IPMU 2012

Imprecise probabilities - A

Session chair: Enrique Miranda

# Imprecise Bernoulli processes

Jasper De Bock & Gert de Cooman

9 July 2012

# Imprecise Bernoulli processes?

# Defining an imprecise Bernoulli process

A sequence of binary random variables

$$X_1, X_2, \dots, X_n$$

each assuming values in the set

$$\mathscr{X} = \{ H, T \}$$

# Defining an imprecise Bernoulli process

A sequence of binary random variables

$$X_1 , X_2 , \ldots , X_n$$

satisfying the following properties

IDENTICALLY DISTRIBUTED

EPISTEMICALLY INDEPENDENT (IID)

EXCHANGEABLE

# Defining an imprecise Bernoulli process

A sequence of binary random variables

$$X_1, X_2, \ldots, X_n$$

**! IMPLICIT ASSUMPTION !**

a single **Bernoulli experiment $X_i$**
has a **precisely known**
**probability mass function**

# Defining an imprecise Bernoulli process

$P(X_i = H) = \theta$ $P(X_i = T) = 1 - \theta$

with a fixed $\theta \in [0, 1]$

a single **Bernoulli experiment $X_i$**
has a **precisely known**
**probability mass function**

# Imprecise Bernoulli process?

# Defining an imprecise Bernoulli process

A sequence of binary random variables

$$X_1, X_2, \dots, X_n$$

each assuming values in the set

$$\mathscr{X} = \{H, T\}$$

# Defining an imprecise Bernoulli process

A sequence of binary random variables

$$X_1, X_2, \ldots, X_n$$

satisfying the following properties

IDENTICALLY DISTRIBUTED

EPISTEMICALLY INDEPENDENT

EXCHANGEABLE

# Defining an imprecise Bernoulli process

A sequence of binary random variables

$$X_1, X_2, \ldots, X_n$$

satisfying the following properties

IDENTICALLY DISTRIBUTED

EPISTEMICALLY INDEPENDENT

EXCHANGEABLE

# Defining an imprecise Bernoulli process

A sequence of binary random variables

$$X_1, X_2, \dots, X_n$$

satisfying the following properties

IDENTICALLY DISTRIBUTED

EPISTEMICALLY INDEPENDENT

EXCHANGEABLE

# Imprecise ? Bernoulli processes

**Imprecise ?** No precisely known probability distribution!

# Coherent **sets of desirable gambles**

No precisely known probability distribution!

A set $\mathcal{D}$ of desirable gambles

We model a **subject's beliefs** regarding the possible **outcomes $\Omega$ of an experiment** by looking at the **gambles he is willing to accept**

(Peter M. Williams & Peter Walley )

# Coherent sets of desirable gambles

No precisely known probability distribution!

**A set $\mathcal{D}$ of desirable gambles**

Rationality criteria:

COHERENT

C1. *if* $f = 0$ *then* $f \notin \mathcal{D}$
C2. *if* $f > 0$ *then* $f \in \mathcal{D}$
C3. *if* $f \in \mathcal{D}$ *then* $\lambda f \in \mathcal{D}$ *for* $\lambda > 0$
C4. *if* $f_1, f_2 \in \mathcal{D}$ *then* $f_1 + f_2 \in \mathcal{D}$

($f > 0$ iff $f \geq 0$ and $f \neq 0$)

# Coherent sets of desirable gambles

No precisely known probability distribution!

A set $\mathcal{D}$ of desirable gambles

Rationality criteria:

**COHERENT**

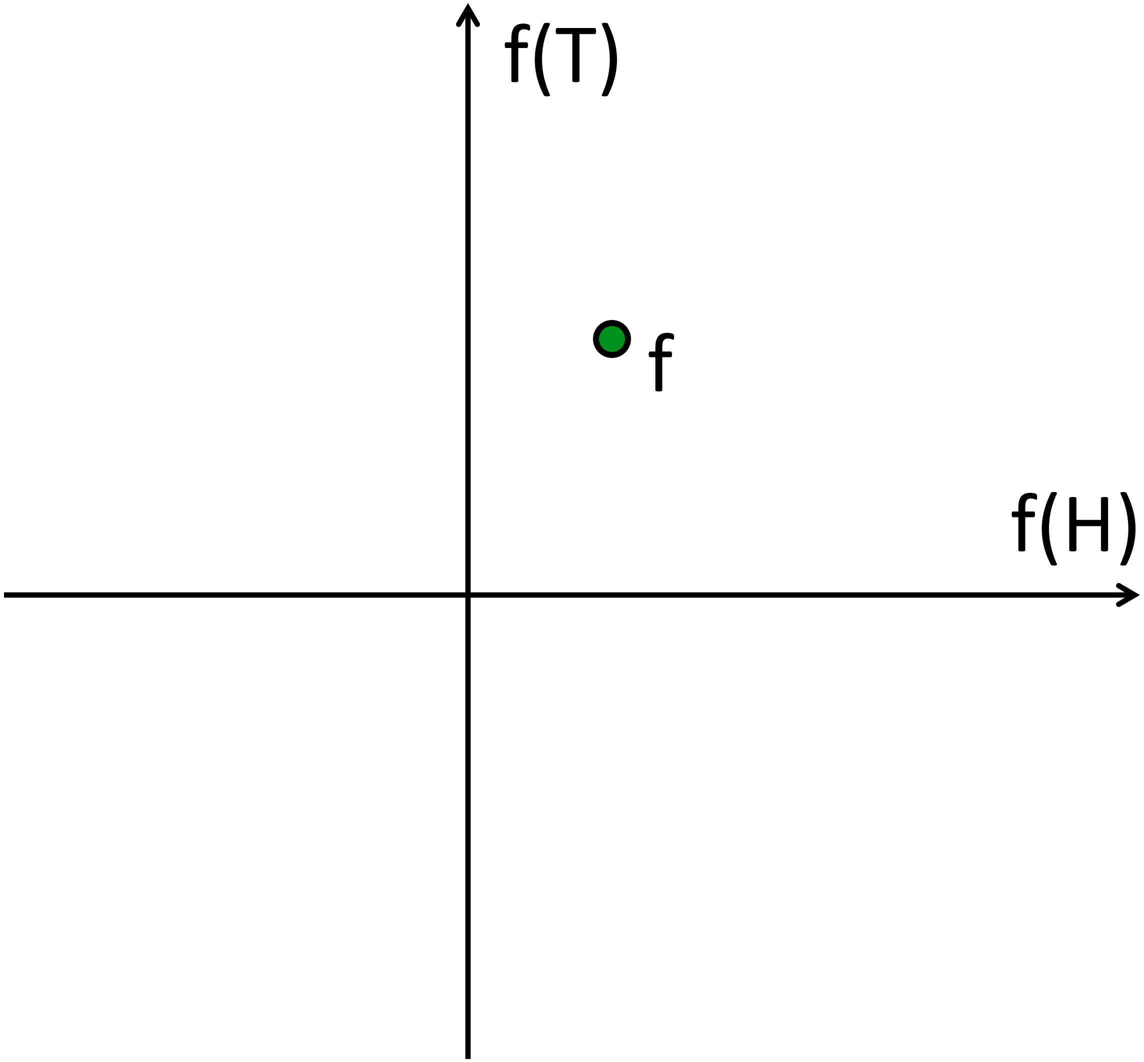
C1. *if* $f = 0$ *then* $f \notin \mathcal{D}$
C2. *if* $f > 0$ *then* $f \in \mathcal{D}$
C3. *if* $f \in \mathcal{D}$ *then* $\lambda f \in \mathcal{D}$ *for* $\lambda > 0$
C4. *if* $f_1, f_2 \in \mathcal{D}$ *then* $f_1 + f_2 \in \mathcal{D}$

($f > 0$ iff $f \geq 0$ and $f \neq 0$)

