New insights in the role of working memory in carry and borrow operations

Psychologica Belgica (2005), 45, 101-121

Ineke Imbo

Stijn De Rammelaere

André Vandierendonck

Ghent University

Contact address: Ineke Imbo

Department of Experimental Psychology

Ghent University

Henri Dunantlaan 2

B – 9000 Ghent

Belgium

Tel: +32 (0)9 2646409

Fax: + 32 (0)9 2646496

e-mail: Ineke.Imbo@UGent.be

Abstract

The present paper provides a state-of-the-art overview concerning the role of working memory in carry and borrow operations in metal arithmetic. The role of the executive working-memory component is discussed, alongside the contribution of the phonological and visuo-spatial working-memory components. Moreover, a broad view on various carry characteristics (such as the number of carry/borrow operations and the value of the carry) and various operations (addition, subtraction, and multiplication) is provided. Finally, some ideas for further research are offered.

The role of working memory in carry and borrow operations in mental arithmetic

Most people use mental arithmetic in their daily lives, for example to calculate the amount to pay in a restaurant, to check an account balance, to determine how much a price in Euros would cost in their 'old' money system, or to estimate how many time there is left before a meeting. These arithmetic problems encountered in daily life are more often complex (i.e., involving multi-digit numbers, e.g., 12+43; 78-34; 6x14) than simple (i.e., involving single-digit numbers, e.g., 3+5; 9-2; 4x8). The frequent use of mental arithmetic in our daily life notwithstanding, not much is known about the functional mechanisms that are at the heart of this cognitive process. In particular, questions concerning the more complex forms of mental arithmetic remain unanswered. Whereas solutions to simple forms of mental arithmetic are often retrieved from long-term memory (e.g., Cooney, Swanson, & Ladd, 1988; Siegler, 1988), complex forms of mental arithmetic also require other processes. For those processes used in complex mental arithmetic, such as the temporary storage of intermediate results, the use of problem-solving skills, or the use of rule-based procedures (e.g., Geary, 1994; Geary & Widaman, 1987; Hope & Sherill, 1987), people rely on their working memory, as was shown in an early result by Hitch (1978). The present paper further pursues the role of working memory (WM) in complex arithmetic, and more specifically in carry and borrow operations.

The role of WM in mental arithmetic

The WM model

WM is a capacity-limited system that is responsible for storing and processing information in a variety of cognitive tasks. In the present study, as in most of the work on WM usage in mental arithmetic, the WM model of Baddeley and Hitch (1974; Baddeley, 1986, 1992; Baddeley & Logie, 1999) is used as a conceptual framework. This multi-componential model of WM comprises three components: a central-executive component and two subordinate slave systems. The executive WM component is responsible for the supervision and coordination of the two slave systems, being the phonological loop and the visuo-spatial sketchpad (Baddeley, 1986; Gilhooly, Logie, Wetherick, & Wynn, 1993; Logie, 1993). The phonological loop stores and manipulates phonologically coded verbal information (Baddeley & Logie, 1992; Baddeley, Thomson, & Buchanan, 1975; Salamé & Baddeley, 1982), whereas the visuo-spatial sketchpad performs the same function for visually and spatially coded information (Baddeley & Lieberman, 1980; Farmer, Berman, & Fletcher, 1986; Logie, 1986, 1989, 1991). The executive WM component is also responsible for task coordination, task switching, selective attention, and processes involving long-term memory such as holding and maintaining information (e.g., Baddeley, 1996).

The most frequently used method to investigate the role of WM in cognitive tasks is the selective-interference paradigm, which is based on a dual-task methodology: participants perform the task of interest (i.e., the primary task) in combination with a task loading a particular WM component (i.e., the secondary task). If the primary and the secondary task rely on the same WM component, performance on the primary task will get worse as the secondary task becomes more demanding. Generally, secondary-task performance also decreases under dual-load conditions compared to secondary-task-only conditions. By using the selective-interference paradigm, it is possible to isolate the roles of the different WM components in each primary task.

The role of WM in mental arithmetic

Many empirical studies elucidated the role of WM in mental arithmetic (see DeStefano & LeFevre, 2004, for an extensive review). The *executive* WM component has been shown to play an important role in simple additions and multiplications (Ashcraft, Donley, Halas, & Vakali, 1992; De Rammelaere, Stuyven, & Vandierendonck, 1999, 2001; De Rammelaere & Vandierendonck, 2001; Hecht, 2002; Lemaire, Abdi, & Fayol, 1996). Logie, Gilhooly, and

Wynn (1994) were the first to show that the executive WM component is also crucial to perform complex forms of mental arithmetic. More recently, the crucial role of this WM component in complex arithmetic problems has been confirmed, both for additions (Fürst & Hitch, 2000) and multiplications (Seitz & Schumann-Hengsteler, 2000, 2002).

Since the *phonological* loop is able to maintain phonological representations of intermediate results, this component would be especially important for the maintenance of accuracy during calculations (e.g., Hitch, 1978; Logie & Baddeley, 1987; Logie et al., 1994). As maintaining intermediate results is especially required in complex arithmetic problems, previous research indeed has confirmed that the phonological loop is indispensable in complex additions, multiplications, and subtractions (e.g., Fürst & Hitch, 2000; Heathcote, 1994; Noël, Désert, Aubrun, & Seron, 2001; Seitz & Schumann-Hengsteler, 2000, 2002; Trbovich & LeFevre, 2003; Seyler, Kirk, & Ashcraft, 2003), but not in simple ones (e.g., De Rammelaere et al., 1999, 2001; Seitz & Schumann-Hengsteler, 2000, 2002; Seyler et al., 2002; but see Lee & Kang, 2002, for an exception).

Until now, the role of the *visuo-spatial* sketchpad in mental arithmetic remains unclear. In most studies, no evidence was found for a role of this memory component in mental arithmetic (e.g., Noël et al., 2001; Seitz & Schumann-Hengsteler, 2000). However, the visuo-spatial sketchpad may be used under specific conditions (e.g., when participants are encouraged to use a visual problem representation), in specific populations (e.g., in highly skilled participants), or with specific presentation modalities (e.g., when arithmetic problems are presented vertically) – see DeStefano and LeFevre (2004) for an elaboration on these issues.

