Robustifying the Viterbi Algorithm

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A Toy Example
Weather estimation

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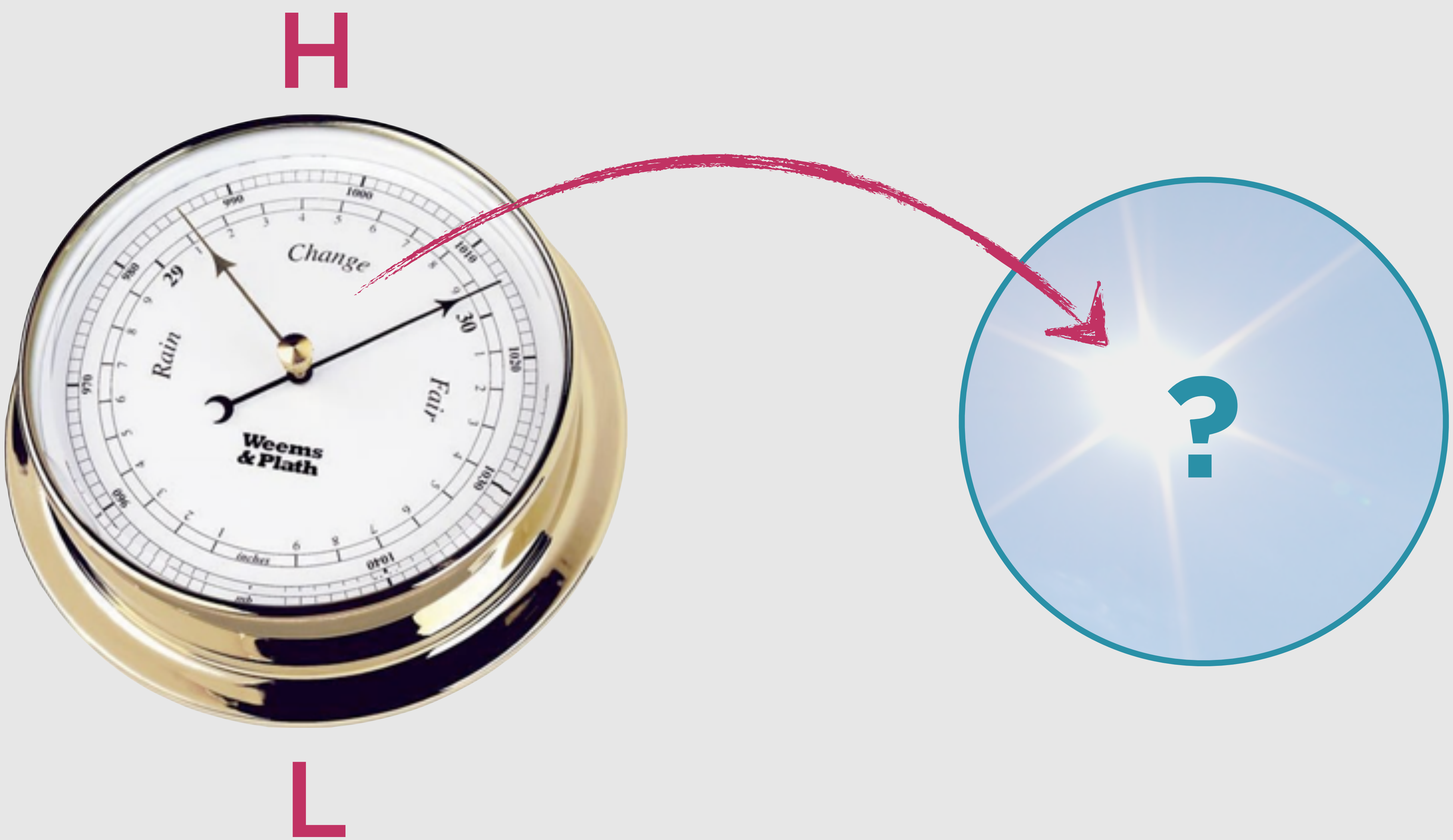

A Toy Example

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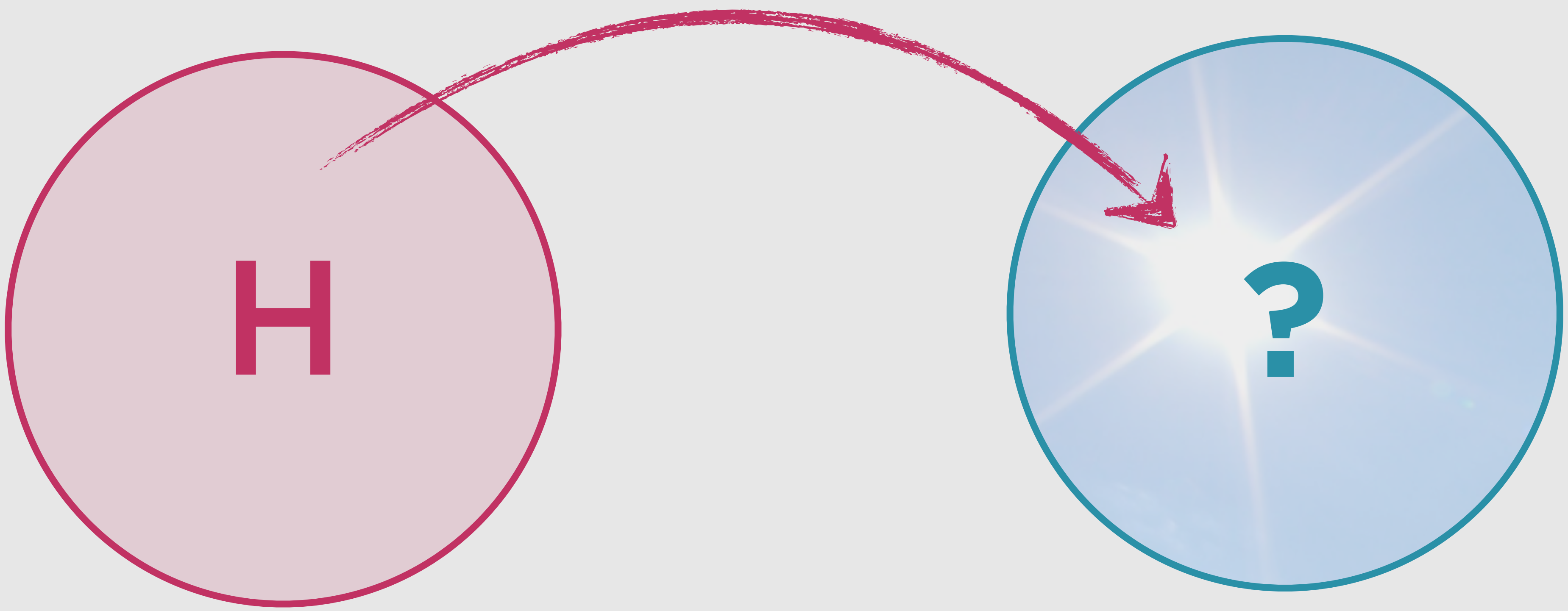

