Imprecise Bernoulli processes

Jasper De Bock & Gert de Cooman

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Imprecise Bernoulli process?
Defining an imprecise Bernoulli process

A sequence of binary random variables

\[ X_1, X_2, \ldots, X_n \]

each assuming values in the set

\[ \mathcal{X} = \{ H, T \} \]
Defining an imprecise Bernoulli process

A sequence of binary random variables

\[ X_1, X_2, \ldots, X_n \]

satisfying the following properties

IDENTICALLY DISTRIBUTED
EPISTEMICALLY INDEPENDENT (IID)
EXCHANGEABLE
Defining an imprecise Bernoulli process

A sequence of binary random variables

\[ X_1, X_2, \ldots, X_n \]

! IMPLICIT ASSUMPTION!

A single Bernoulli experiment \( X_i \) has a precisely known probability mass function
Defining an imprecise Bernoulli process

\[ P(X_i = H) = \theta \quad P(X_i = T) = 1 - \theta \]

with a fixed \( \theta \in [0, 1] \)

a single Bernoulli experiment \( X_i \)
has a precisely known
probability mass function
Imprecise Bernoulli process?
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Imprecise Bernoulli processes?
Imprecise Bernoulli processes?

No precisely known probability distribution!
Coherent sets of desirable gambles

No precisely known probability distribution!

A set $\mathcal{D}$ of desirable gambles

We model a subject’s beliefs regarding the possible outcomes $\Omega$ of an experiment by looking at the gambles he is willing to accept

(Peter M. Williams & Peter Walley)
Coherent sets of desirable gambles

No precisely known probability distribution!

A set $\mathcal{D}$ of desirable gambles

Rationality criteria:

C1. if $f = 0$ then $f \notin \mathcal{D}$
C2. if $f > 0$ then $f \in \mathcal{D}$
C3. if $f \in \mathcal{D}$ then $\lambda f \in \mathcal{D}$ for $\lambda > 0$
C4. if $f_1, f_2 \in \mathcal{D}$ then $f_1 + f_2 \in \mathcal{D}$

($f > 0$ iff $f \geq 0$ and $f \neq 0$)
Coherent sets of desirable gambles

No precisely known probability distribution!

A set $\mathcal{D}$ of desirable gambles

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$(f > 0$ iff $f \geq 0$ and $f \neq 0)$
Coherent sets of desirable gambles

A single Bernoulli Experiment \( X \)

\[ \Omega = \mathcal{X} \]
Coherent sets of desirable gambles

A single Bernoulli Experiment $\mathcal{X}$

$\Omega = \mathcal{X}$

A coherent set $\mathcal{D}_1$ of desirable gambles on $\mathcal{X}$
Coherent sets of desirable gambles

a sequence of Bernoulli experiments

\[ X_1, X_2, \ldots, X_n \]

\[ \Omega = \mathcal{X}^n \]

Example: \((H, T, H, \ldots, H, T)\)

\(n\) outcomes
Coherent sets of desirable gambles

A sequence of Bernoulli experiments

\[ X_1, X_2, \ldots, X_n \]

\[ \Omega = \mathcal{X}^n \]

Example: \((H, T, H, \ldots, H, T)\)

A coherent set \( \mathcal{D}_n \) of desirable gambles on \( \mathcal{X}^n \)
Defining an imprecise Bernoulli process

A sequence of binary random variables

\[ X_1, X_2, \ldots, X_n \]

satisfying the following properties

IDENTICALLY DISTRIBUTED
EPISTEMICALLY INDEPENDENT
EXCHANGEABLE
Defining an imprecise Bernoulli process

A sequence of binary random variables

\[ X_1, X_2, \ldots, X_n \]

satisfying the following properties

IDENTICALLY DISTRIBUTED
EPISTEMICALLY INDEPENDENT
EXCHANGEABLE?
Exchangeability
**Exchangeability** (defining property)

We assess $X_1, X_2, \ldots, X_n$ to be exchangeable.
We assess $X_1, X_2, \ldots, X_n$ to be exchangeable.

