

Endogenous Wage Indexation and Aggregate Shocks

ONLINE APPENDIX

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This Online Appendix describes the full derivation of the model and provides further details on the paper's results and additional analyses.

Contents

A	New Keynesian Model with Sticky Wages and Prices	3
A.1	Households and Wage Setting	3
A.1.1	Consumption and Savings	4
A.1.2	Wage Setting	4
A.2	Firms	8
A.2.1	Final good producer	8
A.2.2	Intermediate-good firms	8
A.3	Government and Monetary Policy	10
A.4	Shocks	11
A.5	Equilibrium	11
B	Welfare	15
B.1	Social welfare	15
B.2	Private welfare	16
B.2.1	A second-order approximation to labor disutility	17
B.3	Welfare costs measures	19
B.4	Numerical illustration	20
B.5	An example using comparative statics	22
C	New-Keynesian Wage Phillips Curve with fixed indexation	24

D	Explaining the time-varying wage indexation in the US: additional results	26
D.1	Wage indexation effects as predicted by the simple model	26
D.2	Robustness checks	28
D.2.1	Robustness check for non-constant trend inflation	28
D.2.2	Robustness check for zero price indexation	28
D.3	Counterfactual analysis	30
E	Endogenous wage indexation and firm profits	33
F	Endogenous wage indexation in a more complex DSGE model with financial frictions	36

A New Keynesian Model with Sticky Wages and Prices

The model consists of 5 types of agents: the final goods producer, intermediate goods producers, consumers, workers, and the fiscal and monetary authorities. The complete version of the model is used in Section 5.2 of the main text and Sections D2-3 and E in this Appendix, where we calibrate it for the United States (US). A stripped-down version of the model, with sticky wages, flexible prices, and no habit formation, is used in Sections 2-4 of the main text to gather intuition about the mechanism we are studying.

A.1 Households and Wage Setting

The economy is inhabited by a continuum of differentiated households, indexed by $i \in [0, 1]$. Household i is endowed with a unique labor type, $\ell_{i,t}$, which allows it to set its own wage using monopolistic power. Household i selects consumption, $c_{i,t}$, one-period-maturity bond holdings, $b_{i,t}$, and a nominal wage, $W_{i,t}$, in order to maximize its expected discounted lifetime utility

$$\mathbb{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \mathcal{U}(c_{i,T}, \ell_{i,T}), \quad (\text{A.1})$$

subject to the sequence of budget constraints

$$c_{i,t} + \frac{b_{i,t}}{(1 + R_t) \exp(\varepsilon_{b,t})} \leq (1 + \tau_w) \frac{W_{i,t}}{P_t} \ell_{i,t} + \frac{b_{i,t-1}}{1 + \pi_t} + \frac{\Upsilon_{i,t}}{P_t}, \quad (\text{A.2})$$

and *no Ponzi* schemes. \mathbb{E}_t is the expectation operator conditional on the available information in period t . R_t is the risk-free nominal interest rate, τ_w is a labor-income subsidy calibrated to eliminate any non-stochastic steady-state distortion in labor allocations generated by workers' monopolistic power, P_t denotes the price of the final good, $\pi_t \equiv P_t/P_{t-1} - 1$ is the inflation rate, $\Upsilon_{i,t}$ is a lump sum including taxes, Arrow-Debreu state-contingent securities, and profits from monopolistic firms, and $\varepsilon_{b,t}$ is a stochastic disturbance with mean zero that creates a spread between the return on bonds and the risk-free rate (cf. risk spread shock in Smets and Wouters, 2007). Preferences are separable between consumption and labor:

$$\mathcal{U}(c_{i,t}, \ell_{i,t}) \equiv \frac{(c_{i,t} - \gamma^h c_{i,t-1})^{1-\sigma} - 1}{1 - \sigma} - \psi \frac{\ell_{i,t}^{1+\omega}}{1 + \omega}, \quad (\text{A.3})$$

where γ^h is a parameter controlling external habits, $\sigma^{-1} > 0$ is the inter-temporal elasticity of substitution, ω^{-1} is the Frisch elasticity of labor supply, ψ is a normalizing constant that ensures that labor equals $\frac{1}{3}$ at the deterministic steady-state. We assume for simplicity that households are divided into two units: a consumer and a worker. The former chooses consumption demand and

bond holdings, while the latter sets the nominal wage knowing that the elapsed time between wage re-optimizations is a stochastic process. Notice that the presence of state-contingent securities ensures that all households begin a period with the same wealth and therefore choose the same level of consumption. Therefore, we can drop the subscript i in the first-order conditions (FOCs) of $b_{i,t}$, and $c_{i,t}$.

A.1.1 Consumption and Savings

The consumer's problem is

$$\max_{c_T, b_T} E_t \left\{ \sum_{T=t}^{\infty} \beta^{T-t} \left[\frac{(c_T - \gamma^h c_{T-1})^{1-\sigma} - 1}{1-\sigma} + \lambda_T \left(\frac{b_{T-1}}{1 + \pi_T} + \frac{\Upsilon_{i,T}}{P_T} - c_{i,T} - \frac{b_T}{(1 + R_T) \exp(\varepsilon_{b,T})} \right) \right] \right\},$$

where λ_t is the Lagrange multiplier of the household problem. Notice that this variable also serves as the shadow marginal utility of wealth and that it is common to all households due to the presence of Arrow-Debreu securities. Finally, λ_t is also a signaling device for income effects in the economy. An increase in λ_t implies that households become poorer and, if workers' labor supply does not change, households will afford a lower consumption level.

The first order conditions (FOCs) of b_t and c_t , are respectively given by

$$1 = \beta E_t \left\{ \frac{\lambda_{t+1} (1 + R_t) \exp(\varepsilon_{b,T})}{\lambda_t (1 + \pi_{t+1})} \right\}, \quad (\text{A.4})$$

$$(c_t - \gamma^h c_{t-1})^{-\sigma} - \beta \gamma^h E_t \left\{ (c_{t+1} - \gamma^h c_t)^{-\sigma} \right\} = \lambda_t. \quad (\text{A.5})$$

A.1.2 Wage Setting

Labor packer. Following Erceg *et al.* (2000), we assume that a competitive employment agency builds an aggregate labor input from a set of differentiated labor types $\ell_{i,t}$, for $i \in [0, 1]$, according to the following CES technology

$$\ell_t = \left(\int_0^1 \ell_{i,t}^{(\theta_w - 1)/\theta_w} di \right)^{\theta_w / (\theta_w - 1)}, \quad (\text{A.6})$$

where $\theta_w > 1$ is the elasticity of substitution between any two labor types. Profit maximization by the labor intermediary yields the demand for type- i labor

$$\ell_{i,t} = \left(\frac{W_{i,t}}{W_t} \right)^{-\theta_w} \ell_t \quad \forall i, \quad (\text{A.7})$$

while the aggregate nominal wage obeys

$$W_t = \left(\int_0^1 W_{i,t}^{1-\theta_w} di \right)^{1/(1-\theta_w)}. \quad (\text{A.8})$$

Wage setter. Similar to Calvo (1983), we assume that in each period a worker re-calibrates his labor contract with a probability $1 - \alpha_w$. In the re-calibration, the worker chooses a wage indexation scheme for updating his wage in non-re-optimizing periods and an optimal wage level that maximizes his or her utility. There are two indexation schemes in the economy, namely δ^{past} and δ^{trend} . The former updates wages using the previous period inflation rate, while the latter uses the central bank's inflation target, which in the model corresponds to trend inflation. Formally

$$\delta_{t,T}^{past} = (1 + \pi_{T-1}) \delta_{t,T-1}^{past} \quad \text{and} \quad \delta_{t,T}^{trend} = (1 + \pi_T^*) \delta_{t,T-1}^{trend},$$

$\forall T > t$, and $\delta_{t,t}^k = 1$ for $k \in \{past, trend\}$; t is the period where the last optimization occurred. For the ease of exposition purposes, it is convenient to first describe the well known wage-setting problem given the choice of $\delta_{t,T}^k$. The selection of the indexation scheme is described afterwards. Thus, given a $\delta_{t,T}^k$, a worker sets his wage according to (dropping subscript i , as any worker maximizing at t and with a rule $\delta_{t,T}^k$ will choose the same wage)

$$W_t^{k,*} \in \arg \max_{W_t^k} E_t \left\{ \sum_{T=t}^{\infty} (\beta \alpha_w)^{T-t} \left(\lambda_T (1 + \tau_w) \frac{\delta_{t,T}^k W_t^k}{P_T} \ell_{t,T}^k - \psi \frac{(\ell_{t,T}^k)^{1+\omega}}{1 + \omega} \right) \right\}, \quad (\text{A.9})$$

subject to

$$\ell_{t,T}^k = \left(\frac{\delta_{t,T}^k W_t^k}{W_T} \right)^{-\theta_w} \ell_T. \quad (\text{A.10})$$

The FOC with respect to $W_t^{k,*}$ is

$$r w_t^{k,*} \equiv \frac{W_t^{k,*}}{W_t} = \psi \frac{\theta_w}{(\theta_w - 1)(1 + \tau_w)} \frac{E_t \left\{ \sum_{T=t}^{\infty} (\beta \alpha_w)^{T-t} (\ell_{t,T}^k)^{1+\omega} \right\}}{E_t \left\{ \sum_{T=t}^{\infty} (\beta \alpha_w)^{T-t} \lambda_T \left(\frac{\delta_{t,T}^k}{\pi_{t,T}^w} \right) w_T \ell_{t,T}^k \right\}} \quad (\text{A.11})$$

where $r w_t^{k,*}$ is the relative wage of workers using indexation rule k , $\pi_{t,T}^w \equiv W_T/W_t$ is gross wage inflation between periods t and T and the labor-specific demand $\ell_{t,T}^k$ takes the form

$$\ell_{t,T}^k = \left(\frac{\delta_{t,T}^{w,k}}{\pi_{t,T}^w} r w_t^{k,*} \right)^{-\theta_w} \ell_T.$$

Notice that to eliminate the wage markup $\mu_w \equiv \theta_w/(\theta_w - 1)$, the labor subsidy must equal $1 + \tau_w = \mu_w$. To get a simplified wage-setting equation, replace the labor-specific demand and the calibrated

labor subsidy into the optimal wage-setting equation and re-arrange to obtain

$$\left(rw_t^{k,\star}\right)^{1+\omega\theta_w} = \psi \frac{\text{num}_{k,t}^w}{\text{den}_{k,t}^w}, \quad (\text{A.12})$$

where

$$\begin{aligned} \text{num}_{k,t}^w &= [\ell_t]^{1+\omega} + \beta\alpha_w \mathbb{E}_t \left\{ \left(\frac{1 + \pi_{t+1}^w}{\delta_{t,t+1}^k} \right)^{\theta_w(1+\omega)} \text{num}_{k,t+1}^w \right\}, \\ \text{den}_{k,t}^w &= \lambda_t w_t \ell_t + \beta\alpha_w \mathbb{E}_t \left\{ \left(\frac{1 + \pi_{t+1}^w}{\delta_{t,t+1}^k} \right)^{\theta_w-1} \text{den}_{k,t+1}^w \right\}, \end{aligned}$$

and $\pi_t^w \equiv W_t/W_{t-1} - 1$ is the wage inflation rate.

Indexation-rule selection. Let ξ_t denote the time t total proportion of workers who have selected past-inflation indexation, independently of their last contract negotiation. In short, ξ_t represents the degree of *aggregate indexation* to past inflation in time t . Furthermore, let Σ be an information set describing the distribution of stochastic shocks in the economy. Finally, let Ξ_t denote the vector containing current and expected future levels of aggregate indexation, such that $\Xi_t = \{\mathbb{E}_t \xi_{t+i}\}_{i=0}^{\infty}$. We can now formalize workers' indexation-rule decision as follows: If worker i can re-negotiate its labor contract in time t , he or she selects the rule that maximizes his or her conditional expected utility, i.e.

$$\delta_{i,t}^*(\xi_t, \Sigma) \in \arg \max_{\delta_i \in \{\delta^{trend}, \delta^{past}\}} \mathbb{W}_{i,t}(\delta_i, \Xi_t, \Sigma) \text{ subject to } \wp(\Theta, \xi_t, \Sigma), \quad (\text{A.13})$$

where

$$\mathbb{W}_{i,t}(\delta_i, \Xi_t, \Sigma) = \mathbb{E}_t \left\{ \sum_{T=t}^{\infty} (\beta\alpha_w)^{T-t} \mathcal{U}(c_T(\xi_T, \Sigma), \ell_{i,T}(\delta_i, \xi_T, \Sigma)) \right\}, \quad (\text{A.14})$$

where Θ is the full set of structural parameters of the model, and $\wp(\cdot)$ represents a system of equations that summarizes all relevant general-equilibrium constraints that determine the allocation of the economy at any given period. Notice that $\mathbb{W}_{i,t}$ is constrained by the expected duration of the labor contract since the effective discount factor for a worker is $\beta\alpha_w$. In addition, state-contingent securities ensure that individual consumption equals the aggregate level. Indeed, a worker's individual indexation-rule choice has a negligible effect on aggregate quantities since a single worker counts for a infinitesimal proportion with respect to the aggregate. In contrast, aggregate consumption does depend on aggregate indexation ξ_t and the current economic regime Σ . Thus, for worker i , ξ_t and Σ are given, so he or she selects the indexation rule δ_i that maximizes

his or her individual expected utility. The latter brings the possibility of coordination failure, since workers do not internalize how their own choice affects the aggregate. The externality implies that what is good for a worker, may not be good for all workers.

