

Group Theory and Symmetry

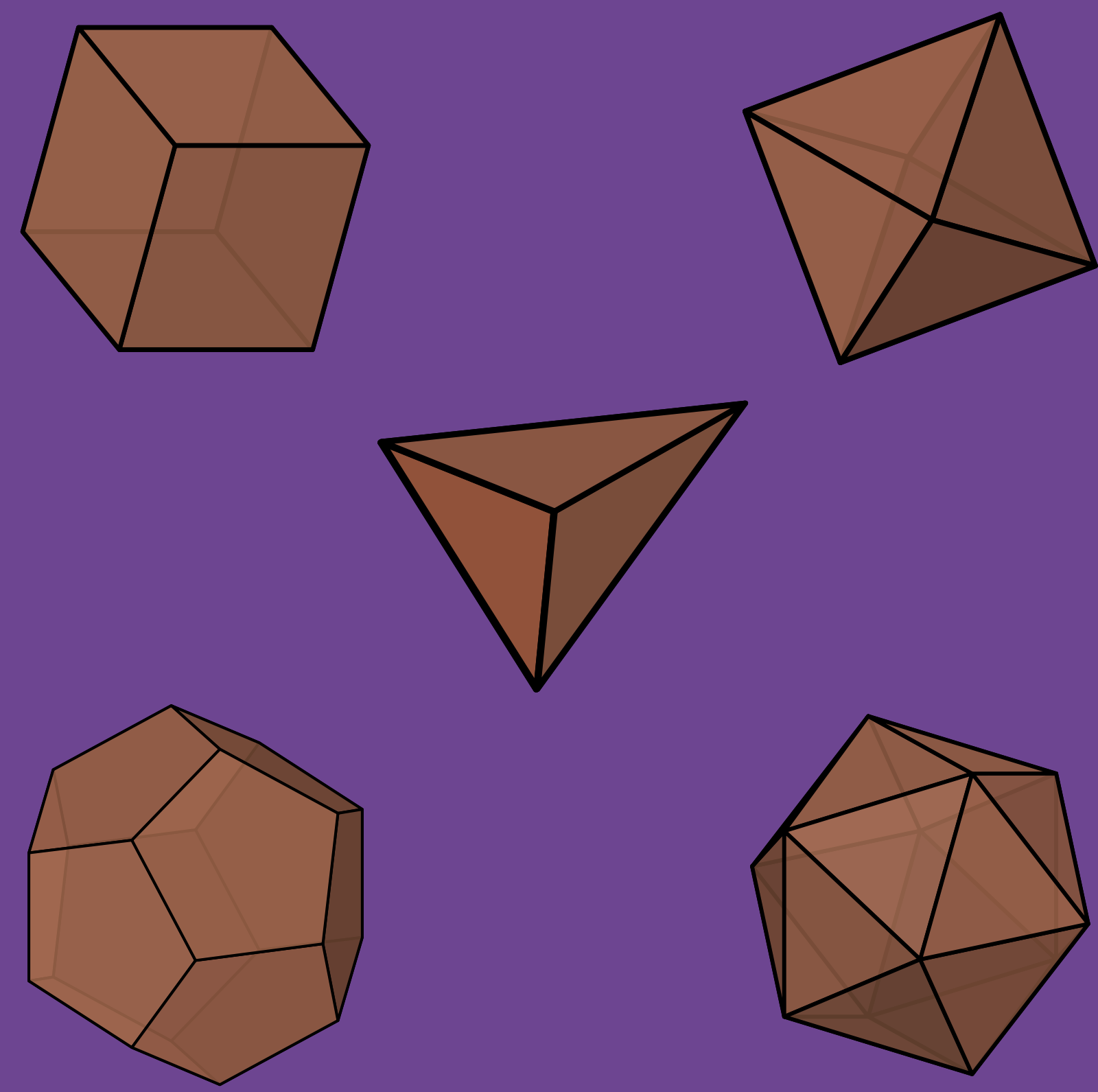
Group theory is the abstract study of symmetry. By defining a "symmetry group" axiomatically, we can study what possible kinds of symmetries an object can have.

\mathbb{Z}

C_n

S_n

1



SL_n

Algebraic Groups

- Algebraic groups: matrix groups defined by polynomial equations.
- Geometric structure: defined on group by "continuity" of polynomials.