"The order of the variables does not matter"
Exchangeability (defining property)

We assess $X_1, X_2, \ldots, X_n$ to be exchangeable

“The order of the variables does not matter”

For any gamble $f$ on the outcome of the sequence, we are willing to exchange it for any permuted version of this gamble, in which the order of the variables in the argument has been changed.
Exchangeability (defining property)

We assess $X_1, X_2, \ldots, X_n$ to be exchangeable

**Motivation:** For precise binomial processes, exchangeability is implied by the IID property
We assess $X_1, X_2, \ldots, X_n$ to be exchangeable

**Motivation:** For precise binomial processes, exchangeability is implied by the IID property

How to impose this property?
Exchangeability (defining property)

Exchangeability and sets of desirable gambles

Gert de Cooman  Erik Quaeghebeur
Exchangeability (defining property)

Gamble \( f \) on \( \mathcal{X}^n \)

2\(^n\)-dimensional space

\( B_n^n \)

(n+1)-dimensional space

Polynomial function \( B_n^n(f) \) on \([0,1]\) (degree \( \leq n \))
Exchangeability (defining property)

Gamble $f$ on $\mathcal{X}^n$

2^n-dimensional space

$B^n_n$ (n+1)-dimensional space

Polynomial function $B^n_n(f)$ on $[0,1]$ (degree $\leq n$)

$#H(x)$ times $H$
#$T(x)$ times $T$
$(H, T, H, ..., H, T)$

$\mathcal{X}^n$
**Exchangeability** (defining property)

- Gamble \( f \) on \( \mathcal{X}^n \)
  - \( 2^n \)-dimensional space
- Transformation
  - \( B_n^n \)
  - \( (n+1) \)-dimensional space

\forall \theta \in [0,1]:

\[ \theta \#H(x)(1-\theta)\#T(x) \]

Polynomial function \( B_n^n(f) \) on \( [0,1] \) (degree \( \leq n \))
**Exchangeability (defining property)**

Gamble $f$ on $\mathcal{X}^n$

2$^n$-dimensional space

$B_n^n$

(n+1)-dimensional space

Polynomial function $B_n^n(f)$ on $[0,1]$ (degree $\leq n$)

\[ \forall \theta \in [0,1]: \]

$\theta \ #H(x) (1-\theta) #T(x)$

$f(x)p(x)$

$\#H(x)$ times $H$

$\#T(x)$ times $T$

$(H, T, H, ..., H, T)$
Exchangeability (defining property)

Gamble \( f \) on \( \mathcal{X}^n \)

2\(^n\)-dimensional space

\( B_n^N \)

(n+1)-dimensional space

Polynomial function \( B_n^N(f) \) on [0,1] (degree \( \leq n \))

\( \forall \theta \in [0,1]: \)

\[
\sum_{X \in \mathcal{X}^n} f(x)p(x)
\]

\[ \theta \#H(x)(1-\theta)\#T(x) \]
Exchangeability (defining property)

Gamble $f$ on $\mathcal{X}^n$

$2^n$-dimensional space

$\mathcal{B}_n^n$

$(n+1)$-dimensional space

Polynomial function $\mathcal{B}_n^n(f)$ on $[0,1]$ (degree $\leq n$)

$\forall \theta \in [0,1]:$

$$\mathcal{B}_n^n(f)(\theta) = \sum_{x \in \mathcal{X}^n} f(x)p(x)$$

$\theta \#H(x)(1-\theta)^\#T(x)$

#H(x) times H

#T(x) times T

(H, T, H, ..., H, T)
Exchangeability (defining property)

\[ \forall \theta \in [0,1]: \]
\[ B^n(\theta) = \sum_{x \in \mathcal{X}^n} f(x)p(x) \]
\[ \theta \#H(x)(1 - \theta)\#T(x) \]
**Exchangeability** (defining property)

\[
\forall \theta \in [0,1]: \quad B_n^n(f)(\theta) = \sum_{X \in \mathcal{X}^n} f(x)p(x) \quad \theta \times \#H(x) \times (1-\theta) \times \#T(x)
\]

probability of heads

\[
\text{probability of the sequence } x
\]
Exchangeability (defining property)

\[ \forall \theta \in [0,1]: \quad \text{Bn}^n (f)(\theta) = \sum_{X \in \mathcal{X}^n} f(x)p(x) \]

Precise Expected Value!