Labor market aggregation. The degree of aggregate indexation ξ_t is determined as follows: each period, only a fraction $1 - \alpha_w$ of workers re-optimize their wages. Let χ_t denote the time t proportion of workers from subset $(1 - \alpha_w)$ that selects δ^{past} . Accordingly, ξ_t is given by

$$\xi_t = (1 - \alpha_w) \sum_{h=0}^{\infty} \chi_{t-h} (\alpha_w)^h, \quad (\text{A.15})$$

which recursively can be written as $\xi_t = (1 - \alpha_w) \chi_t + \alpha_w \xi_{t-1}$. Without loss of generality, assume that workers are sorted according to the indexation rule they have chosen, so workers in the interval $i \in I_t^{past} = [0, \xi_t]$ use δ^{past} , while those in the interval $i \in I_t^{trend} = [\xi_t, 1]$ use δ^{trend} . Measures of wage dispersion for each of the two sectors can be computed by adding up total hours worked, which are determined by the set of labor-specific demands. Therefore, we have that $\int_{i \in I_t^k} \ell_{i,t} di = \ell_t \text{disp}_{k,t}^w$, where $\text{disp}_{k,t}^w = \int_{i \in I_t^k} \left(\frac{W_{i,t}}{W_t} \right)^{-\theta_w} di$. Recursive expressions for the wage dispersion measures are given by

$$\text{disp}_{k,t}^w = (1 - \alpha_w) \tilde{\chi}_t^k \left(r w_t^{k,*} \right)^{-\theta_w} + \alpha_w \left(\frac{1 + \pi_t^w}{\delta_{t-1,t}^k} \right)^{\theta_w} \text{disp}_{k,t-1}^w, \quad (\text{A.16})$$

$$\text{where } \tilde{\chi}_t^k = \begin{cases} \chi_t & \text{if } k = \text{past} \\ 1 - \chi_t & \text{if } k = \text{trend} \end{cases}. \quad (\text{A.17})$$

Finally, given the Dixit-Stiglitz technology of the labor intermediary, the aggregate wage level is given by $W_t^{1-\theta_w} = \int_0^1 W_{i,t}^{1-\theta_w} di$. This expression can be rewritten in terms of the sum of relative wages within each indexation-rule sector, which are given by $\tilde{w}_t^k \equiv \int_{i \in I_t^k} \left(\frac{W_{i,t}}{W_t} \right)^{1-\theta_w} di$. Thus, it follows that

$$1 = \tilde{w}_t^{past} + \tilde{w}_t^{trend}, \quad \text{and} \quad (\text{A.18})$$

$$\tilde{w}_t^{past} = (1 - \alpha_w) \chi_t \left[r w_t^{1,*} \right]^{1-\theta_w} + \alpha_w \left(\frac{1 + \pi_t^w}{\delta_{t-1,t}^{past}} \right)^{\theta_w-1} \tilde{w}_{t-1}^1, \quad (\text{A.19})$$

$$\tilde{w}_t^{trend} = (1 - \alpha_w) (1 - \chi_t) \left[r w_t^{2,*} \right]^{1-\theta_w} + \alpha_w \left(\frac{1 + \pi_t^w}{\delta_{t-1,t}^{trend}} \right)^{\theta_w-1} \tilde{w}_{t-1}^2. \quad (\text{A.20})$$

Notice that these weights may change over time due to variations in rw_t^k and χ_t . The recursive law of motion of \tilde{w}_t^k is given by

$$\tilde{w}_t^k = (1 - \alpha_w) \tilde{\chi}_t^k \left[rw_t^{k,*} \right]^{1-\theta_w,t} + \alpha_w \left(\frac{1 + \pi_t^w}{\delta_{t-1,t}^k} \right)^{\theta_w-1} \tilde{w}_{t-1}^k. \quad (\text{A.21})$$

The rest of the model is quite standard, so we describe it briefly.

A.2 Firms

A.2.1 Final good producer

A perfectly competitive firm produces a homogenous good, y_t by combining a continuum of intermediate goods, $y_{j,t}$ for $j \in [0, 1]$, using the following CES production function

$$y_t = \left(\int_0^1 y_{j,t}^{\frac{\theta_p-1}{\theta_p}} dj \right)^{\frac{\theta_p}{\theta_p-1}},$$

where $\theta_p > 1$ is the price elasticity of demand for intermediate good j . Profit maximization yields the typical set of input-specific demand functions:

$$y_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\theta_p} y_t \quad \forall j,$$

where $P_{j,t}$ denotes the price of the type- j intermediate good. The aggregate price level compatible with a zero-profit condition and the particular shape of the production function is:

$$P_t = \left(\int_0^1 P_{j,t}^{1-\theta_p} dj \right)^{\frac{1}{1-\theta_p}}.$$

A.2.2 Intermediate-good firms

Each intermediate good is produced by a single monopolistic firm using the linear technology

$$y_{j,t} = A \exp(z_t) n_{j,t},$$

where $n_{j,t}$ is the composite labor input, A is a normalizing constant that ensures that the detrended output at the deterministic steady state equals one, and z_t is a permanent technology shock that obeys

$$z_t = z_{t-1} + \nu_{z,t}, \quad (\text{A.22})$$

where $\nu_{z,t}$ is a zero-mean white noise. Each period, an intermediate firm re-optimizes its price with a fixed probability $1 - \alpha_p$. If the firm is unable to re-optimize in period T , then its price is

updated according to a rule-of-thumb of the form $P_{j,T} = \delta_{t,T}^p P_{j,t}$, where $t < T$ denotes the period of last reoptimization and $\delta_{t,T}^p = (1 + \pi_T^*)^{1-\gamma_p} (1 + \pi_{t-1})^{\gamma_p} \delta_{t,T-1}$ for $T > t$ and $\delta_{t,t}^p = 1$.¹ The firm sets $P_{j,t}$ by maximizing its profits, so

$$P_{j,t}^* \in \arg \max_{P_{j,t}} \mathbb{E}_t \left\{ \sum_{T=t}^{\infty} (\beta \alpha_p)^{T-t} \frac{\lambda_{t+1}}{\lambda_t} \left[\frac{\delta_{t,T}^p P_{j,t}}{P_T} y_{j,t,T} - (1 - \tau_p) S(y_{j,t,T}) \right] \right\},$$

$$\text{subject to } y_{j,t,T} = \left(\frac{\delta_{t,T}^p P_{j,t}}{P_T} \right)^{-\theta_p} y_T,$$

where the real cost function is given by $S(y_{j,t}) = w_t y_{j,t} / (A \exp(z_t))$, $\theta_p > 1$ is the price elasticity of demand for intermediate good j , and τ_p is a fiscal subsidy aimed to remove allocation distortions associated with the monopolistic power of intermediate firms. The subsidy is set such that $1 - \tau_p = (\theta_p - 1) / \theta_p$.

The FOC of the optimal intermediate-good price P_t^* is given by:²

$$\frac{P_t^*}{P_t} = \frac{\text{num}_t^p}{\text{den}_t^p},$$

where

$$\text{num}_t^p = \frac{w_t y_t}{A \exp(z_t)} + \beta \alpha_p \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{1 + \pi_{t+1}}{\delta_{t,t+1}^p} \right)^{\theta_p} \text{num}_{t+1}^p \right\},$$

$$\text{den}_t^p = y_t + \beta \alpha_p \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{1 + \pi_{t+1}}{\delta_{t,t+1}^p} \right)^{\theta_p - 1} \text{den}_{t+1}^p \right\}.$$

Price dispersion and labor demand. Since the aggregate price level equals $P_t = \left(\int_0^1 P_{j,t}^{1-\theta_p} dj \right)^{\frac{1}{1-\theta_p}}$, it follows that

$$1 = (1 - \alpha_p) (p_t^*)^{1-\theta_p, t} + \alpha_p \left(\frac{1 + \pi_t}{\delta_{t-1,t}^p} \right)^{\theta_p - 1}.$$

The aggregate labor demand is (using the definition of the production function and the input-specific demand)

$$\int_0^1 n_{j,t} dj = \frac{y_t}{A \exp(z_t)} \text{disp}_t^p,$$

where $\text{disp}_t^p = \int_0^1 \left(\frac{P_{j,t}}{P_t} \right)^{-\theta_p} dj > 1$. In recursive form, this equation becomes:

$$\text{disp}_t^p = (1 - \alpha_p) (p_t^*)^{-\theta_p} + \alpha_p \left(\frac{1 + \pi_t}{\delta_{t-1,t}^p} \right)^{\theta_p} \text{disp}_{t-1}^p.$$

¹We could have assumed that firms also endogenously select their price indexation rule. However, we keep the model as simple and tractable as possible to study the determination and implications of wage indexation.

²Notice that each intermediate firm optimizing its price in period t will set the same price, since all firms have identical technology and face similar demand functions.

Labor market and goods market equilibrium. Aggregate supplied hours equal the aggregate labor input times a wage dispersion distortion,

$$\int_0^1 \ell_{i,t} di = \ell_t \sum_k \text{disp}_{k,t}^w.$$

The labor composite is partitioned or distributed among all intermediate firms, according to their labor-specific demand, so

$$\ell_t = \int_0^1 n_{j,t} dj.$$

Finally, aggregating the labor specific demand across firms yields aggregate output as a function of aggregate labor and price dispersion:

$$y_t = \frac{1}{\text{disp}_t^p} A \exp(z_t) \ell_t. \quad (\text{A.23})$$

Flexible prices. In Sections 3 to 5 in the paper, we assume that producer prices are flexible, which implies that $\alpha_p = 0$. If such a case, it is easy to show that $P_{j,t} = P_t$ for all j and t , and therefore $w_t = A \exp(z_t)$. Price dispersion does not exist (i.e., $\text{disp}_t^p = 1$), so we can summarize the production sector of the economy using a single representative firm with technology $y_t = A \exp(z_t) \ell_t$.

A.3 Government and Monetary Policy

Fiscal policy. For simplicity, we assume that the government budget constraint is balanced at all times, so government purchases of the final good g_t plus labor and production subsidies equal lump-sum taxes levied on households:

$$g_t + \tau_w \int_i w_{i,t} \ell_{i,t} di + \tau_p \int_i \frac{w_t y_{j,t}}{A \exp(z_t)} dj = \text{taxes}_t,$$

where taxes_t are an element of the aggregate lump-sum variable $\int_i \Upsilon_{i,t} di$. In Sections 2 to 4 of the main text, we assume that $g_t = 0$, while in Sections 5.2 and Online Appendix sections D2-3 and E, we assume that $g_t = g_y \exp(\varepsilon_{g,t}) y_t$, where $0 < g_y \exp(\varepsilon_{g,t}) < 1$ is the public-spending-to-GDP ratio and $\varepsilon_{g,t}$ is a stochastic disturbance with mean zero.

Monetary policy. Similar to Cogley *et al.* (2010) and Hofmann *et al.* (2012), we assume that the central bank follows the rule below to set the short-run nominal interest rate:

$$1 + R_t = (1 + R_{t-1})^{\rho_R} \left[(1 + R_t^*) \left(\frac{1 + \pi_t}{1 + \pi_t^*} \right)^{a_\pi} \left(\frac{y_t}{y} \right)^{a_y} \left(\frac{y_t}{y_{t-1}} \right)^{a_{\Delta y}} \right]^{1 - \rho_R}, \quad (\text{A.24})$$

where

$$1 + R_t^* = \frac{1 + \pi_{t+1}^*}{\beta},$$

and y is the level of output at the deterministic steady state. This rule has shown good empirical properties, and we use it in Sections 5.2, D2-3, and E, where we calibrate all policy-rule parameters to the estimated values of Hofmann *et al.* (2012). In contrast, in Sections 2 to 4 we assume that ρ_R , a_y , and $a_{\Delta y}$ are all equal to 0 to favor intuition. Finally, the inflation target evolves as

$$\pi_{t+1}^* = \pi_t^* + \nu_{\pi,t+1}.$$

Therefore, inflation-target shocks are permanent.

A.4 Shocks

The model economy face permanent technology and inflation-target shocks, z_t and π_t^* , respectively, and temporary aggregate-demand shocks, $\varepsilon_{b,t}$ and $\varepsilon_{g,t}$, each one following an autoregressive law of motion of order one, such that

$$\varepsilon_{x,t} = \rho_x \varepsilon_{x,t-1} + \nu_{x,t},$$

where $0 \leq \rho_x < 1$ for $x \in \{b, g\}$. In turn, $\nu_{x,t}$ are white noise processes with standard deviation σ_x for $x \in \{b, z, g, \pi\}$.

A.5 Equilibrium

The resource constraint is given by

$$y_t = c_t + g_t, \tag{A.25}$$

The equilibrium of this economy is characterized by a set of prices $\{P_t, P_{j,t}, W_t, W_{i,t}, R_t\}$ and a set of quantities $\{y_t, g_t, c_{i,t}, b_{i,t}, n_{j,t}, \ell_t, \ell_{i,t}\}$, for all i and j , such that all markets clear at all times, and agents act consistently according to the maximization of their utility and profits. Notice that in equilibrium $c_t = \int_0^1 c_{i,t} di$, $\int_0^1 \ell_{i,t} di = \int_0^1 n_{j,t} dj$, $\int_0^1 b_{i,t} di = 0$.