The role of WM in carrying and borrowing

Carrying and borrowing

Carrying and borrowing are additional solution steps which are often needed in complex arithmetic tasks. For example, in 37+14, a carry operation is needed. Since the sum of the unit digits exceeds 10, a 1 has to be carried from the units to the tens. Although almost all studies concerning the carry operation were executed on addition problems, this operation also appears in multiplication problems. Geary, Widaman, and Little (1986) indeed showed that carry operations in multiplication problems resemble those in addition problems. In 27x6 for example, the multiplication of the units (7x6) gives 42. The 2 of the units has to be kept active in WM as a result, while the 4 of the tens has to be added to the result of the multiplication of 2x6. This 4 thus has to be carried from the units to the tens. Almost no studies investigated the borrow operation in subtractions, although severe problems with the borrow procedure have been observed in children (e.g., Brown & Burton, 1978) and in brain-damaged patients (e.g., Sandrini, Miozzo, Cotelli, & Cappa, 2003). A borrow operation is needed, for example, in the subtraction 24-6. Since 4 minus 6 is less than zero, borrowing is required: a 1 has to be borrowed from the tens to the units.

An apparent characteristic of carry and borrow operations is that they not only involve declarative knowledge (such as fact retrieval), but also procedural knowledge (Ashcraft, 1992; Sokol, McCloskey, Cohen, & Aliminosa, 1991). On this basis one may expect that arithmetic performance is slower and less accurate for problems that require a carry or borrow operation as compared to those that do not. Several authors indeed showed that the time that is needed to mentally calculate the solution of complex arithmetic problems increases when a carry or borrow operation has to be performed (e.g., Ashcraft & Faust, 1994; Ashcraft & Kirk, 2001; Ashcraft & Stazyk, 1981; Dansereau & Gregg, 1966; Faust, Ashcraft, & Fleck, 1996; Hamann & Ashcraft; 1985; Widaman, Geary, Cormier, & Little, 1989). After a step in which a digit has to be carried or borrowed, there has to be an extra step in which this information is put into WM. In a later step, this information has to be retrieved from WM. When this information is lost from WM, errors emerge. Inefficient carry or borrow procedures indeed have been shown to be one of the most frequent causes of errors in mental arithmetic in children, adults, and brain-damaged patients (e.g., Brown & Burton, 1978; Fürst & Hitch, 2000; Hitch, 1978; Noël et al., 2001; Sandrini et al., 2003).

Even though the great influence of carrying and borrowing on latency and accuracy of arithmetic performance is very important, WM contributions to these operations have not been studied intensively. Only a few studies explicitly investigated which WM components play a role in carry and borrow operations. The present paper evaluates the existing empirical research on the role of WM in carrying and borrowing, and complements it with more recent observations partly based on the master's thesis of the first author. These observations are assembled in two papers, one about carrying in additions (Imbo, Vandierendonck, & De Rammelaere, 2005a), and one about carrying in multiplications and borrowing in subtractions (Imbo, Vandierendonck, & Vergauwe 2005b). The role of the executive, phonological, and visuo-spatial WM components in carrying and borrowing are discussed successively, with each time a review of experimentally based evidence and a discussion section in which explanations are sought for the observed results.

The executive WM component

Experimental evidence. A first issue is whether the executive WM component is needed to perform carry operations. In order to prove such a role for the executive WM component in carry operations, an interaction between executive WM load and the number of carry operations should be observed. Logie et al. (1994) were the first who systematically investigated the role of WM in mental arithmetic and carrying. They observed that additions were harder to solve as the number of carry operations increased. However, they did not observe an interaction between the number of carry operations and executive WM load, and conclude that WM may have no specific role in carrying. Fürst and Hitch (2000) also found that problems were harder as the number of carry operations increased. Moreover, contrary to Logie et al. (1994), these researchers did observe an interaction between the number of carry operations and executive WM load, indicating that executive processes contribute to carrying. Both Ashcraft and Kirk (2001) and Imbo et al. (2005a) found a main effect of number of carry operations as well, and an interaction with executive WM load (see Figure 1a). In contrast to Logie et al. (1994) but in accordance with Fürst and Hitch (2000), they suggest that executive processes are important for carrying. Finally, Seitz and Schumann-Hengsteler (2002) observed that error rates increased under executive WM load when the sums required carry operations. Taken all together, most studies argue for a role of the executive WM component in the carry operation in complex additions. However, several questions remain unanswered: (1) Is the executive WM component also crucial in carrying and borrowing in other arithmetic operations such as multiplication and subtraction?, and (2) Is the number of carry operations the only variable determining the significance of the executive WM component, or are other variables also involved? Recent studies elaborated on these questions, and observed some new and interesting findings, which are discussed below.

Concerning the first question, Imbo et al. (2005b) investigated the role of the executive WM component in the borrow operation in subtractions, and in the carry operation in multiplications. In a first subtraction experiment, the problems consisted of two 2-digit numbers (e.g., 64-16) with the number of borrow operations being zero or one, whereas in a second subtraction experiment, the stimuli consisted of two 4-digit numbers (e.g., 6542-1638) with zero, one, two or three borrow operations. The executive WM component was loaded by means of a secondary task in which participants had to decide whether tones were high or low (i.e., a choice reaction task; Szmalec, Vandierendonck, & Kemps, 2005). The results were similar in both experiments. Both reaction times and error rates increased with the number of borrow operations. Moreover, the rises in reaction times and error rates were significantly larger when WM was executively loaded, showing that the executive WM component is crucial to perform borrow operations (see Figures 2a and 2b).

Two subsequent experiments were meant to examine the role of the executive WM component in the carry operation in multiplications. Stimuli were multiplication problems in which a 2-digit or a 3-digit number had to be multiplied with a 1-digit number (e.g., 32x8; 113x6). The number of carry operations was one or two. Results showed that multiplications with two carry operations were solved more slowly and less accurately than multiplications with only one carry operation. However, no interaction between executive WM load and number of carry operations was observed, indicating that the executive WM component did not play a crucial role in the carry operation in multiplications (see Figure 3a).

Concerning the second question (i.e., the involvement of other variables besides the number of carry operations in determining the role of WM), Imbo et al. (2005a, 2005b) report a number of experiments in which not only the number of carry operations was manipulated, but also the value that had to be carried. For example, in the addition 175+261+182, a 2 has to be carried from the tens to the hundreds, and in the multiplication 17x6, a 4 has to be carried from the units to the tens. Results showed that both number and value slowed reaction and resulted in larger error rates. Indeed, performance decreased linearly as the value of the carry increased. Moreover, an executive WM load x value interaction was observed in additions (Imbo et al., 2005a), indicating that the executive WM component is not only needed to perform more carry operations, but also to perform carry operations with larger values (see Figure 1b).

Discussion. It is clear that the executive WM component plays a crucial role in complex arithmetic, and especially in carry and borrow operations. Calculation was observed to be slower and less accurate under executive memory loads in complex additions (e.g., Fürst & Hitch, 2000; Imbo et al., 2005a; Logie et al., 1994; Seitz & Schumann-Hengsteler, 2002), complex multiplications (Seitz & Schumann-Hengsteler, 2000, 2002), and complex subtractions (Imbo et al., 2005b). Moreover, the role of the executive WM component grew even larger when more carry or borrow operations had to be executed. This role in carrying and borrowing may be explained by the controlling function of the executive WM component. When a carry or borrow operation has to be executed, a conflict occurs between this carry or borrow sequence and the no-carry or no-borrow sequence (which is the ‘normal’ order of operations during calculating). It is the role of the executive WM component to inhibit the no-carry or no-borrow sequence and to plan and execute the sequence containing carry or borrow operations (see e.g., Fürst & Hitch, 2000; Imbo et al., 2005a for elaborations on this issue).

