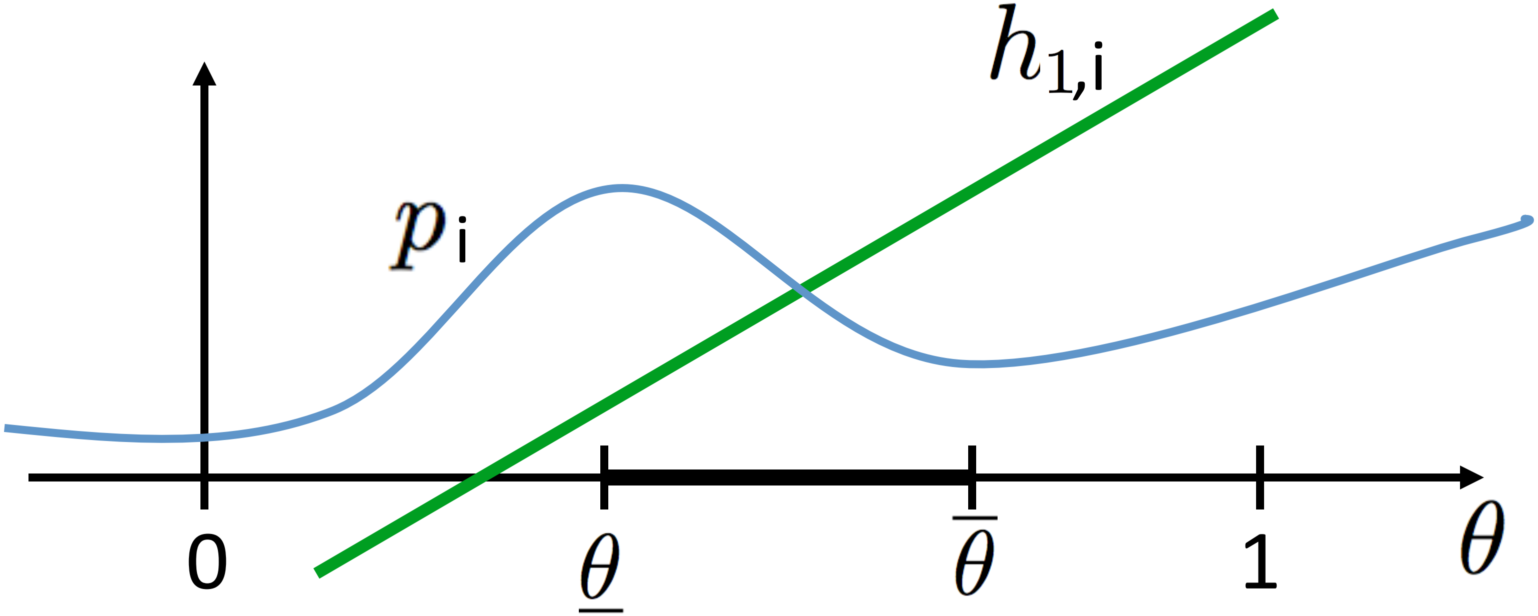
$$\Rightarrow p\, h_1 \in \mathcal{H}_\infty \quad p \in \mathcal{V}^+$$