The model has an stochastic trend due to the technology shock process. Therefore, we divide all trending real variables by $\exp(z_t)$, except for λ_t , which we multiply by $\exp(z_t)$, to preserve a stable saddle path:

$$y_t^s \equiv \frac{y_t}{\exp(z_t)}, \quad c_t^s \equiv \frac{c_t}{\exp(z_t)}, \quad g_t^s \equiv \frac{g_t}{\exp(z_t)}, \quad w_t^s \equiv \frac{w_t}{\exp(z_t)}, \quad \lambda_t^s \equiv \lambda_t \exp(z_t).$$

Tables 1 and 2 summarize the equilibrium conditions of the simple model used in Sections 2 to 4 and the complete model used in Sections 5.2 (and D2-3 and E), respectively. In addition, Table 3 describes the calibration used for each model. The simple model is calibrated using values typically found in the ballpark of New Keynesian models. In turn, all the values for the Great Moderation and Great Inflation periods, except for the standard deviation of the inflation target σ_π , are taken from Hofmann *et al.* (2012), while σ_π is taken from Cogley *et al.* (2010). The two papers mentioned estimate a very similar New Keynesian model as the one presented here for the US.

Table 1: Summary of Model used in Sections 2 to 4

Consumption & savings:

$$1 = \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \frac{(1+R_t) \exp(\varepsilon_{b,t})}{1+\pi_{t+1}} \right\} \quad (\text{I})$$

$$c_t^{-\sigma} = \lambda_t \quad (\text{II})$$

Wage setting:

$$\left(r w_t^{*,k} \right)^{1+\omega\theta_w} = \psi \frac{\text{num}_{k,t}^w}{\text{den}_{k,t}^w} \quad (\text{III})$$

$$\text{num}_{k,t}^w = \ell_t^{1+\omega} + \beta \alpha_w \mathbb{E}_t \left\{ \left(\frac{1+\pi_{t+1}^w}{\delta_{t,t+1}^k} \right)^{\theta_w(1+\omega)} \text{num}_{k,t+1}^w \right\} \quad (\text{IV})$$

$$\text{den}_{k,t}^w = \lambda_t w_t \ell_t + \beta \alpha_w \mathbb{E}_t \left\{ \left(\frac{1+\pi_{t+1}^w}{\delta_{t,t+1}^k} \right)^{\theta_w-1} \text{den}_{k,t+1}^w \right\} \quad (\text{V})$$

$$1 = \tilde{w}_t^{\text{past}} + \tilde{w}_t^{\text{trend}} \quad (\text{VI})$$

$$\tilde{w}_t^k = (1 - \alpha_w) \tilde{\chi}_t \left[r w_t^{*,\text{past}} \right]^{1-\theta_w} + \alpha_w \left(\frac{1+\pi_t^w}{\delta_{t-1,t}^k} \right)^{\theta_w-1} \tilde{w}_{t-1}^k \quad (\text{VII})$$

$$\text{disp}_{k,t}^w = (1 - \alpha_w) \tilde{\chi}_t \left(r w_t^{*,k} \right)^{-\theta_w} + \alpha_w \left(\frac{1+\pi_t^w}{\delta_{t-1,t}^k} \right)^{\theta_w} \text{disp}_{k,t-1}^w \quad (\text{VIII})$$

$$\tilde{\chi}_t = \chi_t \text{ if } k = \text{past}; \quad \tilde{\chi}_t = 1 - \chi_t \text{ if } k = \text{trend} \quad (\text{VI})$$

$$\delta_{t-1,t}^{\text{past}} = 1 + \pi_{t-1} \quad \text{and} \quad \delta_{t-1,t}^{\text{trend}} = 1 + \pi_t^* \quad (\text{IX})$$

Price setting:

$$w_t = A \exp(z_t) \quad (\text{X})$$

$$1 + \pi_t = P_t / P_{t-1} \quad (\text{XI})$$

$$P_t y_t - w_t \ell_t = 0 \quad (\text{XII})$$

Aggregation:

$$y_t = \ell_t A \exp(z_t) \quad (\text{XIII})$$

$$\xi_t = (1 - \alpha_w) \chi_t + \alpha_w \xi_{t-1} \quad (\text{XIV})$$

$$1 + R_t = (1 + R_t^*) \left(\frac{1+\pi_t}{1+\pi_t^*} \right)^{\alpha_\pi} \quad (\text{XV})$$

$$y_t = c_t \quad (\text{XVI})$$

Table 2: Summary of Full Non-Linear Model Used in Sections 5.2, D2-3, and E

Consumption & savings:

$$1 = \beta \mathbf{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \frac{(1+R_t) \exp(\varepsilon_{b,t})}{1+\pi_{t+1}} \right\} \quad (\text{I})$$

$$(c_t - \gamma^h c_{t-1})^{-\sigma} - \beta \gamma^h \mathbf{E}_t \left\{ (c_{t+1} - \gamma^h c_t)^{-\sigma} \right\} = \lambda_t \quad (\text{II})$$

Wage setting:

$$(r w_t^{*,k})^{1+\omega \theta_w} = \psi \frac{\text{num}_{k,t}^w}{\text{den}_{k,t}^w} \quad (\text{III})$$

$$\text{num}_{k,t}^w = \ell_t^{1+\omega} + \beta \alpha_w \mathbf{E}_t \left\{ \left(\frac{1+\pi_{t+1}^w}{\delta_{t,t+1}^k} \right)^{\theta_w(1+\omega)} \text{num}_{k,t+1}^w \right\} \quad (\text{IV})$$

$$\text{den}_{k,t}^w = \lambda_t w_t \ell_t + \beta \alpha_w \mathbf{E}_t \left\{ \left(\frac{1+\pi_{t+1}^w}{\delta_{t,t+1}^k} \right)^{\theta_w-1} \text{den}_{k,t+1}^w \right\} \quad (\text{V})$$

$$1 = \tilde{w}_t^{\text{past}} + \tilde{w}_t^{\text{trend}} \quad (\text{VI})$$

$$\tilde{w}_t^k = (1 - \alpha_w) \tilde{\chi}_t [r w_t^{*,\text{past}}]^{1-\theta_w} + \alpha_w \left(\frac{1+\pi_t^w}{\delta_{t-1,t}^k} \right)^{\theta_w-1} \tilde{w}_{t-1}^k \quad (\text{VII})$$

$$\text{disp}_{k,t}^w = (1 - \alpha_w) \tilde{\chi}_t (r w_t^{*,k})^{-\theta_w} + \alpha_w \left(\frac{1+\pi_t^w}{\delta_{t-1,t}^k} \right)^{\theta_w} \text{disp}_{k,t-1}^w \quad (\text{VIII})$$

$$\tilde{\chi}_t = \chi_t \text{ if } k = \text{past}; \quad \tilde{\chi}_t = 1 - \chi_t \text{ if } k = \text{trend} \quad (\text{VI})$$

$$\delta_{t-1,t}^{\text{past}} = 1 + \pi_{t-1} \quad \text{and} \quad \delta_{t-1,t}^{\text{trend}} = 1 + \pi_t^* \quad (\text{IX})$$

Price setting:

$$p_t^* = \frac{\text{num}_t^p}{\text{den}_t^p}, \quad (\text{X})$$

$$\text{num}_t^p = \frac{w_t y_t}{A \exp(z_t)} + \beta \alpha_p \mathbf{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{1+\pi_{t+1}}{\delta_{t,T}^p} \right)^{\theta_p} \text{num}_{t+1}^p \right\} \quad (\text{XI})$$

$$\text{den}_t^p = y_t + \beta \alpha_p \mathbf{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{1+\pi_{t+1}}{\delta_{t,T}^p} \right)^{\theta_p-1} \text{den}_{t+1}^p \right\} \quad (\text{XII})$$

$$1 = (1 - \alpha_p) (p_t^*)^{1-\theta_p} + \alpha_p \left(\frac{1+\pi_t}{\delta_{t-1,t}^p} \right)^{\theta_p-1} \quad (\text{XIII})$$

$$\text{disp}_t^p = (1 - \alpha_p) (p_t^*)^{-\theta_p} + \alpha_p \left(\frac{1+\pi_t}{\delta_{t-1,t}^p} \right)^{\theta_p} \text{disp}_{t-1}^p \quad (\text{XIV})$$

Aggregation:

$$y_t = \ell_t A \exp(z_t) / \text{disp}_t^p \quad (\text{XV})$$

$$\xi_t = (1 - \alpha_w) \chi_t + \alpha_w \xi_{t-1} \quad (\text{XVI})$$

$$1 + R_t = (1 + R_{t-1})^{\rho_R} \left[(1 + R_t^*) \left(\frac{1+\pi_t}{1+\pi_t^*} \right)^{\alpha_p} \left(\frac{y_t}{y} \right)^{\alpha_y} \left(\frac{y_t}{y_{t-1}} \right)^{\alpha_{\Delta y}} \right]^{1-\rho_R} \quad (\text{XVII})$$

$$y_t = c_t + g_t \quad (\text{XVIII})$$

Table 3: Calibration used in the paper

		Sections 3 to 4: Simple Model	Section 5.2: Great Moderation	Section 5.2: Great Inflation
<i>Deep parameters</i>				
β	Subjective discount factor	.99	.99	.99
σ	Intertemp. elasticity of subst.	1	1	1
ω	Inverse of Frisch elasticity	1	2	2
θ_w	Elasticity of labor demand	10	10	10
θ_p	Elasticity of interm. goods	-	10	10
γ^h	Habit formation	-	.37	.71
γ^p	Inflation inertia	-	.17	.8
α_p	Calvo-price rigidity	-	.78	.84
α_w	Calvo-wage rigidity	.5	.54	.64
a_π	Taylor Rule: inflation	1.5	1.35	1.11
a_y	Taylor Rule: output gap	-	.1	.11
$a_{\Delta y}$	Taylor Rule: output gap growth	-	.39	.5
ρ_R	Taylor Rule: smoothing	-	.78	.69
g_y	Public-spending-to-GDP ratio at steady state	-	.2	.2
<i>Shocks</i>				
ρ^g	Autocorr. gov'n't spending	-	.91	.89
ρ^b	Autocorr. risk spread	.5	-	-
σ_z	Std. dev. technology	.5	.31	1.02
σ_g	Std. dev. gov'n't spending	-	3.25	4.73
σ_b	Std. dev. risk spread	.25	-	-
σ_π	Std. dev. inflation target	.5	.049	.081

Note: All the values for the Great Moderation and Great Inflation periods, except for the standard deviation of the inflation target σ_π , are taken from Hofmann *et al.* (2012). In turn, σ_π is taken from Cogley *et al.* (2010). The two papers estimate a very similar New Keynesian model then the one presented here for the US.

B Welfare

As noted in the previous section, a worker's private welfare, which depends on the duration of his labor contract, is different to social welfare - the sum of all workers' welfare - and it is not constrained by the average duration of a labor contract. This difference, along with the fact that individual workers do not internalize the effect of their choice on the aggregate, raises the possibility of coordination failure in the decentralized equilibrium of the economy. In this section, we present formally the welfare functions and the welfare costs associated with a particular regime.

B.1 Social welfare

Define \mathbb{S}_t as the un-weighted sum of instantaneous household utilities:

$$\begin{aligned}
 \mathbb{S}_t &= \int_i \mathcal{U}(c_{i,t}, \ell_{i,t}) di \\
 &= \int_i \left(\frac{(c_{i,t} - \gamma^h c_{i,t-1})^{1-\sigma} - 1}{1-\sigma} - \psi \frac{\ell_{i,t}^{1+\omega}}{1+\omega} \right) di \\
 &= \frac{(c_t - \gamma^h c_{t-1})^{1-\sigma} - 1}{1-\sigma} - \psi \int_i \frac{\ell_{i,t}^{1+\omega}}{1+\omega} di.
 \end{aligned} \tag{B.1}$$

The last line follows from the fact that consumption is equal across households under the model assumptions. Expected social welfare is then defined as:

$$\begin{aligned}
 \text{SW}_t &= \mathbb{E}_t \left\{ \sum_{T=t}^{\infty} \beta_T^{T-t} \mathbb{S}_T \right\} \\
 &= \mathbb{S}_t + \beta \mathbb{E}_t \{ \text{SW}_{t+1} \}.
 \end{aligned}$$

Notice that in equation (B.1) aggregate labor disutility can be decomposed into

$$\int_0^1 \frac{\ell_{i,t}^{1+\omega}}{1+\omega} di = \int_{i \in IR_{past,t}^w} \frac{\ell_{i,t}^{1+\omega}}{1+\omega} di + \int_{i \in IR_{trend,t}^w} \frac{\ell_{i,t}^{1+\omega}}{1+\omega} di,$$

where

$$\int_{i \in IR_{k,t}^w} \frac{\ell_{i,t}^{1+\omega}}{1+\omega} di = \frac{\psi}{1+\omega} \ell_t^{1+\omega} \text{disp}_{k,t}^{\mathbb{S}} di$$

with $\text{disp}_{k,t}^{\mathbb{S}} = \int_{i \in IR_{k,t}^w} \left(\frac{W_{i,t}}{W_t} \right)^{-\theta_w(1+\omega)} di$. Since $\sum_k \text{disp}_{k,t}^{\mathbb{S}} \geq 1$, nominal wage dispersion generates welfare costs because it amplifies the disutility of labor.³ Following the same approach for the

³Showing that $\sum_k \text{disp}_{k,t}^{\mathbb{S}}$ is bounded below by 1 is easy using Jensen's inequality. First, let $rw_i = \frac{W_i}{W}$ denote the relative wage of worker i , where we have dropped the time subindex for simplicity. Then, notice that from the

recursive formulation of $\text{disp}_{k,t}^w$ in (A.16), one can show that:

$$\text{disp}_{k,t}^S = (1 - \alpha_w) \tilde{\chi}_t \left(r w_t^{k,*} \right)^{-\theta_w(1+\omega)} + \alpha_w \left(\frac{1 + \pi_t^w}{\delta_{t-1,t}^k} \right)^{\theta_w(1+\omega)} \text{disp}_{k,t-1}^S. \quad (\text{B.2})$$

