Application of the Nonextensive Statistical Approach for High Energy Particle Collisions

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What is CERN doing? Bizarre clouds over Large Hadron Collider 'prove portals are opening
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"Is it a coincidence they had just started the Awake experiment?"
Why do we need high energy collisions?

Image source: https://www.phy.duke.edu

Image source: arXiv:1308.3328
Why do we need high energy collisions?

- Quark-gluon plasma (QGP): strongly interacting hot, dense matter (and perfect fluid)
- Temperature: $\sim 10^{12}$ K
- Lifetime: $\sim fm/c$
- Hadronization is still "mystery"

Image source: http://icc.ub.edu/
Why high energy collisions?
Why do we need high energy collisions?
Small system, large fluctuations

- High energy physics: new particles from collisions
- Hadron spectra in $pp$ collisions can be described by the Tsallis distribution
- $\pi^+$ spectra in $pp$ collisions depends similarly on $\sqrt{s}$ and on the multiplicity $N$

Small system, large fluctuations

- Spectrum:
  - low $p_T$ (soft): Boltzmann – Gibbs
  - high $p_T$ (hard): power-law tailed (pQCD)
  - the whole range is difficult

- # of particles in classical atomic matter: $\sim O(10^{24})$
- # of particles produced in heavy ion collisions: $\sim O(10^4 - 10^6)$
- # of particles produced in high energy $pp$ collisions: $\sim O(10 - 100)$
Small system, large fluctuations

- Extensive Boltzmann–Gibbs statistics:

\[
S_{12} = S_1 + S_2 \quad \Rightarrow \quad S_B = - \sum_i p_i \ln p_i
\]

\[
E_{12} = E_1 + E_2
\]

- Non-extensivity due to fluctuations \(\Rightarrow\) generalized entropy

\[
\hat{L}_{12} = \hat{L}_1(S_1) + \hat{L}_2(S_2) \quad \Rightarrow \quad S_q = \frac{1}{q-1} \left(1 - \sum_i p_i^q\right) = - \sum_i p_i \ln_q p_i
\]

\[
L_{12} = L_1(E_1) + L_2(E_2)
\]

\[
S_{12} = S_1 + S_2 + (q - 1)S_1S_2
\]

\[
\ln_q p = \frac{p^{(1-q)} - 1}{1 - q} \quad e_q^p = (1 + (1 - q)p)^{1/(1-q)}
\]

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Small system, large fluctuations

- Boltzmann–Gibbs entropy as $q \to 1$ limit:

\[
S_1 = \lim_{q \to 1} S_q = \lim_{q \to 1} \frac{1}{q - 1} \left( 1 - \sum_i p_i^q \right) \\
= \lim_{q \to 1} \frac{1}{q - 1} \left( 1 - \sum_i p_i e^{(q-1) \ln p_i} \right) \\
= \lim_{q \to 1} \frac{1}{q - 1} \left( 1 - \sum_i p_i \left( 1 + (q - 1) \ln p_i + O((q - 1)^2) \right) \right) \\
= - \sum_i p_i \ln p_i
\]

- Maximizing the Tsallis entropy: the Tsallis–Pareto distribution can be obtained

\[
f(\varepsilon) = \left[ 1 + (q - 1) \frac{\varepsilon}{T} \right]^{-\frac{1}{q-1}}
\]
Small system, large fluctuations

- In high energy collisions: $E$ fixed and $\epsilon \ll E$

$$q = \frac{\langle S'(E)^2 + S''(E) \rangle}{\langle S'(E) \rangle^2} = \frac{\langle N(N-1) \rangle}{\langle N \rangle^2} = 1 - \frac{1}{C} + \frac{\Delta T^2}{T^2}, \quad \frac{1}{T} = \langle S'(E) \rangle, \quad T = \frac{E}{\langle N \rangle}$$

where $C = \frac{dE}{dT} = \frac{E}{T}$

- The Tsallis-entropy: $S_q = \frac{1}{q-1} \left( 1 - \sum_i p_i^q \right)$, if $q \to 1$:

$$\lim_{q \to 1} S_q = - \sum_i p_i \ln p_i = S_{BG}$$

- $q$: the measure of non-extensivity
  - If $q-1$ is large, that means that fluctuations due to small size effects are significant

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Small system, large fluctuations

Program:

- Fit spectra of identified hadrons measured in pp, pA and AA collisions
- Investigate the $\sqrt{s}$ dependency of the fitted parameters (and other dependencies: mass, strangeness content, centrality, multiplicity...)
- Verification of the scale evolution
- Predictions for other collision energies (13-14 TeV) and for other quantities ($v_2$ anizotropic flow...)

