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Endogenous wage indexation and aggregate shocks[☆]

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ABSTRACT

New Keynesian DSGE models assume a constant degree of wage indexation to past inflation, neglecting empirical and institutional evidence of a time-varying degree. We build a DSGE model with utility-maximizing workers that can endogenously choose the wage indexation rule. We find that workers index their wage to past inflation when shocks to productivity and the nominal anchor drive output fluctuations. By contrast, they index wages to the inflation target when aggregate demand shocks dominate. We further show that this decentralized equilibrium is not socially optimal, but explains the time-varying degree of wage indexation in US data very well.

1. Introduction

> *Why bother with a microfounded staggered wage and price setting model if you are just going to add ad hoc lag structure anyway?*
>
> John B. Taylor (2016)

Price and wage inflation can be very persistent. To replicate this feature, New Keynesian dynamic stochastic general equilibrium (DSGE) models feature structural sources of persistence that are either "inherited" from the driving forces of inflation (e.g., the output gap), or an "intrinsic" part of the inflation process (Fuhrer, 2010). For example, in addition to nominal rigidities, a standard assumption is that wages and prices are (partially) indexed to past inflation. In particular, this degree of indexation is hardwired as a constant and policy invariant parameter.[1]

This assumption has been criticized for being specified in an ad-hoc way, rather than being part of the optimization process (Taylor, 2016). Moreover, a constant degree of indexation has been rejected by institutional and empirical evidence, particularly for wages. For instance, in the United States (US), contractual clauses indexing wages to past inflation were prominent during the *Great Inflation* in the late 1970s, whereas such clauses practically disappeared during the *Great Moderation* (we discuss this evidence in Section 5.1).

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1 See Christiano et al. (2005), Smets and Wouters (2007) and Del Negro et al. (2015).

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The degree of wage indexation to past inflation (*wage indexation*, for short) is very important for macroeconomic fluctuations and policymakers. When wage indexation is high, changes in inflation can trigger a reinforcing feedback loop between wages and prices that amplify the effects of shocks on inflation, the so-called second-round effects. Accordingly, larger changes in the policy interest rate are required to bring inflation back to the target. Therefore, the degree of wage indexation is crucial for the inflationary consequences of shocks hitting the economy. In this context, Hofmann et al. (2012) find that the decline of wage indexation between the *Great Inflation* and the *Great Moderation* in the US implies a reduction in the long-run impact of supply and demand shocks on prices of 44% and 39%, respectively.

This paper seeks to explain changes in wage indexation over time. To this end, we build a simple model of aggregate fluctuations with staggered labor contracts that endogenously determines the equilibrium level of wage indexation. Since sticky wages prevent a worker from optimally adjusting his or her labor supply to shocks, a gap emerges between his or her actual labor supply and the utility-maximizing level. The novelty of our model is that in periods when a worker's wage is re-optimized, he or she can choose between indexing the wage to either past inflation or to the central bank's inflation target (i.e., trend inflation, which may vary) until the labor contract can be re-optimized. Thus, a worker's indexation choice is micro-founded in the model since it maximizes the expected utility that can be obtained between the two indexation schemes (subject to the average length of the labor contract and the specific structure of the economy). We also assume that wage setting takes place at a decentralized level, i.e., at the individual worker or firm level, which is consistent with the institutional evidence for wage bargaining in the US.[2] We solve the non-linear model to compute the welfare criterion of workers. The sum of all workers' decisions determines the degree at which nominal wages are indexed to past inflation, which we denote as the degree of *aggregate wage indexation* in the economy. We implement an algorithm that computes the equilibrium level of aggregate wage indexation given the economic regime, i.e., the specific market structures, stochastic shocks, and economic policy frameworks that agents face.

We present three main results. First, at the decentralized equilibrium, workers prefer to index wages to lagged inflation when permanent shocks to technology or to trend inflation drive output fluctuations, which entails a high degree of wage indexation to past inflation. In contrast, when shocks to aggregate demand dominate, workers prefer to index wages to the central bank's inflation target, which leads to low wage indexation.

Second, we find a *coordination failure* among workers' decisions. More precisely, the social planner's solution is to index wages to trend inflation in regimes driven by technology and permanent inflation target shocks and index wages to past inflation in regimes driven by aggregate demand shocks — exactly the opposite to the decentralized equilibrium. Interestingly, the social planner's solution is consistent with the work of Gray (1976) and Fischer (1977) on the socially optimal degree of wage indexation. These studies show that, to reduce output fluctuations, wage indexation should be high when *nominal* or demand-side shocks are important, whereas it should be low when *real* or supply-side shocks dominate.

Third, we show that the decentralized wage indexation equilibrium explains the observed time variation in aggregate wage indexation in the US. Using an empirically suitable model, we show that the decentralized wage indexation equilibrium matches the stylized facts reported in Hofmann et al. (2012): a high degree of wage indexation for the *Great Inflation*, and a low degree for the *Great Moderation*. In the first regime, technology shocks were highly volatile, and trend inflation drifted, while in the second regime, aggregate demand shocks gained relevance relative to other shocks. Counterfactual exercises (shown in the Online Appendix) reveal that the high degree of wage indexation in the 1970s was primarily the result of volatile supply-side shocks, whereas wage indexation vanished when these shocks became less volatile in more recent periods. In contrast, changes in monetary policy, including the anchoring of trend inflation, only played a minor role in determining aggregate wage indexation for the two periods.[3]

One caveat is that we build on a standard New Keynesian DSGE model that does not provide the most detailed description of the labor market. Nevertheless, our model provides insight into workers' wage indexation choices and shows that making a distinction between the centralized and decentralized equilibrium matters both theoretically and empirically.

1.1. Related literature and relevance

This paper contributes to the literature on the optimal degree of wage indexation. Gray (1976) and Fischer (1977), and several extensions reviewed in Cover and VanHoose (2002) and Calmfors and Johansson (2006), search for an optimal wage indexation scheme that minimizes output fluctuations. More recent papers with microfounded DSGE models, such as Cho (2003), Minford et al. (2003), or Amano et al. (2007), look for an optimal wage indexation arrangement that maximizes the welfare of a representative agent, i.e., average social welfare.[4] However, these papers do not differentiate between the decentralized wage indexation equilibrium and the socially optimal level.[5] In contrast, we focus on the optimal choices of individual workers. Specifically, we show that the decentralized wage indexation equilibrium differs from the social optimum, and that it can explain the historical changes in wage indexation in the US.

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2 According to the literature on wage-setting institutions, wage bargaining in the US primarily takes place at the enterprise level (see, e.g., Calmfors and Driffill, 1988; Bruno and Sachs, 1985; Du Caju et al., 2009). The assumption of decentralized wage setting is hence realistic to endogenize wage indexation for the US. There is no involvement of central organizations in bargaining, and no central employer organizations exist.

3 This result relates to De Schryder et al. (2020), who use a panel dataset to estimate a wage Phillips curve equation with interaction effects. Their findings suggest that monetary policy regime shifts were not a crucial driver of wage indexation changes.

4 Minford et al. (2003) provide a comprehensive review of this literature.

5 Earlier theoretical work by Waller and VanHoose (1992) finds that an increase in nominal wage indexation reduces equilibrium trend inflation under discretionary monetary policy. A coordination failure also emerges in their setup because private wage setters do not exploit the potential positive externality from nominal wage indexation.

The endogenous wage indexation mechanism laid out in this paper hinges on the importance of income effects on workers' desired labor supply. Therefore, our work also relates to recent studies that, based on micro data, find that a change in individuals' wealth affects their labor supply.[6] This result is consistent with a scenario in which labor contracts are staggered. Under sticky wages, the preferred indexation rule of a worker is the device through which income effects pass through to his or her actual labor supply.

Overall, our results show that even if the degree of wage indexation appears to be limited in the US and other countries today, this might change if the economic policy framework or the frequency and the nature of shocks change. For example, following the review of its FOMC framework, the FED announced in August 2020 that it was "likely to aim to achieve inflation moderately above 2 percent for some time" following "periods when inflation has been running persistently below 2 percent". Moreover, the COVID-19 pandemic has entailed bouts of persistent supply-side bottlenecks. In the context of our model, a long-lasting deviation of inflation from 2% could be interpreted by households as a changing inflation trend, and the latter as (quasi-)permanent productivity shocks — both of which invoke more wage indexation. Therefore, having a theory on *why wage indexation evolves* matters for our understanding of the business cycle.

The remainder of the paper is organized as follows. Section 2 presents a simple model to gain intuition on the endogenous mechanism. Section 3 analyzes the decentralized equilibrium. Section 4 shows the social planner's choice and discusses the coordination failure among workers. Section 5 applies the wage indexation mechanism to the US experience. Section 6 concludes.

2. A simple model with nominal wage rigidities

This section describes a simple model that intuitively explains why a worker chooses a certain wage indexation rule.[7] The model resembles a real-business-cycle environment without capital but features sticky wages. That is, workers cannot re-optimize their wages in each period since they face labor contracts with a stochastic duration. The novelty of the model is that every worker can select the wage indexation rule that applies to his or her nominal wage between re-optimizing periods. The selected rule maximizes the discounted utility of a worker, conditional on the labor contract's duration.