B.2 Private welfare

The relevant welfare criterion for workers drawn to choose a new indexation rule is their own expected lifetime utility, conditional on the duration of the labor contract:

$$\mathbb{W}_{i,t} = \mathbb{E}_t \left\{ \sum_{T=t}^{\infty} (\beta \alpha_w)^{T-t} \mathcal{U}(c_T, \ell_{i,T}) \right\}. \quad (\text{B.3})$$

Notice that the discount factor takes into account that the labor contract might end each period with probability $1 - \alpha_w$. Since preferences are separable, we can decompose the welfare criterion into two terms, one related to consumption and the other to labor disutility: $\mathbb{W}_{i,t} = \Gamma_t - \Omega_{i,t}$ where

$$\begin{aligned} \Gamma_t &= \mathbb{E}_t \left\{ \sum_{T=t}^{\infty} (\beta \alpha_w)^{T-t} \frac{(c_T - \gamma^h c_{T-1})^{1-\sigma} - 1}{1 - \sigma} \right\}, \text{ and} \\ \Omega_{i,t}(\delta_{i,t}) &= \mathbb{E}_t \left\{ \sum_{T=t}^{\infty} (\beta \alpha_w)^{T-t} \frac{\psi}{1 + \omega} \ell_{i,T}^{1+\omega} \right\}. \end{aligned}$$

Since individual consumption is equal to aggregate consumption, a recursive expression for Γ_t is simply:

$$\Gamma_t = \frac{(c_t - \gamma^h c_{t-1})^{1-\sigma} - 1}{1 - \sigma} + \beta \alpha_w \mathbb{E}_t \{ \Gamma_{t+1} \}. \quad (\text{B.4})$$

For $\Omega_{i,t}(\delta_{i,t})$, we need to arrange further terms to obtain a recursive expression. Let us simplify notation to $\Omega_t^k = \Omega_{i,t}(\delta_{i,t})$ for δ^k . If we insert the labor-specific demand $\ell_{i,t,T}^k = \left(\frac{\delta_{t,T}^k}{\pi_{t,T}^w} r w_t^k \right)^{-\theta_w} \ell_T$ into Ω_t^k , we obtain

$$\Omega_t^k = \frac{\psi}{1 + \omega} \mathbb{E}_t \left\{ \sum_{T=t}^{\infty} (\beta \alpha_w)^{T-t} \left[\left(\frac{\delta_{t,T}^k}{\pi_{t,T}^w} r w_t^k \right)^{-\theta_w} \ell_T \right]^{1+\omega} \right\}.$$

definition of the aggregate wage index, we can write $\bar{r}w \equiv \int_0^1 f(rw_i) di = 1$, where $f(rw_i) = (1/rw_i)^{\theta-1}$. Next, consider the convex function $g(u) = u^{\theta(1+\omega)/(\theta-1)}$, with $\theta/(\theta-1) \geq 1$ for $\theta \geq 1$. The convex transformation of f using g is given by $h(rw_i) = (1/rw_i)^{\theta(1+\omega)}$. Jensen's inequality states that the convex transformation of the mean of a function is less or equal than the mean of the transformed function, i.e., $g(\bar{r}w) \leq \int_0^1 h(rw_i) di$. Since $g(\bar{r}w) = 1$, it follows that $1 \leq \int_i (rw_i)^{-\theta(1+\omega)} di$.

Next, expand the expression and factorize common terms:

$$\Omega_t^k = \frac{\psi}{1+\omega} \mathbb{E}_t \left\{ (rw_t^k)^{-\theta_w(1+\omega)} \left(\begin{array}{l} \ell_t^{1+\omega} \\ + \beta\alpha_w \left[\left(\frac{\delta_{t,t+1}^k}{\pi_{t,t+1}^w} \right)^{-\theta_w} \ell_{t+1} \right]^{1+\omega} \\ + (\beta\alpha_w)^2 \left[\left(\frac{\delta_{t,t+2}^k}{\pi_{t,t+2}^w} \right)^{-\theta_w} \ell_{t+2} \right]^{1+\omega} + \dots \end{array} \right) \right\}.$$

Then, notice that Ω_{t+1}^k is equal to

$$\Omega_{t+1}^k = \frac{\psi}{1+\omega} \mathbb{E}_{t+1} \left\{ (rw_{t+1}^k)^{-\theta_w(1+\omega)} \left(\begin{array}{l} \ell_{t+1}^{1+\omega} \\ + \beta\alpha_w \left[\left(\frac{\delta_{t+1,t+2}^k}{\pi_{t+1,t+2}^w} \right)^{-\theta_w} \ell_{t+2} \right]^{1+\omega} \\ + (\beta\alpha_w)^2 \left[\left(\frac{\delta_{t+1,t+3}^k}{\pi_{t+1,t+3}^w} \right)^{-\theta_w} \ell_{t+3} \right]^{1+\omega} + \dots \end{array} \right) \right\}.$$

So we can write Ω_t^k as :

$$\Omega_t^k = \frac{\psi}{1+\omega} \left[(rw_t^k)^{-\theta_w} \ell_t \right]^{1+\omega} + \beta\alpha_w \mathbb{E}_t \left\{ \left(\frac{1 + \pi_{t+1}^w}{\delta_{t,t+1}^k} \frac{rw_{t+1}^k}{rw_t^k} \right)^{\theta_w(1+\omega)} \Omega_{t+1}^k \right\}. \quad (\text{B.5})$$

B.2.1 A second-order approximation to labor disutility

The indexation criterion prompts workers to choose the contract associated with the lowest labor disutility at the stochastic steady state. For this subsection, we depart from the definition of labor disutility to obtain a second-order approximation, i.e.,

$$\Omega_t^k = \frac{\psi}{1+\omega} \mathbb{E}_t \left(\sum_{T=t}^{\infty} (\beta\alpha_w)^{T-t} [\ell_{t,T}^k]^{1+\omega} \right).$$

Notice that at the steady state, we have that $\ell_{ss,l}^k = \left(\frac{\delta_{ss,l}^k}{\pi_{ss,l}^w} rw_{ss}^k \right)^{-\theta_w} \ell_{ss}$, where $l \in \{0, 1, 2, 3, \dots\}$ is the number of periods since the last re-optimization. If we assume that $\frac{\delta_{ss,l}^k}{\pi_{ss,l}^w} = 1$ for all l , then we have that hours in sector k do not depend on the periods since last re-optimization but only on the relative optimal wages of each labor contract, i.e.

$$\ell_{ss}^k = (rw_{ss}^k)^{-\theta_w} \ell_{ss}.$$

A second-order approximation of term $[\ell_{t,T}^k]^{1+\omega}$ around its steady state reads

$$[\ell_{t,T}^k]^{1+\omega} \approx [\ell_{ss}^k]^{1+\omega} + (\ell_{t,T}^k - \ell_{ss}^k) (1+\omega) [\ell_{ss}^k]^\omega + \frac{1}{2} (\ell_{t,T}^k - \ell_{ss}^k)^2 \omega (1+\omega) [\ell_{ss}^k]^{\omega-1}.$$

It follows that Ω_t^k can be expressed as:

$$\begin{aligned}\Omega_t^k &\approx \frac{\psi}{1+\omega} \mathbb{E}_t \left\{ \sum_{T=t}^{\infty} (\beta\alpha_w)^{T-t} \left\{ \begin{aligned} &[\ell_{ss}^k]^{1+\omega} + (\ell_{t,T}^k - \ell_{ss}^k) (1+\omega) [\ell_{ss}^k]^\omega \\ &+ \frac{1}{2} (\ell_{t,T}^k - \ell_{ss}^k)^2 \omega [\ell_{ss}^k]^{\omega-1} \end{aligned} \right\} \right\}, \\ &\approx \frac{\psi}{1+\omega} \frac{[\ell_{ss}^k]^{1+\omega}}{1-\beta\alpha_w} + \psi \mathbb{E}_t \left\{ \sum_{T=t}^{\infty} (\beta\alpha_w)^{T-t} \left\{ \begin{aligned} &(\ell_{t,T}^k - \ell_{ss}^k) [\ell_{ss}^k]^\omega \\ &+ \frac{1}{2} (\ell_{t,T}^k - \ell_{ss}^k)^2 \omega [\ell_{ss}^k]^{\omega-1} \end{aligned} \right\} \right\}.\end{aligned}$$

In order to find the steady-state value Ω_{ss}^k , we need to apply the unconditional expectation operator to each side of the last expression, which leads to:

$$\mathbb{E} \{ \Omega_t^k \} \equiv \Omega_{ss}^k \approx \mathbb{E} \left\{ \frac{\psi}{1+\omega} \frac{[\ell_{ss}^k]^{1+\omega}}{1-\beta\alpha_w} + \psi \sum_{T=t}^{\infty} (\beta\alpha_w)^{T-t} \left\{ \begin{aligned} &(\ell_{t,T}^k - \ell_{ss}^k) [\ell_{ss}^k]^\omega \\ &+ \frac{1}{2} (\ell_{t,T}^k - \ell_{ss}^k)^2 \omega [\ell_{ss}^k]^{\omega-1} \end{aligned} \right\} \right\},$$

since $\mathbb{E} \{ (\ell_{t,T}^k - \ell_{ss}^k) \} = 0$ and $\text{var} (\ell_{t,T}^k) = \mathbb{E} \{ (\ell_{t,T}^k - \ell_{ss}^k)^2 \}$, it follows that

$$\Omega_{ss}^k \approx \frac{\psi}{1-\beta\alpha_w} \frac{[\ell_{ss}^k]^{1+\omega}}{1+\omega} + \frac{\psi}{2} \omega [\ell_{ss}^k]^{\omega-1} \sum_{T=t}^{\infty} (\beta\alpha_w)^{T-t} \text{var} (\ell_{t,T}^k),$$

From this expression, it is evident that labor disutility at the stochastic steady state is composed by two terms related to the expected level of hours worked as well as its variance. If we maintain the assumption that $\frac{\delta_{ss,l}^k}{\pi_{ss,l}^w} \approx 1$, then the second term can be simplified into $\frac{\psi}{(1-\beta\alpha_w)} \frac{\omega}{2} [\ell_{ss}^k]^{\omega-1} \text{var} (\ell_t^k)$, where the subscript T has being removed. And Ω_{ss}^k can be rewritten as

$$\begin{aligned}\Omega_{ss}^k &\approx \frac{\psi}{1-\beta\alpha_w} (R_{ss}^k + V_{ss}^k), \text{ where} \\ R_{ss}^k &= \frac{[\ell_{ss}^k]^{1+\omega}}{1+\omega}, \\ V_{ss}^k &= \frac{\omega}{2} [\ell_{ss}^k]^{\omega-1} \text{var} (\ell_t^k)\end{aligned}$$

Now, notice that we can use the labor-specific demand to substitute $\ell_{ss}^k = (rw_{ss}^k)^{-\theta_w} \ell_{ss}$ from the expressions above. Further, notice that at the stochastic steady state, relative wages and wage dispersion are altered as follows

$$\begin{aligned}\frac{\text{disp}_{1,ss}^w}{\xi} &= (rw_{ss}^{1,*})^{-\theta_w}, \text{ and} \\ \frac{\text{disp}_{2,ss}^w}{1-\xi} &= (rw_{ss}^{2,*})^{-\theta_w}\end{aligned}$$

given that $\frac{\delta_{ss,l}^k}{\pi_{ss,l}^w} \approx 1$.

Therefore, labor disutility for workers with the past-inflation and trend-inflation are, respectively:

$$\Omega_{ss}^1 \approx \frac{\psi}{1 - \beta\alpha_w} \left(\frac{1}{1 + \omega} \left[\frac{\text{disp}_{1,ss}^w}{\xi} \ell_{ss} \right]^{1+\omega} + \frac{\omega}{2} \left[\frac{\text{disp}_{1,ss}^w}{\xi} \ell_{ss} \right]^{\omega-1} \text{var}(\ell_t^1) \right),$$

$$\Omega_{ss}^2 \approx \frac{\psi}{1 - \beta\alpha_w} \left(\frac{1}{1 + \omega} \left[\frac{\text{disp}_{2,ss}^w}{1 - \xi} \ell_{ss} \right]^{1+\omega} + \frac{\omega}{2} \left[\frac{\text{disp}_{2,ss}^w}{1 - \xi} \ell_{ss} \right]^{\omega-1} \text{var}(\ell_t^2) \right).$$

In the paper, we assume that $\omega \geq 1$. Thus, as wage dispersion increases per sector, or the variance of labor hours in a sector increases, the largest is the unconditional expectation of labor disutility in said sector.

B.3 Welfare costs measures

Lucas (1987) measures welfare costs caused by stochastic disturbances in terms of deterministic steady-state consumption. We take a similar approach here, following the standard practice in DSGE models as suggested by Schmitt-Grohé and Uribe (2007). We assume in what follows that utility in consumption is logarithmic ($\sigma = 1$).