# Coherent sets of desirable gambles

a **single**
Bernoulli
Experiment X

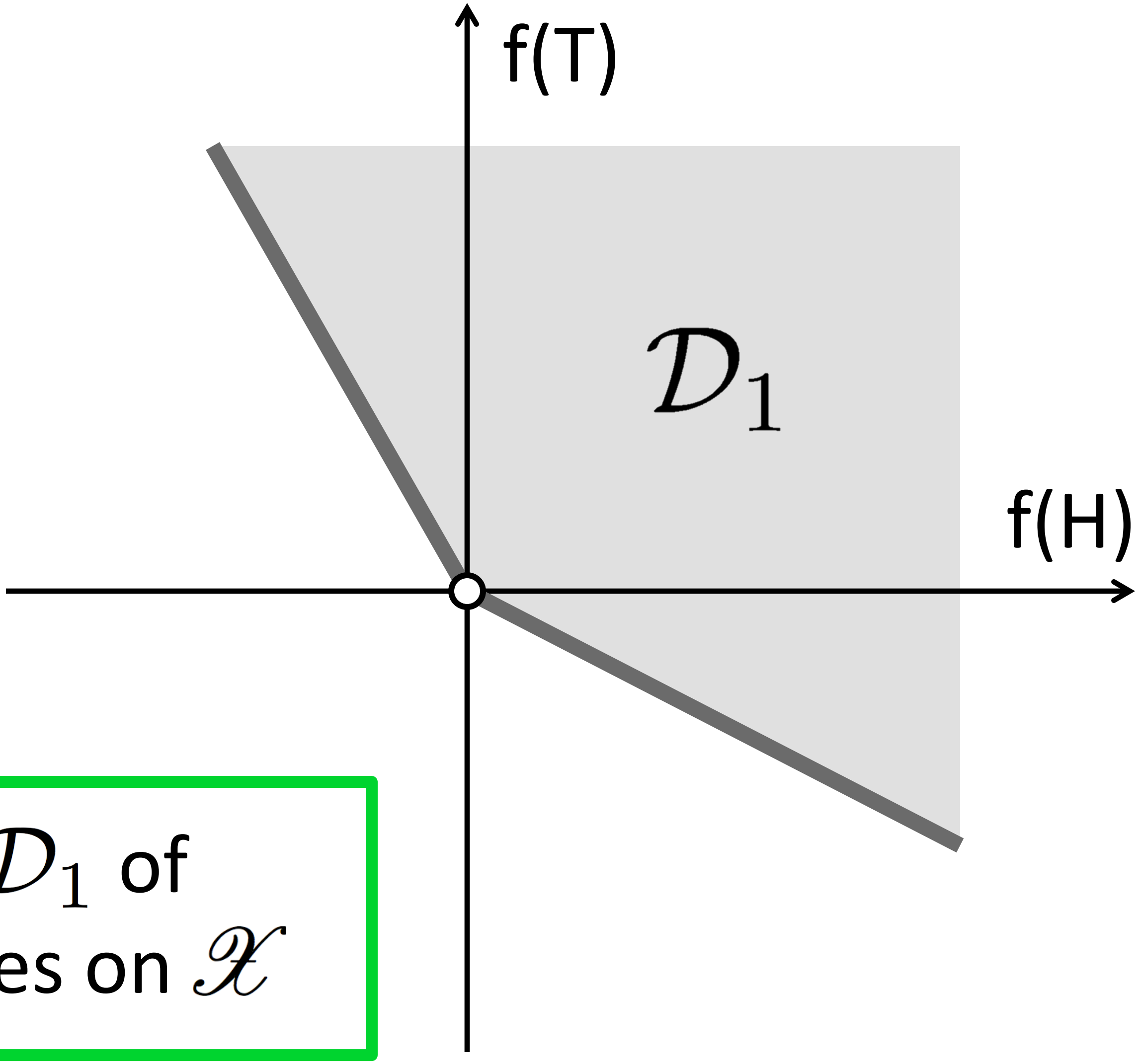
**Ω** = $\mathscr{X}$



# Coherent sets of desirable gambles

a **single** Bernoulli Experiment X

$\boldsymbol{\Omega} = \mathscr{X}$




A coherent set $\mathcal{D}_1$ of desirable gambles on $\mathscr{X}$

# Coherent sets of desirable gambles

a **sequence** of Bernoulli experiments

$$X_1, X_2, \dots, X_n$$

$\mathbf{\Omega} = \mathscr{X}^n$

Example: (H, T, H, ..., H, T)

**n** outcomes

# Coherent sets of desirable gambles

a **sequence** of Bernoulli experiments

$X_1, X_2, \dots, X_n$

$\boldsymbol{\Omega} = \mathscr{X}^n$

Example: (H, T, H, ..., H, T)

**n** outcomes

A coherent set $\mathcal{D}_n$ of desirable gambles on $\mathscr{X}^n$

# Defining an imprecise Bernoulli process

A sequence of binary random variables

$$X_1, X_2, \dots, X_n$$

satisfying the following properties

IDENTICALLY DISTRIBUTED

EPISTEMICALLY INDEPENDENT

EXCHANGEABLE

# Defining an imprecise Bernoulli process

A sequence of binary random variables

$$X_1 , X_2 , \ldots , X_n$$

satisfying the following properties

IDENTICALLY DISTRIBUTED

EPISTEMICALLY INDEPENDENT

EXCHANGEABLE ?

# Exchangeability

# **Exchangeability** (defining property)

We assess $X_1, X_2, \ldots, X_n$ to be **exchangeable**

# **Exchangeability** (defining property)

We assess $X_1, X_2, \ldots, X_n$ to be **exchangeable**

⬍

"The **order** of the variables **does not matter**"

## **Exchangeability** (defining property)

We assess $X_1, X_2, \ldots, X_n$ to be **exchangeable**

⇕

"The **order** of the variables **does not matter**"

⇕

For any **gamble f** on the outcome of the sequence, **we are willing to exchange it for any permuted version of this gamble**, in which the order of the variables in the argument has been changed.

# **Exchangeability** (defining property)

We assess $X_1, X_2, \ldots, X_n$ to be **exchangeable**

**Motivation:** For precise binomial processes, exchangeability is implied by the IID property

## **Exchangeability** (defining property)

We assess $X_1, X_2, \ldots, X_n$ to be **exchangeable**

**Motivation:** For precise binomial processes, exchangeability is implied by the IID property

## How to impose this property?

# **Exchangeability** (defining property)

Exchangeability and **sets of desirable gambles**

Gert de Cooman

Erik Quaeghebeur

## **Exchangeability** (defining property)

Gamble f on $\mathscr{X}^n$

TRANSFORMATION

$2^n$-dimensional space

**$Bn^n$**

(n+1)-dimensional space

Polynomial function **$Bn^n$(f)** on [0,1] (degree ≤ n)

# **Exchangeability** (defining property)

Gamble f on $\mathscr{X}^n$

TRANSFORMATION

$2^n$-dimensional space

**$Bn^n$**

(n+1)-dimensional space

#H(x) times H

#T(x) times T

**(H, T, H, …, H, T)**

x

$x \in \mathcal{X}^n$

Polynomial function **$Bn^n$(f)** on [0,1] (degree ≤ n)

## **Exchangeability** (defining property)

Gamble f on $\mathscr{X}^n$

TRANSFORMATION

$2^n$-dimensional space

**$Bn^n$**

$\forall\theta \in [0,1]$:

#H(x) times H

#T(x) times T

**(H, T, H, …, H, T)**

p(x)

$x \in \mathcal{X}^n$

$\theta^{\#H(x)}(1-\theta)^{\#T(x)}$

(n+1)-dimensional space

Polynomial function **$Bn^n$(f)** on [0,1] (degree ≤ n)

# **Exchangeability** (defining property)

Gamble f on $\mathscr{X}^n$

TRANSFORMATION

$2^n$-dimensional space

**$Bn^n$**

(n+1)-dimensional space

$\forall\theta \in [0,1]$:

#H(x) times H

#T(x) times T

**(H, T, H, …, H, T)**

$$\sum_{x \in \mathcal{X}^n} f(x)p(x)$$

$\theta^{\#H(x)}(1-\theta)^{\#T(x)}$

Polynomial function **$Bn^n(f)$** on [0,1] (degree ≤ n)

# **Exchangeability** (defining property)

Gamble f on $\mathscr{X}^n$

TRANSFORMATION

$2^n$-dimensional space

**$Bn^n$**

(n+1)-dimensional space

#H(x) times H

#T(x) times T

**(H, T, H, …, H, T)**

$\forall\theta \in [0,1]$:

$$\sum_{x \in \mathcal{X}^n} f(x)p(x)$$

$$\theta^{\#H(x)}(1-\theta)^{\#T(x)}$$

Polynomial function **$Bn^n(f)$** on [0,1] (degree ≤ n)

# **Exchangeability** (defining property)

Gamble f on $\mathscr{X}^n$

TRANSFORMATION

$2^n$-dimensional space

**$Bn^n$**

#H(x) times H
#T(x) times T
**(H, T, H, …, H, T)**

$\forall \theta \in [0,1]$:

$$Bn^n(f)(\theta) = \sum_{x \in \mathcal{X}^n} f(x)p(x)$$

$$p(x) = \theta^{\#H(x)}(1-\theta)^{\#T(x)}$$

(n+1)-dimensional space

Polynomial function **$Bn^n(f)$** on [0,1] (degree ≤ n)

# **Exchangeability** (defining property)

probability of heads

||

$\forall\theta \in [0,1]$:

$$\mathrm{Bn}^n(f)(\theta) = \sum_{x \in \mathcal{X}^n} f(x)p(x)$$

#H(x) times H
#T(x) times T
**(H, T, H, …, H, T)**

$\theta^{\#H(x)}(1-\theta)^{\#T(x)}$

# **Exchangeability** (defining property)

probability of heads

||

$\forall\theta \in [0,1]$:

$$\mathrm{Bn}^n(f)(\theta) = \sum_{x \in \mathcal{X}^n} f(x)p(x)$$

#H(x) times H
#T(x) times T
**(H, T, H, …, H, T)**

$$\theta^{\#H(x)}(1-\theta)^{\#T(x)}$$

||

probability of the sequence **x**

# **Exchangeability** (defining property)

probability of heads

||

$\forall \theta \in [0,1]$:

$$\mathrm{Bn}^n(f)(\theta) = \sum_{x \in \mathcal{X}^n} f(x)p(x)$$

||

**Precise Expected Value!**

||

**E(f)**

#H(x) times H
#T(x) times T
**(H, T, H, …, H, T)**

$$\theta^{\#H(x)}(1-\theta)^{\#T(x)}$$

||

probability of the sequence **x**

# **Exchangeability** (defining property)

Gamble f on $\mathscr{X}^n$

TRANSFORMATION

$2^n$-dimensional space

**$Bn^n$**

#H(x) times H

#T(x) times T

**(H, T, H, …, H, T)**

$\forall\theta \in [0,1]$:

$$Bn^n( f )(\theta) = \sum_{x \in \mathcal{X}^n} f(x)p(x)$$

$$\theta^{\#H(x)}(1-\theta)^{\#T(x)}$$

(n+1)-dimensional space

Polynomial function **$Bn^n$(f)** on [0,1] (degree ≤ n)