A Toy Example

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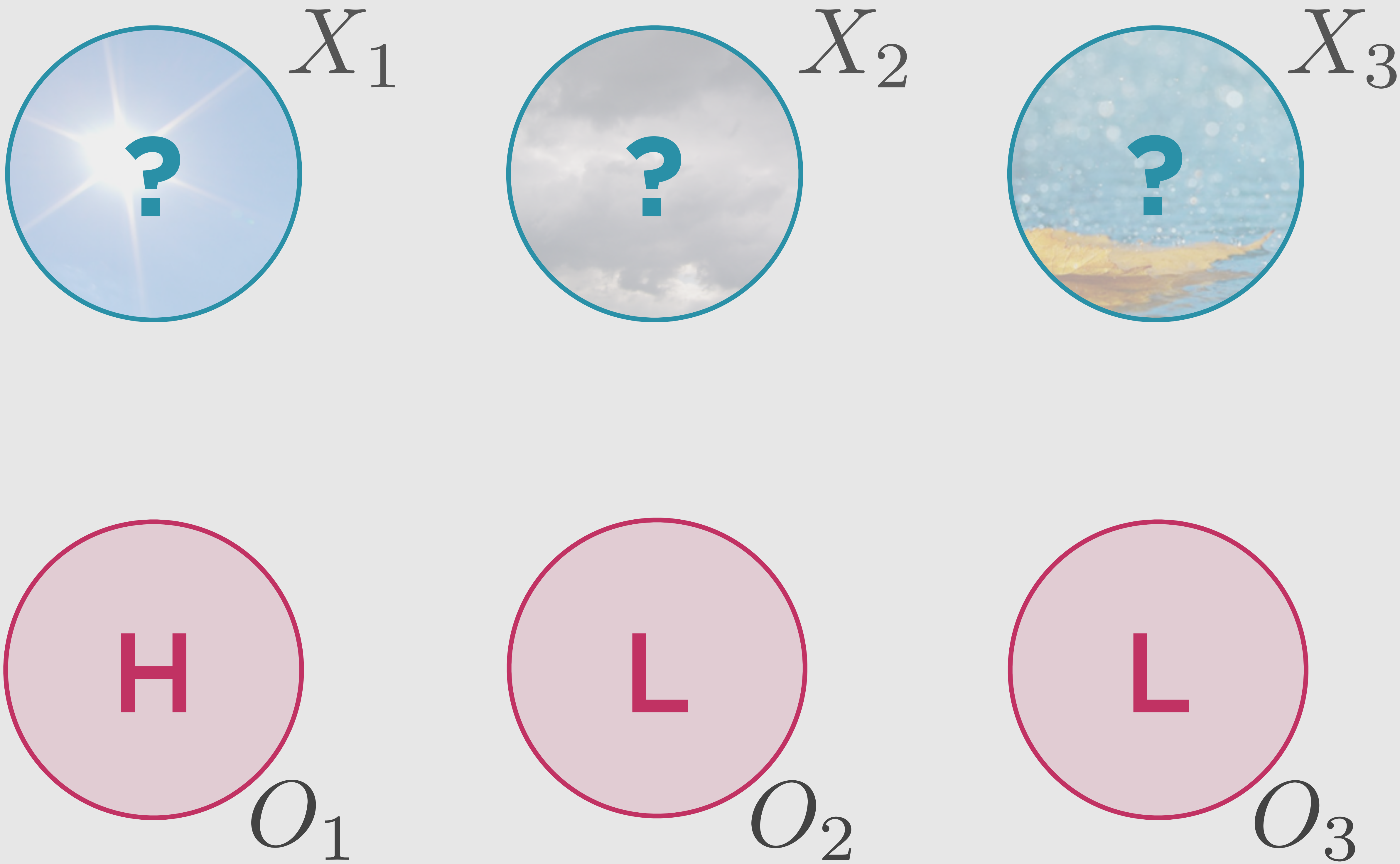
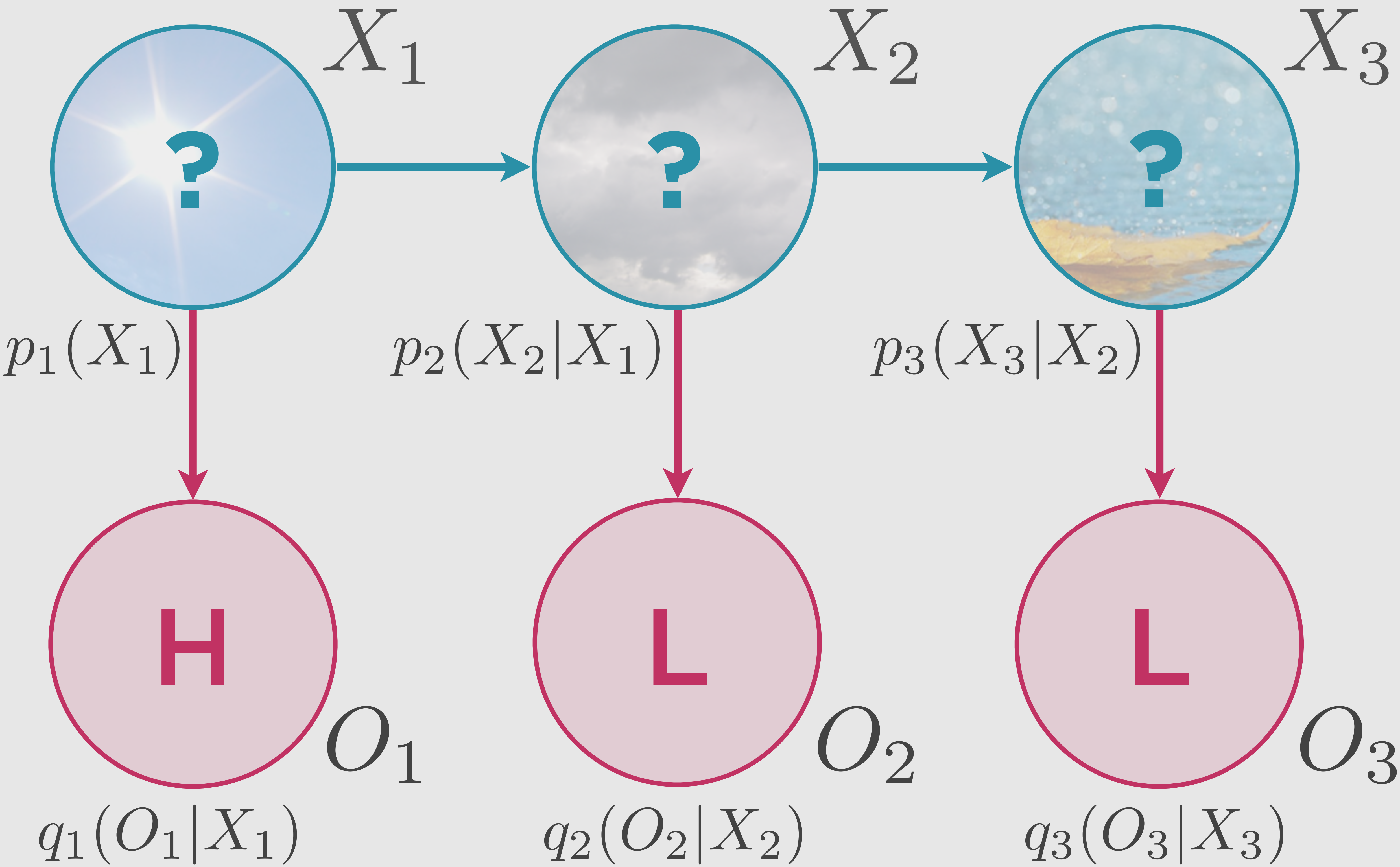


Hidden Markov Model
Local models



Hidden Markov Model

Local models

Local models

$$p_1(X_1) \quad p_2(X_2|X_1) \quad p_3(X_3|X_2)$$

$$q_1(O_1|X_1) \quad q_2(O_2|X_2) \quad q_3(O_3|X_3)$$

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Global model

$$p(X_{1:3}, O_{1:3}) = p_1(X_1)q_1(O_1|X_1)$$

$$\prod_{i=2}^{3} p_i(X_i|X_{i-1})q_i(O_i|X_i)$$

Hidden Markov Model

Estimating the hidden sequence

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$$\arg\max_{x_{1:3}} p(x_{1:3} | o_{1:3})$$

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$$\arg\max_{x_{1:3}} p(x_{1:3}|o_{1:3}) = \arg\max_{x_{1:3}} \frac{p(x_{1:3}, o_{1:3})}{p(o_{1:3})}$$

Hidden Markov Model

Estimating the hidden sequence

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Viterbi algorithm (1967)

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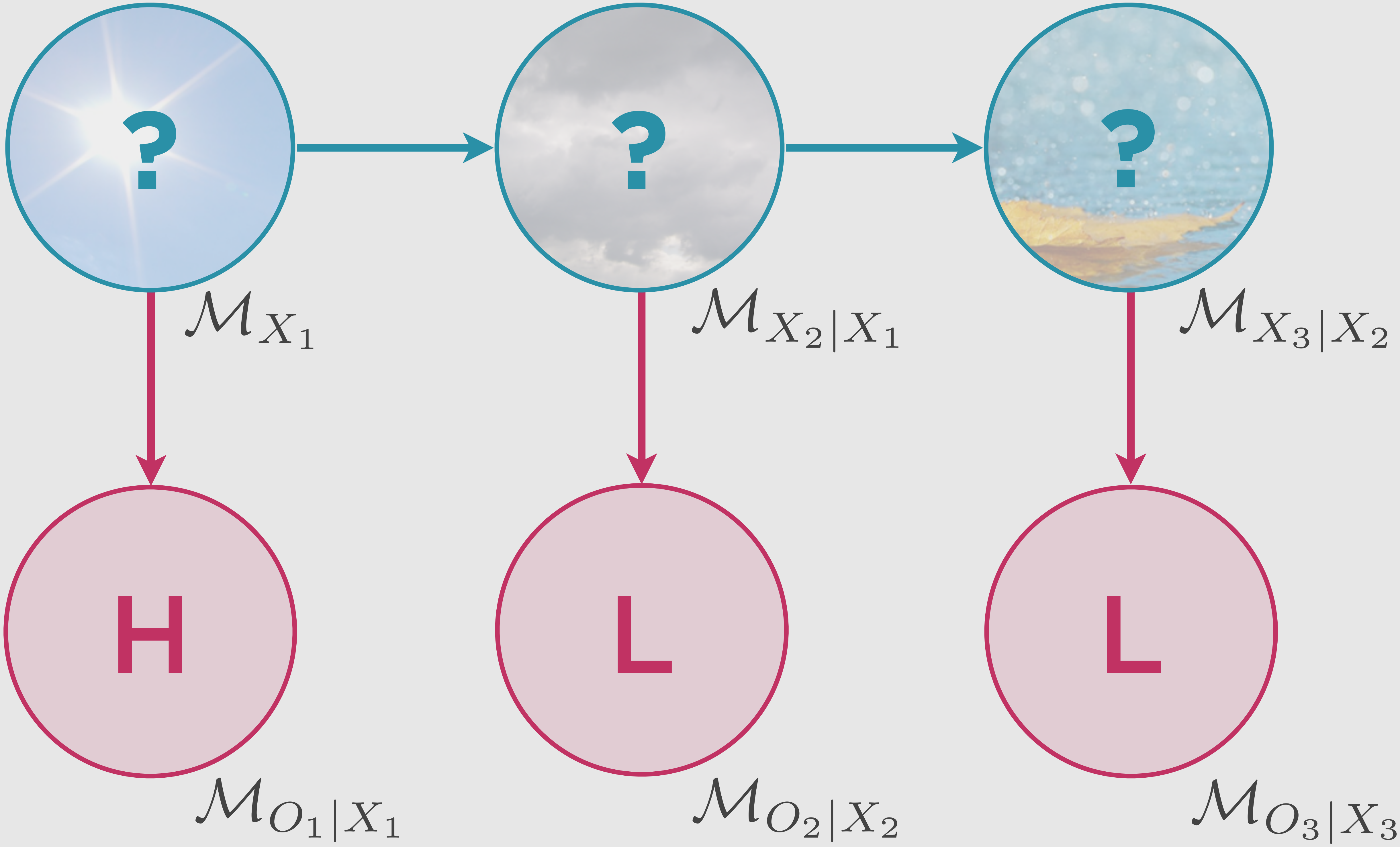
- Complexity $O(nm^2)$
  - *n*: length of the sequence
  - *m*: size of state space

Viterbi algorithm (1967)

- Recursive
- Complexity $O(nm^2)$
  - n: length of the sequence
  - m: size of state space
- Extendible to k-best Viterbi

Imprecise Hidden Markov Model

Local models



Local models

$$\mathcal{M}_{X_1} \quad \mathcal{M}_{X_2|X_1} \quad \mathcal{M}_{X_3|X_2}$$

$$\mathcal{M}_{O_1|X_1} \quad \mathcal{M}_{O_2|X_2} \quad \mathcal{M}_{O_3|X_3}$$

Imprecise Hidden Markov Model
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Global model

$$\mathcal{M} = \left\{ \prod_{i=1}^{3} p_i(X_i|X_{i-1}) q_i(O_i|X_i) : \right.$$
$$(\forall k \in \{1,2,3\})\; p_k(\cdot|X_{k-1}) \in \mathcal{M}_{X_k|X_{k-1}},$$
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Imprecise Hidden Markov Model

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Global model

May contain infinitely many precise models!

$$\mathcal{M} = \left\{ \prod_{i=1}^{3} p_i(X_i|X_{i-1}) q_i(O_i|X_i) : \right.$$
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Partial order

$$x_{1:3} \succ \hat{x}_{1:3} \Leftrightarrow (\forall p \in \mathcal{M})\ p(x_{1:3}|o_{1:3}) > p(\hat{x}_{1:3}|o_{1:3})$$

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Set of maximal solutions

$$\mathrm{opt}_{\max}(\mathcal{X}_{1:3}) \triangleq \{\hat{x}_{1:3} \in \mathcal{X}_{1:3} \colon (\forall x_{1:3} \in \mathcal{X}_{1:3})\ x_{1:3} \not\succ \hat{x}_{1:3}\}$$

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Indecision
There may be multiple maximal solutions.