B4. *if* $p_1, p_2 \in \mathcal{H}$ *then* $p_1 + p_2 \in \mathcal{H}$

$$\Rightarrow \sum_i p_i h_{1,i} \in \mathcal{H}_\infty$$

Imprecise Bernoulli processes

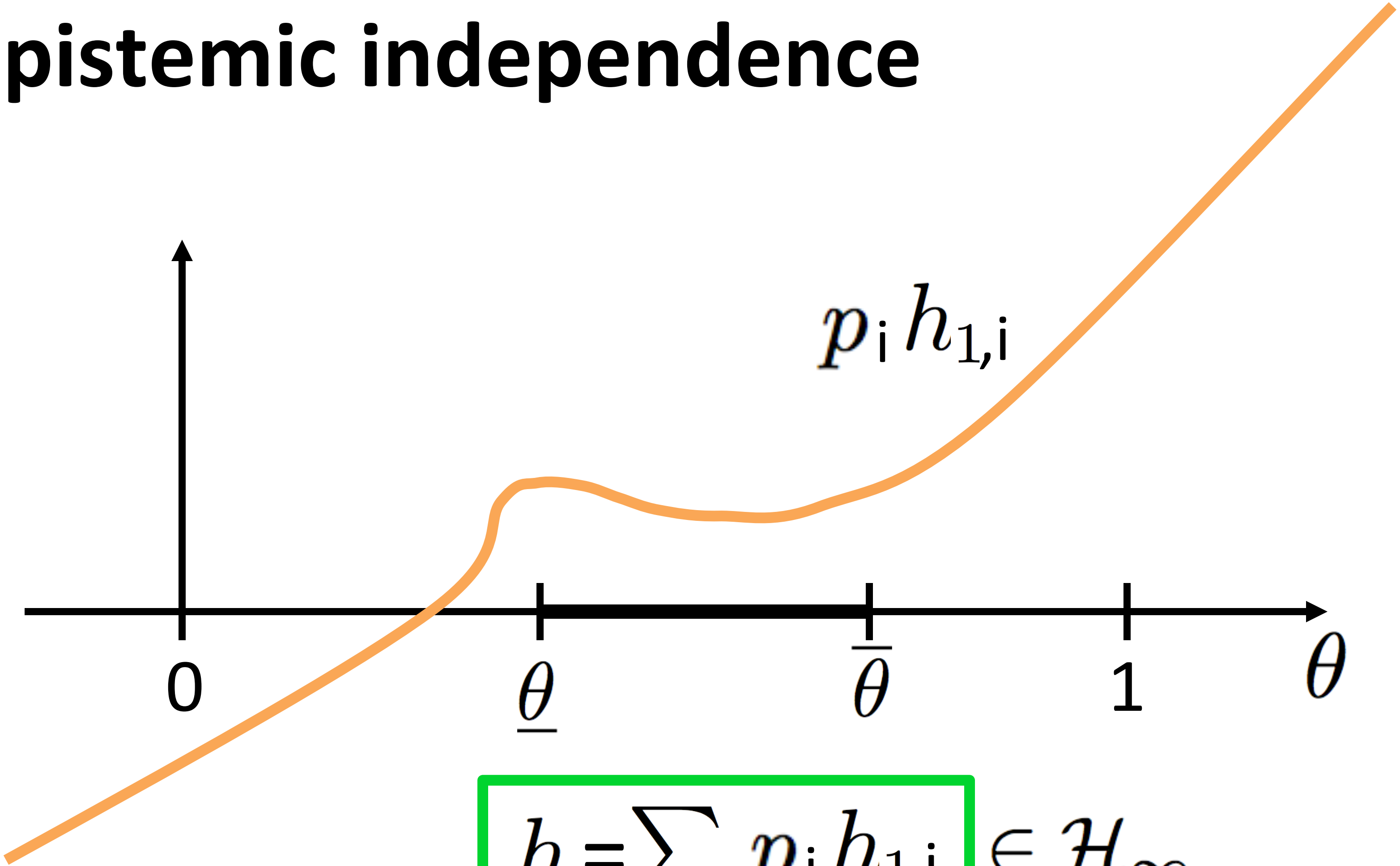
Epistemic independence



$$\boxed{h=\sum_i p_i h_{1,i}} \in \mathcal{H}_\infty$$

Imprecise Bernoulli processes

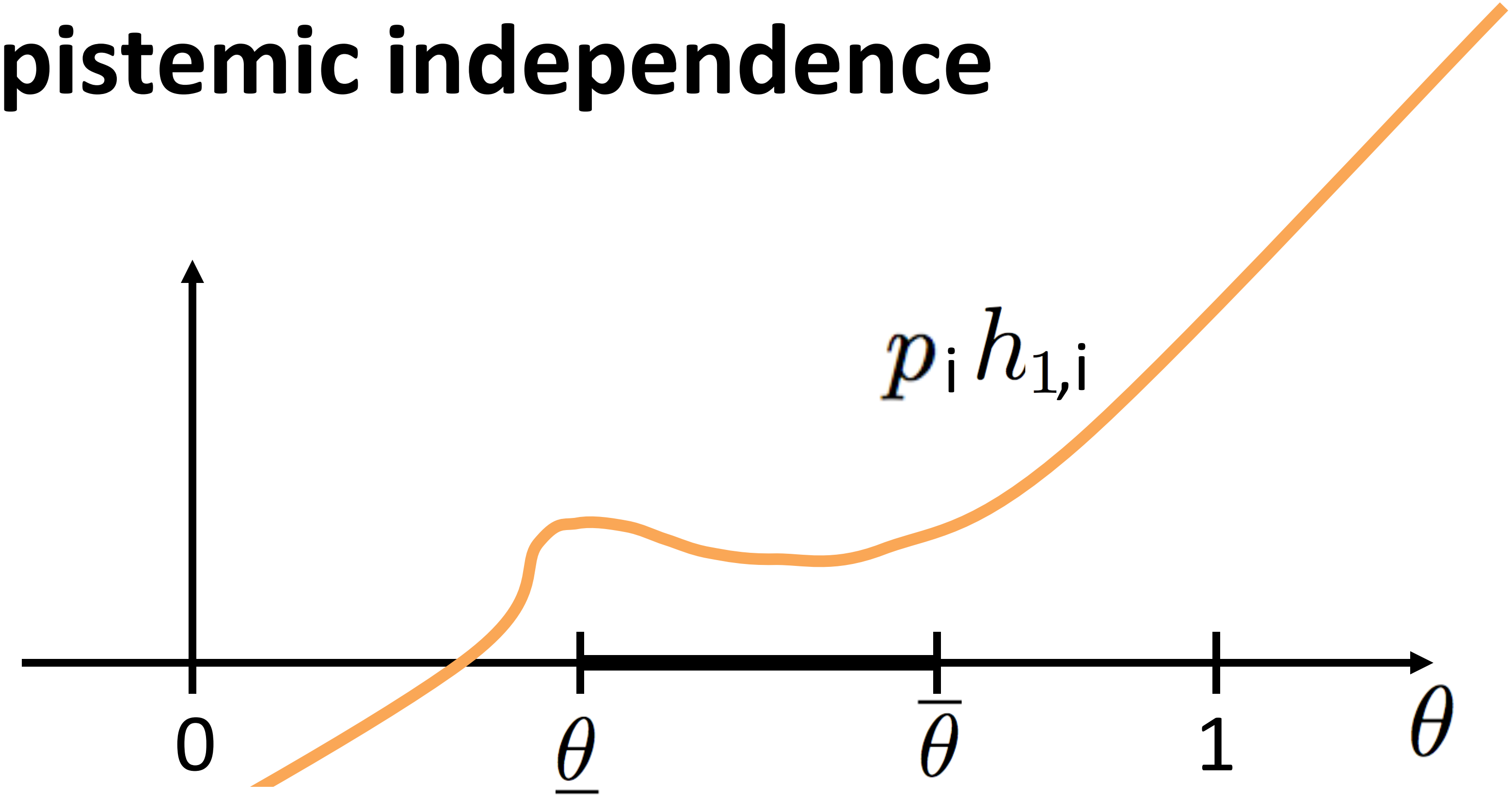