\[ \theta \#H(x)(1-\theta)\#T(x) \]

probability of the sequence \( x \)

\[ \text{probability of heads} \]

\[ \#H(x) \text{ times H} \]

\[ \#T(x) \text{ times T} \]

\[ (H, T, H, ..., H, T) \]
Exchangeability (defining property)

Gamble $f$ on $X^n$

2$^n$-dimensional space

TRANSFORMATION

$B_n^n$

$(n+1)$-dimensional space

Polynomial function $B_n^n(f)$ on $[0,1]$ (degree $\leq n$)
Exchangeability (defining property)

Exchangeable sequence $X_1, X_2, \ldots, X_n$

Coherent set $D_n$ of desirable gambles on $\mathcal{X}^n$
**Exchangeability** (defining property)

Exchangeable sequence $X_1, X_2, \ldots, X_n$

Coherent set $D_n$ of desirable gambles on $\mathcal{X}^n$

Bernstein coherent:

B1. if $p = 0$ then $p \notin \mathcal{H}_n$

B2. if $p \in \mathcal{V}_n^+$, then $p \in \mathcal{H}_n$

B3. if $p \in \mathcal{H}_n$ then $\lambda p \in \mathcal{H}_n$ for $\lambda > 0$

B4. if $p_1, p_2 \in \mathcal{H}_n$ then $p_1 + p_2 \in \mathcal{H}_n$

Set $\mathcal{H}_n$ of polynomial functions (of degree $\leq n$)
Exchangeability (defining property)

Exchangeable sequence $X_1, X_2, \ldots, X_n$

Coherent set $\mathcal{D}_n$ of desirable gambles on $\mathcal{H}^n$

$B_n^n$ and $(B_n^n)^{-1}$

$2^n$-dimensional space

$(n+1)$-dimensional space

Set $\mathcal{H}_n$ of polynomial functions (of degree $\leq n$)
**Exchangeability** (defining property)

**Exchangeable sequence** $X_1, X_2, \ldots, X_n$

Coherent set $D_n$ of desirable gambles on $\mathcal{X}^n$

\[ B_n \rightarrow (B_n)^{-1} \]

2\(^n\)-dimensional space

\[(n+1)\)-dimensional space

Set $\mathcal{H}_n$ of polynomial functions (of degree $\leq n$)
**Exchangeability** (defining property)

**Exchangeable sequence** $X_1, X_2, \ldots, X_n$

Coherent set $D_n$ of desirable gambles on $\mathcal{X}^n$

$\mathbb{B}^n$ \quad (Bn^n)^{-1}

$2^n$-dimensional space

(n+1)-dimensional space

HOW DO WE FIND $\mathcal{H}_n$?

Set $\mathcal{H}_n$ of polynomial functions (of degree $\leq n$)
Defining an imprecise Bernoulli process

A sequence of binary random variables

\[ X_1, X_2, \ldots, X_n \]

satisfying the following properties

IDENTICALLY DISTRIBUTED
EPISTEMICALLY INDEPENDENT
EXCHANGEABLE
Defining an imprecise Bernoulli process

A sequence of binary random variables

\[ X_1, X_2, \ldots, X_n \]

satisfying the following properties

IDENTICALLY DISTRIBUTED?  
EPISTEMICALLY INDEPENDENT  
EXCHANGEABLE
Identical marginal models
For every Bernoulli experiment $X_i$ in the sequence $X_1, \ldots, X_n$, we have the same given marginal model $\mathcal{D}_1$.

A coherent set $\mathcal{D}_1$ of desirable gambles on $\mathcal{X}$.
Identical marginal models

For every Bernoulli experiment $X_i$ in the sequence $X_1, \ldots, X_n$, we have the same given marginal model $\mathcal{D}_1$. Implied by the assumption of exchangeability! (due to symmetry)

A coherent set $\mathcal{D}_1$ of desirable gambles on $\mathcal{X}$
Defining an imprecise Bernoulli process

A sequence of binary random variables

$X_1, X_2, \ldots, X_n$

satisfying the following properties

IDENTICALLY DISTRIBUTED
EPISTEMICALLY INDEPENDENT
EXCHANGEABLE
Defining an imprecise Bernoulli process

A sequence of binary random variables

\[ X_1, X_2, \ldots, X_n \]

satisfying the following properties

(IDENTICALLY DISTRIBUTED) EPISTEMICALLY INDEPENDENT EXCHANGEABLE
Identical marginal models