The model economy is populated by a continuum of households endowed with differentiated labor services, an employment agency that aggregates these services, a competitive final-good producer, and a government in charge of fiscal and monetary policy. We describe the objectives of each agent below.

2.1. Households and wage setting

Each household, indexed by $i \in [0,1]$, is composed of two decision-making units: a consumer and a worker. The former looks for an optimal consumption/savings plan, while the latter is endowed with a unique labor type, $\ell_{i,t}$, and uses its monopolistic power to set its nominal wage, $W_{i,t}$, and an *indexation rule*, $\delta_{i,t}$, that governs how the worker's nominal wage evolves in periods of no labor-contract negotiations. We add a labor income subsidy, τ_w, to eliminate any steady-state distortion in labor allocations generated by the worker's monopolistic power. We introduce heterogeneity in labor types to induce dispersion in wages and labor hours, which will vary depending on the chosen indexation rule.

In order to maximize the household's discounted lifetime utility, the consumer chooses a consumption bundle, $c_{i,t}$, and one-period-maturity bonds, $b_{i,t}$, every period, while the worker sets $W_{i,t}$ and $\delta_{i,t}$ when allowed to do so. The general problem of the household thus looks as follows

$$\max_{c_{i,T}, b_{i,T}, W_{i,T}, \delta_{i,T}} \mathrm{E}_t \left\{ \sum_{T=t}^{\infty} \beta^{T-t} \mathcal{U}\left(c_{i,T}, \ell_{i,T}\right) \right\}, \tag{1}$$

subject to *no Ponzi* schemes, the worker's labor-specific demand (which we define below), and a sequence of budget constraints of the form

$$c_{i,T} + \frac{b_{i,T}}{(1+R_T)\exp\left(d_T\right)} \leq \left(1+\tau_w\right) \frac{W_{i,T}}{P_T} \ell_{i,T} + \frac{b_{i,T-1}}{1+\pi_T} + \frac{Y_{i,T}}{P_T} \qquad \forall T = t, t+1, t+2, \dots, \tag{2}$$

where E_t denotes the expectation operator conditional on information available in period t, R_t is the short-run risk free nominal interest rate, $\pi_t \equiv P_t/P_{t-1} - 1$ is the inflation rate, P_t is the aggregate price level, $Y_{i,t}$ is a variable that includes lump sum taxes and Arrow–Debreu state-contingent securities that ensure that households start each period with equal wealth, and d_t is a temporary aggregate demand shock that creates a wedge between the return on bonds and the risk free rate (similar to the risk-spread shock in Smets and Wouters, 2007). We assume that d_t follows the stationary process $d_t = \rho_d d_{t-1} + \varepsilon_{d,t}$, where $0 \leq \rho_d < 1$, and $\varepsilon_{d,t}$ is a white noise process with standard deviation equal to σ_d. Bonds are in zero net supply.

[6] Algan et al. (2003) find that in France an increase in wealth significantly affects the individuals' labor supply. Daly et al. (2009) notice that, in the initial stages of the global financial crisis in the US, several subgroups of the population increased their labor force participation, and attribute this behavior to wealth declines and reduced access to credit. Finally, Disney and Gathergood (2018) find that an increase in housing prices in the UK lowers labor force participation.

[7] For expositional purposes, the model ingredients are kept as simple as possible in this section. However, the model is augmented with several features of state-of-the-art DSGE models in the empirical application of Section 5.

For simplicity, we assume that a household's instantaneous utility function takes a logarithmic form for consumption and a quadratic form for labor, so

$$\mathcal{U}\left(c_{i,t}, \ell_{i,t}\right) = \log c_{i,t} - \frac{\Psi}{2}\ell_{i,t}^{2},$$

which implies that the Frisch elasticity of labor supply is equal to 1.

The first-order conditions for consumption and savings are quite standard. Therefore, we do not report them here for brevity. The determination of the worker's wage and indexation rule is more involved, so we describe it next.

Labor contracts. Similar to Erceg et al. (2000), we assume that a competitive employment agency combines households' labor hours into an aggregate labor input, ℓ_t, which is then supplied to the final-good producer. The agency uses a CES technology to produce aggregate hours, so that $\ell_t^{(\theta_w-1)/\theta_w} = \int_i \ell_{i,t}^{(\theta_w-1)/\theta_w} \mathrm{d}i$, where $\theta_w > 1$ is the elasticity of substitution between any two labor types. The agency's demand for type-i labor is thus given by

$$\ell_{i,t} = \left(\frac{W_t}{W_{i,t}}\right)^{\theta_w} \ell_t, \tag{3}$$

where $W_t = \left[\int_i W_{i,t}^{1-\theta_w} \mathrm{d}i\right]^{\frac{1}{1-\theta_w}}$ denotes the aggregate nominal wage index.

Labor contract negotiations follow Calvo's price-setting mechanism, which implies that a worker may renegotiate his or her contract each period with probability $1-\alpha_w$. The novelty of our framework is that workers can also select an indexation rule $\delta_{i,t}$ when they re-optimize their wages in a new contract. If allowed to renegotiate in time t, the worker optimally sets $W_{i,t}$ and chooses from a pre-established menu a $\delta_{i,t}$ that dictates how his or her nominal wage must be updated in periods in which no contract negotiation takes place.[8]

To keep the analysis tractable, we assume an indexation-rule menu with only two items: a δ^{trend} rule based on the inflation target of the central bank, and a δ^{past} rule based on past or lagged inflation, so $\delta_{i,t} \in \{\delta_{t,T}^{trend}, \delta_{t,T}^{past}\}$.[9] To see how these rules work, suppose the contract negotiation of worker i happens in period t, and he or she optimally selects the nominal wage $W_{i,t}^{k,\star}$ for contract $k \in \{trend, past\}$. Thus, in period $T \geq t$, worker i's wage is updated to either $W_{i,T}^{trend} = \delta_{t,T}^{trend} W_{i,t}^{trend,\star}$ or $W_{i,T}^{past} = \delta_{t,T}^{past} W_{i,t}^{past,\star}$, where

$$\delta_{t,T}^{trend} = \left(1+\pi_T^{\star}\right)\delta_{t,T-1}^{trend} \text{ and } \delta_{t,T}^{past} = \left(1+\pi_{T-1}\right)\delta_{t,T-1}^{past}, \text{ with } \delta_{t,t}^{k} = 1.$$

The trend inflation rate at time T, $\pi_T^{\star}$, represents the inflation target of the central bank. This target, which everyone knows, can be considered the level at which inflation is expected to converge in the future, once the shocks that affect the economy have vanished. Therefore, rule δ^{trend} indexes wages to the inflation rate that a worker expects to see in the long run. In contrast, rule δ^{past} indexes wages to the inflation rate that a worker has observed in the most recent period.[10]

At the time of a labor-contract negotiation, worker i faces the following problem:

$$\max_{W_{i,t},\ \delta_{i,t}} \quad \mathrm{E}_t\left\{\sum_{T=t}^{\infty}\left(\beta\alpha_w\right)^{T-t}\left[\lambda_T\left(1+\tau_w\right)\frac{\delta_{i,T}W_{i,t}}{P_T}\ell_{i,T} - \frac{\Psi}{2}\ell_{i,T}^{2}\right]\right\}, \tag{4}$$

subject to the labor-specific demand $\ell_{i,T} = \left(\frac{W_T}{\delta_{i,T}W_{i,t}}\right)^{\theta_w}\ell_T$, and the indexation-rule menu, $\delta_{i,T} \in \{\delta_{t,T}^{trend}, \delta_{t,T}^{past}\}$. In the worker's objective function, $\beta\alpha_w$ is the discount rate which takes into account the stochastic duration of the contract. Further, λ_t denotes the marginal utility of income, which rises as income decreases since the utility function is concave. Technically speaking, λ_t is simply the Lagrange multiplier associated with the household budget constraint. Therefore, the first term in Eq. (4) denotes the utility value of labor income, expressed in consumption units, while the second term describes the disutility generated by labor hours. Worker i thus chooses $W_{i,t}$ and $\delta_{i,t}$ to maximize the difference between the expected utility derived from labor income *vs.* the cost imposed by lower leisure.

Notice that we assume that there is no cross-sectional inequality in households' income (due to the existence of state-contingent securities), so λ_t is common to all households. While this assumption simplifies the model's solution, it maintains the role of λ_t as a signaling device of income effects scattered by aggregate shocks in the economy. For instance, suppose that λ_t increases due to a reduction in the household's income. Consequently, the utility value of labor income rises, and worker i would like to increase his or her labor supply. Put differently, as the household becomes poorer, it would like to smooth consumption by adjusting its labor supply. Whether or not it will be able to do so depends on the wage dynamics imposed by the indexation rule chosen by the worker.