Epistemic independence



$$\boxed{h = \sum_i p_i h_{1,i}} \in \mathcal{H}_\infty$$

Imprecise Bernoulli processes

Epistemic independence

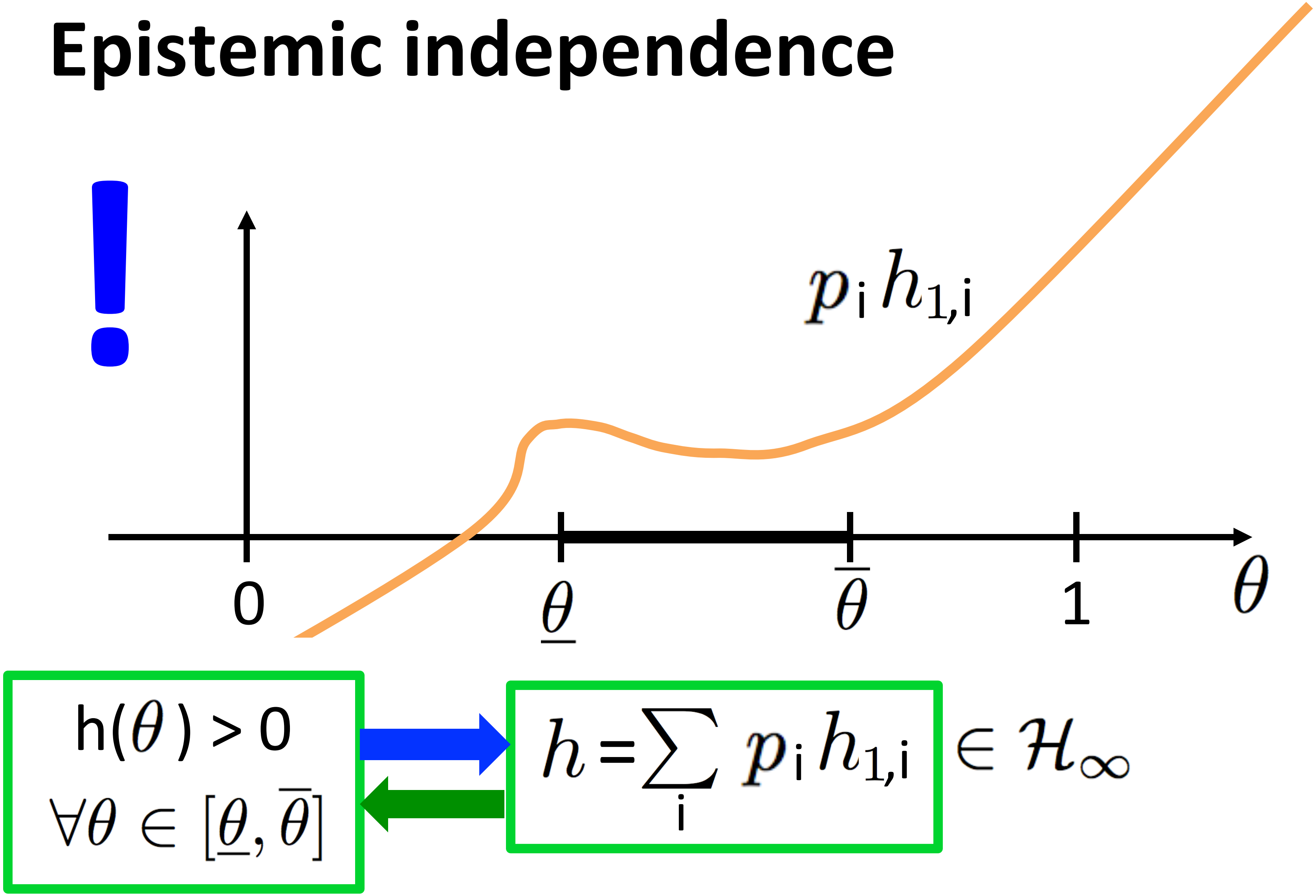


$$h(\theta) > 0 \quad \forall \theta \in [\underline{\theta}, \overline{\theta}]$$

$$h = \sum_i p_i h_{1,i} \in \mathcal{H}_\infty$$

Imprecise Bernoulli processes

Epistemic independence



$h(\theta) > 0$
$\forall \theta \in [\underline{\theta}, \overline{\theta}]$

$$h = \sum_i p_i h_{1,i} \in \mathcal{H}_\infty$$

Imprecise Bernoulli processes

Epistemic independence

! $$\left\{ h : \begin{matrix} h(\theta) > 0 \\ \forall\theta \in [\underline{\theta}, \overline{\theta}] \end{matrix} \right\} \subseteq \mathcal{H}_\infty$$