# **Exchangeability** (defining property)

**Exchangeable sequence $X_1, X_2, \ldots, X_n$**

Coherent set $\mathcal{D}_n$ of desirable gambles on $\mathscr{X}^n$

# **Exchangeability** (defining property)

Exchangeable **sequence $X_1, X_2, \ldots, X_n$**

Coherent set $\mathcal{D}_n$ of desirable gambles on $\mathscr{X}^n$

$\downarrow$ $Bn^n$

**Bernstein coherent:**

B1. *if* $p = 0$ *then* $p \notin \mathcal{H}_n$

B2. *if* $p \in \mathcal{V}_n^+$, *then* $p \in \mathcal{H}_n$

B3. *if* $p \in \mathcal{H}_n$ *then* $\lambda p \in \mathcal{H}_n$ *for* $\lambda > 0$

B4. *if* $p_1, p_2 \in \mathcal{H}_n$ *then* $p_1 + p_2 \in \mathcal{H}_n$

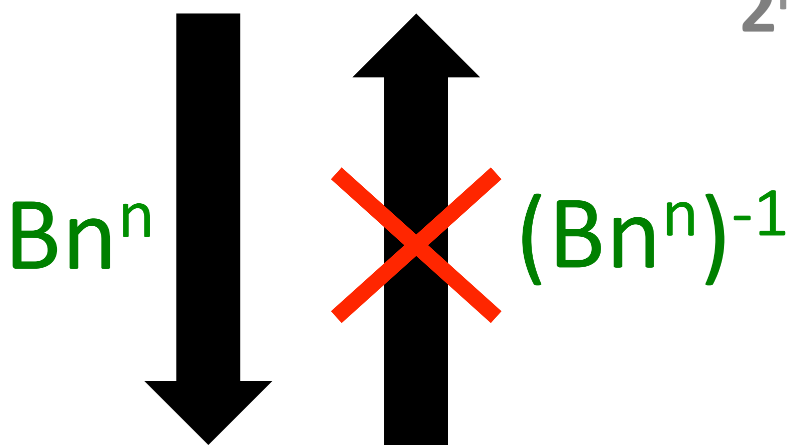
Set $\mathcal{H}_n$ of polynomial functions (of degree $\leq$ n)

# **Exchangeability** (defining property)

Exchangeable **sequence $X_1, X_2, \ldots, X_n$**

Coherent set $\mathcal{D}_n$ of desirable gambles on $\mathscr{X}^n$

**$2^n$-dimensional space**




**(n+1)-dimensional space**

Set $\mathcal{H}_n$ of polynomial functions (of degree ≤ n)

# **Exchangeability** (defining property)

**Exchangeable sequence $X_1, X_2, \ldots, X_n$**

Coherent set $\mathcal{D}_n$ of desirable gambles on $\mathscr{X}^n$

**$2^n$-dimensional space**

$Bn^n$ ↓ ↑ $(Bn^n)^{-1}$

**(n+1)-dimensional space**

Set $\mathcal{H}_n$ of polynomial functions (of degree ≤ n)

# **Exchangeability** (defining property)

**Exchangeable sequence $X_1, X_2, \ldots, X_n$**

Coherent set $\mathcal{D}_n$ of desirable gambles on $\mathscr{X}^n$

**$2^n$-dimensional space**

$Bn^n$

$(Bn^n)^{-1}$

HOW DO WE FIND $\mathcal{H}_n$?

**(n+1)-dimensional space**

Set $\mathcal{H}_n$ of polynomial functions (of degree $\leq n$)

# Defining an imprecise Bernoulli process

A sequence of binary random variables

$$X_1, X_2, \ldots, X_n$$

satisfying the following properties

IDENTICALLY DISTRIBUTED

EPISTEMICALLY INDEPENDENT

EXCHANGEABLE

!

# Defining an imprecise Bernoulli process

A sequence of binary random variables

$$X_1 , X_2 , \dots , X_n$$

satisfying the following properties

IDENTICALLY DISTRIBUTED ?

EPISTEMICALLY INDEPENDENT

EXCHANGEABLE

# Identical marginal models

# Identical marginal models

For every Bernoulli experiment $X_i$ in the sequence $X_1, \ldots, X_n$, we have the same given marginal model $\mathcal{D}_1$

A coherent set $\mathcal{D}_1$ of desirable gambles on $\mathscr{X}$

# Identical marginal models

For every Bernoulli experiment $X_i$ in the sequence $X_1, \dots, X_n$, we have **the same** given **marginal model** $\mathcal{D}_1$

**Implied by the assumption of exchangeability!** (due to symmetry)

A coherent set $\mathcal{D}_1$ of desirable gambles on $\mathscr{X}$

# Defining an imprecise Bernoulli process

A sequence of binary random variables

$$X_1 , X_2 , \dots , X_n$$

satisfying the following properties

IDENTICALLY DISTRIBUTED

EPISTEMICALLY INDEPENDENT

EXCHANGEABLE

# Defining an imprecise Bernoulli process

A sequence of binary random variables

$$X_1, X_2, \dots, X_n$$

satisfying the following properties

(IDENTICALLY DISTRIBUTED)

EPISTEMICALLY INDEPENDENT

EXCHANGEABLE

# Identical marginal models

For every Bernoulli experiment $X_i$ in the sequence $X_1, \ldots, X_n$, we have the same **given marginal model** $\mathcal{D}_1$

A coherent set $\mathcal{D}_1$ of desirable gambles on $\mathscr{X}$

# Identical marginal models

For every Bernoulli experiment $X_i$ in the sequence $X_1, \ldots, X_n$, we have the same **given marginal model** $\mathcal{D}_1$