The optimal choice of a nominal wage given an indexation rule has been widely studied (see Erceg et al., 2000), while the selection of an optimal indexation rule is less familiar. Therefore, we review first the optimality conditions of $W_t^{k,\star}$, highlighting some basic concepts useful to interpret the results of the model, and then we formally present the worker's problem when choosing an optimal indexation rule.

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[8] We assume workers take the frequency of contract negotiations as given, in order to maintain a relevant role for the nominal rigidity in the model.

[9] Wieland (2009) analyzes the indexation decisions of firms in a model with learning and proposes similar indexation rules. However, he does not use an objective-maximizing criterion for choosing the indexation rule. Instead, he uses a forecasting rule for the true process of inflation.

[10] A caveat is that, in practice, wage indexation rules have generally been asymmetric — only raising wages in response to positive inflation (Akerlof, 2007, footnote 79). We do not see this as an important limitation as quarter-on-quarter CPI inflation in the US has only rarely been negative in the post-WWII period and also not for extended periods of time.

Optimal nominal wage selection. Conditional on indexation rule $\delta^k_{t,T}$, the worker's optimal nominal wage is given by

$$\frac{W_t^{k,\star}}{W_t} = \frac{\psi\theta_w}{\left(\theta_w-1\right)\left(1+\tau_w\right)} \frac{\mathrm{E}_t\left\{\sum_{T=t}^{\infty}\left(\beta\alpha_w\right)^{T-t}\left(\ell^k_{t,T}\right)^2\right\}}{\mathrm{E}_t\left\{\sum_{T=t}^{\infty}\left(\beta\alpha_w\right)^{T-t}\lambda_T\left(\delta^k_{t,T}W_t/P_T\right)\ell^k_{t,T}\right\}}. \tag{5}$$

We drop the subindex i because workers with indexation rule $\delta^k_{t,T}$ who can re-optimize in period t will choose the same wage; in turn, $\ell^k_{t,T} = \left[W_T/\left(\delta^k_{t,T}W_{i,t}\right)\right]^{\theta_w}\ell_T$ denotes the time T labor-specific demand for workers in group k who last re-optimized in period $t \le T$. The steady-state distortion generated by a worker's monopolistic power is given by the wage markup $\frac{\theta_w}{\theta_w-1}$. To remove this distortion, we assume that $1+\tau_w \equiv \frac{\theta_w}{\theta_w-1}$. If we apply this definition and replace the labor-specific demand $\ell^k_{t,T}$ into Eq. (5), we obtain an expression for $W_t^{k,\star}$ in terms of only aggregate quantities, such that

$$\left(\frac{W_t^{k,\star}}{W_t}\right)^{1+\theta_w} = \psi\frac{\mathrm{num}_{k,t}}{\mathrm{den}_{k,t}}, \tag{6}$$

where

$$\mathrm{num}_{k,t} \equiv \left(\ell_t\right)^2 + \beta\alpha_w\mathrm{E}_t\left\{\left(\frac{1+\pi^w_{t+1}}{\delta^k_{t,t+1}}\right)^{2\theta_w}\mathrm{num}_{k,t+1}\right\},$$

$$\mathrm{den}_{k,t} \equiv \lambda_t w_t\ell_t + \beta\alpha_w\mathrm{E}_t\left\{\left(\frac{1+\pi^w_{t+1}}{\delta^k_{t,t+1}}\right)^{\theta_w-1}\mathrm{den}_{k,t+1}\right\},$$

$\pi^w_t \equiv \frac{W_t}{W_{t-1}} - 1$ is the wage inflation rate, and $w_t \equiv W_t/P_t$ is the aggregate real wage. Notice that in the case of fully flexible wages ($\alpha_w = 0$), wage dispersion vanishes along with the differences in individual labor supplies. Accordingly, Eq. (6) collapses to the familiar welfare-maximizing condition in which the marginal rate of substitution between consumption and labor, $MRS_{i,t} \equiv \psi\ell_{i,t}/\lambda_t$, equals worker i's real wage $w_{i,t} \equiv W_{i,t}/P_t$. Therefore, worker i's supply of labor hours in the flexible-wage economy is given by

$$\ell^f_{i,t} = \frac{1}{\psi}\lambda^f_t w^f_{i,t}, \tag{7}$$

where the superscript f denotes quantities of the flexible-wage economy, and the individual real wage $w^f_{i,t}$ equals the aggregate real wage w^f_t for all i and t. In this economy, the supply of labor hours increases with the real wage $w^f_{i,t}$, and the income-effect signal λ^f_t. As argued above, this is the case because when $w^f_{i,t}$ and λ^f_t rise, the welfare value of labor income outweighs the costs of a lower time for leisure.

Optimal indexation rule selection. When wages are flexible, households can respond to external shocks by selecting an optimal wage in each period, thereby maximizing welfare. Under staggered wages, by contrast, a worker's wage will not be re-optimized during some periods and will follow an indexation rule instead. In these periods, the worker's effective labor hours might deviate from the optimal labor supply schedule, which entails welfare costs. Therefore, a worker's preferred indexation rule minimizes the welfare costs from wage rigidities.

Let ξ_t denote the share of workers at time t that use the δ^{past} indexation rule, i.e., the aggregate degree of wage indexation to past inflation. Furthermore, let vector Ξ_t collect present and future expected levels of aggregate indexation, so $\Xi_t = \mathrm{E}_t\left\{\xi'_{t+h}\right\}_{h=0}^{\infty}$, and let $\wp_t\left(\Xi_t\right)$ summarize all equilibrium conditions that characterize the equilibrium dynamics. Thus, the selection of an optimal wage indexation rule formalizes as follows: when worker i re-optimizes the labor contract, he or she selects the indexation rule that maximizes expected utility, conditional on the present and future level of aggregate indexation in the economy and $\wp_t\left(\Xi_t\right)$, i.e.,

$$\delta^\star_t \in \underset{\delta_i\in\left\{\delta^{trend},\delta^{past}\right\}}{\arg\max}\ \mathbb{W}_{i,t}\left(\delta_i,\Xi_t\right) \text{ subject to } \wp_t\left(\Xi_t\right), \tag{8}$$

where

$$\mathbb{W}_{i,t}\left(\delta_i,\Xi_t\right) = \mathrm{E}_t\left\{\sum_{T=t}^{\infty}\left(\beta\alpha_w\right)^{T-t}\ \mathcal{U}\left(c_{i,T}\left(\xi_T\right),\ \ell_{i,T}\left(\delta_i,\xi_T\right)\right)\right\}. \tag{9}$$

Notice that $\mathbb{W}_{i,t}$ is constrained by the expected duration of the labor contract, as the effective discount factor is $\beta\alpha_w$. Furthermore, because of the state-contingent securities, individual consumption equals the aggregate level and does not depend on the individual indexation choice δ_i. Individual consumption does, in contrast, depend on aggregate indexation ξ_t. Finally, notice that, given worker i's atomistic size relative to the aggregate, the individual choice of an indexation rule has a negligible effect on aggregate indexation. Worker i thus takes ξ_t as given when choosing his or her optimal indexation rule. This can lead to a *coordination failure*, because workers do not internalize how their and others' decisions affect the aggregate. We provide a detailed and more intuitive analysis of a worker's optimal indexation rule selection in Section 3 and discuss the consequences of coordination failure in Section 4.

Labor market aggregation. Each period, only a fraction $1-\alpha_w$ of workers re-optimize their wages. Let χ_t denote the time t proportion of workers from subset $(1-\alpha_w)$ that select δ^{past}. Accordingly, the degree of aggregate indexation ξ_t is given by

$$\xi_t = (1-\alpha_w) \sum_{h=0}^{\infty} \chi_{t-h} (\alpha_w)^h, \tag{10}$$

which can be written recursively as $\xi_t = (1-\alpha_w)\chi_t + \alpha_w \xi_{t-1}$. The equilibrium solution for aggregate wage indexation $\xi^\star$, which is a function of the economic regime Σ, will be characterized in Section 3. We first describe useful measures of wage dispersion and discuss aggregation details of the labor market.