Imprecise Bernoulli processes

Epistemic independence

$$\left\{ h : \begin{array}{c} h(\theta) > 0 \\ \forall \theta \in [\underline{\theta}, \overline{\theta}] \end{array} \right\} \subseteq \mathcal{H}_\infty$$

Smallest **epistemic independent** $\mathcal{H}_\infty$ **?**

$$h \in \mathcal{H}_\infty \begin{array}{l} \Leftrightarrow \theta h \in \mathcal{H}_\infty \\ \Leftrightarrow (1-\theta) h \in \mathcal{H}_\infty \end{array}$$

Imprecise Bernoulli processes

Epistemic independence

$$\left\{ h : \begin{array}{c} h(\theta) > 0 \\ \forall\theta \in [\underline{\theta}, \overline{\theta}] \end{array} \right\} = \mathcal{H}_\infty$$

Smallest **epistemic independent** $\mathcal{H}_\infty$ **?**

$$h \in \mathcal{H}_\infty \begin{array}{l} \Leftrightarrow \theta h \in \mathcal{H}_\infty \\ \Leftrightarrow (1-\theta) h \in \mathcal{H}_\infty \end{array}$$

Imprecise Bernoulli processes

Epistemic independence

$$\left\{ h : \substack{h(\theta) > 0 \\ \forall\theta \in [\underline{\theta}, \overline{\theta}]} \right\} = \mathcal{H}_\infty$$

Smallest **epistemic independent** $\mathcal{H}_\infty$?

$$h \in \mathcal{H}_\infty \begin{array}{l} \Leftrightarrow \theta h \in \mathcal{H}_\infty \\ \Leftrightarrow (1-\theta) h \in \mathcal{H}_\infty \end{array}$$

Set $\mathcal{H}_\infty = \bigcup_{n \in \mathbb{N}_0} \mathcal{H}_n$ of polynomial functions

!

Imprecise Bernoulli processes

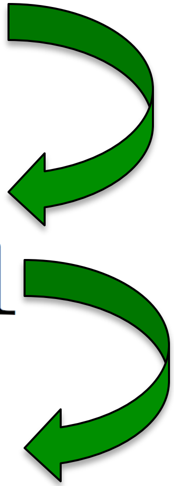
$\mathcal{D}_1$

Imprecise Bernoulli processes

$$\begin{tikzcd}\mathcal{D}_1 \arrow[d, bend left=90] \\ \mathcal{H}_1\end{tikzcd}$$

$p(\theta) := Mn_n(f \mid \theta)$

Imprecise Bernoulli processes



$\mathcal{D}_1$

$p(\theta) := \mathrm{Mn}_n(f \mid \theta)$

$\mathcal{H}_1$

smallest epistemic independent set of polynomials

$$\mathcal{H}_\infty = \left\{ h : \begin{array}{c} h(\theta) > 0 \\ \forall \theta \in [\underline{\theta}, \overline{\theta}] \end{array} \right\}$$

Imprecise Bernoulli processes

$\mathcal{D}_1$

$p(\theta) := \mathrm{Mn}_n(f \mid \theta)$

$\mathcal{H}_1$

smallest epistemic independent set of polynomials

$$\mathcal{H}_\infty = \left\{ h : \begin{matrix} h(\theta) > 0 \\ \forall \theta \in [\underline{\theta}, \overline{\theta}] \end{matrix} \right\}$$

exchangeability

Family of sets $\mathcal{D}_n$ of desirable gambles on $\mathscr{X}^n$

(for all $n \in \mathbb{N}_0$)