Social welfare in the deterministic steady state is given by:

$$\begin{aligned} \text{SW}_d(c_d, \ell_d) &\equiv \frac{1}{1 - \beta} \mathcal{U}(c_d, \ell_d), \\ &= \frac{1}{1 - \beta} \left(\log(c_d (1 - \gamma^h)) - \frac{\psi}{1 + \omega} \ell_d^{1+\omega} \right). \end{aligned}$$

Notice that this level does not depend on ξ . In contrast, at the stochastic steady state, social welfare will vary with ξ , the distribution of shocks (summarized by vector Σ), and current policy practices.

$$\text{SW}_{ss}(\xi, \Sigma) \equiv \text{E} \left\{ \sum_{T=t}^{\infty} \beta_T^{T-t} \int_0^1 \mathcal{U}(c_{i,t}, \ell_{i,t}) di \right\}.$$

When shocks appear, $\text{SW}_{ss} \leq \text{SW}(c_d, \ell_d)$. Let the fraction $0 \leq ce < 1$ denote welfare costs as a proportion of deterministic consumption c_d , such that $(1 - ce)c_d$ makes the social planner indifferent between the deterministic environment and the stochastic one, i.e.

$$\begin{aligned} \text{SW}_{ss}(\xi, \Sigma) &= \text{SW}_d((1 - ce) c_d, \ell_d) \\ &= \frac{1}{1 - \beta} \left(\log((1 - ce) c_d (1 - \gamma^h)) - \frac{\psi}{1 + \omega} \ell_d^{1+\omega} \right). \end{aligned}$$

Solving for ce yields

$$ce = 1 - \exp((\mathbb{S}\mathbb{W}_{ss}(\xi, \Sigma) - \mathbb{S}\mathbb{W}_d(c_d, \ell_d)) \times (1 - \beta)). \quad (\text{B.6})$$

For a worker, the benchmark welfare is the one that is conditional on the expected duration of the labor contract, and which value is maximum at the deterministic steady state, i.e.,

$$\mathbb{W}_d(c_d, \ell_d) = \frac{1}{1 - \beta\alpha_w} \mathcal{U}(c_d, \ell_d).$$

Notice that this benchmark is equal amid agents and does not depend on ξ neither. Similarly to social welfare, in the stochastic long-run equilibrium, individual welfare will depend on ξ , Σ , current policy practices, and the indexation rule a worker δ^k decides upon.

$$\mathbb{W}_{ss}(\xi, \Sigma, \delta^k) = \mathbb{E} \left\{ \sum_{T=t}^{\infty} (\beta\alpha_w)^{T-t} \mathcal{U}(c_{i,t}, \ell_{i,t}) \right\}.$$

Let ce^k denote the percentage decrease in deterministic steady-state consumption that makes a worker with indexation rule δ^k indifferent between the deterministic and the stochastic environment. Formally, ce^k is implicitly given by

$$\begin{aligned} \mathbb{W}_{ss}(\xi, \Sigma, \delta^k) &= \mathbb{W}_d((1 - ce)c_d, \ell_d). \\ &= \frac{1}{1 - \beta\alpha_w} \left(\log((1 - ce)c_d(1 - \gamma^h)) - \frac{\psi}{1 + \omega} \ell_d^{1+\omega} \right). \end{aligned}$$

Solving for ce^k yields

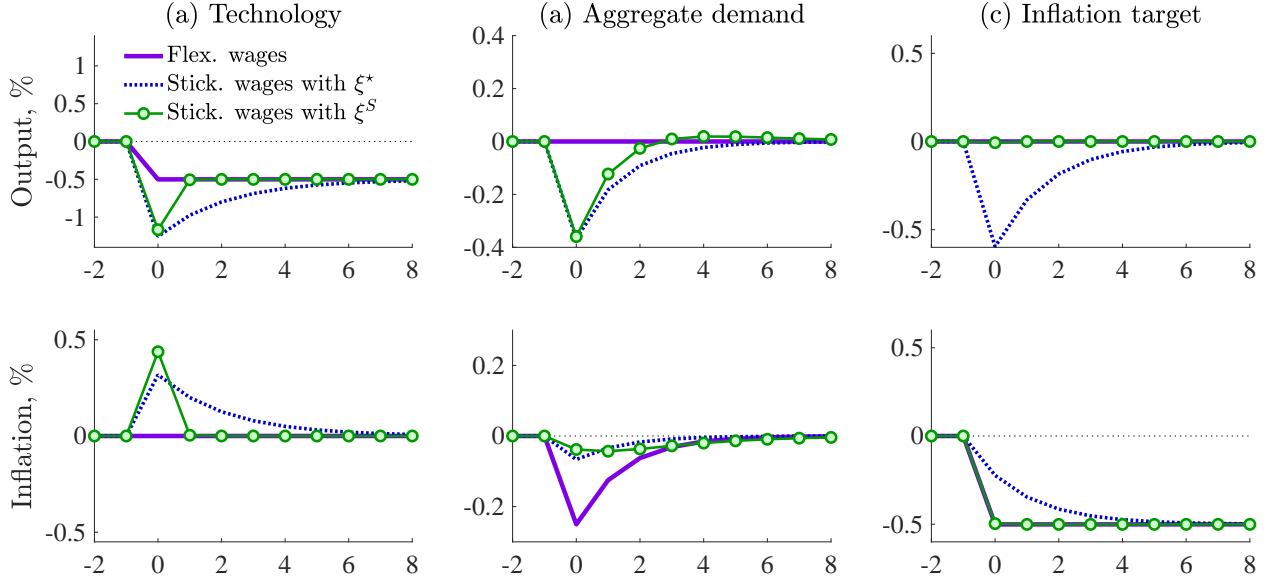
$$ce^k = 1 - \exp((\mathbb{W}_{ss}(\xi, \Sigma, \delta^k) - \mathbb{W}_d(c_d, \ell_d)) \times (1 - \beta\alpha_w)). \quad (\text{B.7})$$

B.4 Numerical illustration

To complement the analysis of Section 4 from the main text, Figure 1 displays the impulse responses of output (first row) and inflation (second row) to shocks in technology, aggregate demand, and the inflation target for the flexible-wage economy (plain lines), and two versions of the sticky-wage economy: one that follows the decentralized equilibrium (ξ^* , dotted lines), and another one that follows the social planner's preferred degree of wage indexation (ξ^S , lines with circles). In turn, Table 4 presents the discounted percent deviations of output and inflation from their efficient levels for each of the shocks. The results confirm that output and inflation rejoin their efficient levels quicker when workers coordinate towards the social planner's indexation choice. Output and inflation are indeed more persistent in the decentralized equilibrium.

The decentralized level of aggregate indexation and the planner's choice differ because workers take the level of aggregate indexation as given and neglect how their own indexation choice affects

Figure 1: Impulse responses for different ξ s: decentralized equilibrium vs social planner's choice.



Note: Each row shows selected impulse response to an aggregate shock, indicated at the left of the row. For the technology shock, $\xi^* = 1$ and $\xi^S = 0$; for the aggregate demand shock, $\xi^* = 0$ and $\xi^S = 1$; and for the inflation-target shock, $\xi^* = 0.8$ and $\xi^S = 0$.

Table 4: Discounted deviations from efficient levels.

Shock	Output		Inflation	
	ξ^*	ξ^S	ξ^*	ξ^S
Technology	2	0.67	0.84	0.44
Aggregate demand	0.72	0.60	0.36	0.35
Inflation target	1.32	0	0.62	0

Note: Each number is expressed as a percentage deviation from the efficient level, i.e., the level the variable would obtain if wages were perfectly flexible. For output, we compute $\sum_{t=0}^{\infty} \beta^t |\hat{y}_t - \hat{y}_t^f|$, while for inflation we compute $\sum_{t=0}^{\infty} \beta^t |\hat{\pi}_t - \hat{\pi}_t^f|$. For the technology shock, $\xi^* = 1$ and $\xi^S = 0$; for the aggregate demand shock, $\xi^* = 0$ and $\xi^S = 1$; and for the inflation-target shock, $\xi^* = 0.8$ and $\xi^S = 0$.

ξ . The atomistic size of workers and the lack of a coordination technology among them effectively exert a negative externality into the decentralized equilibrium. The nature of this externality implies that, even if the economy would start at ξ^S , workers would have incentives to change their indexation rules because, at the margin, they think they could reach a higher personal welfare.

B.5 An example using comparative statics

In order to understand a worker's preference towards an indexation rule, we introduce the following thought experiment. Suppose worker i' has a flexible-wage contract, while all other workers in the economy face staggered wages. Worker i' 's supply of labor hours has the same functional form as in the flexible-wage economy, in the sense that the marginal rate of substitution between consumption and labor equals the real wage:

$$\frac{\psi \ell_{i',t}^*}{\lambda_t} = w_{i',t}^* \tag{B.8}$$

where $w_{i',t}^*$ is worker i' 's optimal real wage. For short, we refer to the above expression as the *MRS condition*, which results from imposing $\alpha_w = 0$ in equation (A.11). This condition determines what we call the *desired labor hours* of worker i' , namely $\ell_{i',t}^*$. If the worker supplies that amount of hours given the real wage $w_{i',t}^*$ and the income effect λ_t , the welfare of household i' is maximized.⁴

Now suppose that an aggregate shock reduces the income of all households, so that λ_t increases. For any given wage, equation (B.8) suggests that worker i' would like to increase his or her labor hours in response to an increase in λ_t . Since the worker faces a labor demand equal to $\ell_{i',t} = (w_t/w_{i',t})^{\theta_w} \ell_t$, he or she must choose $w_{i',t}^*$ strategically. Imposing $\alpha_w = 0$ in equation (A.12), worker i' 's optimal real wage equals

$$w_{i',t}^* = \left(\frac{1}{M_t} \right)^{\frac{1}{1+\theta_w}} w_t \tag{B.9}$$

where M_t represents an efficiency wedge in the labor market due to staggered nominal wages, and it is defined as the ratio between the aggregate real wage and the *average* marginal rate of substitution of the staggered-wage economy, such that

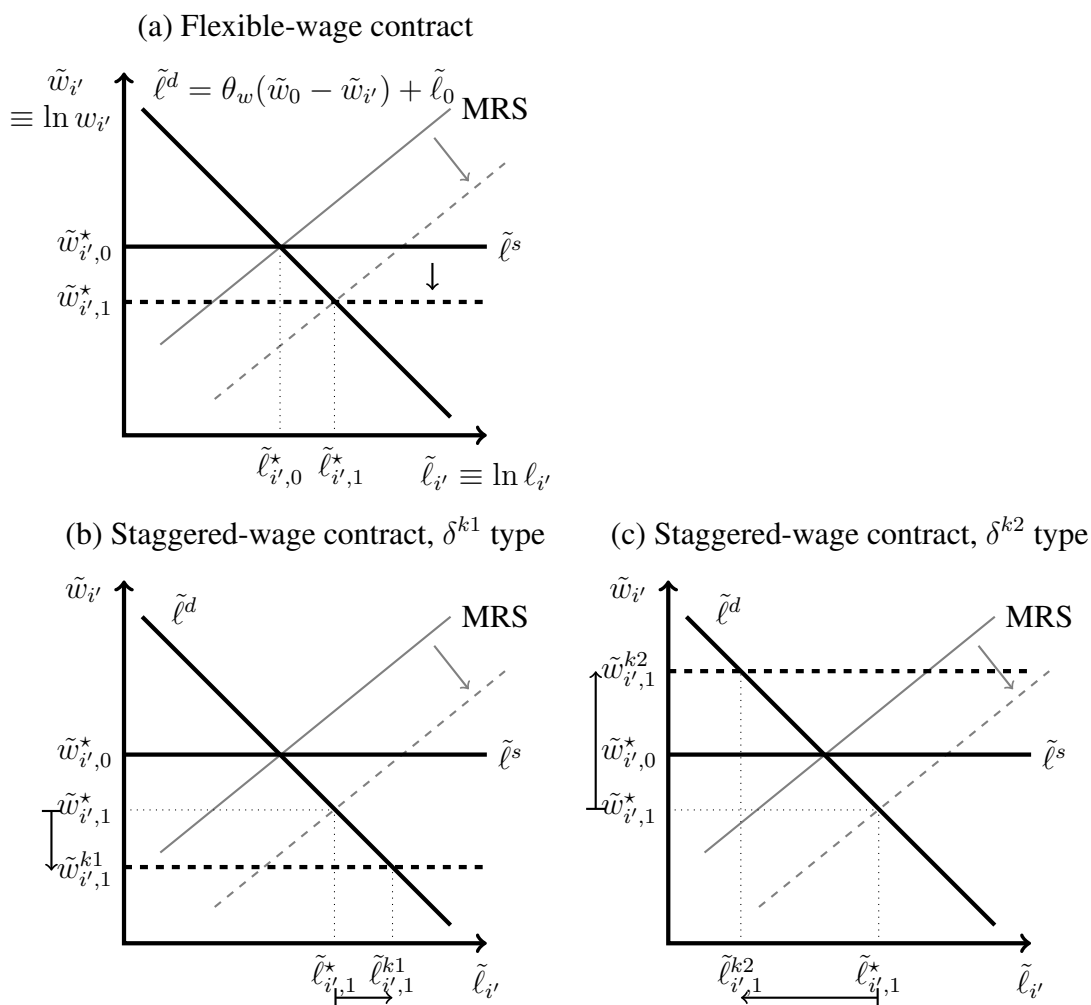
$$M_t \equiv \lambda_t \frac{w_t}{\psi \ell_t}$$

M_t can also be thought of as the *average* wage markup in the economy. According to equation (B.9), for a given aggregate real wage w_t and aggregate labor hours ℓ_t , an increase in λ_t implies

⁴Notice that even if worker i' has a flexible-wage contract, he or she receives the income-effect signal λ_t of the staggered-wage economy. Therefore, when aggregate shocks hit the economy, $\ell_{i',t}^*$ will differ from the flexible-wage economy hours, $\ell_{i',t}^f$.

that worker i' would like to cut his or her real wage relative to the aggregate level w_t . By doing so, labor hours will increase by the right magnitude as suggested by the MRS condition (see equation B.8).

