Bayesian X-ray Computed Tomography using a Three-level Hierarchical Prior Model

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Abstract

The ill-posed Computed Tomography (CT) reconstruction problem has been widely studied via a linear forward model: $g = Hf + \epsilon$, where $f$ represents the object, $g$ the projection data, $H$ the forward projection matrix and $\epsilon$ the errors. In this paper, we propose a novel three-level hierarchical structured prior model for a 3D CT reconstruction problem. With this model, the object and its wavelet transformation are estimated simultaneously, meanwhile, the relationship between them are considered. Particularly, the objects we considered are piece-wise continuous, which lead to the property that its haar wavelet coefficients are sparse. The heavy-tailed Generalized Student-t distribution with its Infinite Gauss Scaled Mixture (IGSM) model is used to model these sparse wavelet coefficients, which are then estimated via a Bayesian inference method. The following graphical model and relations summarize the proposed probabilist model:

$$p(f, z, v_z, v_{\epsilon}, v_{\xi}|g) \propto p(g|f, v_{\epsilon}) p(f|z, v_{\xi}) p(z|v_z) \cdot p(v_z|\alpha_{z0}, \beta_{z0}) p(v_{\epsilon}|\alpha_{\epsilon0}, \beta_{\epsilon0})$$

Where $z$ represents the wavelet coefficients of the object $f$, $D$ the corresponding transform dictionary, $v_z$ variance of $z$, $v_{\epsilon}$ variance of noise $\epsilon$ and $v_{\xi}$ variance of noise $\xi$. Joint Maximum A Posterior (JMAP) and Variational Bayesian Approach (VBA) estimators are then applied to this model. Results will be shown with both simulated and real-world data. Comparison with Total Variation (TV) and other state-of-art methods will be given in this paper.

Keywords: Computed Tomography (CT), Bayesian Method, Hierarchical Model, Sparsity, Student-t distribution, Inverse Problem, Variational Bayesian Approach (VBA), Joint Maximum A Posterior (JMAP), Infinite Gaussian Scaled Mixture (IGSM)