Without loss of generality, assume that workers are sorted according to the indexation rule they have chosen. Workers in the interval $i \in I_t^{past} = [0, \xi_t]$ use δ^{past}, while those in the interval $i \in I_t^{trend} = [\xi_t, 1]$ use δ^{trend}. Measures of wage dispersion for each of the two sectors can be computed by adding up total hours worked, given by the set of labor-specific demands. Hence, we have $\int_{i\in I_t^k} \ell_{i,t} \mathrm{d}i = \ell_t \mathrm{disp}_{\mathrm{w},t}^k$, where $\mathrm{disp}_{\mathrm{w},t}^k = \int_{i\in I_t^k} \left(\frac{W_{i,t}}{W_t}\right)^{-\theta_w} \mathrm{d}i$. Recursive expressions for the wage dispersion measures are given by

$$\mathrm{disp}_{\mathrm{w},t}^k = (1-\alpha_w)\chi_t^k \left(rw_t^{k,\star}\right)^{-\theta_w} + \alpha_w \left(\frac{1+\pi_t^w}{\delta_{t-1,t}^k}\right)^{\theta_w} \mathrm{disp}_{\mathrm{w},t-1}^k, \tag{11}$$

$$\text{where } \chi_t^k = \begin{cases} \chi_t & \text{if } k = past \\ 1-\chi_t & \text{if } k = trend, \end{cases} \tag{12}$$

and $rw_t^{k,\star} \equiv \frac{W_t^{k,\star}}{W_t}$. Finally, given the Dixit–Stiglitz technology of the labor intermediary, the aggregate wage level is given by $W_t^{1-\theta_w} = \int_0^1 W_{i,t}^{1-\theta_w} \mathrm{d}i$. This expression can be rewritten in terms of the sum of relative wages within each indexation-rule sector, which are given by $\tilde{w}_t^k \equiv \int_{i\in I_t^k} \left(\frac{W_{i,t}}{W_t}\right)^{1-\theta_w} \mathrm{d}i$. Thus, it follows that

$$\tilde{w}_t^{past} + \tilde{w}_t^{trend} = 1.$$

Notice that these weights may change over time due to variations in rw_t^k and χ_t. The recursive law of motion of $\tilde{w}_t^k$ is given by

$$\tilde{w}_t^k = (1-\alpha_w)\chi_t^k \left[rw_t^{k,\star}\right]^{1-\theta_w} + \alpha_w \left(\frac{1+\pi_t^w}{\delta_{t-1,t}^k}\right)^{\theta_w-1} \tilde{w}_{t-1}^k. \tag{13}$$

The rest of the model is standard, so we describe it briefly.

2.2. Final-good producer

A perfectly competitive firm produces the final good y_t using a linear technology on aggregate labor hours, so that:

$$y_t = A\exp(z_t)\,\ell_t,$$

where z_t is a permanent productivity shock that follows the process $\Delta z_t = \gamma + \varepsilon_{z,t}$, with γ representing potential growth, and $\varepsilon_{z,t}$ denoting a white noise process with standard deviation σ_z. The linear technology on labor implies that the firm's demand for the aggregate labor input is completely flat, and so the real wage fluctuates only with productivity even if the labor market features nominal rigidities, i.e.,

$$w_t = A\exp(z_t).$$

2.3. Economic policy

We assume that the government budget constraint is balanced at all times, there is no public debt, and government spending is equal to zero.[11] Therefore, the labor subsidies provided by the government are entirely financed through lump-sum taxes levied on households.

To close the economy, we assume in this simple version of the model that the central bank sets its policy interest rate according to

$$1+R_t = \frac{1+\pi_t^\star}{\beta}\left(\frac{1+\pi_t}{1+\pi_t^\star}\right)^{a_\pi},$$

where the inflation target $\pi_t^\star$ may vary over time according to the law of motion $\pi_t^\star = \pi_{t-1}^\star + \varepsilon_{\pi,t}$. Therefore, any change in $\pi_t^\star$ is permanent.

[11] In Section 5 we relax this assumption and allow for government-spending shocks.

2.4. Equilibrium and model solution

Equilibrium in the goods market satisfies the resource constraint, so $y_t = c_t \equiv \int_0^1 c_{i,t} di$. In equilibrium, there exists a set of prices $\{\lambda_t, P_t, W_t, W_{i,t}, R_t\}_i$ and a set of quantities $\{y_t, g_t, c_{i,t}, b_{i,t}, \ell_t, \ell_{i,t}, \chi_t\}_i$ for all i such that all markets clear at all times, and agents maximize their utility and profits.

To determine the equilibrium level of indexation, we define a fine grid of indexation values $\xi \in [0,1]$ and compute the individual expected welfare costs related to the two indexation choices in each point. To solve for the stochastic steady state of the model and rank the indexation rules according to their related welfare, we use a second-order perturbation method as proposed by Schmitt-Grohé and Uribe (2004).[12] The model is solved using Dynare (Adjemian et al., 2011), version 4.5.7, and the pruning algorithm of Andreasen et al. (2018).

Calibration. We use a textbook calibration for the model described above. As such, a period is a quarter, $\beta = 0.99$, $\theta_w = 10$, $a_\pi = 1.5$, and $\alpha_w = 0.5$; the value for the last parameter implies that the average duration of a wage contract is only 2 periods, so the model displays only a moderate amount of nominal rigidities. We normalize the parameters A and ψ such that output and labor equal to 1 and $\frac{1}{3}$ at the non-stochastic steady state, respectively. Further, we calibrate the size of shocks to match a 0.5% permanent drop in productivity, an unexpected rise in the returns of bonds of 25 basis points, and a 50 basis points fall in the inflation target. For simplicity, we normalize the rate of potential growth γ to zero and assume that aggregate demand shocks are mildly persistent (so $\rho_d = 0.5$).

3. Aggregate indexation in the decentralized equilibrium

This section characterizes the equilibrium level of aggregate wage indexation that prevails in the long run for a given set of aggregate shocks. We show that workers decide to index wages to past inflation when technology and trend inflation shocks explain a large proportion of output fluctuations. When demand-side shocks dominate the aggregate dynamics, workers prefer to index to the central bank's inflation target. We demonstrate how the gap between the desired and the actual labor supply drives our results.[13]

3.1. General equilibrium effects and comparative dynamics

We now present the aggregate dynamics of the economy with staggered wages to three aggregate shocks: a permanent reduction in productivity, a temporary increase in aggregate demand, and a permanent reduction in the inflation target. Let Σ denote a vector containing information about the volatility and persistence of aggregate shocks, i.e., $\Sigma \equiv \{\rho_d, \sigma_z, \sigma_d, \sigma_\pi\}$. We assume that in each scenario, the economy faces only one type of shock, so that the technology-shock regime is represented by a vector $\Sigma^z = \{0, \sigma_z, 0, 0\}$; similarly, the aggregate demand shock regime has a $\Sigma^d = \{\rho_d, 0, \sigma_d, 0\}$, and the inflation target shock regime has a $\Sigma^{\pi^\star} = \{0, 0, 0, \sigma_\pi\}$. Finally, in all scenarios, we set the level of aggregate indexation to past inflation $\xi = 0.5$.

Technology shock. Fig. 1 shows the impulse responses of flexible-wage and sticky-wage economies to a 0.50% permanent fall in productivity. At the starting point $t = -1$, the economies have not faced a shock for a long time. In period $t = 0$, a single shock occurs. Panels (a) to (f) show how aggregate quantities react to this shock in the flexible and staggered wage economies. Panels (g) to (k) focus on the behavior of the sub-populations of workers who index either to past or trend inflation. In particular, panel (g) compares the expected and desired hours worked for households re-optimizing their wage contracts at the impact period ($t = 0$) in the sticky-wage economy. In turn, panels (h) to (k) show all possible trajectories of hours and relative wages ($W_{i,t}/W_t$) that the two subgroups expect from $t = 0$ onwards. In these dispersion plots, the size of circles is proportional to the subgroups' population size, which is determined by the Calvo price-setting mechanism. We first discuss what happens to the economy with flexible wages and then turn to that with sticky wages.

Under flexible wages, the negative productivity shock causes output and the real wage to decrease immediately to their new steady-state levels (see plain lines in panels (a) to (f)). Marginal utility of income λ_t^f increases because households have less resources to spend for consumption and leisure. Concerning the aggregate labor market, lower productivity implies a permanent drop in aggregate labor demand, while the negative income effect prompts households to increase their labor supply. In equilibrium, the shifts in aggregate labor demand and supply balance, such that the real wage is lower and aggregate hours remain unchanged. In the flexible-wage economy, the necessary decline of the real wage is achieved by a drop of nominal wages while prices do not change. The transition dynamics occur immediately after the shock.

The transition to a new equilibrium is much slower when households cannot freely adjust their wages in the staggered wage economy. These dynamics are shown by the dashed lines in panels (a) to (f) of Fig. 1. Since the technology in labor is linear, it remains the case that aggregate labor demand falls, which lowers the real wage. However, this variable's adjustment now comes from an increase in prices rather than a decrease in nominal wages. Nominal wage rigidities prevent households from increasing their labor supply sufficiently in response to their permanent income loss. As a result, aggregate hours fall, and output drops more

[12] The stochastic steady state is also referred to as the "ergodic mean in the absence of shocks", or EMAS (see Born and Pfeifer, 2014) or the "risky steady state" (see Juillard, 2011).

[13] We focus on how wage indexation depends on the relative importance of shocks by changing their volatility. Investigating the role of uncertainty (second-moment) shocks, which could in principle also be relevant, is beyond the scope of this paper.