$$f \in \mathcal{D}_1$$

$\mathrm{Bn}^1$ ↓

$$\mathrm{Bn}^1(f) \in \mathcal{H}_1$$

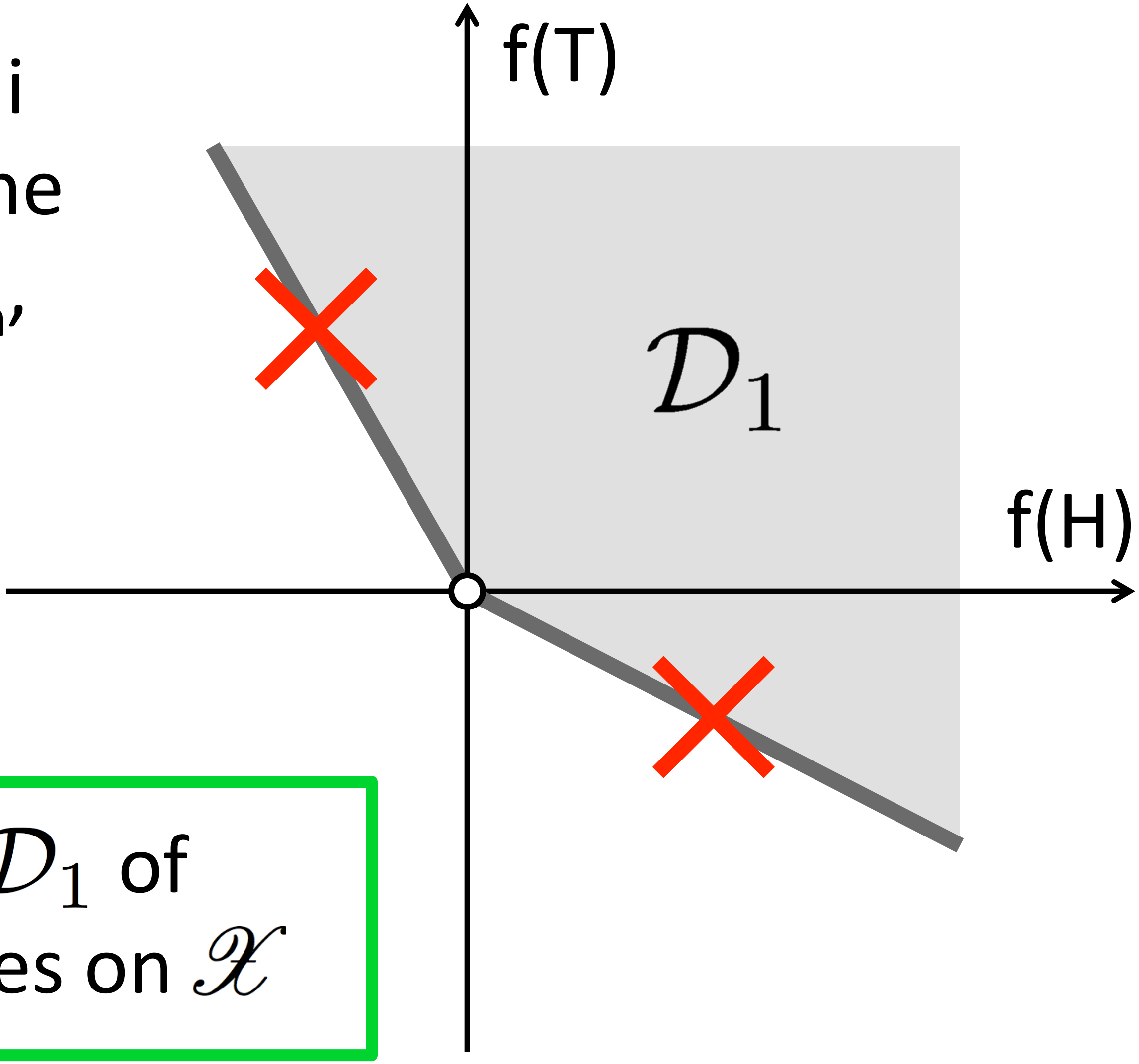
=

linear polynomials in $\mathcal{H}_n$

A coherent set $\mathcal{D}_1$ of desirable gambles on $\mathscr{X}$

# Identical marginal models

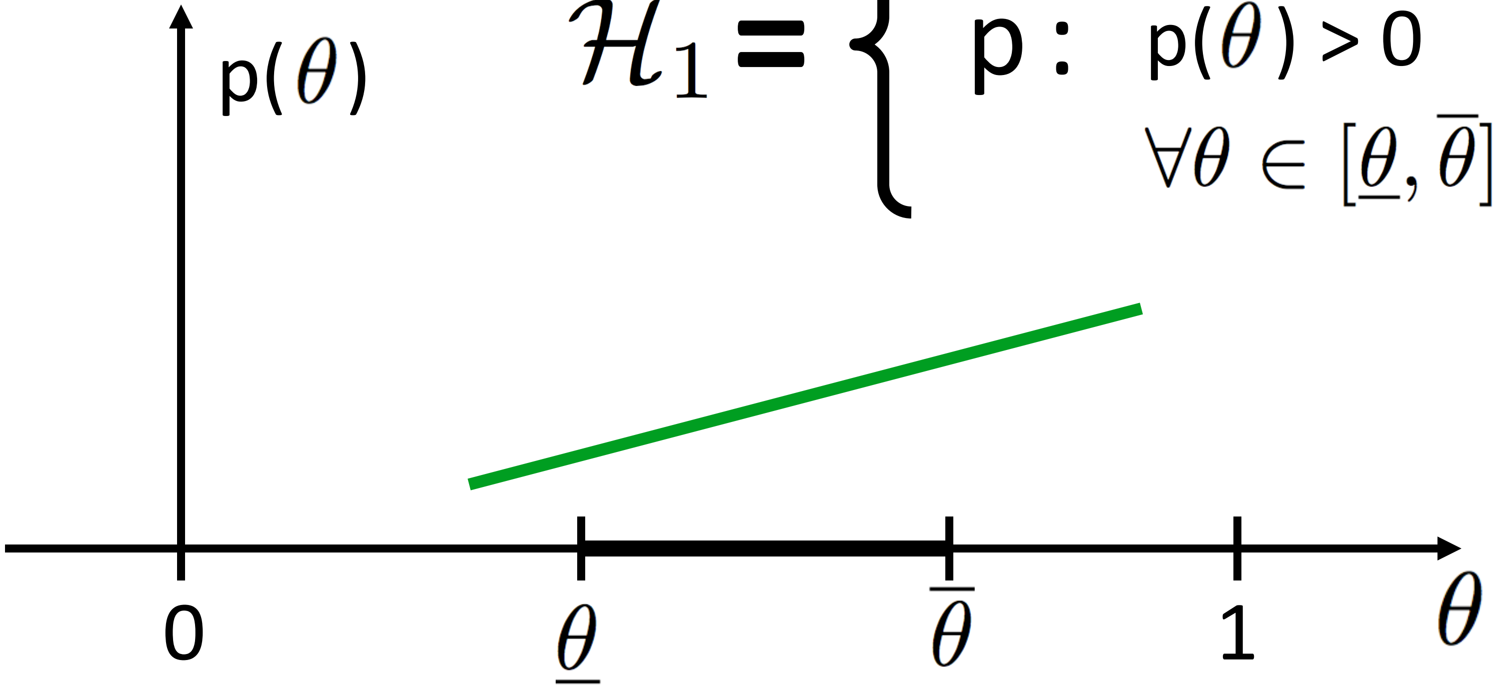
For every Bernoulli experiment $X_i$ in the sequence $X_1, \ldots, X_n$, we have the same **given marginal model** $\mathcal{D}_1$




A coherent set $\mathcal{D}_1$ of desirable gambles on $\mathscr{X}$

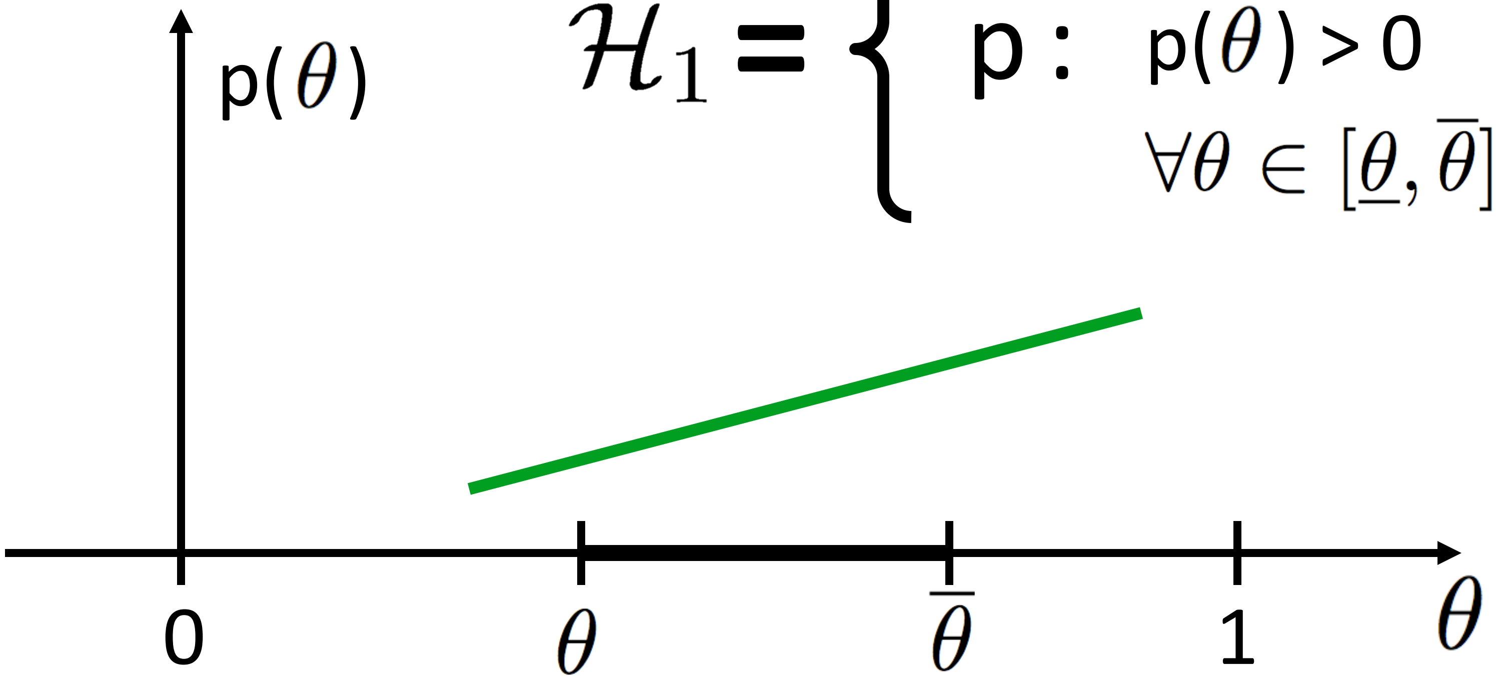
# Identical marginal models

$$\mathcal{H}_1 = \left\{ p : \begin{array}{l} \text{p is linear} \\ p(\theta) > 0 \\ \forall\theta \in [\underline{\theta}, \overline{\theta}] \end{array} \right\}$$



# Identical marginal models

$$\mathcal{H}_1 = \left\{ p : \begin{array}{l} \text{p is linear} \\ p(\theta) > 0 \\ \forall \theta \in [\underline{\theta}, \overline{\theta}] \end{array} \right\}$$




Set $\mathcal{H}_n$ of polynomial functions (of degree ≤ n) **?**

# Defining an imprecise Bernoulli process

A sequence of binary random variables

$$X_1, X_2, \ldots, X_n$$

satisfying the following properties

(IDENTICALLY DISTRIBUTED)

EPISTEMICALLY INDEPENDENT !

EXCHANGEABLE

# Epistemic independence

# **Epistemic independence** (defining property)

We assess $X_1$, $X_2$, … , $X_n$ to be

**epistemically independent**

# **Epistemic independence** (defining property)

We assess $X_1, X_2, \ldots, X_n$ to be **epistemically independent**

$$\Updownarrow$$

Learning the value of any number of variables does not change our beliefs about any subset of the remaining, unobserved variables.

$$X_1, X_2, X_3, X_4, X_5, \ldots, X_{n-1}, X_n$$

$$X_{n-1} = \mathbf{H}, \quad X_n = \mathbf{H}$$

# **Epistemic independence** (defining property)

We assess $X_1, X_2, \ldots, X_n$ to be **epistemically independent**

⬍

Learning the value of any number of variables does not change our beliefs about any subset of the remaining, unobserved variables.