However, data also implicated a role of the executive WM when carrying higher values. Since we are more used to carry small values, the same executively based mechanisms will be needed in order to inhibit the carry operation with small values and to execute the carry operation with higher values. Problem-size effects could have played a role as well (e.g., Ashcraft, 1992, 1995; Ashcraft & Battaglia, 1978; Butterworth, Zorzi, Girelli, & Jonckheere, 2001; Geary, 1996; Groen & Parkman, 1972). Given that mental arithmetic gets harder as the numbers get larger, it would be reasonable to assume that carrying high values requires more executive WM resources than carrying small values. However, these and alternative explanations to explain the role of the executive WM component in carrying should be put to further investigation.

The phonological loop

Experimental evidence. Hitch (1978) was one of the first researchers investigating carrying in mental arithmetic. He observed longer reaction times and lower accuracies when carry operations had to be executed. According to him, a carry operation creates intermediate information which has to be maintained in the phonological loop in the device of an extra calculation step. In the next calculation step, this intermediate information has to be read out in order to produce a correct answer. This reasoning emphasizes the significance of the phonological loop, since loss of the intermediate information results in erroneous performances. However interesting this first result is, not much research had been carried out in order to investigate the role of the phonological loop in carrying. In the same way as for the executive WM component, we have to search for an interaction between phonological WM load and the number of carry operations in order to prove its role in carrying.

Logie et al. (1994) were the first to investigate the role of the phonological loop in carrying. Although they observed larger reaction times and error rates on additions with more carry operations, this effect did not interact with phonological WM load. Fürst and Hitch (2000) also investigated the effects of phonological WM load on complex additions. They observed that the increase in errors with the number of carries just failed to reach significance under phonological WM load (p = .08). Based on this result, Fürst and Hitch (2000, p. 779) suggested that “the phonological loop could play a minor role in supporting carrying”. Furthermore, Ashcraft and Kirk (2001) observed an interaction between phonological WM load and carrying: the increase in error rates under phonological WM load was larger for carry problems than for non-carry problems. Moreover, this pattern was larger when the phonological WM load was higher. However, since their secondary task (i.e., letter recall with 2 or 6 letters), meant to load the phonological loop, could also have loaded the executive WM component, no univocal conclusions can be drawn. Finally, the role of the phonological loop in carrying was also examined by Seitz and Schumann-Hengsteler (2002). They did not observe a significant interaction between phonological WM load and the number of carry operations either. Based on all these results, the phonological loop would play no role at all in carrying. More recently however, we conducted several additional experiments in order to further investigate the role of the phonological loop. We did not only question the role of the phonological loop in the carry operation in additions and multiplications, but also in the borrow operation in subtractions. Furthermore, since Fürst and Hitch (2000) suggested that the phonological loop could be used to store the amount to be carried, we not only looked for an interaction between phonological WM load and the number of carry/borrow operations but also for an interaction between phonological WM load and the value of the carry. Indeed, as carrying higher values requires more counting steps, more phonological WM resources might be needed to perform these processes accurately (e.g., Logie & Baddeley, 1987).

Both the number of carry operations and the value to be carried were manipulated in two experiments with complex additions (Imbo et al., 2005). The phonological loop was loaded by means of articulatory suppression. Accuracy data of the second experiment showed an interaction between phonological WM load and the number of carries. Accuracy decreased as more carry operations had to be performed – an effect that was enhanced under phonological WM load (see Figure 1a). Furthermore, in both experiments, an interaction between phonological WM load and the value of the carry was observed. In the first experiment, the differences in reaction times between problems where a 1 had to be carried and problems where a 2 had to be carried grew larger under phonological WM load. In the subsequent experiment, the growth in error percentages with the value of the carry was higher when WM was phonologically loaded than when it was not (see Figure 1b). Taken together, these results are the first to show a role for the phonological loop in carrying.

Imbo et al. (2005b) also investigated the role of the phonological loop in the borrow operation in subtractions, and in the carry operation in multiplications. The subtraction experiments showed that the phonological loop is used to solve complex subtractions. Indeed, error rates (but not reaction times) increased significantly under phonological WM load. Moreover, in one subtraction experiment, an interaction between phonological WM load and number of borrow operations was observed, indicating that the borrow operation relies on the phonological loop (see Figure 2b). The value that had to be borrowed was not manipulated in the subtraction experiments. In the experiments with multiplication problems, neither influence of a phonological WM load was observed, nor an interaction between phonological WM load and carry characteristics (i.e., the number of carry operations and the value to be carried; see Figures 3a and 3b).

Discussion. Previous research has shown that the phonological loop is required in complex additions (e.g., Fürst & Hitch, 2000; Imbo et al., 2005a; Logie et al., 1994; Noël et al., 2001; Seitz & Schumann-Hengsteler, 2002) and complex subtractions (Imbo et al., 2005b). Up until now, evidence indicates that the phonological loop would play no role in complex multiplications (e.g., Imbo et al., 2005b; Seitz & Schumann-Hengsteler, 2002; but see Seitz & Schumann-Hengsteler, 2000, for an exception). Although it seems worthwhile to design studies to challenge the absence of this effect, researchers should be aware that manipulating the complexity of multiplications is rather limited. Multiplication problems in which a 3-digit number has to be multiplied with a 1-digit number are solvable (Imbo et al., 2005b), but mentally calculating more complex multiplication problems (e.g., 134x36) may become impossible in dual-task conditions.

Recent results also showed that the phonological loop plays a role in carrying and borrowing. In additions and subtractions, effects of a phonological WM load were larger as more carry or borrow operations had to be executed, respectively. This can easily be explained as follows: as more carry or borrow operations have to be performed, more intermediate results have to be kept temporarily in WM. The phonological WM component is most suited for this maintenance. Furthermore, interference effects may provide an additional explanation. As more carry or borrow operations have to be processed, more doubts may arise so as to whether the present operation has already been performed as the values that have been encountered. For example, participants might be confused when in the previous problem a 2 had to be carried whereas in the present problem a 3 has to be carried. The prevention of such confusion errors would rely on the phonological WM component as well.