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x_{1:n} \succ \hat{x}_{1:n} &\Leftrightarrow (\forall p \in \mathcal{M})\ p(x_{1:n}|o_{1:n}) > p(\hat{x}_{1:n}|o_{1:n}) \\
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&\Leftrightarrow (\forall p \in \mathcal{M})\ \frac{p(x_{1:n}, o_{1:n})}{p(\hat{x}_{1:n}, o_{1:n})} > 1
\end{aligned}
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Imprecise Hidden Markov Model

Rewriting the solution set

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What if $p(\hat{x}_{1:n}, o_{1:n})$ becomes zero?

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Can be calculated in advance

Imprecise Hidden Markov Model

Rewriting the solution set

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$$\Updownarrow$$

$$\max_{x_{1:n} \in \mathcal{X}_{1:n}} \prod_{k=1}^{n} \chi_k(x_k, x_{k-1}, \hat{x}_k, \hat{x}_{k-1}) \omega_k(x_k, \hat{x}_k, o_k) \leq 1$$

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How do we calculate this set?

Imprecise Hidden Markov Model

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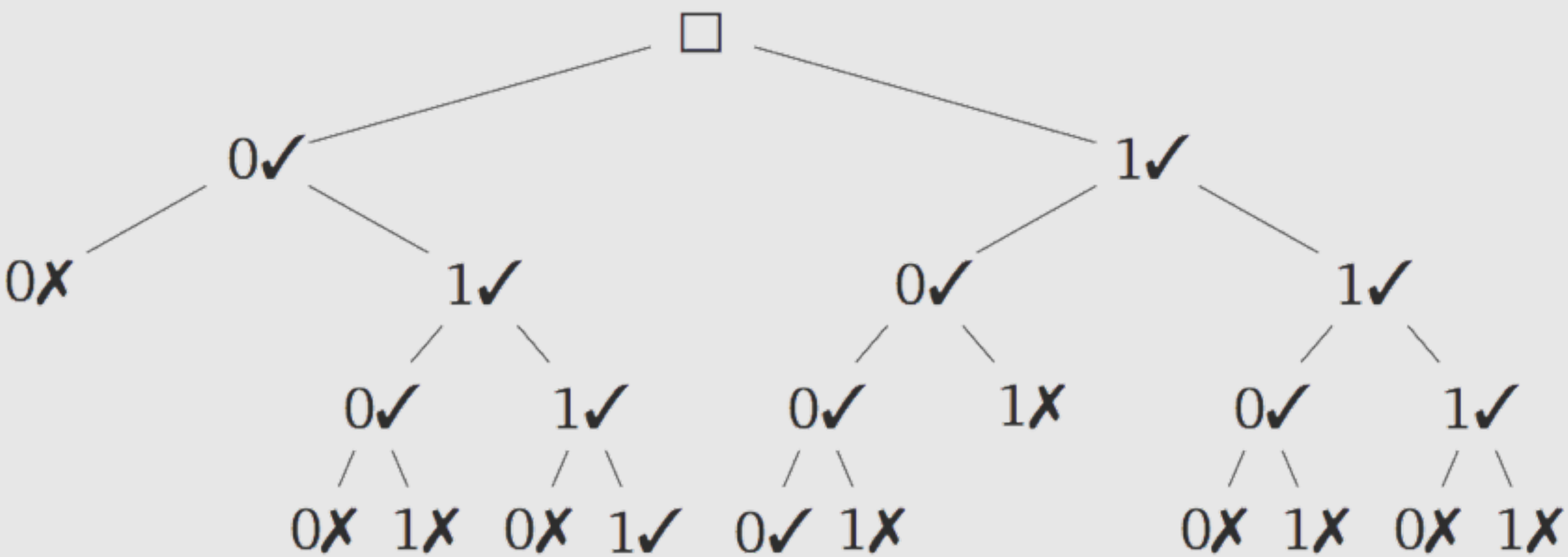
How do we calculate this set?

MaxiHMM algorithm

MaxiHMM algorithm

General overview

MaxiHMM algorithm



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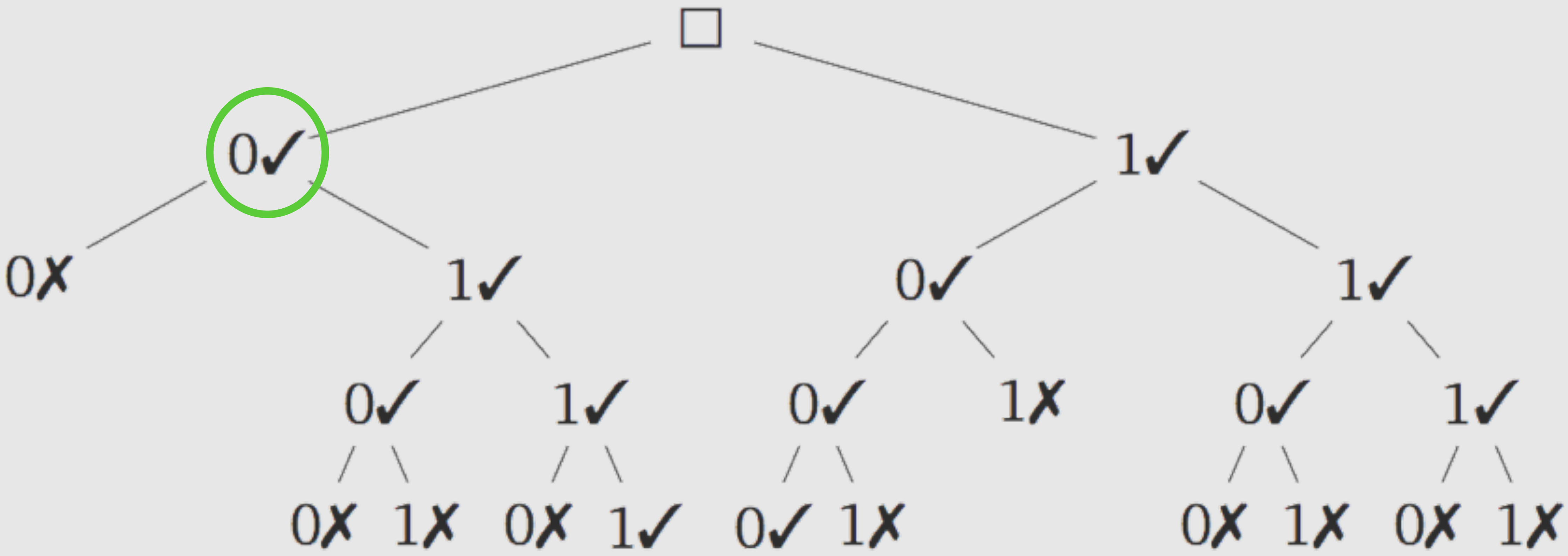

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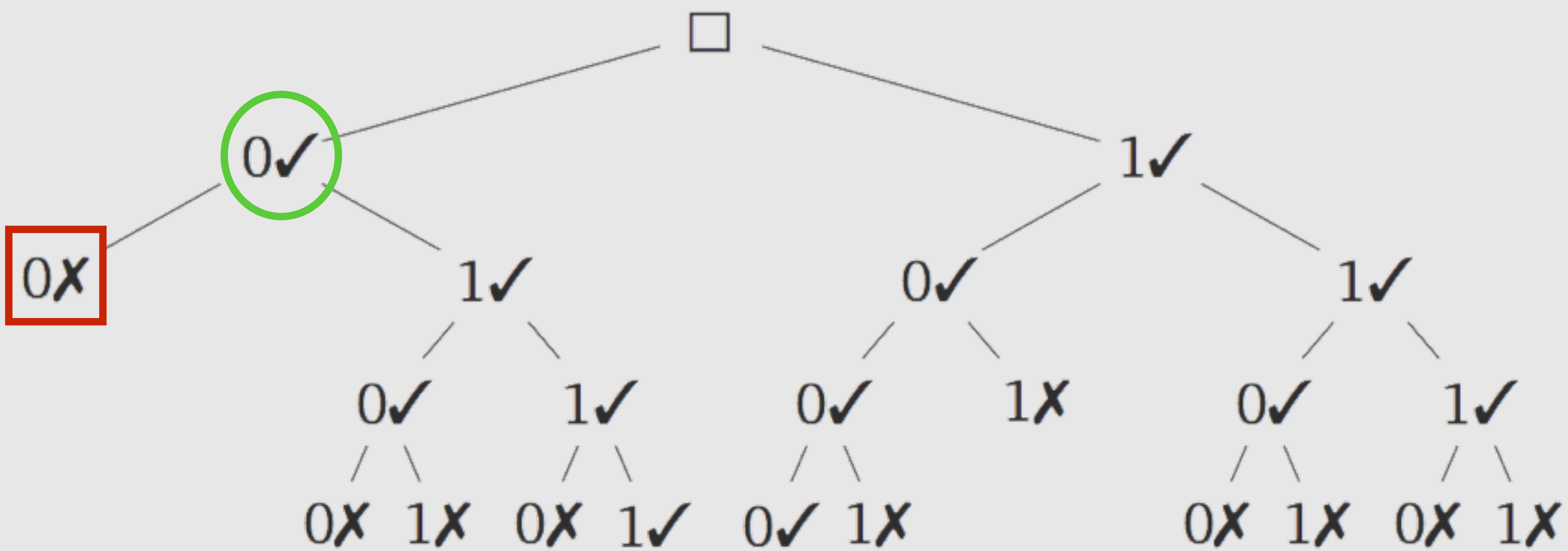