$\underline{X_1, X_2, X_3}, X_4, X_5, \ldots, X_{n-1}, X_n$

$X_{n-1} = H$, $X_n = H$

# **Epistemic independence** (defining property)

We assess $X_1$ , $X_2$ , … , $X_n$ to be
**epistemically independent**

**Motivation:** Epistemic independence is a weaker assessment than the strong independence that is usually assumed

## **Epistemic independence** (defining property)

We assess $X_1$, $X_2$, … , $X_n$ to be **epistemically independent**

**Motivation:** Epistemic independence is a weaker assessment than the strong independence that is usually assumed

Set $\mathcal{H}_n$ of polynomial functions (of degree ≤ n) **?**

# **Epistemic independence** (defining property)

We assess $X_1, X_2, \ldots, X_n$ to be
**epistemically independent**

$$p \in \mathcal{H}_n \quad \Longleftrightarrow \quad \theta p \in \mathcal{H}_n$$

$$p \in \mathcal{H}_n \quad \Longleftrightarrow \quad (1-\theta) p \in \mathcal{H}_n$$

(degree ≤ n-1)

Set $\mathcal{H}_n$ of polynomial functions (of degree ≤ n)

**?**

# An imprecise Bernoulli process

# An imprecise Bernoulli process

## Exchangeability:

**Set $\mathcal{H}_n$ of polynomial functions**

Bernstein coherent:

B1. *if* $p = 0$ *then* $p \notin \mathcal{H}_n$
B2. *if* $p \in \mathcal{V}_n^+$, *then* $p \in \mathcal{H}_n$
B3. *if* $p \in \mathcal{H}_n$ *then* $\lambda p \in \mathcal{H}_n$ *for* $\lambda > 0$
B4. *if* $p_1, p_2 \in \mathcal{H}_n$ *then* $p_1 + p_2 \in \mathcal{H}_n$

## Epistemic independence:

$p \in \mathcal{H}_n$ (degree ≤ n-1) $\Leftrightarrow \theta p \in \mathcal{H}_n$

$p \in \mathcal{H}_n$ (degree ≤ n-1) $\Leftrightarrow (1-\theta) p \in \mathcal{H}_n$

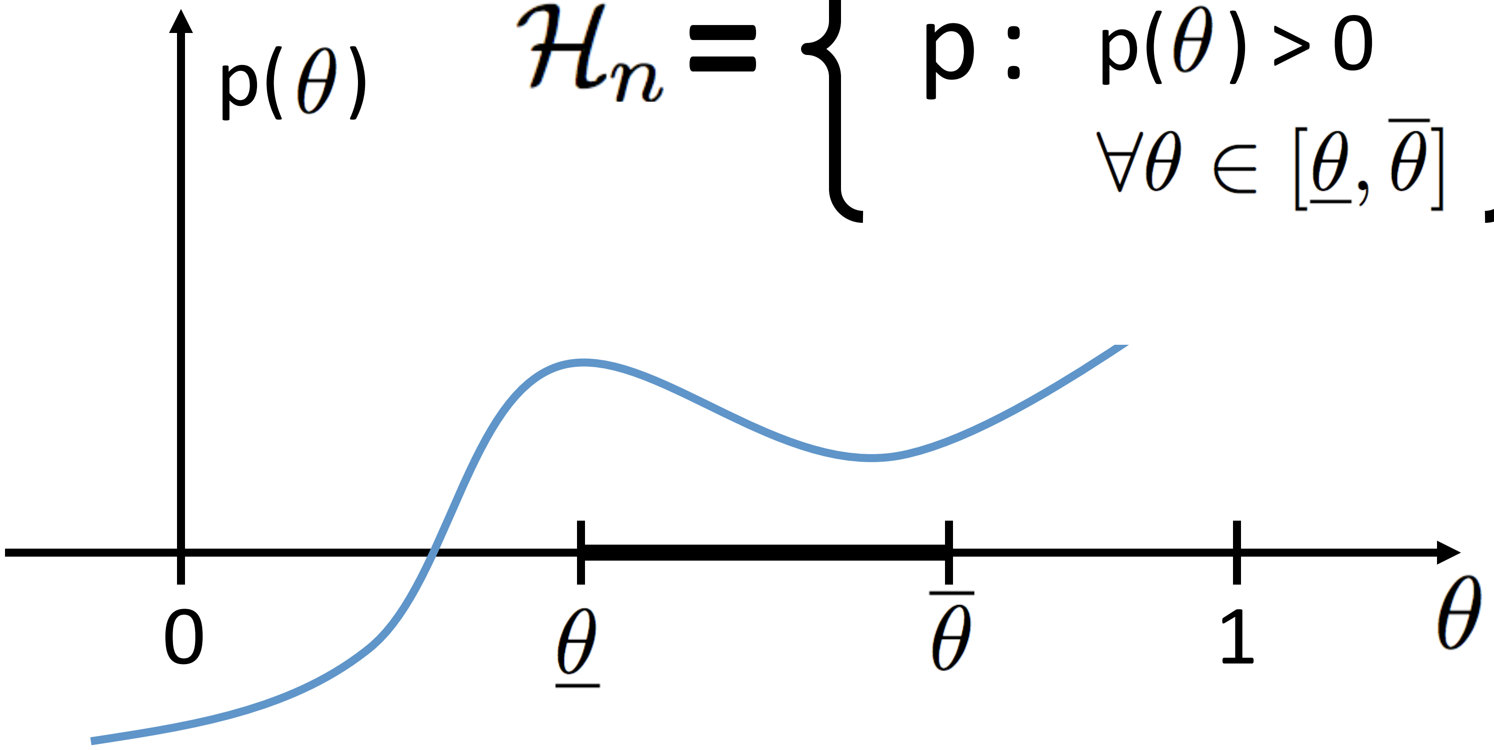
$\mathcal{H}_1$ = The linear polynomials in $\mathcal{H}_n$

**We are looking for the smallest such set $\mathcal{H}_n$**
(most conservative inferences)

# An imprecise Bernoulli process

$$\mathcal{H}_n = \left\{ p : \begin{array}{l} \deg(p) \leq n \\ p(\theta) > 0 \\ \forall \theta \in [\underline{\theta}, \overline{\theta}] \end{array} \right\}$$

**!**



# An imprecise Bernoulli process

$$\mathcal{H}_n = \left\{ p : \begin{array}{l} \deg(p) \leq n \\ p(\theta) > 0 \\ \forall \theta \in [\underline{\theta}, \overline{\theta}] \end{array} \right\}$$ **!**

$(Bn^n)^{-1}$ **Exchangeability**

$\mathcal{D}_n$

**Coherent set of desirable gambles on** $\mathscr{X}^n$
**(exchangeable and epistemic independent)**

**= IMPRECISE BERNOULLI PROCESS**

# How is this useful?

# Lower (and upper) prevision

# Lower (and upper) prevision

A set $\mathcal{D}_n$ of desirable gambles on $\mathscr{X}^n$

➡ associated **lower prevision** $\underline{P}_{\mathcal{D}_n}$

# Lower (and upper) prevision

A set $\mathcal{D}_n$ of desirable gambles on $\mathscr{X}^n$

➡ associated **lower prevision** $\underline{P}_{\mathcal{D}_n}$

For every gamble f on $\mathscr{X}^n$ :

$$\underline{P}_{\mathcal{D}_n}(f) := \sup\{\mu \in \mathbb{R} : f - \mu \in \mathcal{D}_n\}$$

supremum acceptable buying price

# Lower (and upper) prevision

A set $\mathcal{D}_n$ of desirable gambles on $\mathscr{X}^n$

➡ associated **lower prevision** $\underline{P}_{\mathcal{D}_n}$

For every gamble f on $\mathscr{X}^n$ :

$$\underline{P}_{\mathcal{D}_n}(f) := \sup\{\mu \in \mathbb{R} : f - \mu \in \mathcal{D}_n\}$$

supremum acceptable buying price

In a similar way: **upper prevision** $\overline{P}_{\mathcal{D}_n}$

infimum acceptable selling price

# Lower (and upper) prevision

A set $\mathcal{D}_n$ of desirable gambles on $\mathscr{X}^n$

➡ associated **lower prevision** $\underline{P}_{\mathcal{D}_n}$

For every gamble f on $\mathscr{X}^n$ :

$$\underline{P}_{\mathcal{D}_n}(f) := \sup\{\mu \in \mathbb{R} : f - \mu \in \mathcal{D}_n\}$$

supremum acceptable buying price

# Lower (and upper) prevision

A set $\mathcal{D}_n$ of desirable gambles on $\mathscr{X}^n$

➡ associated **lower prevision** $\underline{P}_{\mathcal{D}_n}$

For every gamble f on $\mathscr{X}^n$ :

$$\underline{P}_{\mathcal{D}_n}(f) := \sup\{\mu \in \mathbb{R} : f - \mu \in \mathcal{D}_n\}$$

supremum acceptable buying price

What happens for our specific set $\mathcal{D}_n$ ?

# Lower (and upper) prevision

A set $\mathcal{D}_n$ of desirable gambles on $\mathscr{X}^n$

➡ associated **lower prevision** $\underline{P}_{\mathcal{D}_n}$

For every gamble f on $\mathscr{X}^n$ :

$$\underline{P}_{\mathcal{D}_n}(f) := \sup\{\mu \in \mathbb{R} : f - \mu \in \mathcal{D}_n\}$$

supremum acceptable buying price

↕

$\mathcal{H}_n$

What happens for our specific set $\mathcal{D}_n$ ?