Figure 2: Labor market for worker i' under a staggered wage contract.



Note: The figure shows the effect of an increase in λ_t on the desired labor hours of worker i' , given a constant level for the aggregate real wage w_0 and aggregate labor hours ℓ_0 . As λ_t increases, the desired labor supply schedule of worker i' , denoted by the MRS condition, shifts rightwards. Under a flexible-wage contract, the worker may again achieve the desired labor hours given the change in λ_t if he or she sets the real wage equal to $w_{i',1}^*$. Under a staggered wage contract, the dynamics of worker i' 's wage and labor hours depend on the indexation rule chosen. In the example above, the most preferred indexation rule is δ^{k1} because it minimizes the gap between desired and effective labor hours.

Figure 2 displays graphically the effect of an increase in λ_t on worker i' 's desired labor hours and the optimal real wage. Panel (a) in the figure shows worker i' 's labor market under a flexible-wage contract, in which the employment agency's labor demand schedule is given by ℓ^d (in logs). Worker i' 's labor supply schedule is represented by ℓ^s , which is completely flat because the worker

sets a wage and pledges to work whatever the employment agency asks for that wage. In turn, the MRS curve denotes the MRS condition expressed in equation (B.8) (also in logs). In the case of a fully-flexible wage contract, every period worker i' can set a wage such that the MRS condition is satisfied. In Figure 2, the initial equilibrium is given by point $(\ell_{i',0}^*, w_{i',0}^*)$. After an increase in λ_t , the MRS curve shifts rightwards. In order to fulfill this condition again and avoid any welfare costs, worker i' must reduce his or her wage to the point where the MRS curve cuts again the employment agency's labor demand, which happens at the point $(\ell_{i',1}^*, w_{i',1}^*)$.

Now assume that we inform worker i' that he or she cannot afford a fully-flexible wage contract, but rather has to settle with a staggered wage contract that can be personalized by choosing an indexation rule from the menu $\{\delta^{k1}, \delta^{k2}\}$. Under this setting, worker i' selects both an indexation rule and a corresponding optimal nominal wage while considering how the chosen indexation rule affects the dynamics of his or her wage and labor hours. Panels (b) and (c) in Figure 2 depict a hypothetical scenario for the wages and labor hours under each indexation rule. Similar to the case of the fully-flexible wage contract above, we assume that a recessionary shock shifts the MRS curve rightwards, so desired labor hours increase to $\ell_{i',1}^*$. In panel (b), the δ^{k1} contract implies that worker i' 's effective labor hours $\ell_{i',1}^{k1}$ would be slightly above the desired level, while the real wage would be slightly below the unrestricted welfare-maximizing real wage $w_{i',1}^*$. In contrast, panel (c) shows that under the δ^{k2} contract, worker i' 's effective labor hours $\ell_{i',1}^{k2}$ would be below the desired level, and the real wage above $w_{i',1}^*$. The latter means that worker i' has better chances to smooth consumption under the δ^{k1} contract than under the δ^{k2} contract. In other words, in this example, the δ^{k1} rule achieves lower welfare costs than the δ^{k2} rule, and, thus, the former is the most preferred indexation rule.

The twist of this example is that any worker in the staggered wage economy is worker i' , and so every worker chooses the indexation rule that minimizes the expected welfare cost. We next show how this intuition maps into the dynamic model after a series of shocks, and then we rank the rules δ^{trend} and δ^{past} according to the welfare they generate to an individual worker at the stochastic steady state.

C New-Keynesian Wage Phillips Curve with fixed indexation

If the proportion of workers indexing their wages to past inflation is exogenous and fixed, so $\xi_t = \gamma^w \in [0, 1]$ for all t , then the log-linearized equation for wage inflation collapses to a typical New-Keynesian wage Phillips curve of the form

$$\hat{\omega}_t^w = \kappa^w m_t^w + \beta E_t \{ \hat{\omega}_{t+1}^w \}, \quad (\text{C.1})$$

where

$$\begin{aligned}\widehat{\omega}_t^w &\equiv \widehat{\pi}_t^w - \gamma^w \widehat{\pi}_{t-1} - (1 - \gamma^w) \widehat{\pi}_t^*, \\ m_t^w &\equiv \omega_w \widehat{\ell}_t - \widehat{\lambda}_t - \widehat{w}_t, \text{ and} \\ \kappa^w &\equiv \frac{(1 - \beta\alpha_w)(1 - \alpha_w)}{\alpha_w(1 + \omega_w\theta_w)}.\end{aligned}$$

To prove it, we first need to log-linearize the wage-setting equilibrium conditions (i.e., III to VIII in Tables 1 and 2), which give

$$\widehat{r\widehat{w}}_t^{*,past} = (1 - \beta\alpha_w) \kappa_0^w m_t^w + \beta\alpha_w \mathbf{E}_t (\widehat{\pi}_{t+1}^w - \widehat{\pi}_t) + \beta\alpha_w \mathbf{E}_t \widehat{r\widehat{w}}_{t+1}^{*,past}, \quad (\text{C.2})$$

$$\widehat{r\widehat{w}}_t^{*,trend} = (1 - \beta\alpha_w) \kappa_0^w m_t^w + \beta\alpha_w \mathbf{E}_t (\widehat{\pi}_{t+1}^w - \widehat{\pi}_{t,t+1}^*) + \beta\alpha_w \widehat{\mathbf{E}}_t \widehat{r\widehat{w}}_{t+1}^{*,trend} \quad (\text{C.3})$$

$$0 = \gamma^w \left(\widehat{w}_t^{past} \right) + (1 - \gamma^w) \left(\widehat{w}_t^{trend} \right) \quad (\text{C.4})$$

$$\widehat{w}_t^{past} = (1 - \alpha_w) \left((1 - \theta_w) \widehat{r\widehat{w}}_t^{*,past} \right) + \alpha_w \left((\theta_w - 1) (\widehat{\pi}_t^w - \widehat{\pi}_{t-1}) + \widehat{w}_{t-1}^{past} \right) \quad (\text{C.5})$$

$$\widehat{w}_t^{trend} = (1 - \alpha_w) (1 - \theta_w) \widehat{r\widehat{w}}_t^{*,trend} + \alpha_w \left((\theta_w - 1) (\widehat{\pi}_t^w - \widehat{\pi}_t^*) + \widehat{w}_{t-1}^{trend} \right) \quad (\text{C.6})$$

Substituting (C.5) and (C.6) into (C.4) results in

$$\left\{ \begin{array}{l} -\gamma^w (1 - \alpha_w) (1 - \theta_w) \widehat{r\widehat{w}}_t^{*,past} \\ -\gamma^w \alpha_w \left((\theta_w - 1) (\widehat{\pi}_t^w - \widehat{\pi}_{t-1}) + \widehat{w}_{t-1}^{past} \right) \end{array} \right\} = \left\{ \begin{array}{l} (1 - \gamma^w) (1 - \alpha_w) (1 - \theta_w) \widehat{r\widehat{w}}_t^{*,trend} \\ + (1 - \gamma^w) \alpha_w \left((\theta_w - 1) (\widehat{\pi}_t^w - \widehat{\pi}_t^*) + \widehat{w}_{t-1}^{trend} \right) \end{array} \right\}. \quad (\text{C.7})$$

After simplifying (C.7) by using (C.4) in order to eliminate \widehat{w}_{t-1}^{past} and $\widehat{w}_{t-1}^{trend}$ terms, we obtain

$$\widehat{\pi}_t^w - (1 - \gamma^w) \widehat{\pi}_t^* - \gamma^w \widehat{\pi}_{t-1} = \frac{1 - \alpha_w}{\alpha_w} \left(\gamma^w \widehat{r\widehat{w}}_t^{*,past} + (1 - \gamma^w) \widehat{r\widehat{w}}_t^{*,2} \right). \quad (\text{C.8})$$

Substituting (C.2) and (C.3) into the above expression gives

$$\left(\widehat{\pi}_t^w - (1 - \gamma^w) \widehat{\pi}_t^* - \gamma^w \widehat{\pi}_{t-1} \right) \frac{\alpha_w}{1 - \alpha_w} = \left\{ \begin{array}{l} (1 - \beta\alpha_w) \kappa_0^w m_t^w + \\ \beta\alpha_w \mathbf{E}_t \left(\widehat{\pi}_{t+1}^w - (1 - \gamma^w) \widehat{\pi}_{t,t+1}^* - \gamma^w \widehat{\pi}_t \right) + \\ \beta\alpha_w \mathbf{E}_t \left(\chi \widehat{r\widehat{w}}_{t+1}^{*,past} + (1 - \chi) \widehat{r\widehat{w}}_{t+1}^{*,trend} \right). \end{array} \right\} \quad (\text{C.9})$$

After iterating one period forward (C.8) and substituting it into the last term on the right hand side of (C.9), plus some manipulations, we obtain (C.1).

D Explaining the time-varying wage indexation in the US: additional results

This section provides some additional results that complement the analysis from Section 5 in the main text. First, we elaborate on why the stylized facts of changes in US wage indexation are consistent with a changing degree of wage indexation in the simple model presented in Section 2. Next, we present a robustness check of our endogenous wage indexation predictions when trend inflation is allowed to be non-constant. Finally, we conduct a counterfactual analysis to detect the drivers of wage indexation changes over time.

D.1 Wage indexation effects as predicted by the simple model

Figure 3 summarizes Hofmann *et al.*'s findings in terms of the time-varying correlations between i) price inflation and its lag (blue line), and ii) wage inflation and lagged price inflation (black line), as calculated from the TVP-BVAR model of their paper. The red markers show the annual COLA index. Periods when COLA values are high (low) correspond to a strong (weak) degree of co-movement between lagged price inflation on the one hand, and current wage and price inflation on the other hand.⁵ This subsection shows that this evidence is consistent with a changing degree of wage indexation in the simple model presented in Section 2 of the main text.

As we argued in Section B.5, nominal wage rigidities constrain the aggregate labor supply to adjust freely to shocks. A first-order approximation to the wage-setting equation (A.12), and the aggregate nominal wage index yields a wage Phillips curve of the form:

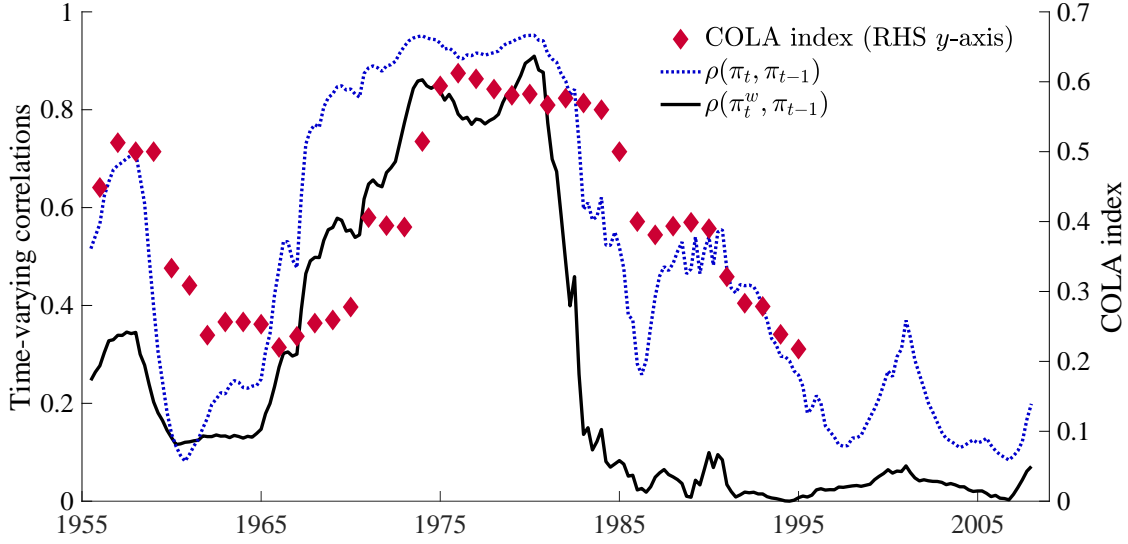
$$\hat{\pi}_t^w = \xi \hat{\pi}_{t-1} - \kappa \hat{M}_t + \beta E_t \{ \hat{\pi}_{t+1}^w - \xi \hat{\pi}_t \} + \eta_t^w, \quad (\text{D.1})$$

where hatted variables denote the percent deviation from a variable's non-stochastic steady-state level, $\kappa \equiv \frac{(1-\beta\alpha_w)(1-\alpha_w)}{\alpha_w(1+\theta_w)}$, $\eta_t^w \equiv (1-\xi)(1-\beta)\pi_t^*$, and $\hat{M}_t = \hat{w}_t + \hat{\lambda}_t - \hat{\ell}_t$ denotes the economy's average wage markup (see equation B.9). In this approximation, we assume that aggregate indexation ξ remains fixed.

The sluggishness in the labor market translates into an upward-sloping Phillips curve for prices, even if prices are flexible. Using the equilibrium conditions of the economy, the above wage

⁵A similar result is obtained when the implied slope in $\pi_t^w = \alpha + \beta\pi_{t-1}^p + \epsilon_t$ is calculated from the TVP-BVAR output as $\beta = \text{cov}(\pi_t^w, \pi_{t-1}^p) / \text{var}(\pi_{t-1}^p)$.