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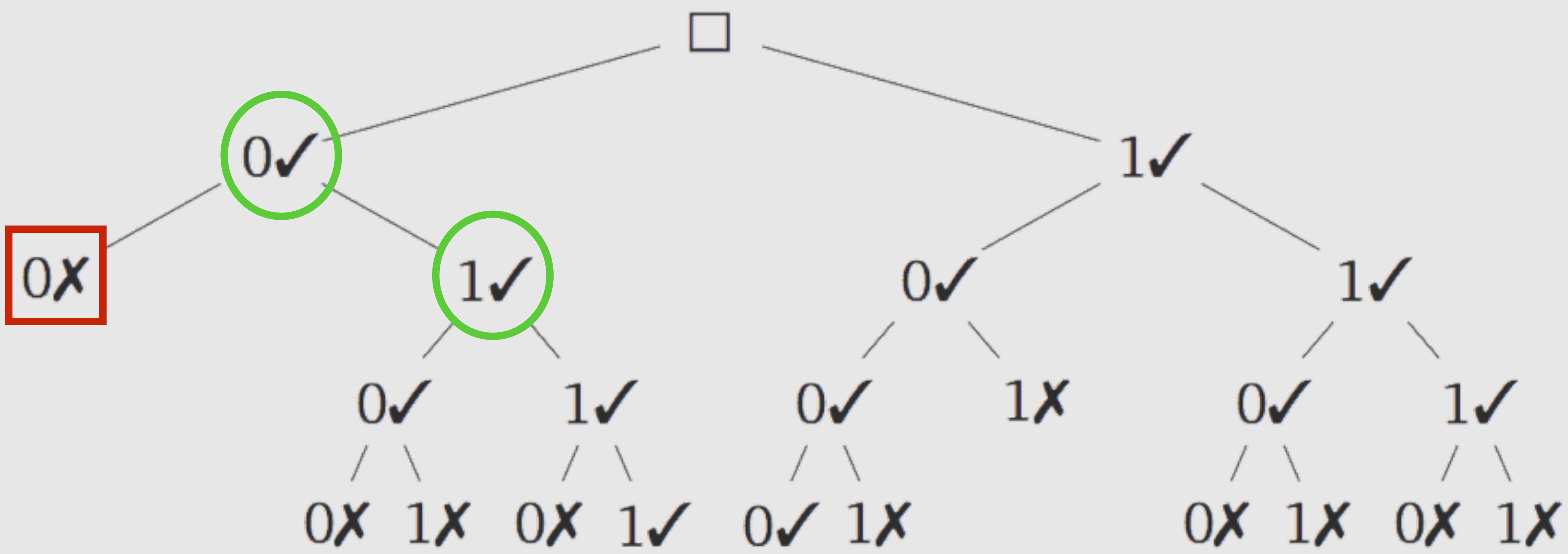

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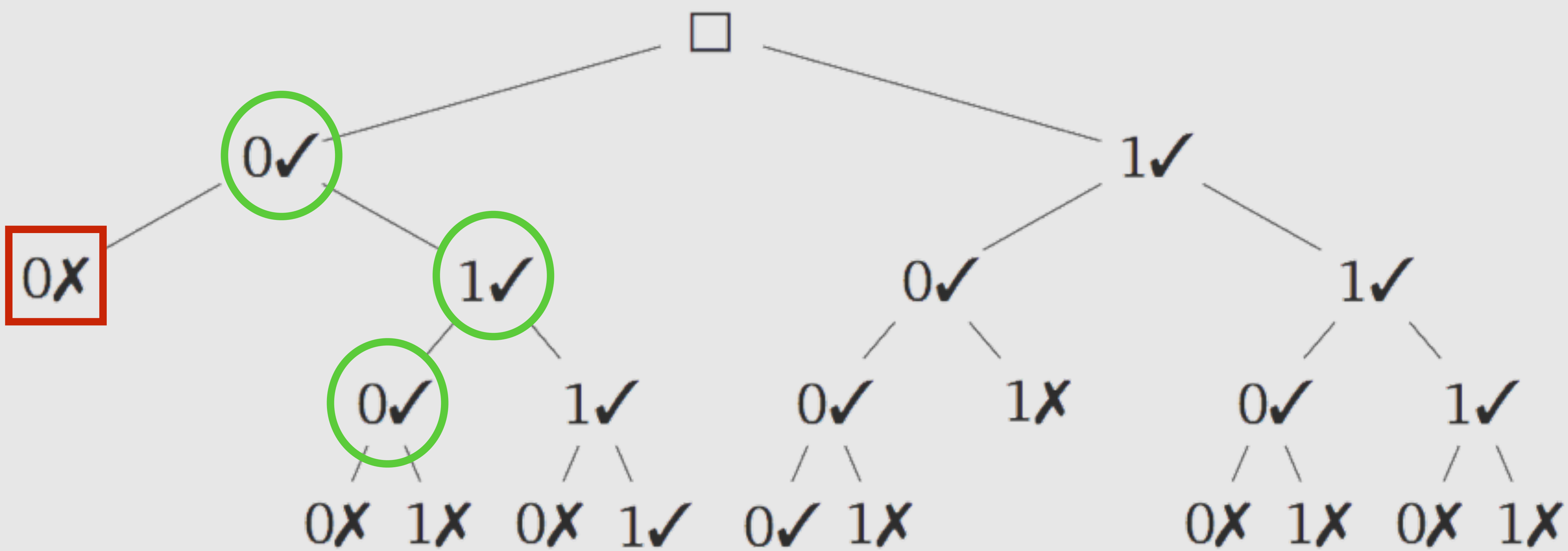

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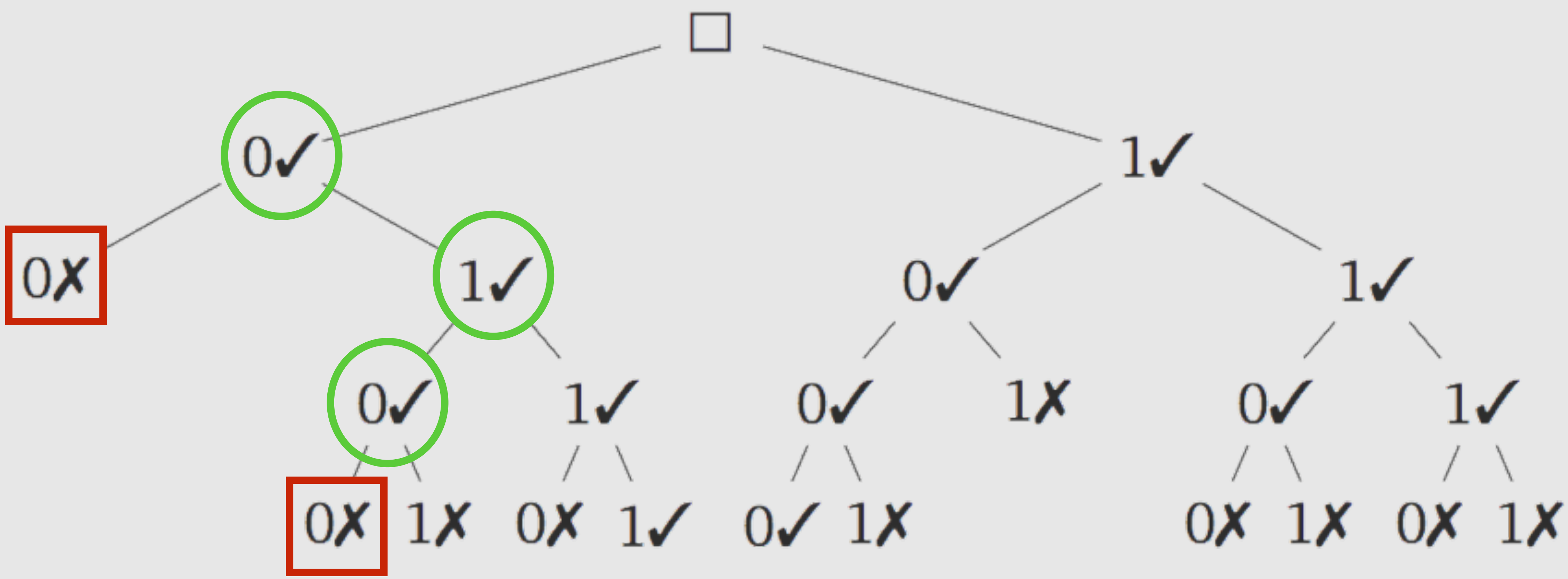

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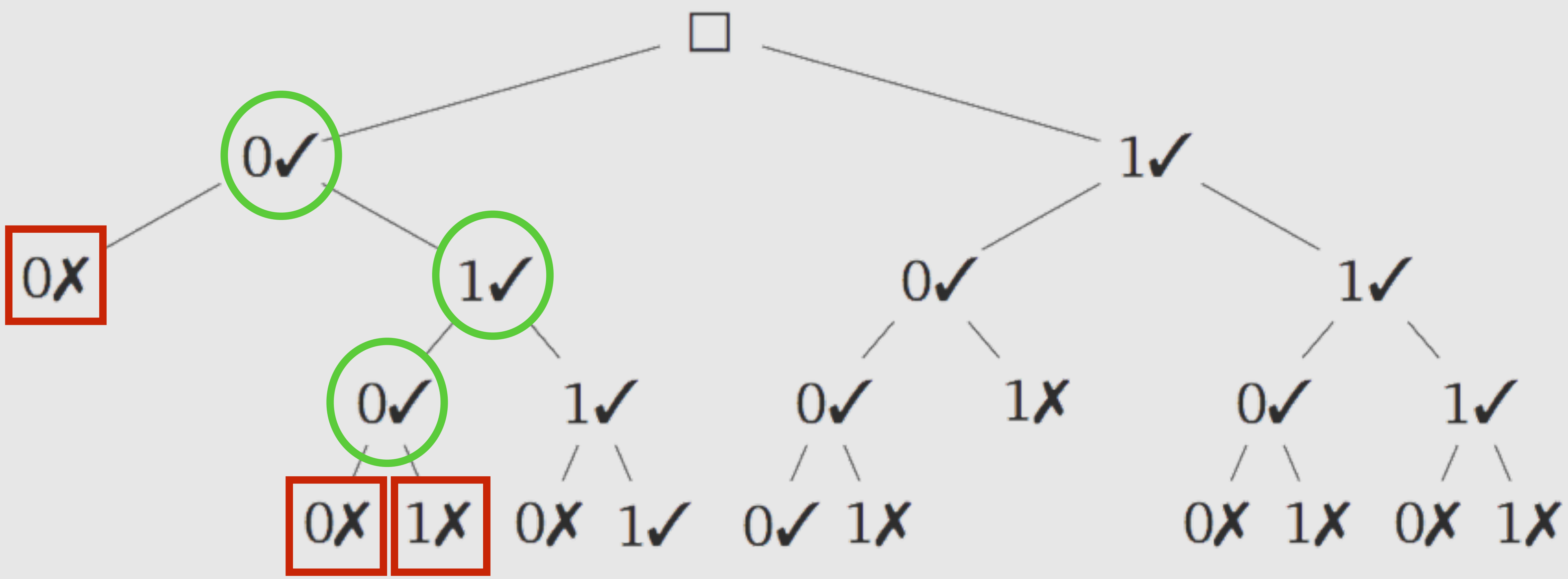