For every Bernoulli experiment $X_i$ in the sequence $X_1, \ldots, X_n$, we have the same given marginal model $\mathcal{D}_1$

A coherent set $\mathcal{D}_1$ of desirable gambles on $\mathcal{X}$
For every Bernoulli experiment $X_i$ in the sequence $X_1, \ldots, X_n$, we have the same given marginal model $\mathcal{D}_1$

A coherent set $\mathcal{D}_1$ of desirable gambles on $\mathcal{X}$

\[ f \in \mathcal{D}_1 \]

\[ \text{Bn}^1 \]

\[ \text{Bn}^1(f) \in \mathcal{H}_1 \]

\[ |\| \]

linear polynomials in $\mathcal{H}_n$
For every Bernoulli experiment $X_i$ in the sequence $X_1, \ldots, X_n$, we have the same given marginal model $D_1$.

A coherent set $D_1$ of desirable gambles on $\mathcal{X}$. 
Identical marginal models

\[ \mathcal{H}_1 = \{ p : p(\theta) > 0 \quad \forall \theta \in [\underline{\theta}, \overline{\theta}] \} \]

\[ p(\theta) \]

\[ 0 \quad \underline{\theta} \quad \overline{\theta} \quad 1 \]

$p$ is linear
Identical marginal models

\[ \mathcal{H}_1 = \left\{ p : \begin{array}{l} p \text{ is linear} \\ p(\theta) > 0 \\ \forall \theta \in [\underline{\theta}, \overline{\theta}] \end{array} \right\} \]

Set \( \mathcal{H}_n \) of polynomial functions (of degree \( \leq n \))
Defining an imprecise Bernoulli process

A sequence of binary random variables

\[ X_1, X_2, \ldots, X_n \]

satisfying the following properties

(IDENTICALLY DISTRIBUTED)

EPISTEMICALLY INDEPENDENT!

EXCHANGEABLE
Epistemic independence
Epistemic independence (defining property)

We assess $X_1, X_2, \ldots, X_n$ to be epistemically independent
Epistemic independence (defining property)

We assess $X_1, X_2, ..., X_n$ to be epistemically independent.

Learning the value of any number of variables does not change our beliefs about any subset of the remaining, unobserved variables.

$X_1, X_2, X_3, X_4, X_5, ..., X_{n-1}, X_n$
Epistemic independence (defining property)

We assess $X_1, X_2, \ldots, X_n$ to be epistemically independent

\[ \begin{align*}
X_1, X_2, X_3, X_4, X_5, \ldots, X_{n-1}, X_n \\
\text{\underbrace{\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad
We assess $X_1, X_2, \ldots, X_n$ to be epistemically independent

**Motivation:** Epistemic independence is a weaker assessment than the strong independence that is usually assumed
We assess $X_1, X_2, \ldots, X_n$ to be epistemically independent

**Motivation:** Epistemic independence is a weaker assessment than the strong independence that is usually assumed

Set $\mathcal{H}_n$ of polynomial functions (of degree $\leq n$)
Epistemic independence (defining property)

We assess $X_1, X_2, \ldots, X_n$ to be epistemically independent

Set $\mathcal{H}_n$ of polynomial functions (of degree $\leq n$)

$p \in \mathcal{H}_n \iff \theta p \in \mathcal{H}_n \iff (1 - \theta) p \in \mathcal{H}_n$ (degree $\leq n-1$)
An imprecise Bernoulli process
An imprecise Bernoulli process

Exchangeability:
Set $\mathcal{H}_n$ of polynomial functions
Bernstein coherent:

Epistemic independence:

We are looking for the smallest such set $\mathcal{H}_n$
(most conservative inferences)
An imprecise Bernoulli process

\[ \mathcal{H}_n = \left\{ p : \begin{array}{l} \deg(p) \leq n \\ p(\theta) > 0 \\ \forall \theta \in [\underline{\theta}, \overline{\theta}] \end{array} \right\} \]
An imprecise Bernoulli process

\[ \mathcal{H}_n = \left\{ p : \begin{array}{l} \text{deg}(p) \leq n \\ p(\theta) > 0 \\ \forall \theta \in [\underline{\theta}, \overline{\theta}] \end{array} \right\} \]