Imprecise Bernoulli processes

Link with sensitivity analysis

For every gamble f on $\mathscr{X}^n$:

$$\underline{E}(f) := \sup\{\mu \in \mathbb{R} : f - \mu \in \mathcal{D}_n\}$$

Supremum acceptable buying price

Imprecise Bernoulli processes

Link with sensitivity analysis

For every gamble f on $\mathscr{X}^n$:

$$
\begin{aligned}
\underline{E}(f) &:= \sup\{\mu \in \mathbb{R} : f - \mu \in \mathcal{D}_n\} \\
&= \sup\{\mu \in \mathbb{R} : p - \mu \in \mathcal{H}_\infty\}
\end{aligned}
$$

$$\left[\, p(\theta) = \mathrm{Mn}_n(\, f \mid \theta \,) \,\right]$$

Imprecise Bernoulli processes

Link with sensitivity analysis

For every gamble f on $\mathscr{X}^n$:

$$
\begin{aligned}
\underline{E}(f) &:= \sup\{\mu \in \mathbb{R} : f - \mu \in \mathcal{D}_n\} \\
&= \sup\{\mu \in \mathbb{R} : p - \mu \in \mathcal{H}_\infty\}
\end{aligned}
$$

$$p - \mu \in \mathcal{H}_\infty \Longleftrightarrow p(\theta) > \mu \quad \forall \theta \in [\underline{\theta}, \overline{\theta}]$$

$$\left[\, p(\theta) = \mathrm{Mn}_n(f \mid \theta) \,\right]$$

Imprecise Bernoulli processes

Link with sensitivity analysis

For every gamble f on $\mathscr{X}^n$:

$$
\begin{aligned}
\underline{E}(f) &:= \sup\{\mu \in \mathbb{R} : f - \mu \in \mathcal{D}_n\} \\
&= \sup\{\mu \in \mathbb{R} : p - \mu \in \mathcal{H}_\infty\} \\
&= \sup\{\mu \in \mathbb{R} : \mathrm{p}(\theta) > \mu \ \forall\theta \in [\underline{\theta}, \overline{\theta}]\}
\end{aligned}
$$

$$\left[\, \mathrm{p}(\theta) = \mathrm{Mn}_n(\, \mathrm{f} \mid \theta \,) \,\right]$$

Imprecise Bernoulli processes

Link with sensitivity analysis

For every gamble f on $\mathscr{X}^n$:

$$\begin{aligned}\underline{E}(f) &:= \sup\{\mu \in \mathbb{R} : f - \mu \in \mathcal{D}_n\} \\ &= \sup\{\mu \in \mathbb{R} : p - \mu \in \mathcal{H}_\infty\} \\ &= \sup\{\mu \in \mathbb{R} : \mathrm{p}(\theta) > \mu \ \forall \theta \in [\underline{\theta}, \overline{\theta}]\} \\ &= \min\{\mathrm{p}(\theta) : \theta \in [\underline{\theta}, \overline{\theta}]\}\end{aligned}$$

$$\left[\, \mathrm{p}(\theta) = \mathrm{Mn}_n(\, f \mid \theta \,) \,\right]$$

Imprecise Bernoulli processes

Link with sensitivity analysis

For every gamble f on $\mathscr{X}^n$:

$$\begin{aligned}
\underline{E}(f) &:= \sup\{\mu \in \mathbb{R} : f - \mu \in \mathcal{D}_n\} \\
&= \sup\{\mu \in \mathbb{R} : p - \mu \in \mathcal{H}_\infty\} \\
&= \sup\{\mu \in \mathbb{R} : \mathrm{p}(\theta) > \mu \ \forall \theta \in [\underline{\theta}, \overline{\theta}]\} \\
&= \min\{\mathrm{p}(\theta) : \theta \in [\underline{\theta}, \overline{\theta}]\} \\
&\boxed{= \min\{\mathrm{Mn}_n(f|\theta) : \theta \in [\underline{\theta}, \overline{\theta}]\}}
\end{aligned}$$

Imprecise Bernoulli processes

Link with sensitivity analysis

Sensitivity analysis:

$$\overline{E}(f) = \max\{\mathrm{Mn}_n(f|\theta) : \theta \in [\underline{\theta}, \overline{\theta}]\}$$
$$\underline{E}(f) = \min\{\mathrm{Mn}_n(f|\theta) : \theta \in [\underline{\theta}, \overline{\theta}]\}$$

Imprecise Bernoulli processes

EXCHANGEABILITY

+

EPISTEMIC INDEPENDENCE

↓

SENSITIVITY ANALYSIS