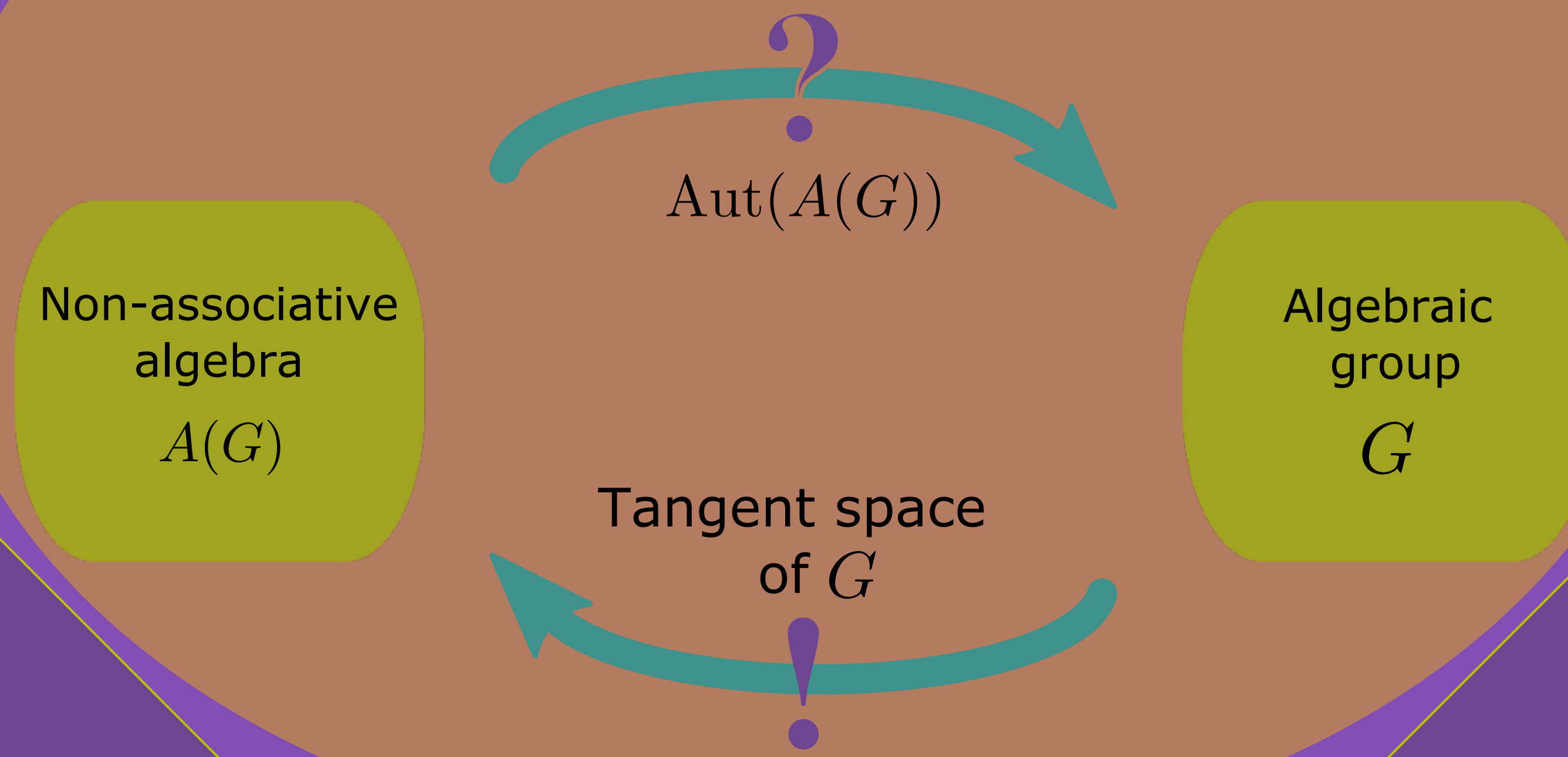
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SO_n

SP_n

Five families of exceptional groups: G_2, F_4, E_6, E_7 and E_8 .

Non-associative algebras for exceptional groups



- Relation to exceptional groups:
- Lie algebras as tangent spaces
 - The octonion algebra with automorphism group G_2 .

Non-associative algebras

Examples include:

- Lie algebras
- Malcev algebras
- Jordan algebras
- Structurable algebras

4

3

$$\mathbb{Z} \rightarrow GL_2(\mathbb{C}): a \mapsto \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}$$

Representation theory

- Representation theory: study of linear actions of a group on objects.
- Characters and irreducible representations: powerful tools to look at groups as linear symmetries.