\( \frac{1}{(Bn^n)^{-1}} \)

Exchangeability

\[ \mathcal{D}_n \]

Coherent set of desirable gambles on \( \mathcal{X}^n \)
(exchangeable and epistemic independent)

= IMPRECISE BERNOULLI PROCESS
How is this useful?
Lower (and upper) prevision
Lower (and upper) prevision

A set $\mathcal{D}_n$ of desirable gambles on $\mathcal{X}^n$ associated lower prevision $\underline{P}_{\mathcal{D}_n}$
Lower (and upper) prevision

A set $\mathcal{D}_n$ of desirable gambles on $\mathcal{X}^n$  
\[ \text{associated lower prevision} \quad \underline{P}_{\mathcal{D}_n} \]

For every gamble $f$ on $\mathcal{X}^n$:
\[ \underline{P}_{\mathcal{D}_n}(f) := \sup\{\mu \in \mathbb{R} : f - \mu \in \mathcal{D}_n\} \]

supremum acceptable buying price
Lower (and upper) prevision

A set $\mathcal{D}_n$ of desirable gambles on $\mathbb{X}^n$ associated **lower prevision** $\underline{P}_{\mathcal{D}_n}$

For every gamble $f$ on $\mathbb{X}^n$:

$$\underline{P}_{\mathcal{D}_n}(f) := \sup\{\mu \in \mathbb{R} : f - \mu \in \mathcal{D}_n\}$$

supremum acceptable buying price

In a similar way: **upper prevision** $\overline{P}_{\mathcal{D}_n}$

infimum acceptable selling price
A set $\mathcal{D}_n$ of desirable gambles on $\mathcal{X}^n$ associated lower prevision $P_{\mathcal{D}_n}$

For every gamble $f$ on $\mathcal{X}^n$:

$$P_{\mathcal{D}_n}(f) := \sup \{ \mu \in \mathbb{R} : f - \mu \in \mathcal{D}_n \}$$

supremum acceptable buying price
Lower (and upper) prevision

A set $\mathcal{D}_n$ of desirable gambles on $\mathcal{X}^n$ associated lower prevision $P_{\mathcal{D}_n}$

For every gamble $f$ on $\mathcal{X}^n$:

$$P_{\mathcal{D}_n}(f) := \sup\{\mu \in \mathbb{R} : f - \mu \in \mathcal{D}_n\}$$

suppremum acceptable buying price

What happens for our specific set $\mathcal{D}_n$?
Lower (and upper) prevision

A set $D_n$ of desirable gambles on $\mathcal{X}^n$ associated lower prevision $P_{D_n}$

For every gamble $f$ on $\mathcal{X}^n$:

$$P_{D_n}(f) := \sup\{\mu \in \mathbb{R} : f - \mu \in D_n\}$$

Supremum acceptable buying price

What happens for our specific set $D_n$?
Lower (and upper) prevision

A set $\mathcal{D}_n$ of desirable gambles on $\mathcal{X}^n$ associated with lower prevision $P_{\mathcal{D}_n}$

For every gamble $f$ on $\mathcal{X}^n$:

$P_{\mathcal{D}_n}(f) := \sup \{ \mu \in \mathbb{R} \mid f - \mu \in \mathcal{D}_n \}$

supremum acceptable buying price

$B_n(f) - \mu \in \mathcal{H}_n$
A set $$\mathcal{D}_n$$ of desirable gambles on $$\mathcal{H}^n$$ associated lower prevision $$\underline{P}_{\mathcal{D}_n}$$

For every gamble $$f$$ on $$\mathcal{H}^n$$:

$$\underline{P}_{\mathcal{D}_n}(f) := \sup\{\mu \in \mathbb{R} : f - \mu \in \mathcal{D}_n\}$$

supremum acceptable buying price

$$= \sup\{\mu \in \mathbb{R} : \text{Bn}^n(f) - \mu \in \mathcal{H}_n\}$$
Lower (and upper) prevision

\[
\sup\{\mu \in \mathbb{R} : B_n^n(f) - \mu \in \mathcal{H}_n\}
\]
Lower (and upper) prevision

$$
\sup\{\mu \in \mathbb{R} : B_n^\mu(f) - \mu \in \mathcal{H}_n\}
$$
Lower (and upper) prevision