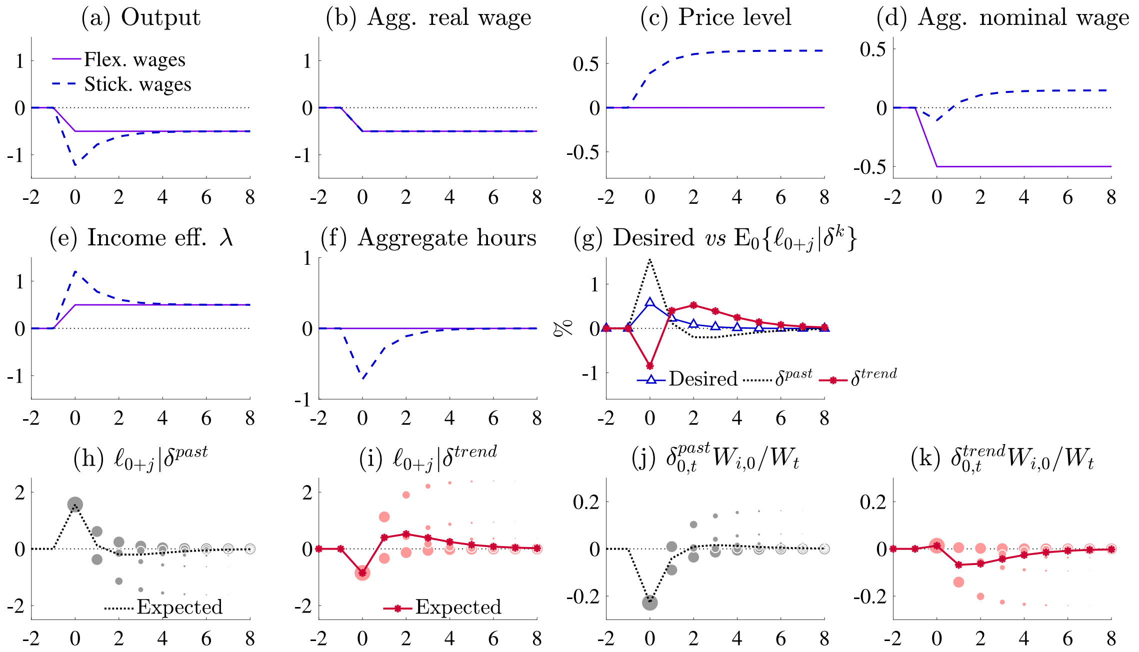

Fig. 1. Impulse responses to a permanent downfall in productivity. **Note:** Panels (a) to (f) show the impulse responses of selected variables in the flexible and staggered wage economies to the aggregate shock considered. Panels (g) to (k) focus on the behavior of the sub-populations of workers who index to either past or trend inflation. The variables shown are: (a) output, (b) aggregate real wage, (c) price of the final good, (d) general nominal wage index, (e) Lagrange multiplier of the households' budget constraint, (f) aggregate hours worked, (g) desired and expected hours worked of a worker negotiating his or her labor contract at the impact period, i.e., worker i', (h and j) the full set of hours and the nominal-wage ratio that worker i' might face under δ^{past} indexation, respectively, (i and k) *ibidem*, but under δ^{trend} indexation.

strongly than in the case of flexible wages. This implies that the negative income effect, as portrayed by λ_t, is larger when wages are sticky. Since households are poorer under sticky wages, their desired labor supply increases on impact and then decreases slowly, as shown by the line with triangles in panel (g). As such, workers will select the indexation rule that helps them close the gap between their expected and desired hours worked. In panel (g), we observe that, from the perspective of workers choosing their wage contract at the impact period, the gap is smaller if they select the δ^{past} contract (black dotted line) as opposed to the δ^{trend} contract (red line with circles).

The rationale behind this result is the following. Nominal wages under the δ^{past} contract tend to overshoot the general wage index W_t in periods of no re-optimization (see panel (j)), since inflation temporarily rises due to the shock.[14] Moreover, demand for specific labor $\ell_{i,t}$ is also pushed down by a lower aggregate labor demand ℓ_t (see Eq. (3)). A higher relative wage implies that a worker's hours fall. However, desired hours worked have *increased*, not decreased. Therefore, a worker with a past inflation rule and re-optimizing in period $t = 0$ will cut his or her wage to counter both the expected increase in relative wages in non-re-optimizing periods and the lower aggregate labor demand. By strategically setting their nominal wage, workers with δ^{past} contracts can raise working hours in the impact period, which reduces the gap between desired and expected hours worked.

In contrast, wages under the δ^{trend} contract tend to undershoot W_t in no re-optimizing periods. The δ^{trend} contract updates nominal wages according to trend inflation, which remains fixed at zero. Thus, setting a very low $W_0^{trend,\star}$ in $t = 0$ implies that expected hours worked will be too high in subsequent periods (as relative wages will be too low). To keep the expected path of hours worked as close as possible to the desired level, $W_0^{trend,\star}$ is raised to offset future expected decreases in relative wages. As a result, $W_0^{trend,\star}$ cannot counter the drop in aggregate hours, so hours worked under the δ^{trend} contract fall in period $t = 0$. This raises the gap between desired and actual hours worked in that period and makes the δ^{trend} contract the least appealing for workers' point of view after a technology shock.[15]

Aggregate demand shock. Fig. 2 shows the impulse responses to an unexpected increase in the spread between the return on bonds and the risk-free rate. This shock stimulates households to save more and consume less, which lowers aggregate demand. Since productivity remains steady, so do aggregate labor demand and the real wage. Under flexible wages, lower aggregate demand causes equally sized decreases in wage and price inflation, which induces stability in output, the income-effect signal λ, and hours

[14] This effect is clearer for the subgroup of households who never got the chance to re-optimize their wages, not shown in the graph.

[15] These effects prevail even if the proportion of households indexing to past inflation indexation is zero ($\xi = 0$). In this scenario, it remains the case that cutting $W_0^{trend,\star}$ below the aggregate W_0 increases the odds of having excessively low future relative wages. In turn, expected hours worked will be too high compared to the future desired labor supply.

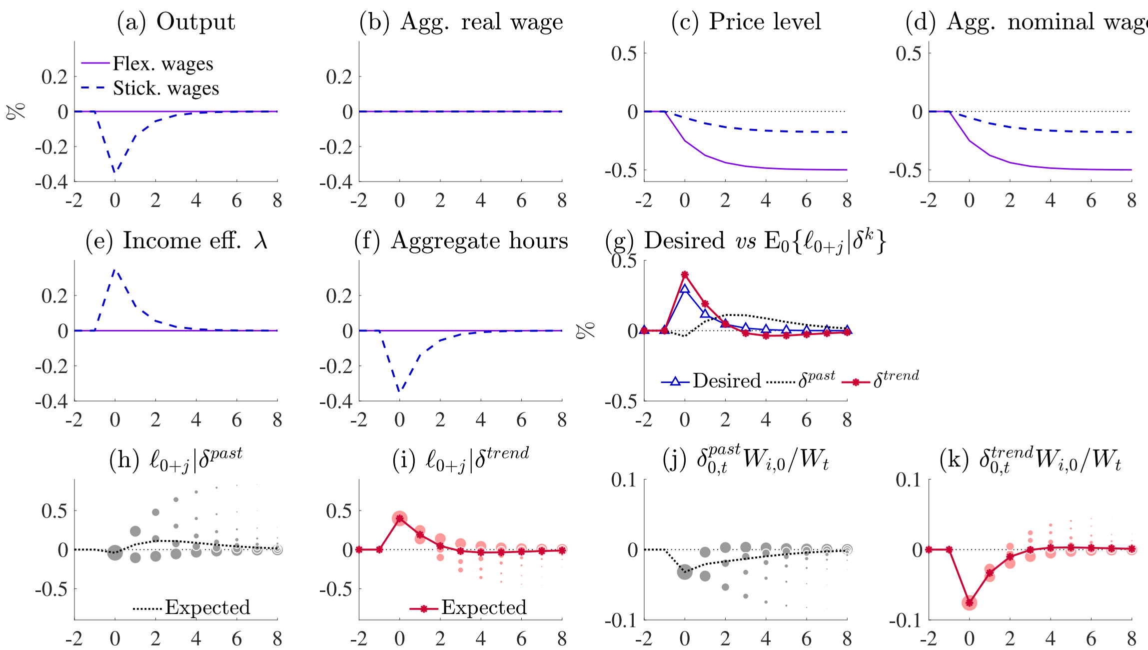

Fig. 2. Impulse responses to a negative aggregate demand shock. **Note:** Panels (a) to (f) show the impulse responses of selected variables in the flexible and staggered wage economies to the aggregate shock considered. Panels (g) to (k) focus on the behavior of the sub-populations of workers who index to either past or trend inflation. The variables shown are: (a) output, (b) aggregate real wage, (c) price of the final good, (d) general nominal wage index, (e) Lagrange multiplier of the households' budget constraint, (f) aggregate hours worked, (g) desired and expected hours worked of a worker negotiating his or her labor contract at the impact period, i.e., worker i', (h and j) the full set of hours and the nominal-wage ratio that worker i' might face under δ^{past} indexation, respectively, (i and k) *ibidem*, but under δ^{trend} indexation.

worked. However, when wages are sticky, output and aggregate hours plummet because wages and prices cannot adjust sufficiently downwards. As a result, λ_t rises due to negative-income effects, which increases households' desired labor supply.