In complex additions, the effects of a phonological WM load also grew larger as higher-valued digits had to be carried. Indeed, as the phonological loop is used to store intermediate results, its role in the current experiments would be specialized in storing the values that had to be carried. These observations also confirmed the suggestion of Fürst and Hitch (2000) that the phonological loop would be used to store the amount to be carried. It is clear that still various manipulations (of both number and value) should be done in order to further clarify the role of the phonological loop in carrying and borrowing. Doing so, researchers should also be aware to use a secondary task loading the phonological loop without loading the executive WM component.

The visuo-spatial sketchpad

Experimental evidence. Not much research has been carried out concerning the role of the visuo-spatial sketchpad in complex arithmetic problems, and in particular its role in carrying or borrowing. Effects of visuo-spatial WM load on carrying in addition problems have not been observed up until now (e.g., Logie et al., 1994; Seitz & Schumann-Hengsteler, 2000). To our knowledge, only one study investigated the effects of visuo-spatial WM load in complex subtractions (Imbo et al., 2005b). In this study, the passive matrix tap task (Quinn, 1994) was used to interfere in spatial processing. However, the visuo-spatial sketchpad appeared not to play a role in borrowing either, at least not in the presentation conditions studied thus far.

Discussion. We are convinced that the evidence is too sparse to draw any conclusions about the role of the visuo-spatial sketchpad in complex arithmetic and in carrying and borrowing. The only suggestion we would like to make is that future research is indispensable.

General Discussion

The execution of both carry and borrow operations relies on WM components. Based on the combination of past research and more recent findings, some provisional conclusions can be drawn. First, both the executive and the phonological WM components are needed to perform the carry operation in additions and the borrow operation in subtractions. So far, there are no indications that these WM components would be used to perform the carry operation in multiplications, however. Second, people rely even more heavily on their executive and phonological WM components as the *number* of carry or borrow operations grows. Third, people do also rely more heavily on their executive and phonological WM components as the *value* to be carried is higher. And finally, the visuo-spatial sketchpad does not seem to be needed in carrying or borrowing.

There were some indications that the executive and the phonological WM components might differ somewhat in their specific roles. For example, (a) executive WM loads had larger impacts than phonological WM loads, (b) effects of an executive WM load were more salient in latencies while effects of a phonological WM load were more salient in accuracies, and (c) the number of carry or borrow operations tended to rely more heavily on the executive WM component whereas the value to be carried tended to rely more heavily on the phonological WM component. However, since these effects were not found consistently across all experiments, a more general conclusion would be that both WM components run parallel.

This review thus clearly shows that many issues remain unanswered. We further discuss the interaction between WM loads and the carry or borrow operation and end with providing some new ideas for further research.

The load x carry interaction

In complex arithmetic. It is clear that carrying and borrowing are operations that require much effort to be solved fast and correctly. Indeed – apart from the role of WM – in all studies where both carry/borrow problems and non-carry/non-borrow problems had to be solved, the former were solved slower and less accurately than the latter. But why is solving carry/borrow problems so much harder than solving non-carry/non-borrow problems? In line with previous research, we claim that arithmetic problems with carry or borrow operations need greater WM demands than arithmetic problems without such operations. Indeed, several studies showed interactions between executive WM load and carry or borrow demands (e.g., Ashcraft & Kirk, 2001; Fürst & Hitch, 2000; Imbo et al., 2005a, 2005b; Seitz & Schumann-Hengsteler, 2002), and between phonological WM load and carry or borrow demands as well (e.g., Imbo et al., 2005a, 2005b). These results reveal that both executive and phonological WM components are needed to perform carry and borrow operations. However, more convincing evidence could be obtained by manipulating both variables in an even more detailed way, for example by using different levels of WM load and a more fine-grained carry/borrow complexity.

Some researchers already addressed this issue. The number of carry operations for example, was manipulated in a fairly detailed way in some studies. This number was zero, one, or two in the study of Fürst and Hitch (2000), and one, two or three in the study of Imbo et al. (2005a). Both studies showed that reaction times and error rates increased linearly with the number of carries. Moreover, they also showed that the interference of executive WM load grew larger as the number of carries was higher. Yet, studies in which a more fine-grained variation in both the number of carries and the WM load were included still have to be executed. Future research should elaborate on this issue, using problems with a variable number of carry or borrow operations, and a more thoroughly manipulated WM load. This should not only be done for the executive WM component, but also for the phonological loop and the visuo-spatial sketchpad. More detailed ideas are provided further in this paper.

In simple arithmetic. In their review, DeStefano & LeFevre (2004) mention another point of debate: although carrying also occurs in simple-arithmetic problems (e.g., 9 + 4 vs. 6 + 3), a load x carry interaction has never been observed there. Since in simple additions the difference between easy and hard problems is often made by the presence of a carry operation, this would mean that the increased problem difficulty associated with carrying is based on the carry operation *in se*, and not to the greater demands of retrieving larger values for carry problems compared to non-carry problems (e.g., 13 vs. 9). As De Stefano and LeFevre (2004) note, this issue is critical for understanding the source of executive WM demands in solving arithmetic problems. Although there are no studies that explicitly resolved this issue, we believe that the largest fraction of executive WM resources is used to perform the carry operation *in se*. Indeed, in simple arithmetic, an interaction between executive WM load and problem size has hardly ever been found, indicating that the role of the executive WM component does not grow larger as problem size increases (e.g., De Rammelaere et al., 1999, 2001; De Rammelaere & Vandierendonck, 2001, 2003; Lemaire et al., 1996). Since the carry x load interaction has been found frequently in complex arithmetic (as noted above), we would argue that the main role of the executive WM component in carrying is coordinating the increased number of steps and manipulating larger numbers, but not retrieving larger numbers. However, additional evidence would be welcome. Since the difference between easy and hard simple-arithmetic sums is defined based on the presence of a carry operation, it is almost impossible to disentangle both effects (i.e., the problem-size effect and the carry effect). This is one of the key challenges for future research.

Building further on recent findings

As noted above, one of the ideas to implement in future research is to use more fine-grained manipulations of both the number of carry operations and the WM load in order to further investigate the role of the various WM components in carrying. For the number of carries one may use any number between zero and the total number of stimulus digits minus one. For example, in an addition problem with 4-digit numbers like 1564+2657, the maximum amount of carry operations is three. It will be obvious that increasing the total number of digits may exceed the WM capacity limit at some point. A more fine-grained manipulation of the WM loads is less obvious, however. It is known that the executive WM component can be divided in various subfunctions such as shifting, updating, and inhibition (e.g., Lehto, 1996; Miyake, Friedman, Emerson, Witzki, Howerter, & Wager, 2000; Ward, Roberts, & Philips, 2001, see also Vandierendonck, 2000a, 2000b). Contrasting primary-task performance (i.e., mental arithmetic) with different secondary tasks loading one or more of these subfunctions would provide detailed information about the specific role of the executive WM component in carrying and borrowing (see Deschuyteneer & Vandierendonck, 2005, for an application of this method in simple arithmetic). Concerning the phonological and visuo-spatial WM loads, researchers may think of using a preload with different amounts of letters/words or asterisks/blocks, respectively.