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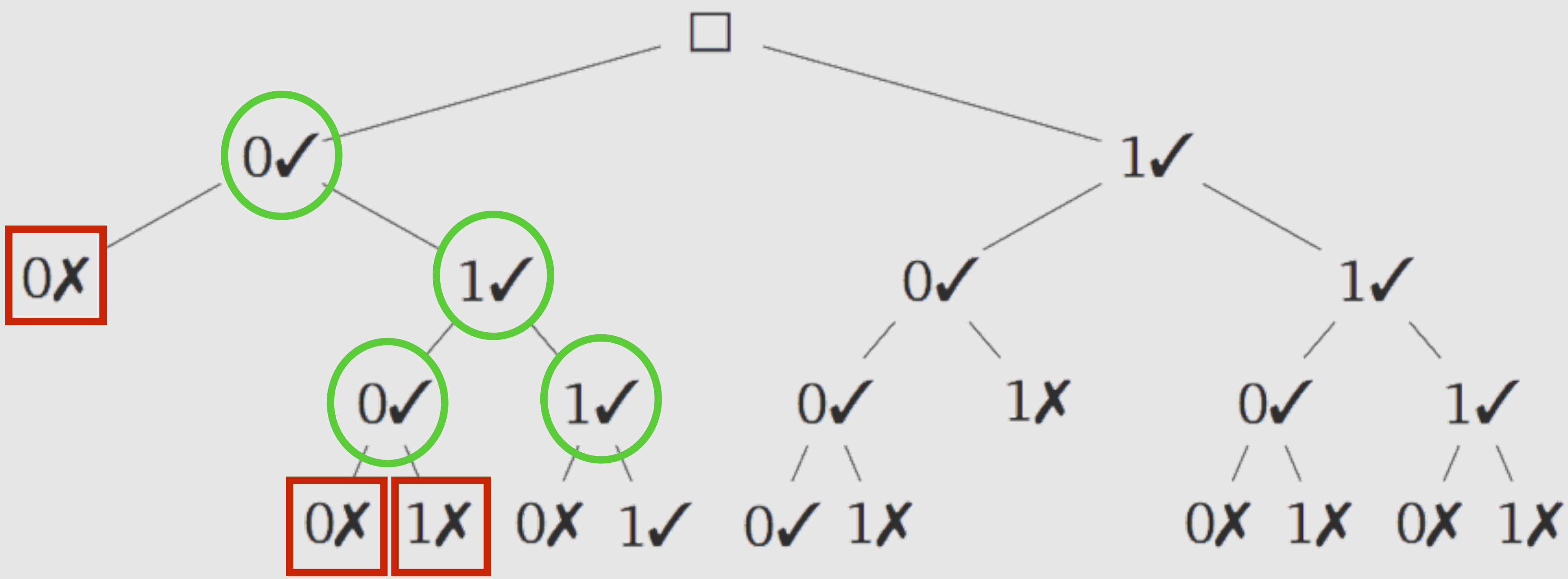

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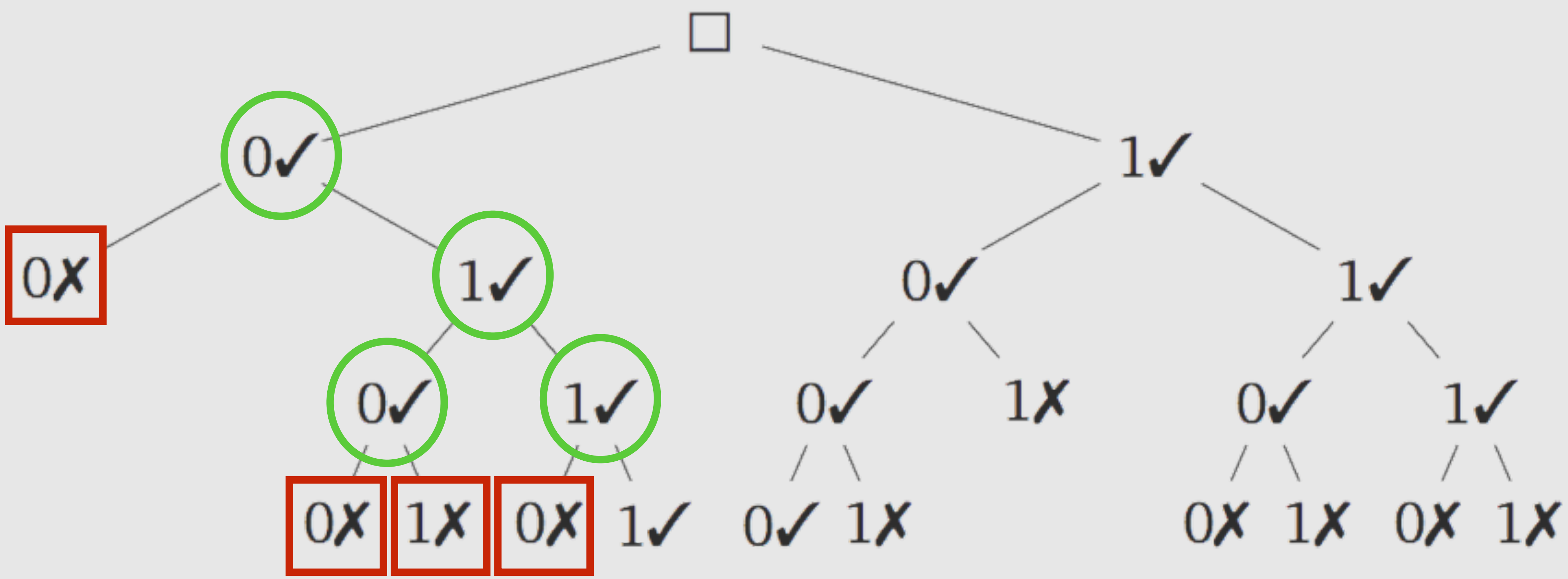

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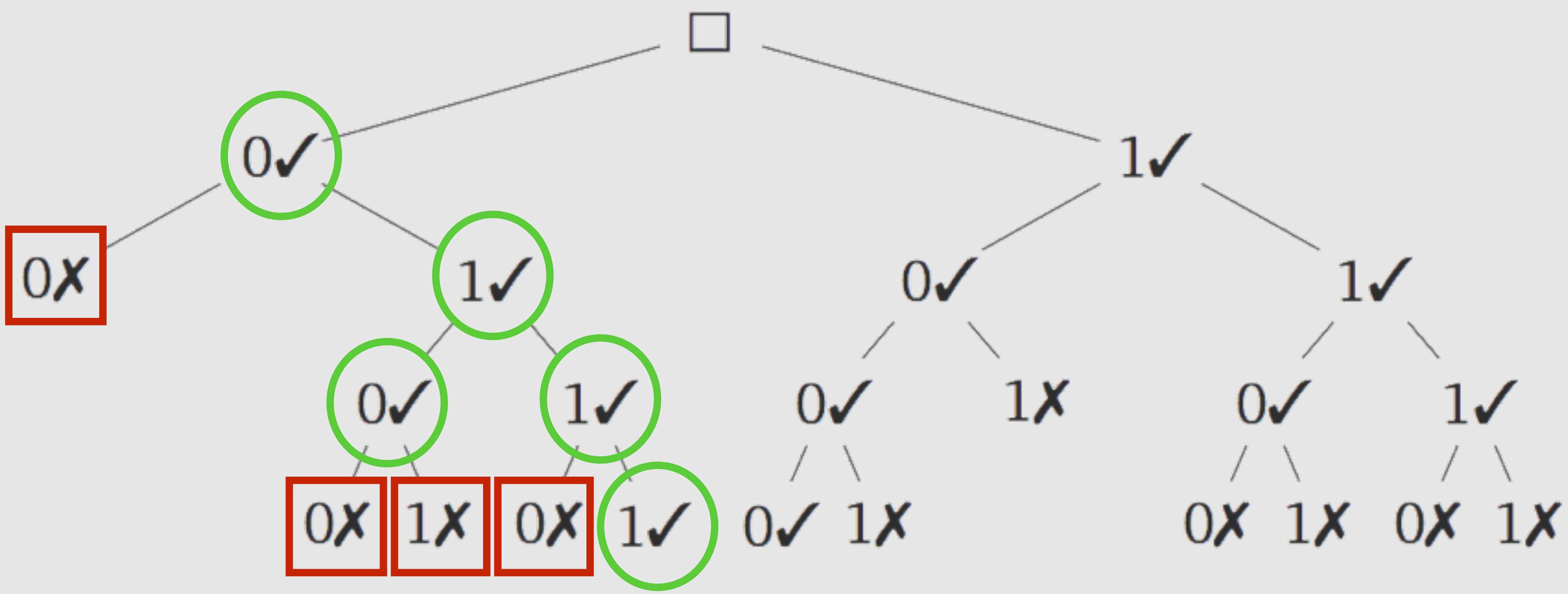