In this scenario, the δ^{trend} contract minimizes the gap between expected and desired hours worked. This choice is again driven by nominal wage dynamics in periods of no re-optimization. Since prices and the general wage index fall, nominal wages under the δ^{trend} contract tend to overshoot W_t in no re-optimizing periods. Workers who can re-optimize in period $t = 0$ will cut $W_0^{trend,\star}$ to balance expected increases in their future relative wages. As a result, hours worked under the δ^{trend} contract rise on impact and get closer to the desired hours worked. In contrast, wages under the δ^{past} contract tend to undershoot W_t in no re-optimizing periods because they index to falling inflation. As such, $W_0^{past,\star}$ cannot be reduced significantly in the impact period: doing so would prompt very low relative wages in the future and too many hours worked. Therefore, $W_0^{past,\star}$ is set to reduce the chances of overshooting the future path of desired hours worked, at the cost of not offsetting the current fall in the labor-specific demand in period $t = 0$. Hence, hours worked under the δ^{past} contract fall on impact and create a larger gap between actual and desired hours worked.

Inflation target shock. Fig. 3 shows the responses to a permanent and unexpected decrease in the inflation target. In the flexible-wage scenario, prices and wages adjust immediately to the new nominal anchor, and there are no effects on real quantities. But staggered wages imply again a slow transition to the new equilibrium. As in the previous case, the real wage remains stable. Since price and wage inflation move slowly to their new equilibria, the central bank increases its policy rate R_t to cut aggregate demand and reduce the inflation gap $\pi_t - \pi_t^\star$ (not shown in the figure). The costly disinflation translates into a drop in output and hours worked, which generates negative income effects that raise the desired labor supply.

From the individual perspective, a household finds that the gap between desired and expected hours worked is smaller under the δ^{past} contract. As before, the expected path of relative wages explains this choice. Wages under the δ^{trend} (δ^{past}) contract tend to undershoot (overshoot) W_t in no re-optimizing periods because wages drop faster when indexed to trend inflation compared to past inflation. As a result, $W_0^{past,\star}$ can be lowered to counter expected increases in future relative wages, while the same cannot be said for $W_0^{trend,\star}$. Therefore, workers with the δ^{past} contract reduce the gap between expected and desired hours worked over the contract's likely duration.

3.2. Welfare analysis of a single worker

We now turn to a worker's welfare implications of selecting a particular wage indexation rule. Let $\mathbb{W}_{i,ss}\left(\delta^k, \xi, \Sigma\right) \equiv \mathrm{E}\{\mathbb{W}_{i,t}\}$ represent the welfare a worker expects to obtain in the long run if he or she chooses the indexation rule δ^k, where $\mathbb{W}_{i,t}$ is given by Eq. (9), ξ is a given level of aggregate indexation, and Σ is a given configuration of aggregate shocks.

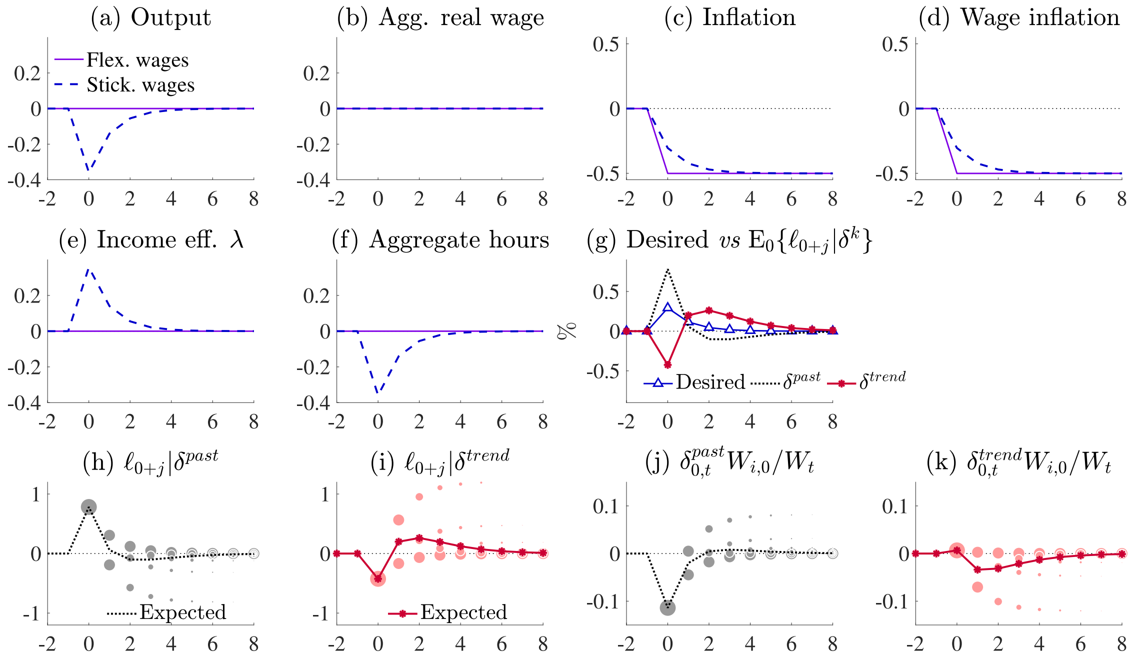

Fig. 3. Impulse responses to a permanent reduction in the inflation target. **Note:** Panels (a) to (f) show the impulse responses of selected variables in the flexible and staggered wage economies to the aggregate shock considered. Panels (g) to (k) focus on the behavior of the sub-populations of workers who index to either past or trend inflation. The variables shown are: (a) output, (b) aggregate real wage, (c) price inflation, (d) nominal wage inflation, (e) Lagrange multiplier of the households' budget constraint, (f) aggregate hours worked, (g) desired and expected hours worked of a worker negotiating his or her labor contract at the impact period, i.e., worker i', (h and j) the full set of hours and the nominal-wage ratio that worker i' might face under δ^{past} indexation, respectively, (i and k) *ibidem*, but under δ^{trend} indexation.

If there were no shocks in the economy, consumption and labor (and thus welfare) would be invariant to ξ and δ^k. Define this scenario as the deterministic regime $\Sigma_d = 0$. In such a case, welfare is simply determined by

$$\mathbb{W}_d \equiv \frac{1}{1-\beta\alpha_w}\mathcal{U}\left(c_d, \ell_d\right),$$

where c_d and ℓ_d denote the levels of consumption and hours worked in the deterministic scenario. It is common in the literature to measure the welfare costs from stochastic regimes in terms of proportional losses in deterministic steady-state consumption (see Schmitt-Grohé and Uribe, 2007). Let ce^k for $k \in \{past, trend\}$ denote the percentage reduction in c_d that leaves a worker with indexation rule δ^k indifferent between the deterministic regime and the stochastic one. Formally, given δ^k, ξ and Σ, ce^k solves the following equation:

$$\mathbb{W}_{i,ss}\left(\delta^k, \xi, \Sigma\right) = \frac{1}{1-\beta\alpha_w}\mathcal{U}\left(\left(1-ce^k\right)c_d, \ell_d\right).$$

Put differently, ce^k measures the expected welfare cost associated with indexation rule δ^k in the economic regime Σ and under aggregate wage indexation ξ. Worker i's preferred δ^k is the one with the lowest ce^k. Adding up all the decisions of workers gives us the decentralized equilibrium of aggregate wage indexation to past inflation, which we denote by $\xi^\star$.

The corner solution $\xi^\star = 0$ is achieved when, for any $\xi \in [0,1]$, the trend inflation indexation rule yields the lowest welfare costs (i.e. $ce^{trend} < ce^{past}$). Similarly, $\xi^\star = 1$ when $ce^{trend} > ce^{past}$ for any $\xi \in [0,1]$. An interior solution exists if there is at least one $0 < \xi < 1$ for which $ce^{trend} = ce^{past}$; in such a case, workers are indifferent between the two indexation rules. Next, we show that $\xi^\star$ is an *equilibrium state* and is *globally stable* when the economy faces shocks to technology, aggregate demand, and the inflation target.

Fig. 4 shows the long-run welfare costs associated to δ^{trend} indexation (ce^{trend} is the plain line) and those related to δ^{past} indexation (ce^{past} is the dashed line) for the economic regimes Σ^z, Σ^d, and $\Sigma^{\pi^\star}$ described in the previous section. The figure shows that there is a corner solution in the first two cases. That is, for any level of ξ, worker i has a clear preference. He or she chooses past inflation indexation when permanent technology shocks drive the economy. However, when aggregate demand shocks drive the economy, trend inflation indexation is preferred.[16] In contrast, the permanent trend inflation shock regime has an interior solution, since for

[16] We have analyzed different types of aggregate demand shocks, such as government spending, preferences, or high-frequency monetary policy shocks (i.e., a temporary deviation from the policy rule). In all cases, we find similar results.