Not only has the *number* of carry operations been shown to increase WM interference, but also the *value* to be carried. Although not often investigated in previous research (but see Imbo et al., 2005a, 2005b), this variable is easy to manipulate. The value to be carried may vary between zero and the quantity of numbers to be added minus one. For example, in 128+149+238, where *three* numbers have to be added, the maximum value to carry is *2*. Indeed, 8 + 9 + 8 = 25, meaning that a 2 had to be carried from the units to the tens. Future research may study the role of the various WM components in carrying higher-valued digits more deeply. Indeed, up until now, the results are equivocal: both the executive WM component and the phonological WM component would be needed more as the number of carry operations or the value to be carried grows. Moreover, future research may find out why carrying higher-valued digits in multiplications did not rely on executive or phonological WM components. An extra idea for future research is manipulating the variability of the value to be carried. Up until now, this value was kept constant within one problem (e.g., all digits to be carried had value 2). However, it is plausible that carrying digits with variable values (e.g., carrying a 1, a 2, and a 3) is more difficult than carrying digits with all the same value. Another idea that never has been implemented is manipulating the value to be borrowed. However, it is not totally clear how this should be done. Moreover, even when it would be possible, it is still highly questionable whether people would ever encounter such problems in real-life situations.

The evidence concerning the role of the visuo-spatial sketchpad in carrying and borrowing is very sparse. Based on these results, this WM component would play no role in carrying or borrowing. However, we believe that further research is needed to definitively reject the role of the visuo-spatial sketchpad in carrying and borrowing.

The role of WM in the carry operation in multiplications is very sparse as well. Up until now, we would conclude that no WM component is used to perform carry operations in multiplications. This sounds quite implausible. Probably no effect has been found because the problems used were simple rather than complex. Moreover, West-European people are often intensively trained in memorizing multiplications (Seitz & Schumann-Hengsteler, 2000, 2002), which could have reduced the effects.

Finally, it is important to notice that research on carrying and borrowing is not limited to ‘pure’ cognitive approaches. As carry and borrow operations rely on the executive WM component, other coincidental processes may also suffer when these executive WM components are loaded. For example, people with a high math anxiety are bad in mental arithmetic, and particularly when a carry operation has to be performed (Faust et al., 1996). Since the strongest evidence that math anxiety did affect arithmetic performance came from the comparison of carry versus no-carry problems, the number of carry operations would be an effective manipulation in order to increase task complexity and the demands on WM. Indeed, both the anxious, intrusive thoughts and the carry procedure are competing for available executive WM components (Eysenck & Calvo, 1992). Consequently, carry and borrow procedures would not only suffer from (relevant) math anxiety, but also from irrelevant anxiety (e.g., in persons with an anxiety disorder) – a hypothesis that still has to be tested. Future research may thus manipulate carry and borrow characteristics (such as the number of carry/borrow operations and the value to be carried) in order to investigate effects of various types of variables (e.g., emotional, developmental, cognitive, etc.) on arithmetic performance.

References

Ashcraft, M. H. (1992). Cognitive arithmetic: A review of data and theory. *Cognition, 44, 75-106.*

Ashcraft, M. H. (1995). Cognitive psychology and simple arithmetic: A review and summary of new directions. *Mathematical Cognition, 1, 3-34.*

Ashcraft, M. H., & Battaglia, J. (1978). Cognitive arithmetic: Evidence for retrieval and decision processes in mental addition. *Journal of Experimental Psychology: Human Learning and Memory, 4, 527-538.*

Ashcraft, M. H., Donley, R. D., Halas, M. A., & Vakali, M. (1992). Working memory, automaticity, and problem difficulty. In J. I. D. Campbell (Ed.), *The nature and origins of mathematical skills* (pp. 301-329). Amsterdam: Elsevier.

Ashcraft, M. H., & Faust, M. W. (1994). Mathematics anxiety and mental arithmetic performance: An exploratory investigation. *Cognition and Emotion, 8, 97-125.*

Ashcraft, M. H., & Kirk, E. P. (2001). The relationships among working memory, math anxiety, and performance. *Journal of Experimental Psychology: General, 130, 224-237.*

Ashcraft, M. H., & Stazyk, E. K. H. (1981). Mental addition: A test of three verification models. *Memory & Cognition, 9, 185-196.*

Baddeley, A. D. (1986). *Working Memory.* Oxford: Oxford University Press.

Baddeley, A. D. (1992). Is working memory working? The fifteenth Bartlett Lecture. *Quarterly Journal of Experimental Psychology, 44, 1-31.*

Baddeley, A. D. (1996). Exploring the central executive. *Quarterly Journal of Experimental Psychology, 49A, 5-28.*

Baddeley, A. D., Hitch, G. J. (1974). Working Memory. In G. Bower (Ed.). *The Psychology of Learning and Motivation (vol 8, pp. 47-90).* New York: Academic Press.

Baddeley, A. D., & Lieberman, K. (1980). Spatial working memory. In R. S. Nickerson (Ed.), *Attention and performance VIII* (pp. 521-539). Hillsdale, NJ: Erlbaum.

Baddeley, A. D., & Logie, R. H. (1992). Auditory imagery and working memory. In D. Reisberg (Ed.), *Auditory imagery* (pp. 179-197). Hillsdale, NJ: Erlbaum.

Baddeley, A. D., & Logie, R. H. (1999). Working memory: The multi-component model. In A. Miyake & P. Shah (Ed.), *Models of working memory: Mechanisms of active maintenance and executive control* (pp. 28-61). New York: Cambridge University Press.