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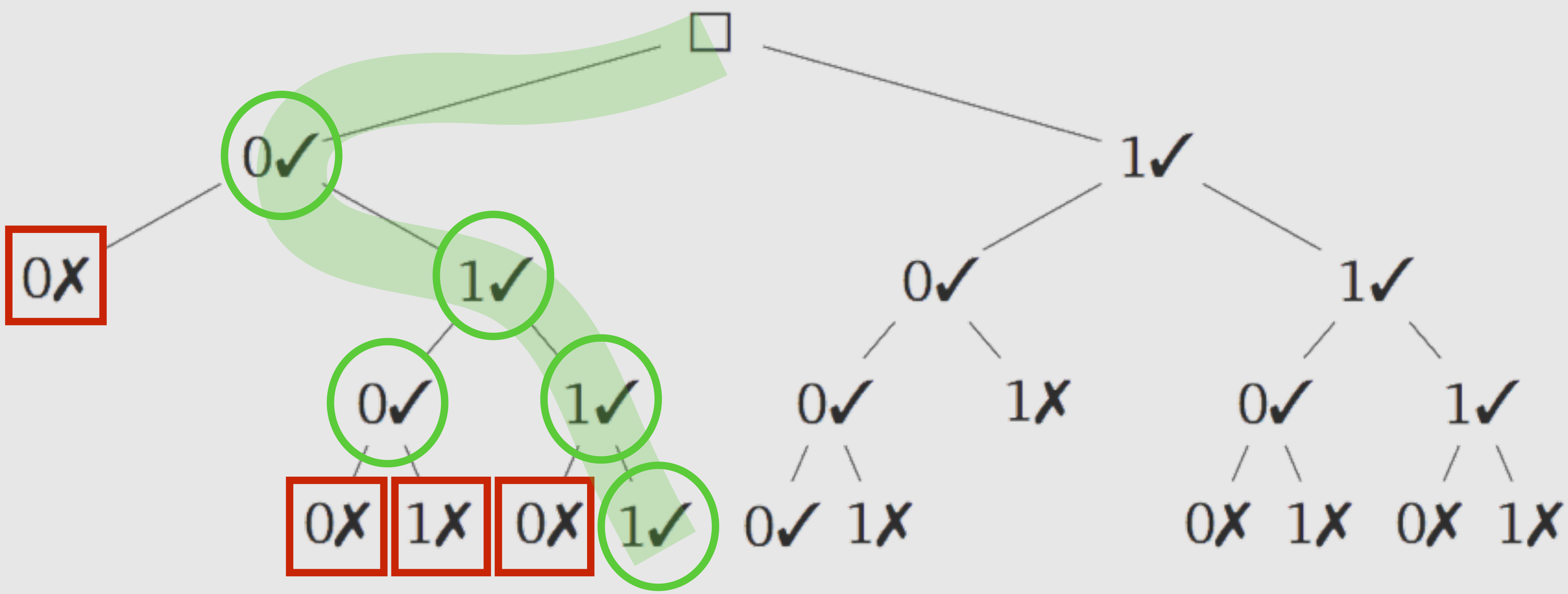

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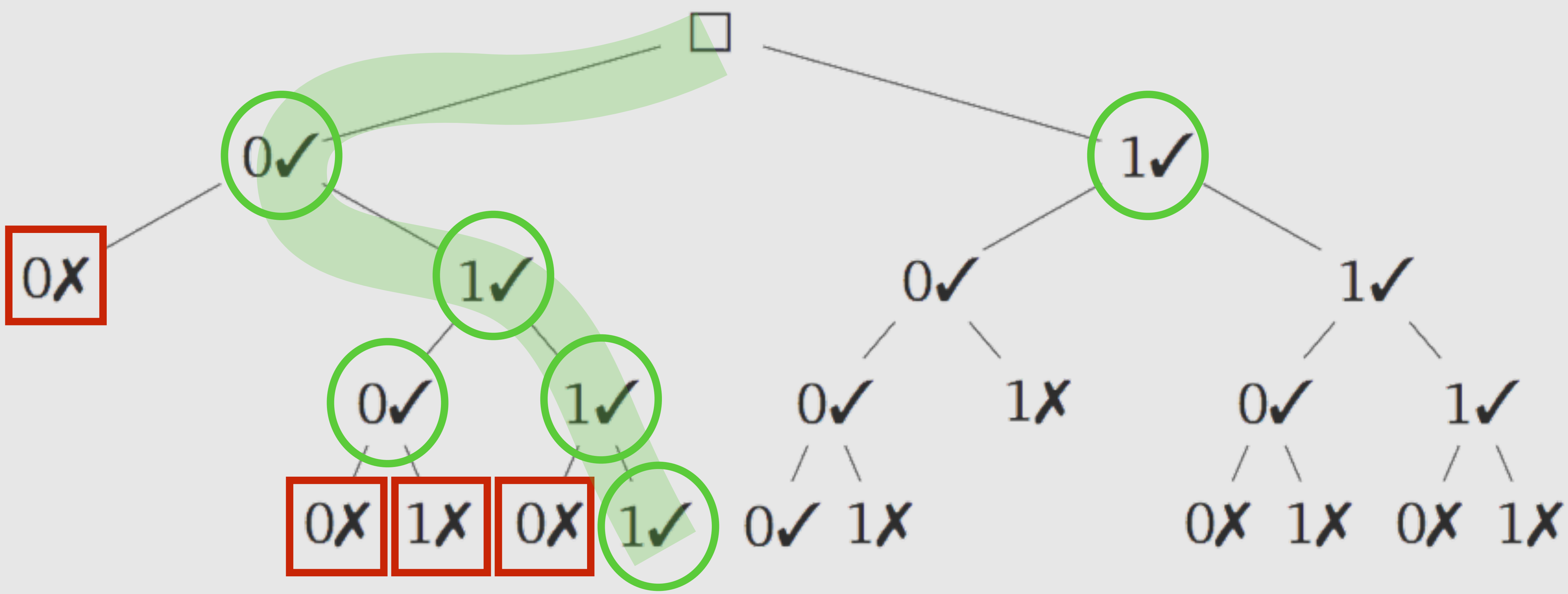

MaxiHMM algorithm

General overview

We recursively define two parameters:

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$$
\begin{aligned}
\gamma_k(x_k, \hat{x}_k) &\triangleq \min_{\hat{x}_{k+1} \in \mathcal{X}_{k+1}} \max_{x_{k+1} \in \mathcal{X}_{k+1}} \chi_{k+1}(x_{k+1}, x_k, \hat{x}_{k+1}, \hat{x}_k) \\
&\qquad \omega_{k+1}(x_{k+1}, \hat{x}_{k+1}, o_{k+1}) \gamma_{k+1}(x_{k+1}, \hat{x}_{k+1}) \\
\gamma_n(x_n, \hat{x}_n) &\triangleq 1
\end{aligned}
$$

MaxiHMM algorithm

General overview

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$$\gamma_k(x_k, \hat{x}_k) \triangleq \min_{\hat{x}_{k+1} \in \mathcal{X}_{k+1}} \max_{x_{k+1} \in \mathcal{X}_{k+1}} \chi_{k+1}(x_{k+1}, x_k, \hat{x}_{k+1}, \hat{x}_k) \omega_{k+1}(x_{k+1}, \hat{x}_{k+1}, o_{k+1}) \gamma_{k+1}(x_{k+1}, \hat{x}_{k+1})$$

$$\gamma_n(x_n, \hat{x}_n) \triangleq 1$$

… and …

$$\delta_k(x_k, \hat{x}_{1:k}) \triangleq \max_{x_{k-1} \in \mathcal{X}_{k-1}} \chi_k(x_k, x_{k-1}, \hat{x}_k, \hat{x}_{k-1}) \omega_k(x_k, \hat{x}_k, o_k) \delta_{k-1}(x_{k-1}, \hat{x}_{1:k-1})$$

$$\delta_1(x_1, \hat{x}_1) \triangleq \chi_1(x_1, \hat{x}_1) \omega(x_1, \hat{x}_1, o_1)$$

MaxiHMM algorithm

General overview

We want to be able to check whether:

$$(\forall \hat{x}_{1:n} \in \mathrm{opt}_{\max}(\mathcal{X}_{1:n}))\ \hat{x}_{1:k} \neq \hat{x}^*_{1:k}$$

some initial segment

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some initial segment

$$\Uparrow$$

$$\max_{x_k \in \mathcal{X}_k} \delta_k(x_k, \hat{x}^*_{1:k})\gamma_k(x_k, \hat{x}^*_k) > 1$$

MaxiHMM algorithm

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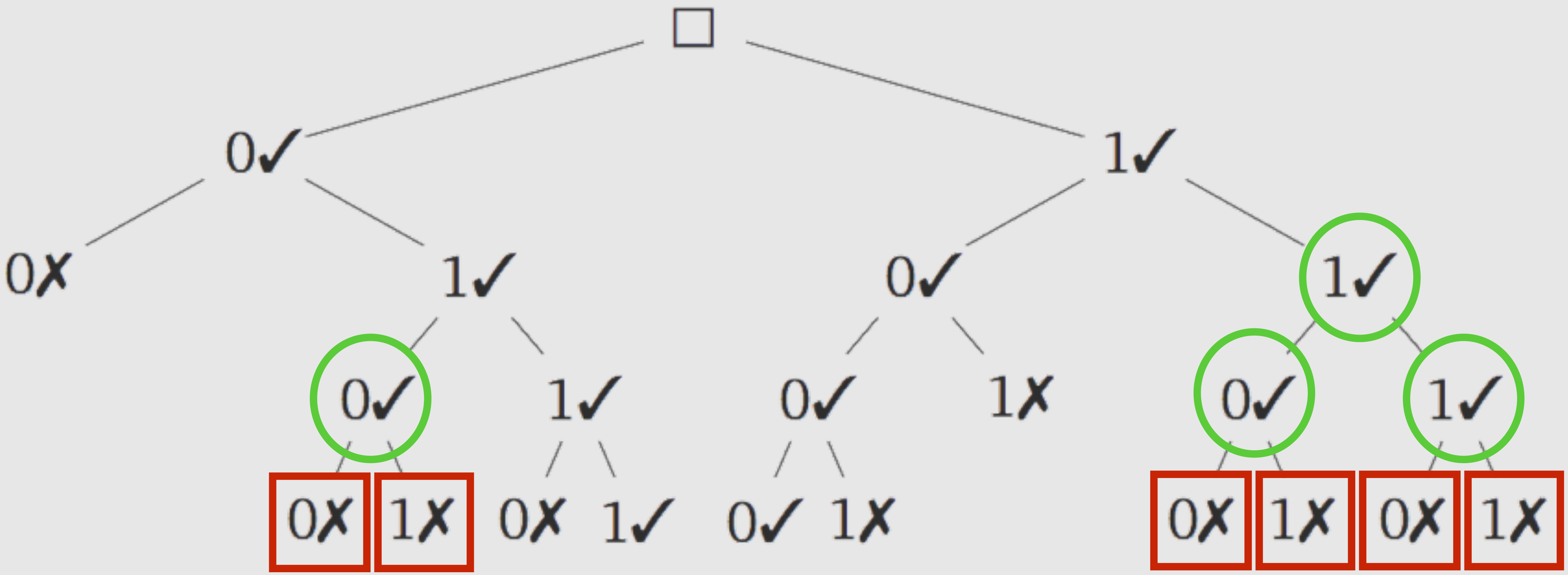