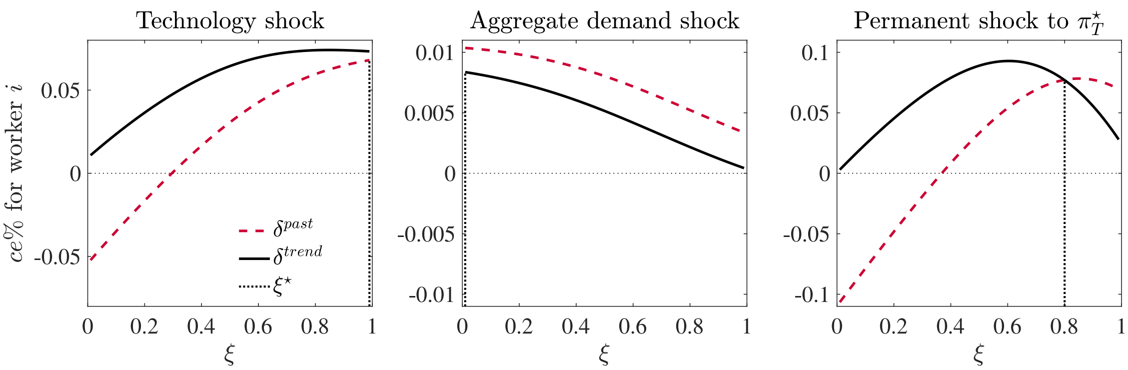

Fig. 4. Private welfare costs for different economic regimes. **Note:** The figure shows the private welfare costs associated with each indexation rule conditional on specific shocks. The decentralized equilibrium level of aggregate indexation to past inflation is signaled in the x-axis by the dotted line.

$\xi = 0.80$ we have that $ce^{trend} = ce^{past}$. Further, notice that $\xi^\star$ is an equilibrium for all regimes since workers have no incentive to change their rule at this level of aggregate indexation. Also, $\xi^\star$ is globally stable because for any initial $\xi_0 \neq \xi^\star$, workers choose the contract with the lowest expected costs, and so aggregate indexation ξ_t converges eventually to $\xi^\star$ in the long run.[17] Therefore, the decentralized equilibrium leads to a high aggregate indexation to past inflation in regimes Σ^z and $\Sigma^{\pi^\star}$, and to a low aggregate indexation to past inflation in regime Σ^d.

4. The social planner's preferred aggregate indexation level

The equilibrium aggregate wage indexation $\xi^\star$ described above corresponds to a set of uncoordinated decisions among workers, taken in a decentralized manner, and where each worker considers everybody else's decision as exogenous. At the same time, each worker judges his or her own decision as too small to affect the aggregate. We now show that the decentralized equilibrium does not generally reflect the socially desired level of aggregate wage indexation.

Assume that the social planner cannot remove the nominal rigidities in the economy, but can still choose the wage indexation rule for each worker. When doing so, the planner internalizes the overall effect of nominal wage dynamics on output and inflation. The preferences of the social planner differ from those of an individual worker in two dimensions. First, the effective discount factor of the planner does not depend on the duration of a labor contract. Second, the planner takes an aggregate perspective and adds up the welfare of all workers. As such, social welfare is given by

$$\mathbb{SW}_{ss}(\xi, \Sigma) \equiv \mathrm{E}\left\{ \max_{\{c_{i,t+j}, \ell_{i,t+j}, W_{i,t+j}\}_i} \mathrm{E}_t \sum_{j=1}^{\infty} \beta^j \int_0^1 \mathcal{U}\left(c_{i,t+j}, \ell_{i,t+j}\right) \mathrm{d}i, \text{ s.t. } \wp_t\left(\Xi_t\right) \right\}.$$

Social welfare, which varies with ξ and Σ, is the sum of every single household welfare in the economy, regardless of their last wage re-optimization. In contrast, the individual measure $\mathbb{W}_{i,ss}\left(\delta^k, \xi, \Sigma\right)$ in Eq. (9) refers only to the welfare of a worker who chose the indexation rule δ^k.

The upper bound of social welfare is achieved when the economy is never hit by shocks, i.e., the deterministic scenario. In all other stochastic regimes, there are social welfare losses, which can be measured in the same way as private welfare costs. Let ce^S denote the reduction in deterministic consumption that leaves the *representative or average* household indifferent between the deterministic and the stochastic regime. Variable ce^S solves the equation

$$\mathbb{SW}_{ss}(\xi, \Sigma) = \frac{1}{1-\beta} \mathcal{U}\left(\left(1 - ce^S\right) c_d, \ell_d\right).$$

The social planner's preferred level of aggregate indexation ξ is the one that minimizes ce^S.

Gray (1976) and Fischer (1977) show that the socially optimal degree of wage indexation depends on the structure of shocks prevailing in the economy, i.e., in vector Σ. They argue that full indexation to past inflation ($\xi = 1$) is optimal when only nominal shocks drive output fluctuations, and that no indexation to past inflation ($\xi = 0$) is optimal when only real shocks prevail.

Fig. 5 shows that Gray and Fischer's results hold in a New Keynesian model such as ours for regimes with either a technology or aggregate demand shock.[18] The preferred level of aggregate indexation to past inflation of a benevolent planner, denoted by ce^S, is the blue line with circles in the figure; the corresponding level of aggregate indexation is ξ^S. Each panel in the figure represents a regime, namely Σ^z, Σ^d, and $\Sigma^{\pi^\star}$. Notably, ce^S lies between the welfare costs associated with the two indexation rules available in

[17] It is worth mentioning that in every single exercise we have performed, either with an interior or a corner solution, $\xi^\star$ is globally stable. Global stability is achieved because when ce^{trend} is greater than ce^{past}, ce^{past} grows faster than ce^{trend}. Formally, we observe that if $ce^{trend} \geq ce^{past}$, then $\partial ce^{past}/\partial\xi \geq \partial ce^{trend}/\partial\xi$. The opposite is also true when $ce^{trend} \leq ce^{past}$; in this case, ce^{trend} grows faster than ce^{past}. It follows that the ce^k's cross only once in the interval $\xi \in [0, 1]$.

[18] This is also the case in Amano et al. (2007).

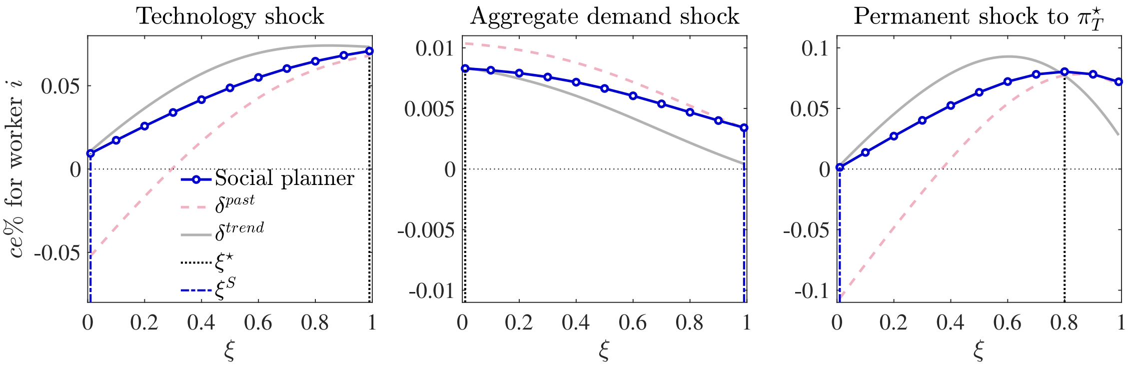

Fig. 5. Welfare costs for different economic regimes. **Note:** The figure shows the *social* welfare costs conditional to specific shocks as a function of aggregate indexation. The private welfare cost valuations are displayed in watercolors.

the economy, ce^{trend} and ce^{past}. As such, ce^S is strictly positive for any value of ξ. It follows that no indexation to past inflation is socially optimal when the economy is driven by permanent technology and inflation target shocks (i.e., $\xi^S = 0$ if $\Sigma = \Sigma^z$ or $\Sigma^{\pi^\star}$). In contrast, full indexation is socially optimal in response to aggregate demand shocks (i.e., $\xi^S = 1$ if $\Sigma = \Sigma^d$).[19]

Equilibrium outcomes $\xi^\star$ and ξ^S differ substantially in all three regimes, opposing each other almost from corner to corner. Gray (1976) reasoned that the socially optimal level of indexation to past inflation should aim to stabilize the real wage, thus avoiding excessive fluctuations in output and inflation. The premise implies that the planner's reference point is the frictionless flexible-wage economy, which delivers the efficient allocation with the highest attainable welfare for households in a stochastic environment. Any deviation from this allocation would entail welfare costs. In Section B.4 of the Online Appendix, we show that deviations of output and inflation from their efficient levels are indeed more persistent under the decentralized equilibrium than the social equilibrium following shocks to technology, demand, or the inflation target.[20]

5. Explaining the time-variation of wage indexation in the US

In the first part of this section, we discuss the stylized facts about changes in wage indexation in the US from the *Great Inflation* to the *Great Moderation*. Then, using an extended model built to fit the US dynamics, we show that the decentralized equilibrium explains the stylized facts. In contrast, the social planner's indexation choices fail to do so.

