Choice Models: From Linear Option Spaces to Sets Of Horse Lotteries

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In recent work [1, 2], we have introduced an axiomatisation for binary and non-binary choice under uncertainty, with an interpretation—or semantics—based on the notion of desirability. This general account of decision under uncertainty is based on the notion of a choice function: a function that selects, from a given set of available options, those options that a subject does not reject. Our basic axiomatisation, and its various extensions, allow for a conservative inference mechanism [1]—notions of consistency and natural extension for choice function assessments—as well as powerful representation theorems [2] in terms of sets of 'atomic' binary models: (general, maximal, lexicographic or strict) sets of desirable gambles, lower and linear previsions, and so on. Our new approach builds on the foundations for choice functions provided by Seidenfeld et al. [4], in whose work the options to choose between are so-called *horse lotteries*: probability mass functions on a finite set of prizes that may depend on the state of the world our subject is uncertain about. But we follow Van Camp et al. [5, 7, 8] in allowing our options to live in an *abstract ordered linear space*.

We first discuss reasons for wanting to do so: it is mathematically more elegant to work with linear spaces rather than convex sets; it allows for incorporating indifference [5, 8]; and it allows us to deal with gambles [9], vector-valued gambles [10], and polynomial gambles in an exchangeability context [3, 6], as only a few of the many imaginable special cases. More importantly, we explain how our axiomatisation, inference and representation results for abstract linear option sets can be made to carry over, via a collection of intricate embedding theorems, to choice models on horse lotteries. To make this work, we have extended ideas by Zaffalon and Miranda [10] for desirability, and carried over to choice functions by Van Camp [5]. This opens up an avenue for extending the seminal work of Seidenfeld et al. [4] in various directions.

References

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