Baddeley, A. D., Thomson, N., & Buchanan, M. (1975). Word length and the structure of short term memory. *Journal of Verbal Learning & Verbal Behaviour, 14, 575-589.*

Brown, J. S., & Burton, R. R. (1978). Diagnostic models for procedural bugs in basic mathematical skills. *Cognitive Science, 2, 155-192.*

Butterworth, B., Zorzi, M., Girelli, L., & Jonckheere, A. R. (2001). Storage and retrieval of addition facts: The role of number comparison. *The quarterly journal of experimental psychology, 54A, 1005-1029.*

Cooney, J. B., Swanson, H. L., & Ladd, S. F. (1988). Acquisition of mental multiplication skill; Evidence for the transition between counting and retrieval strategies. *Cognition and Instruction, 5, 323-345.*

Dansereau, D. F., & Gregg, L. W. (1966). An information processing analysis of mental multiplication. *Psychonomic Science, 6, 71-72.*

De Rammelaere, S., Stuyven, E., Vandierendonck, A. (1999). The contribution of working memory recourses in the verification of simple arithmetic sums. *Psychological Research, 62, 72-77*

De Rammelaere, S., Stuyven, E., Vandierendonck, A. (2001). Verifying simple arithmetic sums and products: Are the phonological loop and the central executive involved? *Memory and Cognition, 29, 267-273.*

De Rammelaere, S., & Vandierendonck, A. (2001). Are executive processes used to solve simple mental arithmetic production tasks? *Current Psychology Letters: Behaviour, Brain & Cognition, 2, 79-82.*

De Rammelaere, S., & Vandierendonck, A. (2003). Number comparison under executive dual-task. *Psychologica Belgica, 43, 259-268.*

Deschuyteneer, M. & Vandierendonck, A. (2005). Are 'input monitoring' and 'response selection' involved in solving simple mental arithmetical sums? *European Journal of Cognitive Psychology, 17, 347-370.*

DeStefano, D., & LeFevre, J.-A. (2004). The role of working memory in mental arithmetic. *European Journal of Cognitive Psychology, 16, 353-386.*

Eysenck, M.W., & Calvo, M.G. (1992). Anxiety and performance: The processing efficiency theory. *Cognition and Emotion, 6, 409-434.*

Farmer, E. W., Berman, J. V. F. & Fletcher, Y. L. (1986). Evidence for a visuo-spatial scratch-pad in working memory. *Quarterly Journal of Experimental Psychology, 38A, 675-688.*

Faust, M. W., Ashcraft, M. H., Fleck, D. E. (1996). Mathematics anxiety effects in simple and complex addition. *Mathematical Cognition, 2, 25-62.*

Fürst, A. J., Hitch, G. J. (2000). Separate roles for executive and phonological components of working memory in mental arithmetic. *Memory & Cognition, 28, 774-782.*

Geary, D. C. (1994). *Childeren's mathematical development: Research and practical applications.* Washington DC: American Psychological Association.

Geary, D. C. (1996). The problem-size effect in mental addition: Developmental and cross-national trends. *Mathematical Cognition, 2, 63-93.*

Geary, D. C., & Widaman, K. F. (1987). Individual differences in cognitive arithmetic. *Journal of Experimental Psychology: General, 116, 154-171.*

Geary, D. C., Widaman, K. F., & Little, T. D. (1986). Cognitive addition and multiplication: Evidence for a single memory network. *Memory & Cognition, 14, 478-487.*

Gilhooly, K. J., Logie, R. H., Wetherick, N., & Wynn, V. (1993). Working memory and strategies in syllogistic reasoning tasks. *Memory & Cognition, 21, 115-124.*

Groen, G. J., & Parkman, J. M. (1972). A chronometric analysis of simple addition. *Psychological Review, 79, 329-343.*

Hamann, M.S., & Ashcraft, M.H. (1985). Simple and complex mental addition across development. *Journal of Experimental Child Psychology, 40, 49-72.*

Heathcote, D. (1994). The role of visual-spatial working memory in the mental addition of multi-digit addends. *Cahiers de Psychologie Cognitive, 13, 207-245.*

Hecht, S. A. (2002). Counting on working memory in simple arithmetic when counting is used for problem solving. Memory and Cognition, 30, 447-455.

Hitch, G. J. (1978). The Role of Short-Term Working Memory in Mental Arithmetic. *Cognitive Psychology, 10, 302-323.*

Hope, J. A., & Sherill, J. M. (1987). Characteristics of skilled and unskilled calculators. Journal of Research in Mathematics Education, 18, 98-111.

Imbo, I., Vandierendonck, A., & De Rammelaere S. (2005a). *The role of working memory in the carry operation of mental arithmetic: Number and value of the carry.* Manuscript submitted for publication.

Imbo, I., Vandierendonck, A., & Vergauwe, E. (2005b). The role of working memory in carrying and borrowing. Manuscript in preparation.

Lee, K.-M., & Kang, S.-Y. (2002). Arithmetic operation and working memory: differential suppression in dual tasks. *Cognition, 83, B63-B68*.

Lehto, J. H. (1996). Are executive function tests dependent on working memory capacity? *Quarterly Journal of Experimental Psychology, 49A, 29-50.*

Lemaire, P., Abdi, H., & Fayol, M. (1996). The role of working memory resources in simple cognitive arithmetic. *European Journal of Cognitive Psychology, 8, 73-103.*

Logie, R. H. (1986). Visuo-spatial processing in working memory. *Quarterly Journal of Experimental Psychology, 38A, 229-247.*

Logie, R. H. (1989). Characteristics of visual short-term memory. *European Journal of Cognitive Psychology, 1, 275-284.*

Logie, R. H. (1991). Visuo-spatial short-term memory: Visual working memory or visual buffer? In C. Cornoldi & M. McDaniels (Eds.), *Imagery and cognition* (pp. 77-102). Berlin: Springer-Verlag.

Logie, R. H. (1993). Working memory in everyday cognition. In G. M. Davies & R. H. Logie (Eds.), *Memory in everyday life* (pp. 173-218). Amsterdam: Elsevier.

Logie, R. H., Baddeley, A. D. (1987). Cognitive processes in counting. *Journal of Experimental Psychology: Learning, Memory, & Cognition, 2, 310-326.*

Logie, R. H., Gilhooly, K. J., Wynn, V. (1994). Counting on working memory in arithmetic problem solving. *Memory & Cognition, 22, 395-410.*

Miyake, A., Friedman, N. P., Emerson, M. J., Witzki, A. H., Howerter, A., & Wager, T. D. (2000). The unity and diversity of executive functions and their contributions to complex “frontal lobe” tasks: A latent variable approach. *Cognitive Psychology, 41, 49-100.*

Noël, M.-P., Désert, M., Aubrun, A. & Seron, X. (2001). Involvement of short-term memory in complex mental calculation. *Memory & Cognition, 29, 34-42.*

Quinn, J.G. (1994). Towards a clarification of spatial processing. *Quarterly Journal of Experimental Psychology, 47, 465-480.*

Salamé, P., & Baddeley, A. D. (1982). Disruption of short-term memory by unattended speech: Implications for the structure of working memory. *Journal of Verbal Learning and Verbal Behavior, 21, 150-164.*

Sandrini, M., Miozzo, A., Cotelli, M., & Cappa, S.F. (2003). The residual calculation abilities of a patient with severe aphasia: Evidence for a selective deficit of subtraction problems. *Cortex, 39, 85-96.*

Seitz, K., & Schumann-Hengsteler, R. (2000). Mental multiplication and working memory. *European Journal of Cognitive Psychology, 12, 552-570.*

Seitz, K., & Schumann-Hengsteler, R. (2002). Phonological loop and central executive processes in mental addition and multiplication. Psychologische Beiträge, 44, 275-302.