# Lower (and upper) prevision

A set $\mathcal{D}_n$ of desirable gambles on $\mathscr{X}^n$

➡ associated **lower prevision** $\underline{P}_{\mathcal{D}_n}$

For every gamble f on $\mathscr{X}^n$ :

$$\underline{P}_{\mathcal{D}_n}(f) := \sup\{\mu \in \mathbb{R} \quad f - \mu \in \mathcal{D}_n\}$$

supremum acceptable buying price

$$\mathrm{Bn}^n(f) - \mu \in \mathcal{H}_n$$

# Lower (and upper) prevision

A set $\mathcal{D}_n$ of desirable gambles on $\mathscr{X}^n$

➡ associated **lower prevision** $\underline{P}_{\mathcal{D}_n}$

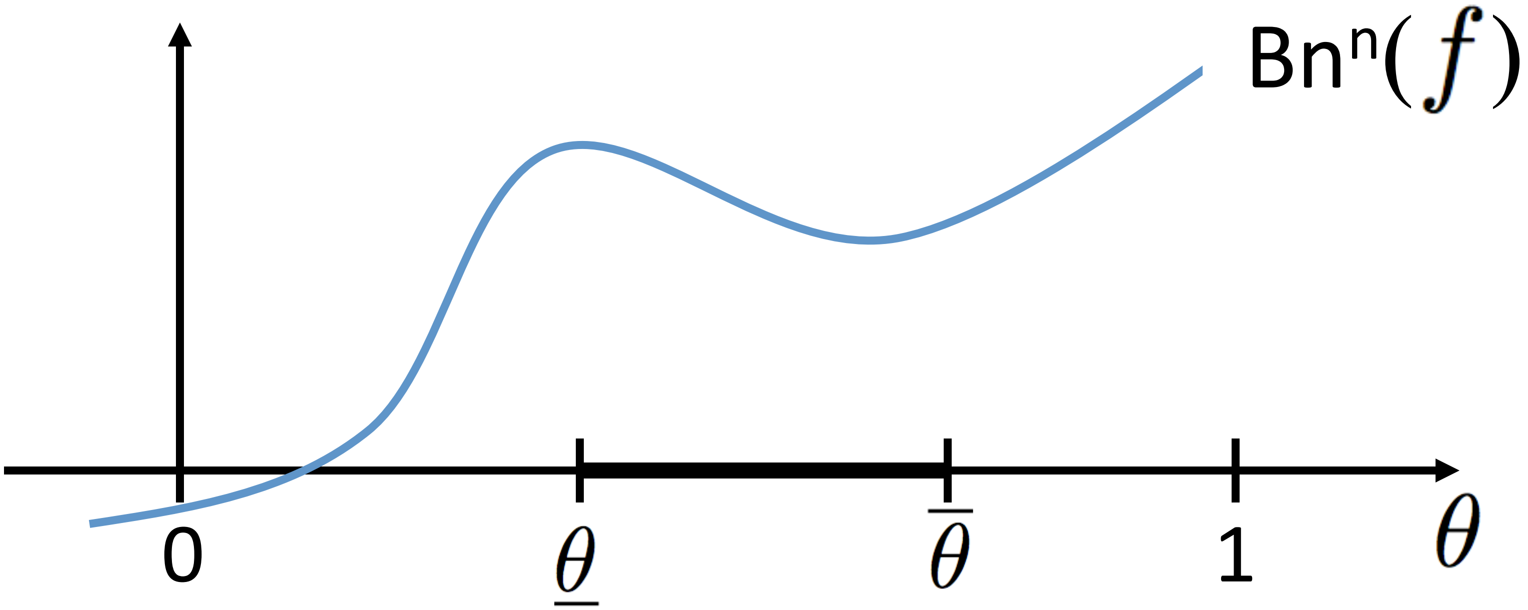
For every gamble f on $\mathscr{X}^n$ :

$$\underline{P}_{\mathcal{D}_n}(f) := \sup\{\mu \in \mathbb{R} : f - \mu \in \mathcal{D}_n\}$$

supremum acceptable buying price

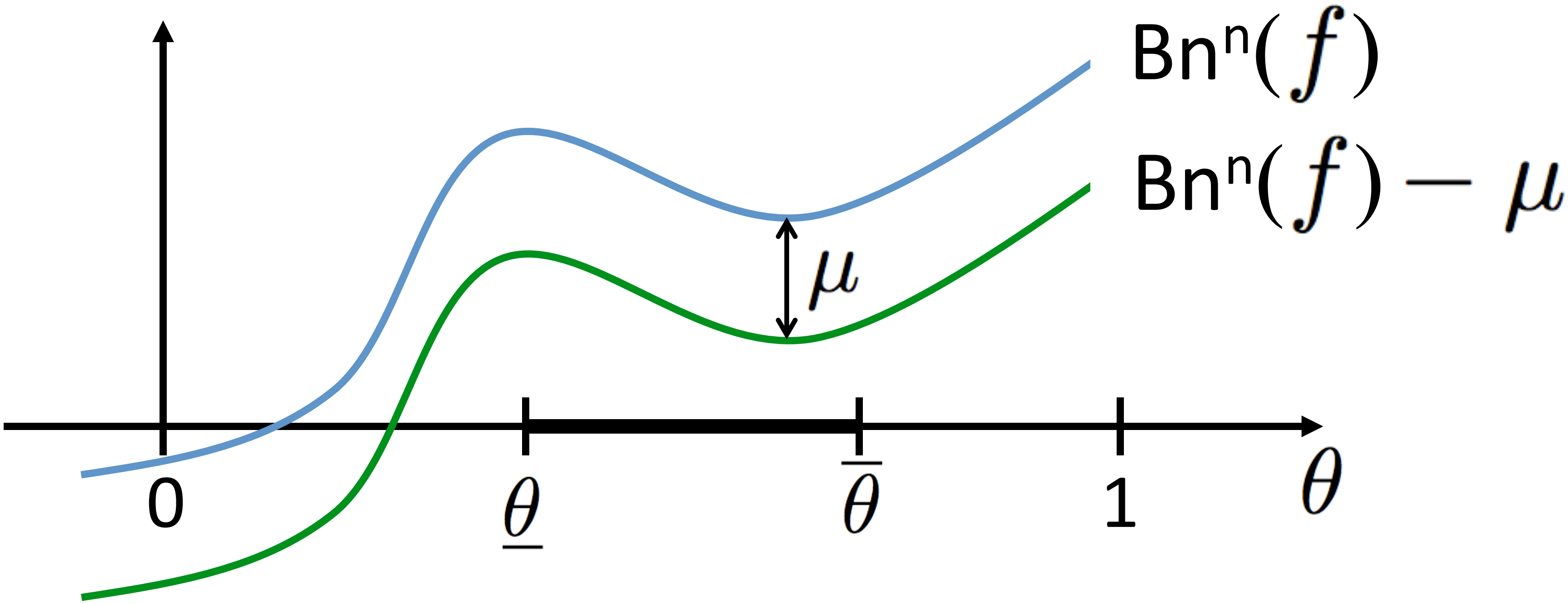
$$= \sup\{\mu \in \mathbb{R} : \mathrm{Bn}^n(f) - \mu \in \mathcal{H}_n\}$$

# Lower (and upper) prevision



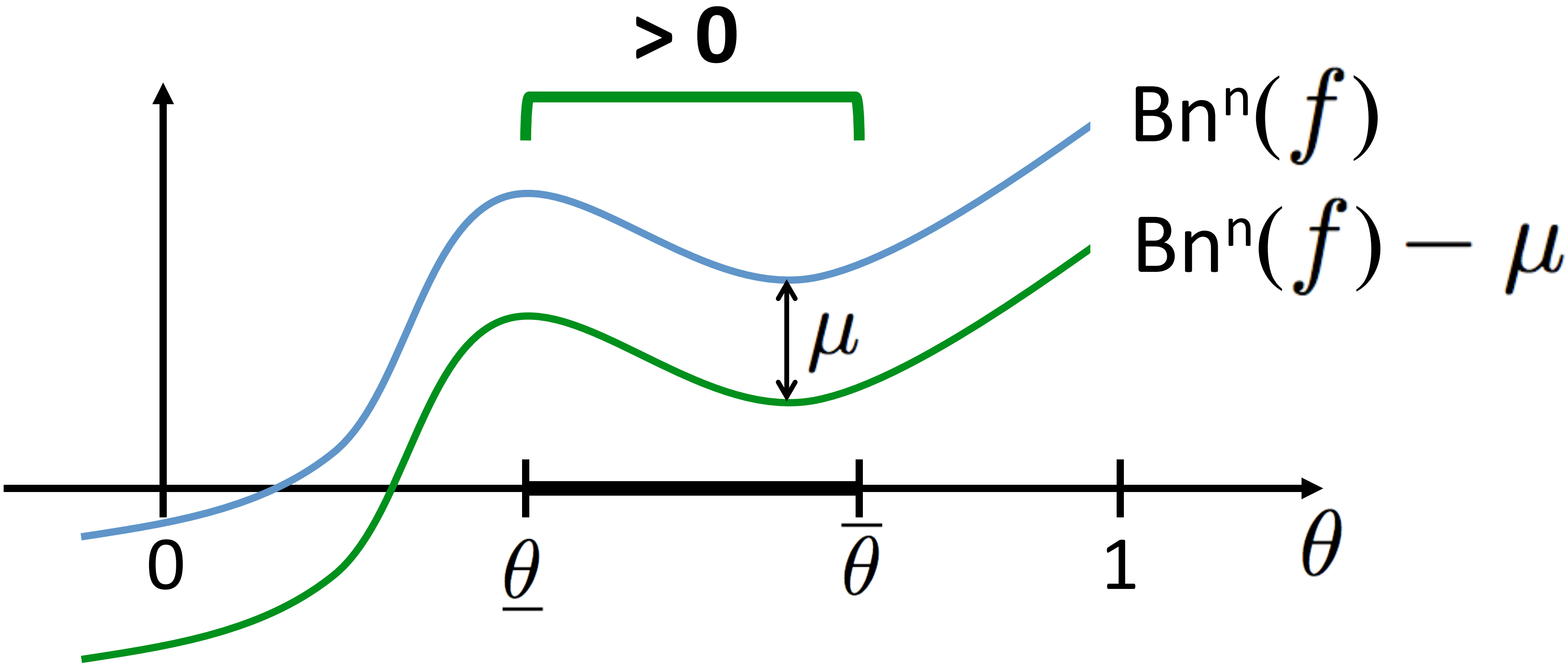

$$\sup\{\mu \in \mathbb{R} : \underline{\mathrm{Bn}^n(f)} - \mu \in \mathcal{H}_n\}$$