Figure 3: COLA index and derived time-varying correlations.



Note: The COLA index gives the proportion of union workers in large collective bargaining agreements with explicit contractual wage indexation clauses. The series is annual from 1956-1995. The time-varying correlations between i) price inflation and its own lag, and ii) wage inflation and lagged price inflation are derived from a time-varying parameter VAR. Sources: Ragan and Bratsberg (2000); Hofmann *et al.* (2012) and own calculations.

Phillips curve can be rewritten as a New-Keynesian Phillips curve (NKPC) for prices:⁶

$$\hat{\pi}_t = \frac{\xi}{1 + \beta\xi} \hat{\pi}_{t-1} + \frac{2\kappa}{1 + \beta\xi} \hat{x}_t + \frac{\beta}{1 + \beta\xi} E_t \hat{\pi}_{t+1} + \eta_t^p, \quad (D.2)$$

where $\eta_t^p \equiv \frac{(1-\xi)(1-\beta\rho_\pi)}{1+\beta\xi} \pi_t^* - \frac{1}{1+\beta\xi} \varepsilon_{z,t}$, and $\hat{x}_t = \hat{y}_t - z_t$ is equal to the flexible-wage output gap, which we define as the log difference between output in the staggered wage economy y_t and the level that would prevail if wages were completely flexible, which in the simple model equals the productivity term z_t .

From the above Phillips curves, we can derive three predictions concerning the effects of wage indexation on the dynamics of wages and prices:

A higher degree of aggregate wage indexation to past inflation implies, ceteris paribus:

1. A stronger link between wage inflation and past inflation (see the first term in equation D.1),
2. A higher intrinsic persistence of inflation (see the first term in equation D.2), and
3. A lower sensitivity of inflation to changes in the output gap (see the second term in equation D.2).

⁶The conditions we use to obtain this equation are the following: the FOC of consumption ($\hat{\lambda}_t = -\hat{y}_t$), labor demand ($\hat{w}_t = z_t$), the production function ($\hat{y}_t = z_t + \hat{\ell}_t$), an accountable definition of wage inflation ($\pi_t^w = w_t - w_{t-1} + \pi_t$), and the exogenous processes for productivity and the inflation target ($z_t = z_{t-1} + \varepsilon_{z,t}$ and $\pi_t^* = \rho_{\pi^*} \pi_{t-1}^* + \varepsilon_{\pi,t}$).

Figure 3 above supports the first two predictions: inflation persistence and the degree of co-movement between wage and past inflation broadly follow the COLA index’s evolution; in particular, the correlations peak when wage indexation was supposedly at its highest level.⁷

D.2 Robustness checks

D.2.1 Robustness check for non-constant trend inflation

The endogenous wage indexation predictions from Section 5.2 in the main text are based on a model extension that is compatible with the empirical analysis of Hofmann *et al.* (2012). However, as their model does not allow for shocks to trend inflation, this subsection performs a robustness check with non-zero volatility of trend inflation shocks. In short, we find that allowing for sensible volatility to trend inflation shocks does not change our predictions for aggregate wage indexation.

Table 5 replicates its counterpart from the main text, but considers two cases for trend inflation. In case 1, trend inflation remains constant ($\sigma_\pi = 0$) - the result discussed in the main text. In case 2, we use the estimated posterior median values for trend inflation volatility from Cogley *et al.* (2010) for the two regimes.⁸ They find that trend inflation volatility is higher in the *Great Inflation* than in the *Great Moderation*.

Adding trend inflation volatility to the analysis does not affect the results. Specifically, allowing for time-varying trend inflation, the ξ^* estimates of case 2 remain at the same levels for both regimes. The rationale of this result is that the trend inflation shocks are relatively small compared to the other shocks in the economy, even during the *Great Inflation*, when trend inflation volatility was twice as high. According to the model, trend-inflation shocks only explain about .79 percent of long-run output fluctuations in the 1974 regime and 1.25 percent in the 2000 regime. Ireland (2007) reports similar numbers.⁹

D.2.2 Robustness check for zero price indexation

Building on the previous subsection, we also check how our wage indexation equilibria change when the price indexation parameter is calibrated to zero. The motivation for this exercise follows from Cogley and Sbordone (2008), who show that the estimated price indexation is equal to zero

⁷Providing evidence for prediction 3 is more complex since changes in the second term of equation (D.2) could also be driven by changes in the frequency of wage renegotiations (parameter α_w), the presence of nominal rigidities on prices, or the smoothness at which agents change their spending patterns.

⁸Cogley *et al.* (2010) estimate a New Keynesian model with sticky prices and flexible wages using Bayesian methods over two sample periods: 1960:Q1-1979:Q3 and 1982:Q4-2006:Q4. We use the estimated σ_{π^*} for both subperiods for respectively the *Great Inflation* and the *Great Moderation* calibrations.

⁹Ireland (2007) estimates a New Keynesian model with a maximum likelihood/Kalman filter approach and finds that the long-run contribution of trend inflation shocks to output fluctuations converges to zero as the horizon increases.

Table 5: Validation exercises.

<i>Common parameters across exercises</i>			
β	Subjective discount factor		.99
σ	Intertemp. elasticity of subst.		1
ω	Inverse of Frisch elasticity		2
θ_w	Elasticity of labor demand		10
θ_p	Elasticity of interm. goods		10
g_y	Public-spending-to-GDP ratio at steady state		.2
<i>Year-specific parameters for:</i>		2000Q1	1974Q1
γ^h	Habit formation	.37	.71
γ^p	Inflation inertia	.17	.8
α_p	Calvo-price rigidity	.78	.84
α_w	Calvo-wage rigidity	.54	.64
a_π	Taylor Rule: inflation	1.35	1.11
a_y	Taylor Rule: output gap	.1	.11
$a_{\Delta y}$	Taylor Rule: output gap growth	.39	.5
ρ_R	Taylor Rule: smoothing	.78	.69
ρ^g	Autocorr. gov'n't spending	.91	.89
σ_z	Std. dev. technology	.31	1.02
σ_g	Std. dev. gov'n't spending	3.25	4.73
$\hat{\xi}$	Estimated wage indexation by HPS	.17	.91
<i>Case I: $\sigma_\pi = 0$</i>			
ξ^*	Implied equilibrium wage indexation	0	.89
ξ^S	Implied social wage optimum	1	0
<i>Case II: $\sigma_\pi > 0$</i>			
σ_π	Std. dev. inflation target	.049	.081
ξ^*	Implied equilibrium wage indexation	0	.89
ξ^S	Implied social wage optimum	1	0

Note: All *common* and *specific* parameter values are extracted from Hofmann *et al.* (2012), who estimated the extended model with US data for 1974Q1 and 2000Q1. For more details about their estimation procedure, see their Section 3.2. The standard deviations for trend inflation are taken from the estimation results of Cogley *et al.* (2010). The *implied* equilibrium wage indexation values are computed using the procedure from our Section 3.

under a persistent process for trend inflation. The only change in predicted indexation values from Table 5, is that ξ^* declines from 0.89 to 0.73 when $\sigma_\pi = 0$, and to 0.75 when $\sigma_\pi > 0$. Yet, these values still indicate high levels of wage indexation that contrast with the social equilibria. Hence, our main conclusions remain unchanged.

D.3 Counterfactual analysis

In this subsection, we conduct a counterfactual analysis to detect the primary drivers of the changes in the equilibrium wage indexation presented above. The exercise is divided into two parts. First, we run a series of counterfactuals, where we take the calibrated parameters for 2000 from Table 5 and then set each parameter one-by-one to its 1974 value.¹⁰ The implied wage indexation equilibrium ξ^* from these counterfactuals is shown in column (1) of Table 6. For the second part, we do the opposite: we start from the 1974 calibration and substitute each parameter with its 2000 value. These results are shown in column (2) of Table 6. The results reported in both columns allow us to assess whether there was a dominant factor explaining changes in ξ^* . We first discuss the effect of changes in the volatility of shocks, followed by the monetary policy rule, and finally, structural changes in habit formation, and the degree of nominal rigidities.

The relative importance of shocks. Several studies have documented a substantial difference in the volatility of aggregate shocks between the *Great Inflation* and the *Great Moderation* (see e.g. Sims and Zha, 2006). The effects of changes in the volatility of shocks on wage indexation are shown in part I of Table 6. Starting from the 2000 parameter values in column (1), substituting the standard deviation of the technology shocks (σ_z) by its 1974 value has a strong effect on wage indexation: ξ^* shifts from 0 to 1. Replacing the volatility of the trend-inflation shock (σ_π) by its 1974 value has a smaller but still substantial impact as ξ^* increases to .6. However, substituting the volatility of government-spending shocks leaves ξ^* at zero. The direction of these changes is consistent with the results reported in Section 3 in the main text. Specifically, we showed that a regime driven by either productivity or permanent inflation target shocks results in an equilibrium where $\xi^* = 1$, whereas a regime driven by demand shocks results in an equilibrium with $\xi^* = 0$. What is surprising, however, is that raising the variance of the inflation-target shock to its 1974 value has a large effect on the predicted degree of wage indexation, while it only has a small effect on the level of wage indexation in the model predictions of Table 5.

A cross-check with column (2) of Table 6 shows that there is no inconsistency. The column

¹⁰The entry in the first row and first column of Table 6 thus corresponds to the 2000 calibration except for σ_z , which is set to its 1974 value. The entry below corresponds to the 2000 calibration with only σ_g at its 1974 value, etc.

Table 6: Counterfactual exercises.

		2000's ξ^* is 0, applying 1974 calibration	1974's ξ^* is .89, applying 2000 calibration
		$\xi^{counterfactual}$ (1)	$\xi^{counterfactual}$ (2)
<i>I - Shocks</i>			
σ_z	Std. dev. tech. shock	1	0
σ_g	Std. dev. dem. shock	0	1
σ_π	Std. dev. inflation target	.6	.89
<i>II - Monetary policy parameters</i>			
a_π	Taylor rule: inflation	0	1
a_y	Taylor rule: output gap	.05	.89
$a_{\Delta y}$	Taylor rule: output gap growth	0	1
ρ_R	Taylor rule: smoothing	0	.94
<i>III - Structural parameters</i>			
γ^h	Habit formation	0	1
γ^p	Inflation inertia	.78	.77
α_p	Calvo-price rigidity	0	.89
α_w	Calvo-wage rigidity	.49	.95
ρ^g	Autocorr. gov'n't spending	0	.85

Note: In this exercise, we keep all parameters at their calibrated values as indicated in the top of columns (1) and (2), and we only change the value of the parameter indicated in each row while other parameters remain equal to their original calibration for the period. Our aim is to evaluate the impact of the change in each parameter on wage indexation.

shows how ξ^* changes from its value of .89 in 1974 when we substitute the volatility of each shock by its value in 2000. Technology shocks are again important, as they drive ξ^* to zero. However, replacing the volatility of trend inflation has no effect, as ξ^* remains .89. It seems thus that technology shocks had such a large variance in 1974 that the variance of trend inflation becomes unimportant, in relative terms. We interpret this result as evidence that the volatility of technology shocks was the key driver of changes in wage indexation over time and not drifting trend inflation. This exercise also illustrates that the consequences of changes in some of the parameters depend on other parameters in the calibration. In this case, the volatility of trend-inflation shocks in 1974 was simply too small to have a relevant effect on wage indexation. Finally, it is clear that changes in the variance of government-spending shocks cannot explain the stylized facts.

Changes in monetary policy. The good-policy hypothesis for the *Great Moderation* asserts that macroeconomic fluctuations became more stable in the post-*Great Inflation* period as a result of a shift in the monetary policy rule (see e.g. Clarida *et al.*, 2000). Such a shift could have changed inflation dynamics and hence wage indexation practices. However, the second part of Table 6 shows that substituting the 2000 policy rule values with their 1974 counterparts has no significant effect on the wage indexation equilibrium. Further, a cross-check with column (2) shows that replacing the 1974 policy rule parameters by their 2000 values increases wage indexation. This exercise clearly shows that changes in the conduct of monetary policy cannot explain the observed changes in wage indexation.¹¹

Structural change. Finally, we check whether other structural adjustments in the economy could have caused changes in wage indexation. It is clear that habit formation (γ^h), Calvo-price rigidity (α_p), and the persistence of demand shocks (ρ_g) cannot explain the stylized facts. In column (1), these parameters do not affect ξ^* , and in column (2), they predict either little change or the wrong direction of the change in ξ^* .

The interpretation of changes in inflation inertia (γ^p) and Calvo-wage rigidity (α_w) is more challenging. Column (1) shows that setting inflation inertia to its 1974 value in the 2000 calibration has a large effect, i.e., ξ^* increases from 0 to .78. However, in column (2), changing this parameter from its 1974 value to its 2000 value only has a small effect on ξ^* , which decreases from .89 to just .77. Therefore, we can conclude that this parameter's effect depends on the entire set of parameters

¹¹This result relates to De Schryder *et al.* (2020), who use a panel dataset to estimate a wage Phillips curve equation with interaction effects. Their results suggest that monetary policy regime shifts were not a crucial driver of wage indexation changes.

in the calibration. Concerning Calvo-wage rigidity, it predicts a moderate increase of ξ^* from 0 to .49 in column (1). However, column (2) predicts that a *decrease* in this parameter leads to an *increase* in ξ^* from .89 to .95, which indicates that non-linearities are at play. We conclude that there is no clear indication that modifications in γ^p and α_w and the other structural parameters have been important contributors to the observed time-variation in wage indexation.