Seyler, D.J., Kirk, E.P., & Ashcraft, M.H. (2003). Elementary subtraction. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 29, 1339-1352.*

Siegler, R. S. (1988). Strategy choice procedures and the development of multiplication skill. *Journal of Experimental Psychology: General, 117, 258-275.*

Sokol, S.M., McCloskey, M., Cohen, N.J., & Aliminosa, D. (1991). Cognitive representations and processes in arithmetic: Inferences from the performance of brain-damaged subjects. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 17, 355-376.*

Szmalec, A., Vandierendonck, A., & Kemps, E. (2005). Response selection involves executive control: Evidence from the selective interference paradigm. *Memory & Cognition, 33, 531-541.*

Trbovich, P. L., & LeFevre, J. A. (2003). Phonological and visual working memory in mental addition. *Memory and Cognition, 31, 738-745.*

Vandierendonck, A. (2000a). Bias and processing capacity in the generation of random time intervals. Cognitive Science Quarterly, 1, 205-233.

Vandierendonck, A. (2000b). Is judgement of random time intervals biased and capacity limited? Psychological Research, 63, 199-209.

Ward, G., Roberts, M. J., & Philips, L. H. (2001). Task-Switching Costs, Stroop-Costs, and Executive Control: a Correlational Study. *Quarterly Journal of Experimental Psychology, 54A, 491-511.*

Widaman, K. F., Geary, D. C., Cormier, O., Little, T. D. (1989). A componential model of mental addition. *Journal of Experimental Psychology: Learning, Memory, & Cognition, 15, 898-919.*

Author note

Ineke Imbo is the recipient of the 2004 Best Thesis Award, and is currently working as a doctoral researcher of the Special Research Fund of Ghent University at the Department of Experimental Psychology (supported by grant no. 011D07803). Stijn De Rammelaere was research assistant of the Fund for Scientific Research – Flanders; and is currently working as a marketing researcher in the private sector. This research was further supported by grant no. 10251101 of the Special Research Fund of Ghent University to André Vandierendonck. Correspondence concerning this article should be addressed to Ineke Imbo, Department of Experimental Psychology, Ghent University, Henri Dunantlaan 2, 9000 B-Ghent, Belgium.

Figure captions

Figure 1

Interactions with Memory Load in complex additions (Imbo et al., 2005a).

Panel a: Number x WM Load Interactions (accuracies).

Panel b: Value x WM Load Interactions (accuracies).

Figure 2

Interactions with Memory Load in complex subtractions (Imbo et al., 2005b).

Panel a: Number x WM Load Interactions (latencies).

Panel b: Number x WM Load Interactions (accuracies).

Figure 3

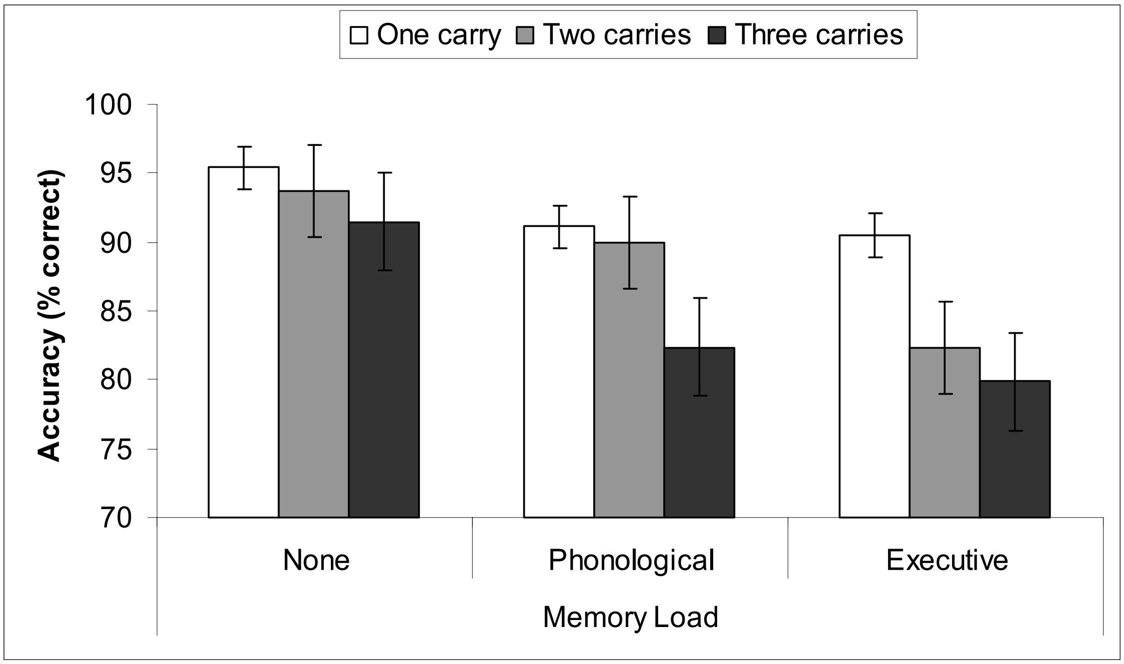
Interactions with Memory Load in complex multiplications (Imbo et al., 2005b).

Panel a: No Number x WM Load Interactions (latencies)

Panel b: No Value x WM Load Interactions (latencies)