MaxiHMM algorithm

General overview

Algorithm 1: MaxiHMM

Data: the local parameters χ_k and ω_k, an output sequence $o_{1:n}$, and the corresponding global parameters γ_k

Result: the set $\mathrm{opt}(\mathcal{X}_{1:n}|o_{1:n})$ of all maximal state sequences

$\mathrm{opt}(\mathcal{X}_{1:n}|o_{1:n}) \leftarrow \emptyset$

for $\hat{x}_1 \in \mathcal{X}_1$ **do**

  for $x_1 \in \mathcal{X}_1$ **do**

    $\delta_1(x_1, \hat{x}_1) \leftarrow \chi_1(x_1, \hat{x}_1)\omega_1(x_1, \hat{x}_1, o_1)$

  if $\boxed{\max_{x_1 \in \mathcal{X}_1} \delta_1(x_1, \hat{x}_1)\gamma_1(x_1, \hat{x}_1) \leq 1}$ **then** `Recur`$(1,\ \hat{x}_1,\ \delta_1(\cdot, \hat{x}_1))$

return $\mathrm{opt}(\mathcal{X}_{1:n}|o_{1:n})$

Procedure `Recur`$(k, \hat{x}_{1:k}, \delta_k(\cdot, \hat{x}_{1:k}))$

if $k = n$ **then**

  add $\hat{x}_{1:n}$ to $\mathrm{opt}(\mathcal{X}_{1:n}|o_{1:n})$ ▷ `We found a solution!`

else

  for $\hat{x}_{k+1} \in \mathcal{X}_{k+n}$ **do**

    $\hat{x}_{1:k+1} \leftarrow (\hat{x}_{1:k}, \hat{x}_{k+1})$ ▷ `Append` $\hat{x}_{k+1}$ `to the end of` $\hat{x}_{1:k}$

    for $x_{k+1} \in \mathcal{X}_{k+1}$ **do**

      $\delta_{k+1}(x_{k+1}, \hat{x}_{1:k+1}) \leftarrow \max_{x_k \in \mathcal{X}_k} \chi_{k+1}(x_{k+1}, x_k, \hat{x}_{k+1}, \hat{x}_k)$
      $\omega_{k+1}(x_{k+1}, \hat{x}_{k+1}, o_{k+1})$
      $\delta_k(x_k, \hat{x}_{1:k})$

    if $\boxed{\max_{x_{k+1} \in \mathcal{X}_{k+1}} \delta_{k+1}(x_{k+1}, \hat{x}_{1:k+1})\gamma_{k+1}(x_{k+1}, \hat{x}_{k+1}) \leq 1}$ **then**

      `Recur`$(k+1,\ \hat{x}_{1:k+1},\ \delta_1(\cdot, \hat{x}_{1:k+1}))$

MaxiHMM algorithm

Properties

MaxiHMM algorithm

- Recursive

MaxiHMM algorithm

Properties

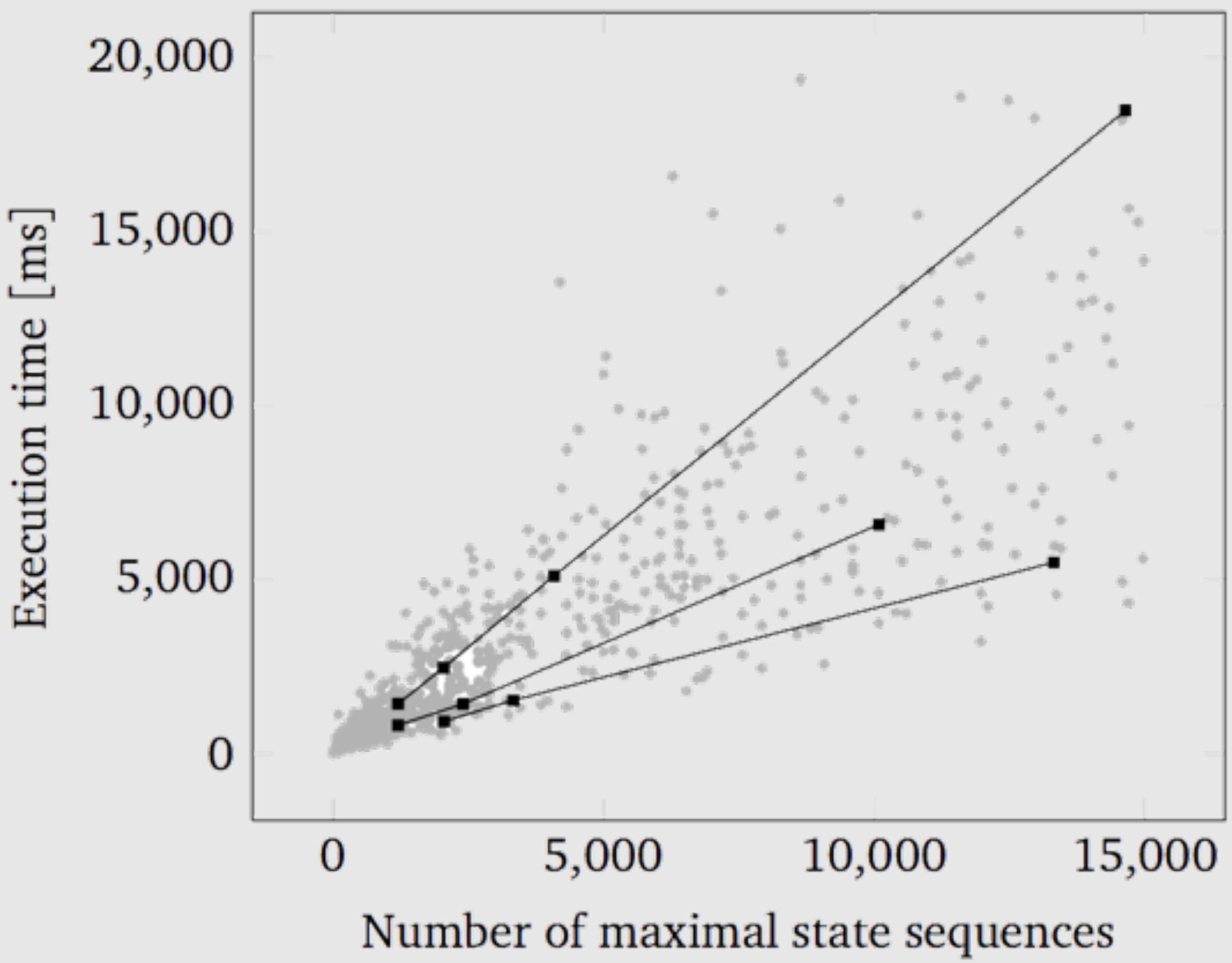
MaxiHMM algorithm

- Recursive
- Heuristic complexity $O(Snm^2)$
  - S: number of solutions
  - n: length of the sequence
  - m: size of state space

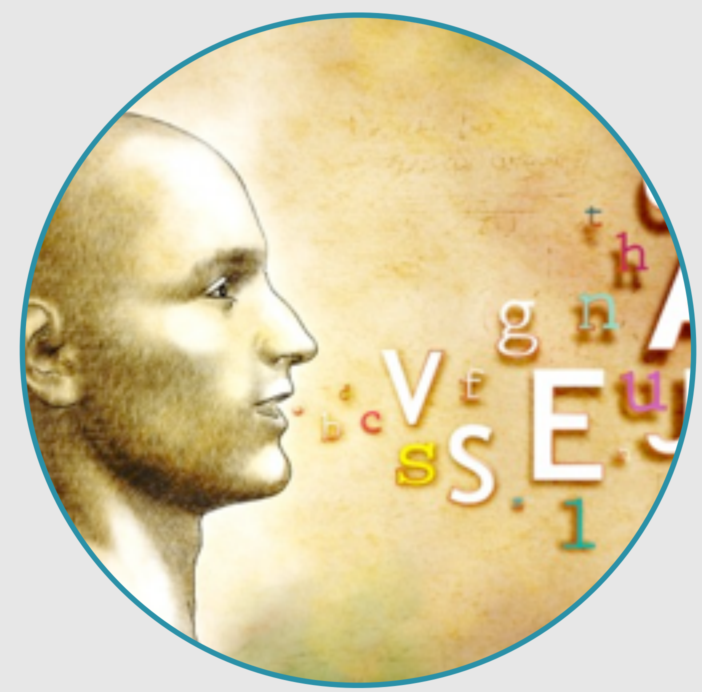
MaxiHMM algorithm

Properties

MaxiHMM algorithm



MaxiHMM algorithm
Applications



OCR

Optical Character Recognition

Willem, die vele bouke maecte,
Daer hi dicken omme waecte,
Hem vernoyde so haerde
Dat die avonture van Reynaerde
In Dietsche onghemaket bleven
- Die Willem niet hevet vulscreven -
Dat hi die vijte van Reynaerde soucken
Ende hise na den Walschen boucken
In Dietsche dus hevet begonnen.