Firms' preferences towards aggregate wage indexation. In this paper, we assume that workers have all bargaining power to settle their preferred indexation rule. Given this assumption, intermediate firms simply accept workers' indexation choice and resume production. However, entrepreneurs might prefer a different indexation scheme than workers, for instance, the one that maximizes their profits. In this exercise, we analyze the relationship between profits of intermediate-good firms and the aggregate degree of wage indexation to past inflation in the US economy. In the next section, we show that with the calibration of 1974, according to the model, firms' profits were maximized with a high level of aggregate indexation ($\xi = 0.88$). In contrast, in 2000 a lower level of aggregate indexation was optimal for firms ($\xi = 0.46$). Therefore, according to the model, in the US of 1974 firms' owners would agree to have a large proportion of workers with a past-inflation indexation for wages, whereas in 2000 owners would prefer to have a lower proportion of workers with past-inflation indexation contracts. The latter means that, regardless of which actor had the largest bargaining power between workers and firms, the model predicts that wage indexation to past inflation would be higher in 1974 than in 2000. The choices of firms' owners regarding the degree of wage indexation to past inflation may be explained by a favorable combination between the standard deviation of profits and the correlation between the latter and real wages.

E Endogenous wage indexation and firm profits

In the main text, we assume that workers have all bargaining power to settle their preferred indexation rule. In such a setting, each period firms simply accept workers' indexation choice and resume production. However, entrepreneurs might prefer a different indexation scheme, for instance the one that maximizes their profits. In this section, we show the relationship between profits of intermediate-good firms and the aggregate degree of wage indexation to past inflation in the economy.

The model used in this exercise corresponds to the one used in the US application of Section 5 in the paper. Therefore, we calibrate the model to the 1974 setting and the 2000 setting. We focus on profits of firms that can change their price optimally each period. Profits are thus defined as

follows:

$$\text{Profits}_t = \left(\frac{P_t^*}{P_t} - w_t \right) \ell_t. \quad (\text{E.1})$$

Figure 4: Firm profits and degree of past-inflation indexation in wages

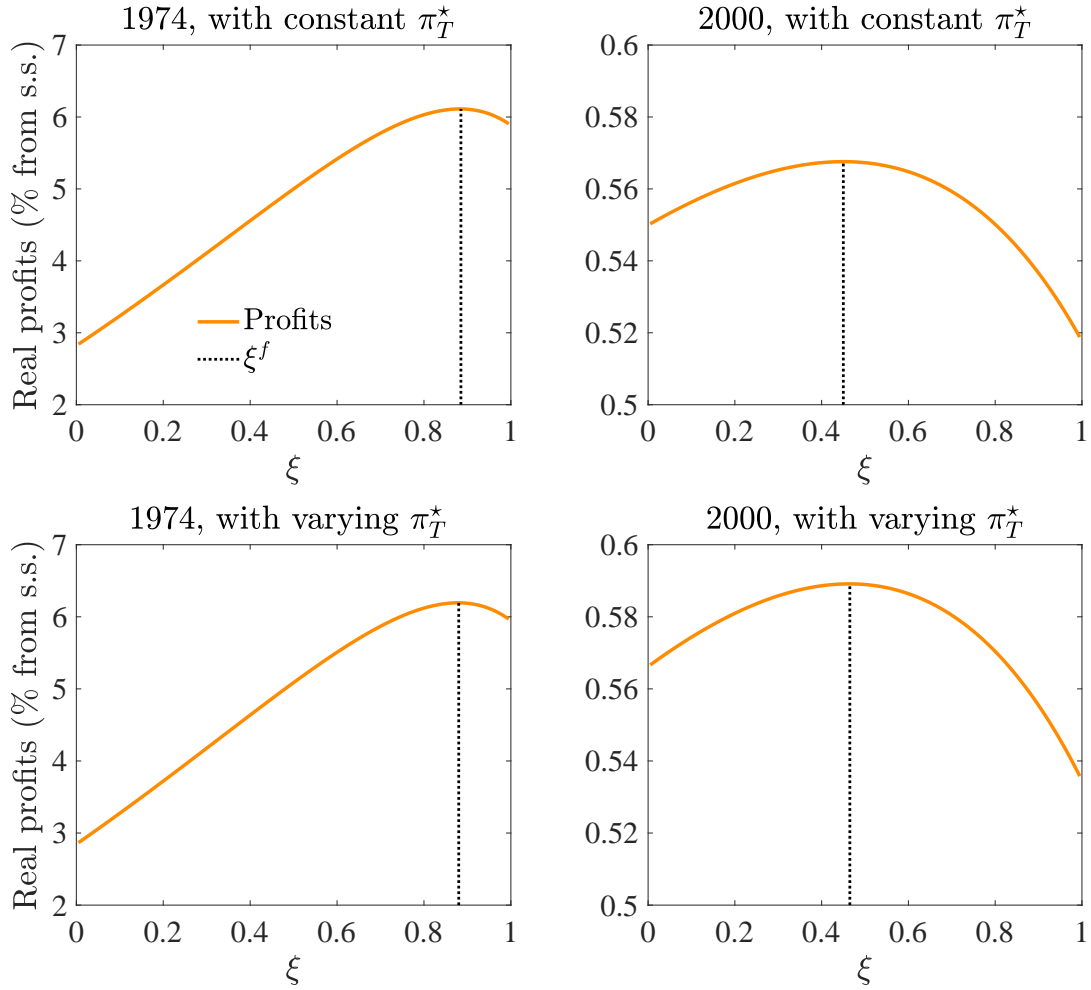


Figure 4 shows the relationship between firms' profits and the aggregate level of past-inflation indexation in wages. The results suggest that in the US of 1974, firms' profits were maximized with a high level of aggregate indexation ($\xi = 0.88$), whereas in 2000, a lower level of aggregate indexation was optimal ($\xi = 0.46$). The results are robust to the degree of variation of trend inflation, as the first and second rows of the figure suggest.

Recall that, according to the model, workers preferred a high degree of past-inflation indexation in wages in 1974, when productivity shocks were quite volatile. In contrast, they favored a lower degree of past-inflation indexation in 2000 when shocks overall were less volatile than in

the 70s. Therefore, according to the model, in the US of 1974 firms' owners would agree to have a large proportion of workers with a past-inflation indexation for wages, whereas in 2000, owners would prefer to have a lower proportion of workers with past-inflation indexation contracts. The latter means that, regardless of which actor had the largest bargaining power to negotiate labor contracts, the model predicts that wage indexation to past inflation would be higher in 1974 than in 2000.

The choices of firms' owners regarding the degree of wage indexation to past inflation may be explained by a favorable combination between the standard deviation of profits and the correlation between the latter and real wages. On the one hand, a lower volatility of profits ensures a relatively more stable stream of dividends to owners. On the other hand, a correlation closer to zero between the growth of both profits and real wages indicates that the former is more isolated from fluctuations in the latter. Table 7 compares these two second-moments derived from a 10,000-period simulation of the model for four different calibration settings belonging to two sets. The first set considers the 1974 calibration, and it varies the degree of aggregate wage indexation from 0.46 to 0.88. The second set performs the same exercise for the 2000 calibration.

Table 7: Profits and wages according to the model

	Calibration of 1974		Calibration of 2000	
	$\xi = 0.46$	$\xi = 0.88^*$	$\xi = 0.46^*$	$\xi = 0.88$
Standard deviation of Profits _t	0.69	0.68	0.42	0.47
Correlation between $\Delta\%Profits_t$ and $\Delta\%w_t$	-0.48	-0.20	-0.06	-0.03

Note: (*) indicates the preferred level of aggregate indexation by firms, according to Figure 4. The table shows that standard deviation of profits, as defined in equation (E.1), and the correlation between the growth rates of profits and the real wage. To obtain these numbers, we simulate the model 10,000 periods using the calibration settings mentioned in the first two rows of the table.

Two results are noteworthy. First, profits are relatively less volatile on the preferred level of aggregate indexation by firms for each of the two calibration sets. And second, for the 1974 calibration, the correlation between the growth of profits and that of real wages decreases when aggregate wage indexation to past inflation increases. In this case, this is the criterion that seems to prevail for firms' choices, since profits standard deviation does not vary much when $\xi = 0.46$ or $\xi = 0.88$. In contrast, in the case of the 2000 calibration, the correlation between the growth of profits and that of real wages is so close to zero that the criterion of profits' volatility seems to prevail. Overall, these results suggest that further research on profits and wage indexation is

needed. However, this topic is beyond the scope of this paper.

F Endogenous wage indexation in a more complex DSGE model with financial frictions

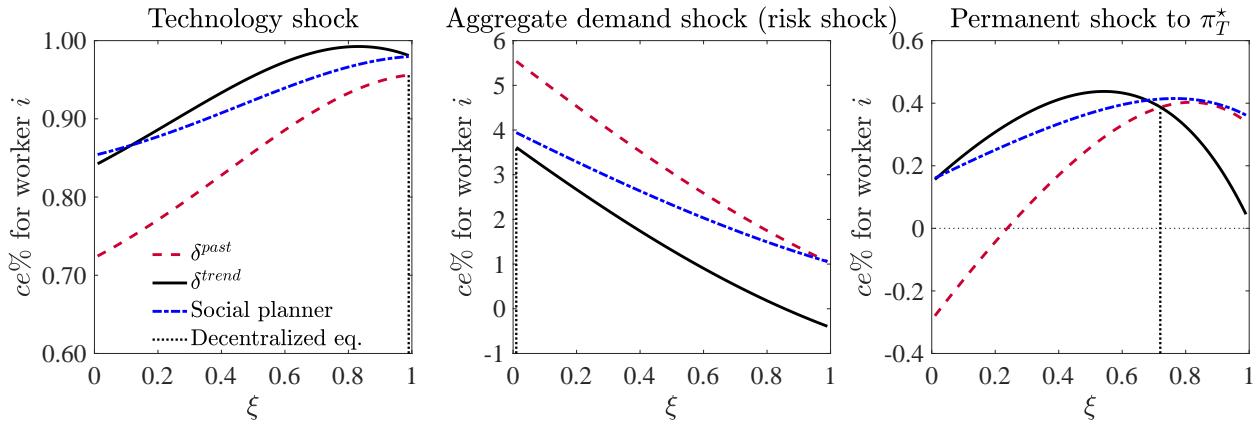
In Section 2 of the paper, we introduced the endogenous wage indexation mechanism in a simple model without capital, in which the only new feature was the selection of an optimal indexation rule by an individual worker. We chose the simple model for expositional purposes. However, such a model omits important real frictions which help to replicate business cycle fluctuations in actual economies. In particular, investment, capital accumulation, and the presence of financial frictions have been shown to be important determinants of output fluctuations (see Christiano *et al.*, 2014).

This section shows that the endogenous wage indexation mechanism featured in the simple model delivers the same predictions in a more complex DSGE model that includes capital and financial frictions. In particular, we use the model described in Carrillo *et al.* (2021), which in turn is based on the one proposed by Christiano *et al.* (2014), who introduce risk shocks into the New Keynesian model with costly state verification in financial intermediation, as in Bernanke *et al.* (1999). The model includes six types of agents: a competitive final-goods producer; a set of non-competitive input producers who set prices in a staggered manner *à la* Calvo; a competitive physical capital producer; entrepreneurs who seek financing for their risky projects and may incur in moral hazard; financial intermediaries who perform costly monitoring of those entrepreneurs who claim to have gone bankrupt; and households who consume, save, and work. Full details are provided in Carrillo *et al.* (2021). To this framework, we add staggered wage-setting *à la* Calvo and allow each worker to individually select a wage indexation scheme for updating his/her wage in no-reoptimizing periods.¹² The two indexation schemes are δ^{past} and δ^{trend} , which are defined as in Section A.1.2 of this Appendix.

Figure 5 shows the long-run welfare costs faced by an individual worker according to the two indexation schemes available. As in the paper, the welfare costs are expressed as compensating lifetime utility-equivalent consumption variations relative to the deterministic steady state (defined as *ce*). Three economic regimes are considered: one driven by permanent technology shocks (left panel); one driven by risk shocks, which affect the volatility of the returns of entrepreneurs' projects, causing financing costs and investment to vary (center panel); and one driven by permanent shocks to trend inflation. Remarkably, as in the simple model, the figure shows a corner solution in the first two cases. For any aggregate indexation level to past inflation ξ , worker i has

¹²For this exercise, we set the Calvo parameter such that workers re-optimize their wages on average once every year.

Figure 5: Welfare costs for different economic regimes in a model with financial frictions



Note: The figure shows the private welfare costs associated with each indexation rule conditional on specific shocks (the δ lines), as well as the average costs perceived by the social planner for the economy as a whole (dashed-and-dotted line). The decentralized equilibrium level of aggregate indexation to past inflation is signaled in the x -axis by the dotted line. The socially preferred level of said indexation is the one that minimizes social costs.

a clear preference: he or she chooses past inflation indexation in the permanent-technology-shock regime and trend inflation indexation in the aggregate-demand-shock regime. As in the simple model, we find similar results with different types of aggregate demand shocks, such as those affecting government spending or inter-temporal consumption. In addition, there is an interior, globally stable solution in the permanent-trend-inflation-shock regime with a high proportion of workers indexing their wages to past inflation.

Figure 5 also displays the welfare costs faced by an *average or representative* household, as perceived by the social planner. Recall that the social planners preferred level of aggregate indexation ξ is the one that minimizes the social welfare cost. As such, as in the simple model, the socially optimum level of wage indexation to past inflation is exactly opposite to the decentralized equilibrium, while it is consistent with the seminal findings of Gray (1976) and Fischer (1977).

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