# Lower (and upper) prevision



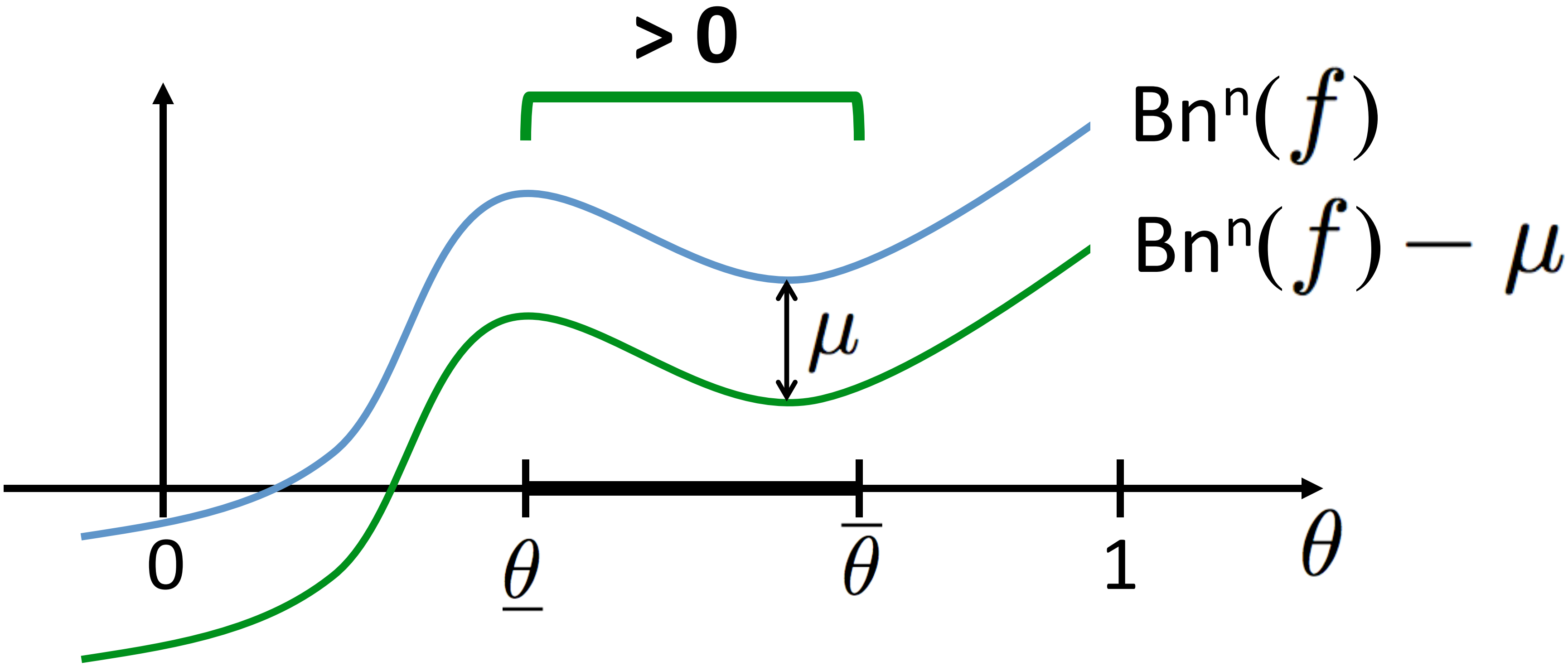

$$\sup\{\mu \in \mathbb{R} : \underline{\mathrm{Bn}^n(f) - \mu} \in \mathcal{H}_n\}$$