\[
f(\varepsilon) = \left[1 + (q - 1) \frac{\varepsilon}{T} \right]^{\frac{-1}{q-1}}
\]

\[
q = \frac{\langle S'(E)^2 + S''(E) \rangle}{\langle S'(E) \rangle^2}
\]

\[
q = \frac{\langle N(N-1) \rangle}{\langle N \rangle^2}
\]

\[
q = 1 + \frac{\Delta T^2}{T^2} - \frac{1}{C}
\]

\[
\frac{E}{\langle N \rangle} = T_{BG}
\]

\[
\frac{E}{\langle N \rangle} = \frac{\int \varepsilon f_{T S}(\varepsilon)}{\int f_{T S}(\varepsilon)}
\]

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Results
Fitted pp→ PID hadron spectra

Data/Fits are good, $\chi^2/\text{NDF} \sim 1$

13 TeV: coming soon...
The evolution of fitted $q$ and $T$ parameters

- Predictions for 13 TeV (spoiler: quite good)
- $q$ is increasing for mesons very similarly, but ~constant for protons (barions)
- $T$ is increasing, but very differently for mesons/barions
  - $T_2$ for kaons and pions is similar, but $T_1$ is not $\rightarrow$ strangeness?
T(q-1): $\sqrt{s} = 62$ GeV
T(q-1): $\sqrt{s} = 200$ GeV

$T(q-1) = A \left(1 + \frac{q-1}{T_0(m_T - m)}\right)^{-q}$

$q_{m-1}/q_{b-1} = 0.939 \pm 0.001$
T(q-1): $\sqrt{s} = 900$ GeV

Fitted T vs q-1, $\sqrt{s}=(62-900)$ GeV

- $\pi^0$
- Barion
- $K^\pm$
- $p/\bar{p}$
- Meson
- $\pi^\pm$
- $K_s^0$
- $\rho^0$

\[ f(m_T) = A \left( 1 + \frac{q-1}{T} (m_T - m) \right)^{q-1} \]

\[ (q_m-1)/(q_b-1) = 0.951 \pm 0.012 \]
$T(q-1): \sqrt{s} = 2760$ GeV

Fitted $T$ vs $q-1$, $\sqrt{s}=(62-2760)$ GeV

- $\pi^0$
- Barion
- $K^\pm$
- $p/\bar{p}$
- Meson
- $\pi^\pm$
- $K_s^0$
- $\rho^0$

$f(m_T)=A\left(1+\frac{q-1}{T}(m_T-m)^2\right)^{\frac{1}{q-1}}$

$(q_{m-1})/(q_{b-1})=1.221\pm0.007$
$T(q-1): \sqrt{s} = 7000$ GeV

Fitted $T$ vs $q-1$, $\sqrt{s}=(62-7000)$ GeV

- $\pi^0$
- Barion
- $K^\pm$
- $p/\bar{p}$
- $\rho^0$

$$f(m_T) = A \left(1 + \frac{q-1}{T(m_T - m)} \right)^{q/4}$$

$$(q_m - 1)/(q_b - 1) = 1.328 \pm 0.014$$
T(q-1), all energies

\[ T \text{ as effective temperature} \]

\[ T_{eff} = T(q) = T_0 + (q - 1)T_v \]

- For AA: \( T_v \) can be negative
  - Soft+hard model...?

\[ f(m) = A \left( 1 + \frac{q-1}{T(m_T-m)} \right)^{q-1} \]

\[ \frac{(q_m-1)}{(q_b-1)} = 0.969 \pm 0.001 \]

\[ \sqrt{s} \]

Scaled $T(q-1)$, all energies

Coalescence:
Hadron distribution $\rightarrow$ quark distribution

For $T_{\text{hadron}} = T_{\text{quark}}$:

$$\frac{q_m - 1}{q_b - 1} \sim \frac{3}{2}$$

Also shows strong $\sqrt{s}$ dependency

Summary

- Maximizing Tsallis-entropy: thermodinamical expressions for particle spectra obtained in high energy $pp$ collisions
  - The fluctuation of the number of particles is large
  - Fitted Tsallis – Pareto distributions describe the spectra very well
  - The $q$ and $T$ parameters show a strong CM energy dependence
  - Physical picture from the $T$ vs $(q-1)$ function
- Similar behaviour in electron-positron, proton-nucleus and nucleus-nucleus collisions
  - Soft+hard model, transverse flow, coalescence…

Thank you!
References