\[ \sup \{ \mu \in \mathbb{R} : B_n^n(f) - \mu \in \mathcal{H}_n \} \]
Lower (and upper) prevision

\[
\sup \left\{ \mu \in \mathbb{R} : B_n^n(f) - \mu \in \mathcal{H}_n \right\}
\]
Lower (and upper) prevision

\[
\sup\{\mu \in \mathbb{R} : B^n(f) - \mu \in \mathcal{H}_n\}
\]
Lower (and upper) prevision

\[ \sup\{\mu \in \mathbb{R} : B_n(f) - \mu \in \mathcal{H}_n\} \]
Lower (and upper) prevision

\[
\sup\{\mu \in \mathbb{R} : B_n^n(f) - \mu \in \mathcal{H}_n}\]

\[
= \min\{B_n^n(f)(\theta) : \theta \in [\underline{\theta}, \overline{\theta}]\}
\]
Lower (and upper) prevision

IMPRECISE BERNULLI PROCESS
associated lower prevision

For every gamble $f$ on $\mathcal{H}^n$:

$$\underline{P}_{D_n}(f) = \min \{ B_{\mathcal{H}^n}(f)(\theta) : \theta \in [\underline{\theta}, \bar{\theta}] \}$$

supremum acceptable buying price
Lower (and upper) prevision

**IMPRECISE BERNOLLI PROCESS**

associated lower prevision and upper prevision

For every gamble $f$ on $\mathcal{F}^n$: 

infimum acceptable selling price

$$
\overline{P}_{\mathcal{D}_n}(f) = \max \{ B^n(f)(\theta) : \theta \in [\underline{\theta}, \overline{\theta}] \} 
$$

supremum acceptable buying price

$$
\underline{P}_{\mathcal{D}_n}(f) = \min \{ B^n(f)(\theta) : \theta \in [\underline{\theta}, \overline{\theta}] \} 
$$
Link with sensitivity analysis
Link with sensitivity analysis

IMPRECISE BERNOULLI PROCESS
associated lower prevision and upper prevision

\[
\overline{P}_{D_n}(f) = \max \{ \text{Bn}^n(f)(\theta) : \theta \in [\theta, \bar{\theta}] \}
\]
\[
\underline{P}_{D_n}(f) = \min \{ \text{Bn}^n(f)(\theta) : \theta \in [\theta, \bar{\theta}] \}
\]

\text{imprecise expected value } \text{E}(f)
Link with sensitivity analysis

IMPRECISE BERNOULLI PROCESS

associated lower prevision and upper prevision

\[ \overline{E}(f) \]

\[ \overline{P_{D_n}}(f) = \max\{B_{n}(f)(\theta) : \theta \in [\underline{\theta}, \bar{\theta}]\} \]

\[ \underline{P_{D_n}}(f) = \min\{B_{n}(f)(\theta) : \theta \in [\underline{\theta}, \bar{\theta}]\} \]

\[ \underline{E}(f) \]

precise expected value \( E(f) \)
Lower and upper prevision associated with an IMPRECISE BERNOULLI PROCESS

MAIN RESULT: Lower and upper expectation obtained by applying SENSITIVITY ANALYSIS to a classical, precise bernoulli process
Why is this important?
Why is this important?

- Strong mathematical assumption: our series of experiments is described by an underlying true probability mass function (satisfying the IID property)
- This model is imprecise only because we do not know which precise model is correct

Lower and upper expectation obtained by applying SENSITIVITY ANALYSIS to a classical, precise Bernoulli process
Why is this important?

Lower and upper prevision associated with an IMPRECISE BERNOULLI PROCESS

- Behavioural assessments: our series of experiments is described by a set of desirable gambles, satisfying exchangeability and epistemic independence (can be expressed in terms of behaviour)
- This model is inherently imprecise
Why is this important?

Lower and upper prevision associated with an IMPRECISE BERNOULLI PROCESS

MAIN RESULT: ！ SURPRISING！

Lower and upper expectation obtained by applying SENSITIVITY ANALYSIS to a classical, precise bernoulli process
Conclusion
Conclusion

EXCHANGEABILITY + EPISTEMIC INDEPENDENCE

SENSITIVITY ANALYSIS BERNOULLI PROCESSES