# Lower (and upper) prevision



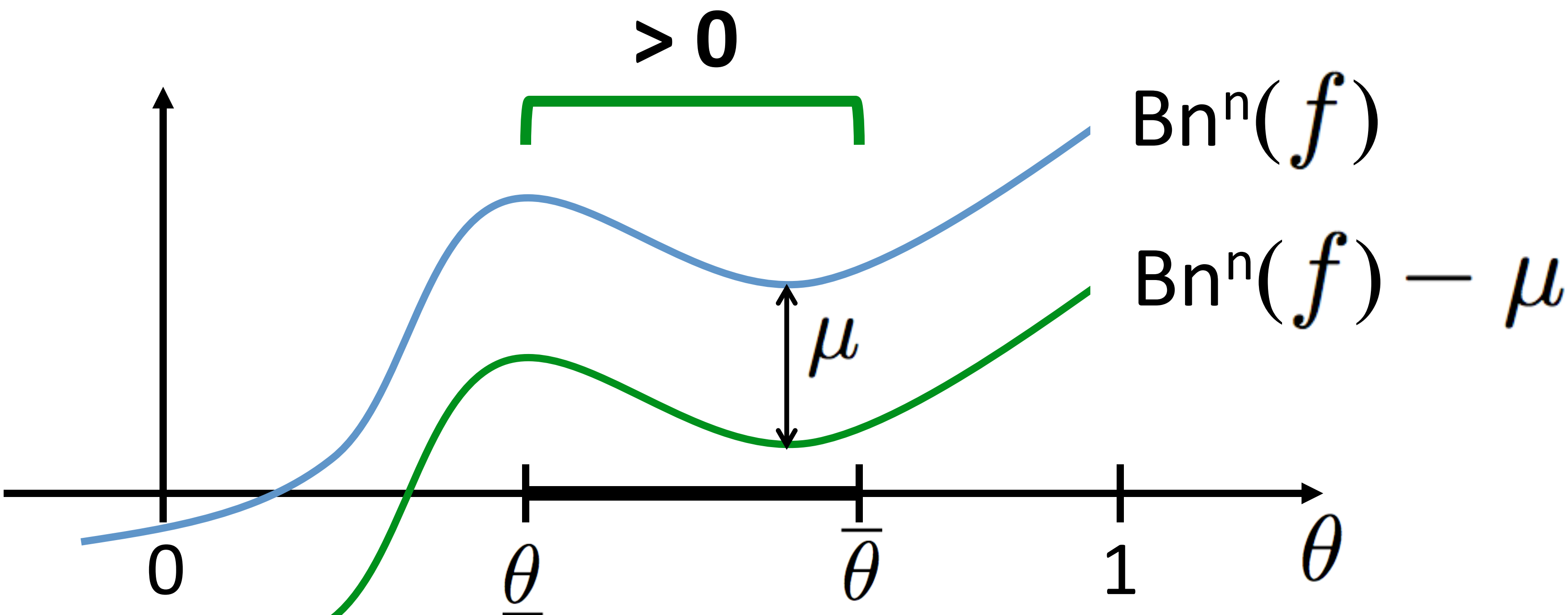

$$\sup\{\mu \in \mathbb{R} : \mathrm{Bn}^n(f) - \mu \in \mathcal{H}_n\}$$

# Lower (and upper) prevision



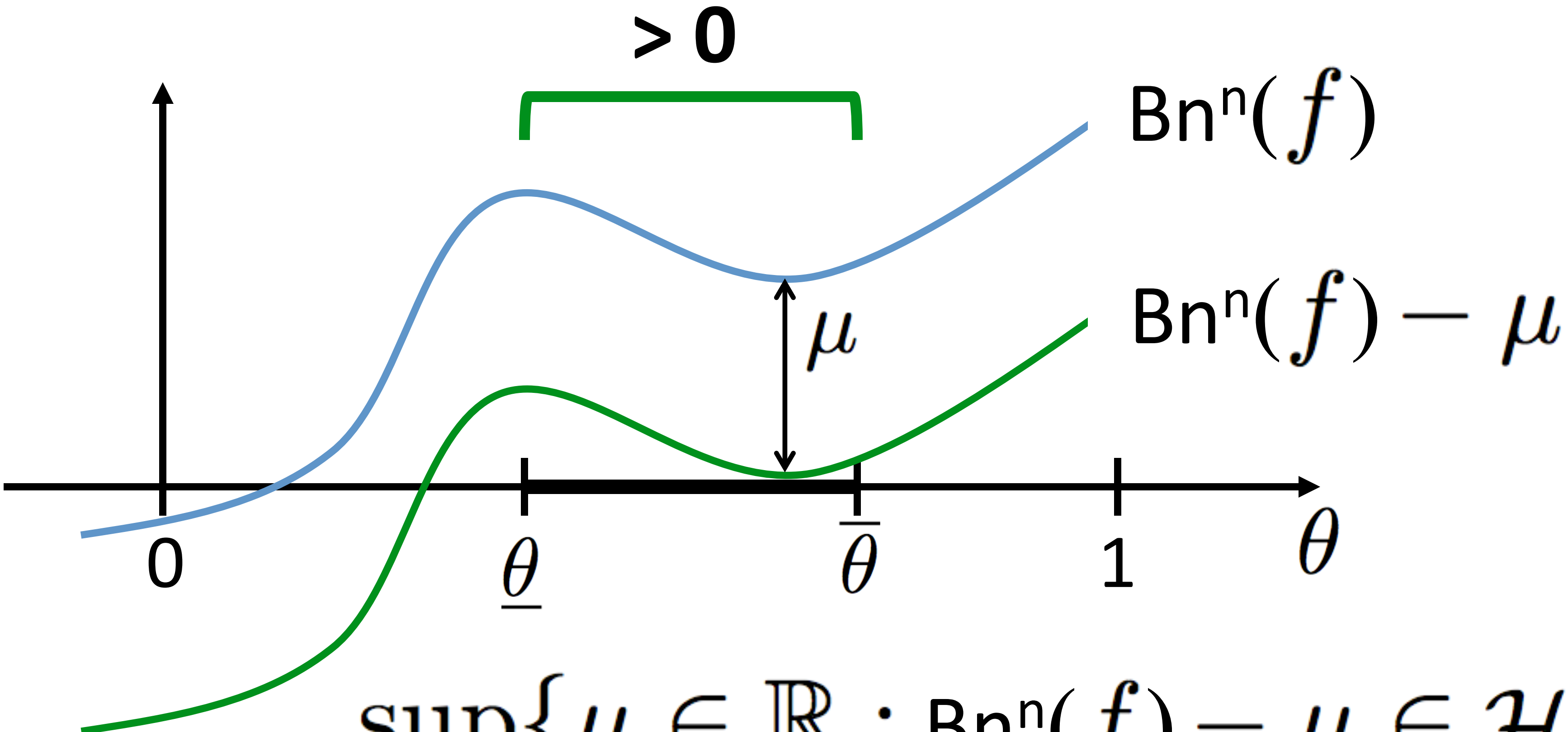

$$\sup\{\mu \in \mathbb{R} : \mathrm{Bn}^n(f) - \mu \in \mathcal{H}_n\}$$

# Lower (and upper) prevision



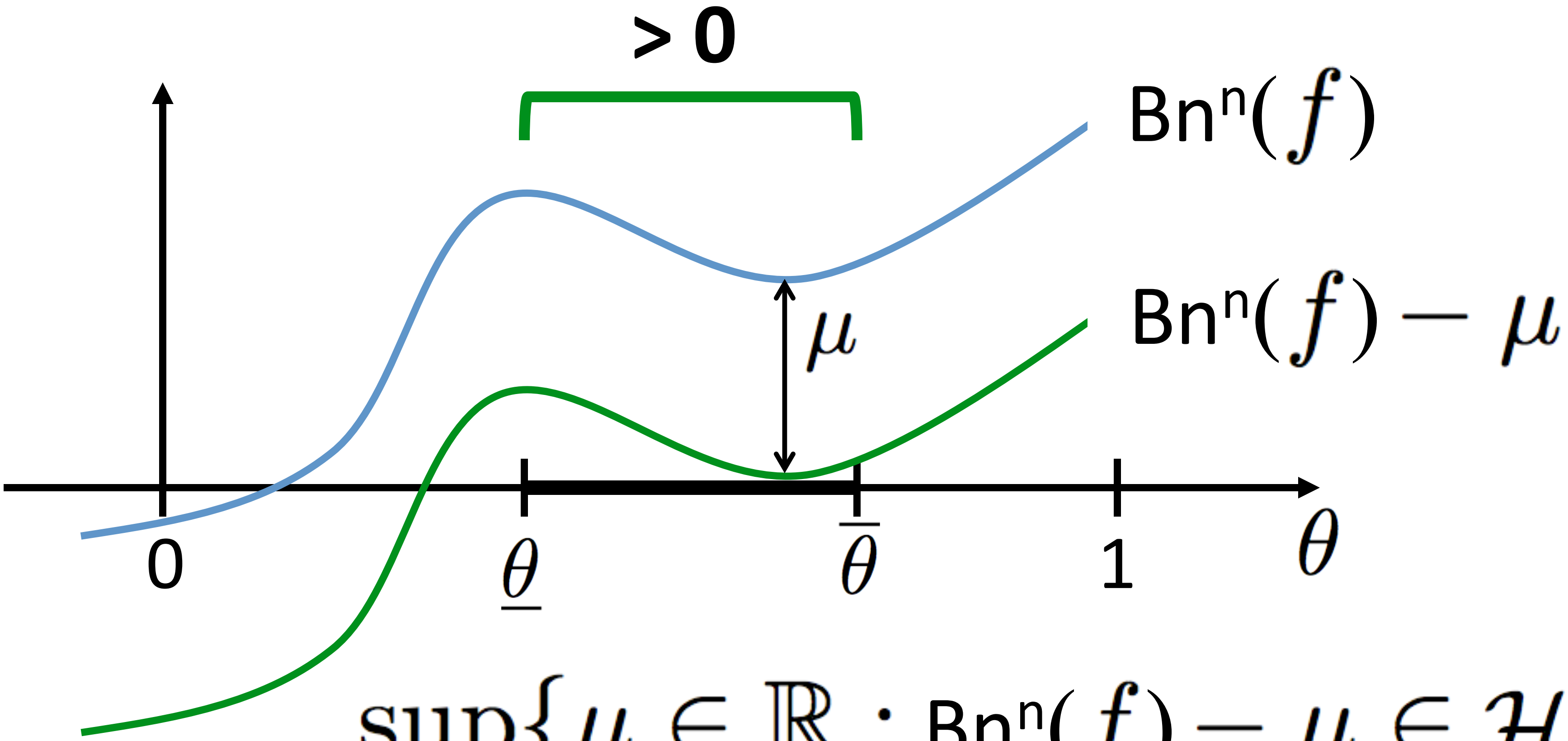

$$\sup\{\mu \in \mathbb{R} : \mathrm{Bn}^n(f) - \mu \in \mathcal{H}_n\}$$

# Lower (and upper) prevision




$$\sup\{\mu \in \mathbb{R} : \mathrm{Bn}^n(f) - \mu \in \mathcal{H}_n\}$$

# Lower (and upper) prevision




$$\sup\{\mu \in \mathbb{R} : \mathrm{Bn}^n(f) - \mu \in \mathcal{H}_n\}$$

$$= \min\{\mathrm{Bn}^n(f)(\theta) : \theta \in [\underline{\theta}, \overline{\theta}]\}$$

# Lower (and upper) prevision

**IMPRECISE BERNOULLI PROCESS**

associated **lower prevision**

For every gamble f on $\mathscr{X}^n$ :

$$\underline{P}_{\mathcal{D}_n}(f) = \min\{\mathrm{Bn}^n(f)(\theta) : \theta \in [\underline{\theta}, \overline{\theta}]\}$$

supremum acceptable buying price

# Lower (and upper) prevision

**IMPRECISE BERNOULLI PROCESS**

associated **lower prevision** and **upper prevision**

For every gamble f on $\mathscr{X}^n$ :

infimum acceptable selling price

$$\overline{P}_{\mathcal{D}_n}(f) = \max\{\mathrm{Bn}^n(f)(\theta) : \theta \in [\underline{\theta}, \overline{\theta}]\}$$

$$\underline{P}_{\mathcal{D}_n}(f) = \min\{\mathrm{Bn}^n(f)(\theta) : \theta \in [\underline{\theta}, \overline{\theta}]\}$$

supremum acceptable buying price

# Link with sensitivity analysis

# Link with sensitivity analysis

**IMPRECISE BERNOULLI PROCESS**

associated **lower prevision** and **upper prevision**

$$\overline{P}_{\mathcal{D}_n}(f) = \max\{\mathrm{Bn}^n(f)(\theta) : \theta \in [\underline{\theta}, \overline{\theta}]\}$$

$$\underline{P}_{\mathcal{D}_n}(f) = \min\{\mathrm{Bn}^n(f)(\theta) : \theta \in [\underline{\theta}, \overline{\theta}]\}$$

$\mathrm{Bn}^n(f)(\theta)$ = precise expected value $\mathrm{E}(f)$

# Link with sensitivity analysis

**IMPRECISE BERNOULLI PROCESS**

associated **lower prevision** and **upper prevision**

$\overline{\mathrm{E}}(f)$

**sensitivity analysis !**

||

$$\overline{P}_{\mathcal{D}_n}(f) = \max\{\mathrm{Bn}^n(f)(\theta) : \theta \in [\underline{\theta}, \overline{\theta}]\}$$

$$\underline{P}_{\mathcal{D}_n}(f) = \min\{\mathrm{Bn}^n(f)(\theta) : \theta \in [\underline{\theta}, \overline{\theta}]\}$$

|| ||

$\underline{\mathrm{E}}(f)$ precise expected value $\mathrm{E}(f)$

## Link with sensitivity analysis

Lower and upper prevision associated with an **IMPRECISE BERNOULLI PROCESS**

**MAIN RESULT:** || !

Lower and upper expectation obtained by applying **SENSITIVITY ANALYSIS** to a classical, precise bernoulli process

# Why is this important?

# Why is this important?

- **Strong mathematical assumption:**
  **our series of experiments is described by an underlying true probability mass function (satisfying the IID property)**
- **This model is imprecise only because we do not know which precise model is correct**

Lower and upper expectation obtained by applying **SENSITIVITY ANALYSIS** to a classical, precise bernoulli process

# Why is this important?

Lower and upper prevision associated with an
**IMPRECISE BERNOULLI PROCESS**

- **Behavioural assessments:**
  **our series of experiments is described by a set of desirable gambles, satisfying exchangeability and epistemic independence (can be expressed in terms of behaviour)**
- **This model is inherently imprecise**

## Why is this important?

Lower and upper prevision associated with an **IMPRECISE BERNOULLI PROCESS**

**MAIN RESULT:** **=** **! SURPRISING !**

Lower and upper expectation obtained by applying **SENSITIVITY ANALYSIS** to a classical, precise bernoulli process

# Conclusion

# Conclusion

**EXCHANGEABILITY**

**+**

**EPISTEMIC INDEPENDENCE**

**⬇**

**SENSITIVITY ANALYSIS**

**BERNOULLI PROCESSES**