Figure 1

Panel a



Panel b

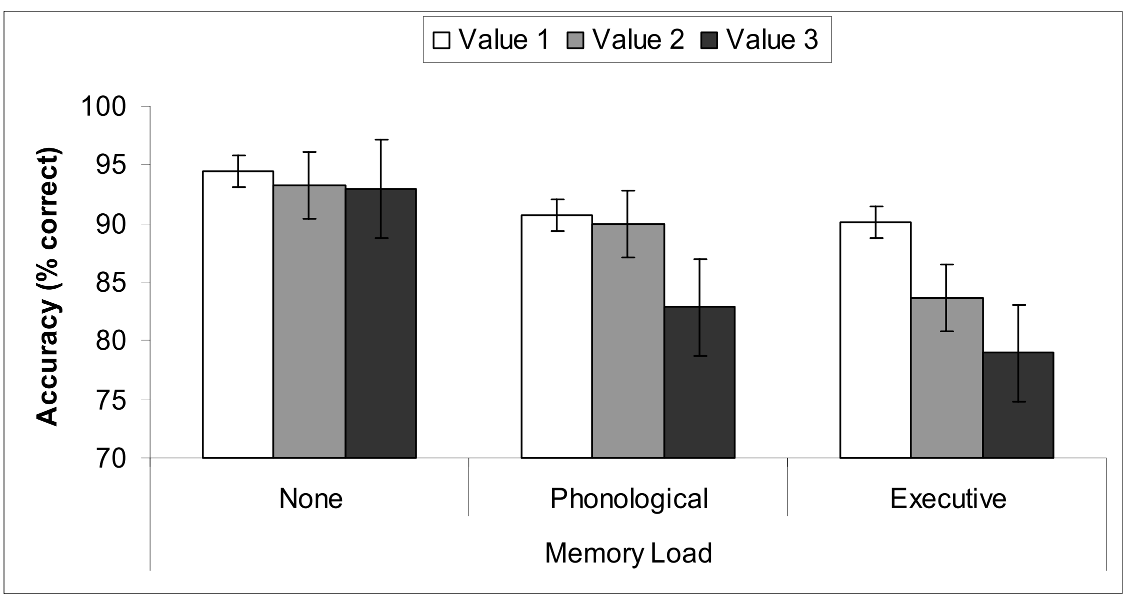
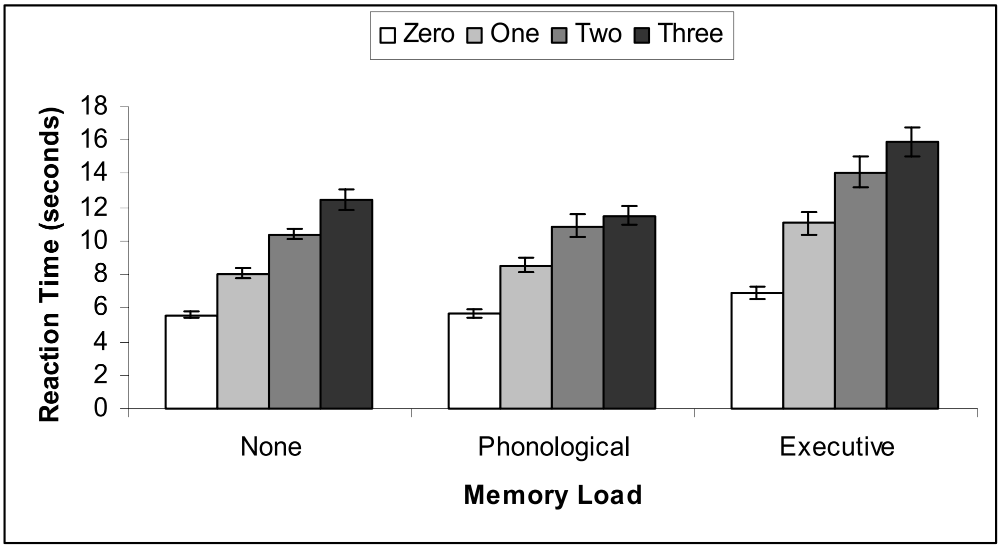


Figure 2

Panel a



Panel b

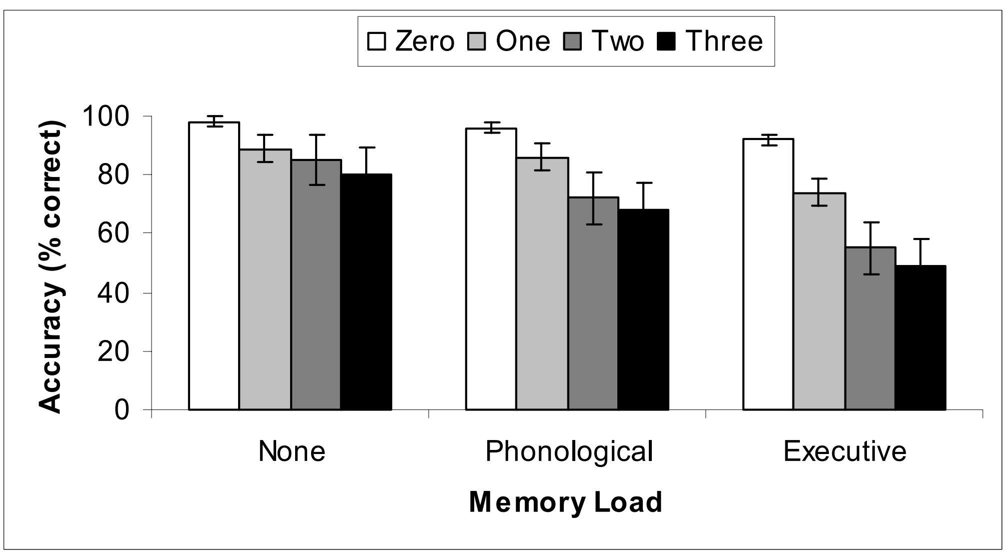
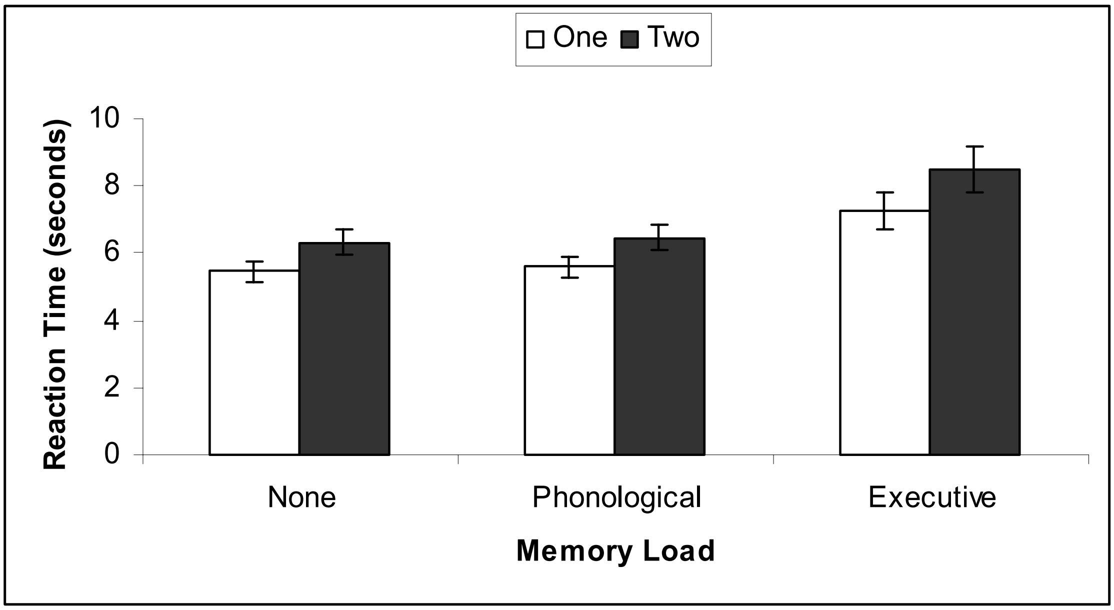


Figure 3

Panel a



Panel b