OCR

Hidden Markov models

Willem, die vele bouke maecte,
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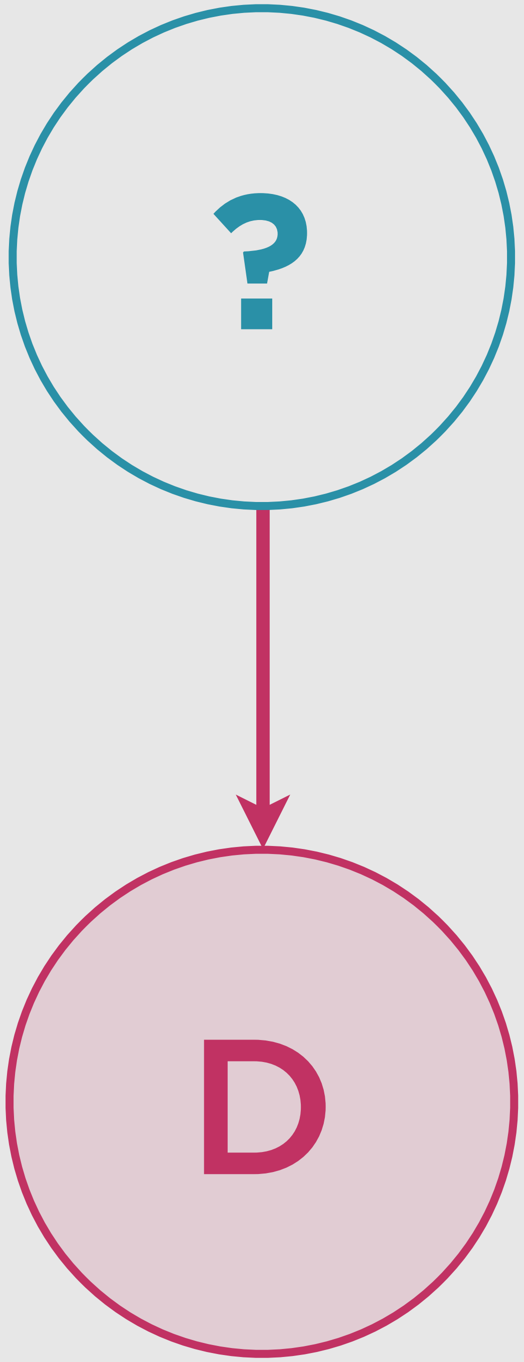
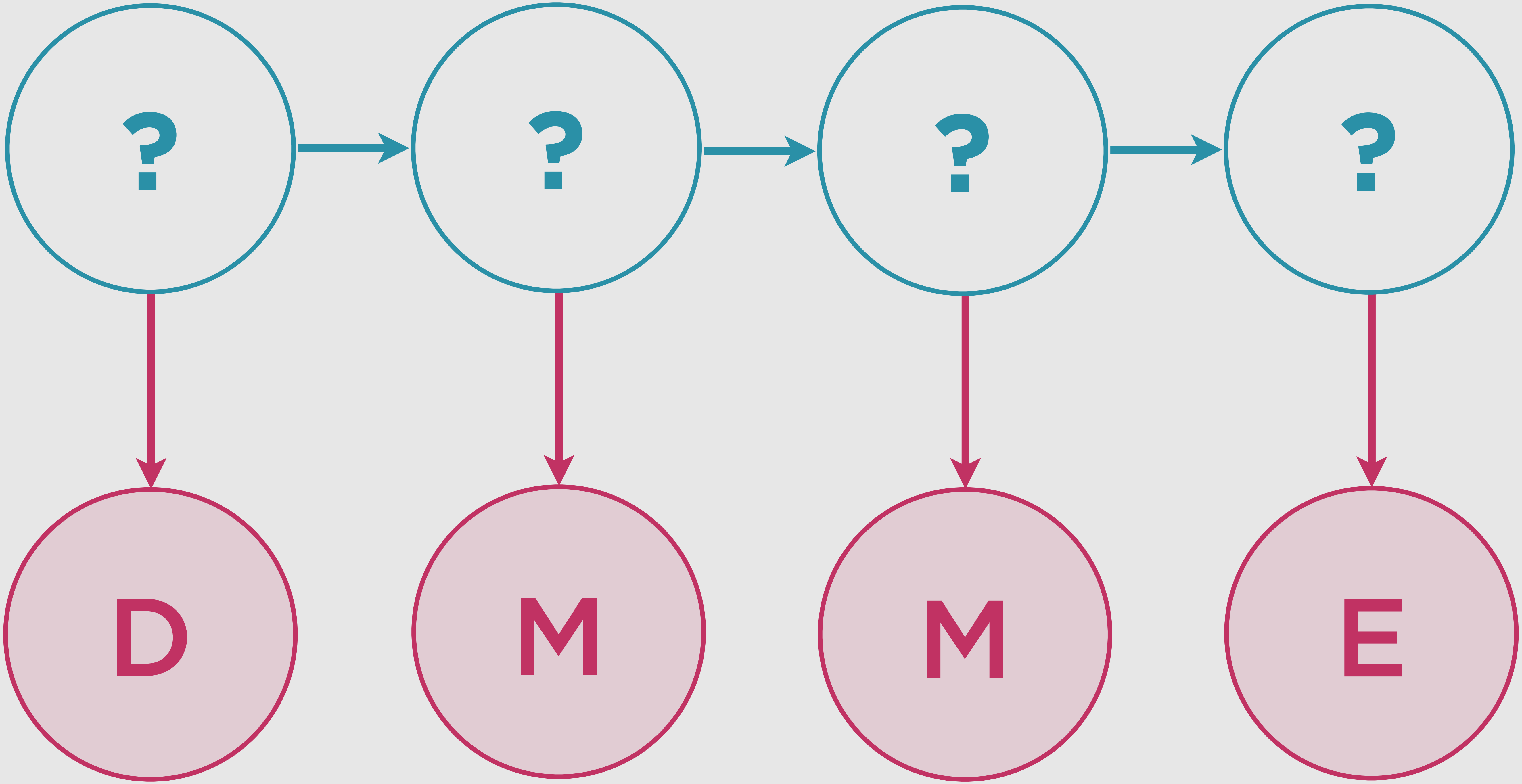


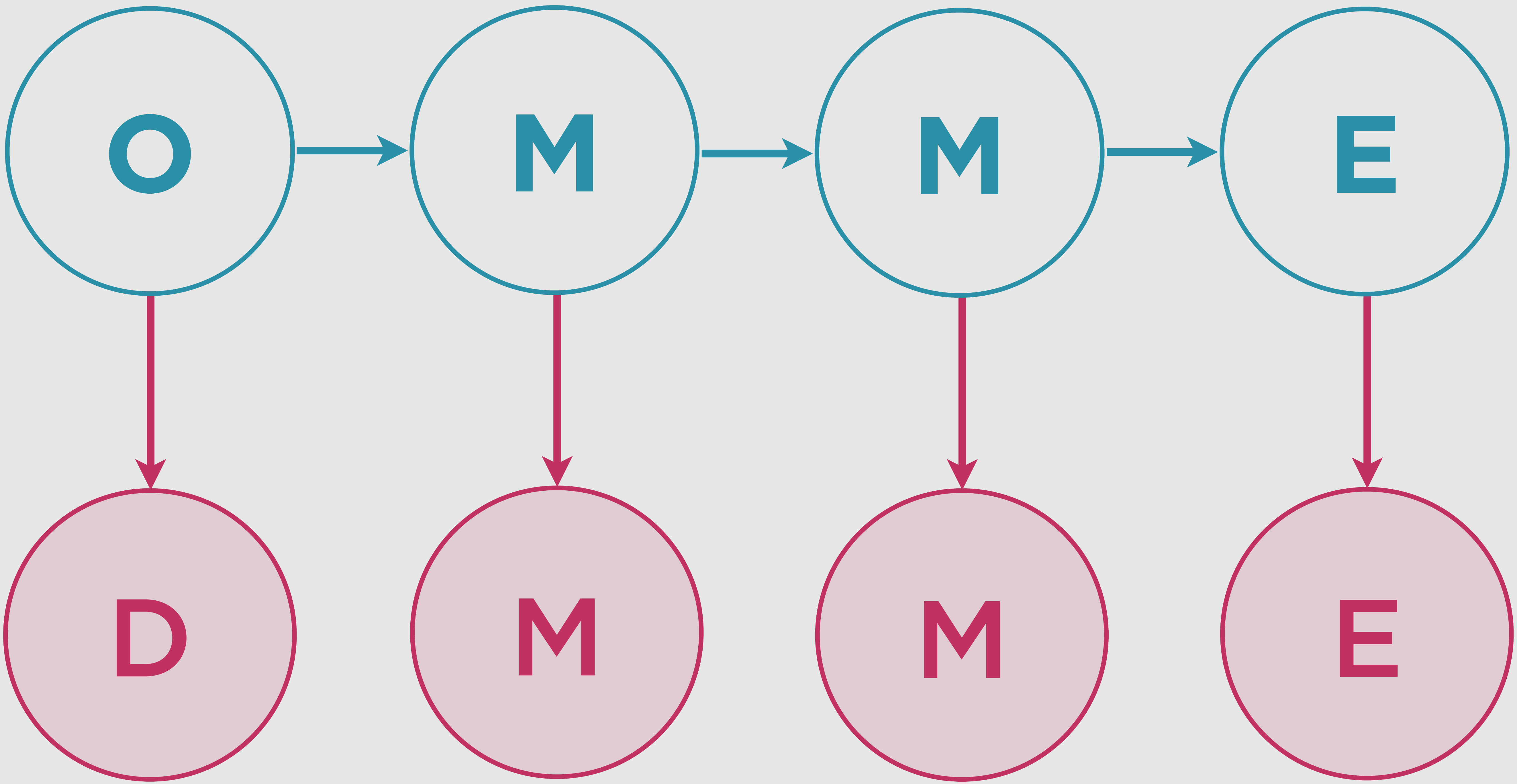
OCR

Hidden Markov models



OCR

Hidden Markov models



Van den Vos Reynaerde

- Medieval Dutch (13th century)

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- Often little data on hand

Van den Vos Reynaerde

- Medieval Dutch (13th century)
- Often little data on hand
- Building very accurate precise models is impossible

Van den Vos Reynaerde

- Medieval Dutch (13th century)
- Often little data on hand
- Building very accurate precise models is impossible
- Use imprecise models

OCR
Results

BEWAENT read as BEWAEHT

3-best Viterbi solutions

BEWAENT BEWAERT BEWAEHT

MaxiHMM solutions

BEWAENT

OCR

Results

UP read as UF

3-best Viterbi solutions

VE OF **UP**

MaxiHMM solutions

OF UF **UP** VE

OCR
Results

UP read as UF

3-best Viterbi solutions

VE OF **UP**

MaxiHMM solutions

OF UF **UP** VE

Multiple solutions are typically returned in cases where the Viterbi algorithm fails to return the correct result

OCR

Results

COMT read as COMT

3-best Viterbi solutions

CONT COMI CONI

MaxiHMM solutions

COMI **COMT** CONT

OCR

Results

COMT read as COMT

3-best Viterbi solutions

CONT COMI CONI

MaxiHMM solutions

COMI **COMT** CONT

Imprecision takes care of problems with small data sets

Questions?