5.1. Evidence of changes in wage indexation in the US

Micro and macro evidence suggests that the degree of wage indexation to past inflation was high during the *Great Inflation*, but was low before and after this period. Institutional evidence is available in the form of private-sector workers' coverage by cost-of-living adjustment (COLA) clauses.[21] This measure was often used as a proxy for the degree of wage indexation to past inflation in the US. The COLA index, discontinued in 1995, measures the proportion of cost-of-living adjustment clauses in major collective bargaining agreements, i.e., contracts covering more than 1000 workers. From the late 1960s onwards, COLA coverage steadily increased from 25% to levels of around 60% in the mid-1980s, after which there was a decline towards 20% in the mid-1990s. Although the sample covers less than 20% of the US labor force, Holland (1988) shows that nonunion wages reacted more to price-level shocks the more indexed were union wage contracts (Devine, 1996). This finding suggests the existence of implicit indexation for *nonunion* wages.

Hofmann et al. (2012) also document considerable time variation in the degree of wage indexation at the macro level. In a first step, they estimate a time-varying parameter Bayesian structural vector autoregressive (TVP-BVAR) model to assess time variation in nominal wage dynamics. In a second step, they estimate a New Keynesian DSGE model's parameters for specific periods (i.e., 1960Q1, 1974Q1, and 2000Q1). In line with the micro evidence, they find a degree of wage indexation of 91% during the *Great Inflation*, compared to 30% and 17% before and after this period.[22]

19 Indexing to past inflation can create individual welfare *gains* (negative welfare costs) for some low values of ξ in the decentralized equilibrium; i.e., when nobody indexes to past inflation ($\xi = 0$) in the Σ^z and $\Sigma^{\pi^\star}$ regimes, worker i has an incentive to change his or her indexation rule towards δ^{past} to gain either consumption or leisure. Yet, switching makes workers end up with a strictly lower welfare level overall. This effect resembles the *paradox of thrift*, although here workers pursue too much protection in terms of indexation instead of savings.

20 The main finding from Sections 3 and 4 remain unchanged when i) we replace price inflation with wage inflation in the central bank's interest rate rule, ii) we impose a proportional labor tax instead of a lump-sum tax, and (iii) we include financial frictions (see Online Appendix Section F).

21 We show the COLA series in the Online Appendix, Section D.1.

22 Ascari et al. (2011) find a similar pattern of time-variation in wage indexation using rolling window techniques in the US. Attey (2016) also finds time variation in wage indexation to past inflation in the US and other developed economies. To preserve space, Section D.1 of our Online Appendix expands on the evidence discussed here and shows that the predictions of the simple model with nominal wage rigidities are consistent with time-varying correlations between wage and price inflation in the US.

Table 1
Validation exercises.

Common parameters across exercises			
β	Subjective discount factor	.99	
σ	Intertemp. elasticity of subst.	1	
ω	Inverse of Frisch elasticity	2	
θ_w	Elasticity of labor demand	10	
θ_p	Elasticity of interm. goods	10	
g_y	Public-spending-to-GDP ratio at steady state	.2	
Year-specific parameters for:		2000Q1	1974Q1
γ^h	Habit formation	.37	.71
γ^p	Inflation inertia	.17	.8
α_p	Calvo-price rigidity	.78	.84
α_w	Calvo-wage rigidity	.54	.64
a_π	Taylor Rule: inflation	1.35	1.11
a_y	Taylor Rule: output gap	.1	.11
$a_{\Delta y}$	Taylor Rule: output gap growth	.39	.5
ρ_R	Taylor Rule: smoothing	.78	.69
ρ^g	Autocorr. govn't spending	.91	.89
σ_z	Std. dev. technology	.31	1.02
σ_g	Std. dev. govn't spending	3.25	4.73
$\hat{\xi}$	Estimated wage indexation by HPS	.17	.91
Predicted indexation outcomes			
$\xi^\star$	Implied equilibrium wage indexation	0	.89
ξ^S	Implied social wage optimum	1	0

Note: All *common* and *specific* parameter values are extracted from Hofmann et al. (2012), who estimated the extended model with US data for 1974Q1 and 2000Q1. For more details about their estimation procedure, see their Section 3.2. The *implied* equilibrium wage indexation values are computed using the procedure from our Section 3.

5.2. Endogenous wage indexation during the Great Inflation and the Great Moderation

This section extends the simple model by adding features that make it compatible with the empirical analysis of Hofmann et al. (2012). These extensions include nominal rigidities in prices, habit formation, a government-spending shock that replaces the risk-spread shock considered in Section 2, and a Taylor rule for the nominal interest rate that includes the output gap and output growth. The full derivation and details of this model are laid out in the Online Appendix, while Table 1 presents the calibrations used for the exercises below.

Hofmann et al. (2012) estimate the extended model for three different periods of post-war US data: 1960Q1, 1974Q1, and 2000Q1. We use the estimated parameters of Hofmann et al. for 1974Q1 and 2000Q1 to represent respectively the *Great Inflation* and the *Great Moderation*. For each of these periods, we compute the endogenous equilibrium wage indexation implied by their estimated parameters. We then compare whether our model-based prediction matches the estimated degree of wage indexation of the authors.

Table 1 shows the parameters for the two periods we consider. Common calibrated parameters for both periods are described in the first part of the table. For the specific parameters of each regime, we take the median values of the estimated posterior distributions of Hofmann et al. (2012). As shown in Table 1, the parameters for each regime exhibit typical patterns found in the literature. For example, the persistence parameters such as habits (γ^h) and inflation inertia (γ^p) were higher during the *Great Inflation*, while the response of the Federal Reserve to inflation deviations in the Taylor rule (a_π) was lower. We also report the estimated degree of wage indexation ($\hat{\xi}$) from Hofmann et al. (2012) for both regimes, which should be compared with our predicted aggregate wage indexation levels. The estimated degree of wage indexation is high in the 1970s and low in the 2000s.

The model predictions for aggregate wage indexation are reported at the bottom of Table 1. The model predicts a decentralized equilibrium aggregate wage indexation $\xi^\star$ of 0 for the *Great Moderation* and .89 for the *Great Inflation*. The model predictions are consistent with the estimated degree of wage indexation from Hofmann et al. (2012) and also the COLA index reported in Section 5.1.

The bottom part of Table 1 also reports the model-based socially optimal rate of aggregate indexation ξ^S. Notice that the social optimum diametrically differs from the decentralized equilibrium. Indeed, the social planner would have opted for high indexation during the *Great Moderation*, and low indexation during the *Great Inflation*. As discussed in Section 4, these are the recommendations of the seminal contributions of Gray (1976) and Fischer (1977), which appear to be at odds with the stylized facts.[23]

Section D.3 of the Online Appendix reports a counterfactual analysis that investigates the drivers of changing wage indexation. By changing the parameters' calibration one by one, we find that the high degree of wage indexation in the 1970s was primarily the result of volatile supply-side shocks (high σ_z), whereas wage indexation vanished when these shocks became less volatile in more recent periods. In contrast, changes in monetary policy, including anchoring trend inflation, only played a minor role in determining aggregate wage indexation for the two periods. This result relates to De Schryder et al. (2020), who use a panel dataset to estimate

[23] In Section D.2 of the Online Appendix, we show that these results are robust to i) allowing for a non-constant value of trend inflation — a feature that is absent in the model of Hofmann et al. (2012) — and (ii) setting price indexation to zero.

a wage Phillips curve equation with interaction effects. Their results suggest that monetary policy regime shifts were not a crucial driver of wage indexation changes.

6. Conclusion

In this paper, we have proposed a microfounded approach to endogenize the degree of wage indexation to past inflation in a New Keynesian DSGE model with sticky wages. In the presence of shocks, staggered labor contracts generate a gap between workers' desired labor supply and the level they actually face. From a worker's perspective, the preferred wage indexation rule is the one that minimizes this gap. We find that workers prefer to index their wages to past inflation when permanent shocks to technology and trend inflation drive output fluctuations. In contrast, when aggregate demand shocks dominate, workers prefer to index their wages to the central bank's inflation target.

Furthermore, we find that wage indexation at the decentralized equilibrium may differ drastically from the socially optimal level. In particular, we find that workers have an incentive to deviate from the social optimum. The resulting decentralized equilibrium is inefficient as workers do not consider the externalities of their decisions.

In the next step, we show that a suitable quantitative model correctly predicts changes in the degree of wage indexation in the US. In particular, the model's decentralized equilibrium predicts a high degree of wage indexation for the *Great Inflation*, and a low degree of wage indexation for the *Great Moderation*, which is consistent with the stylized facts for the US. In the Online Appendix, we explain this result by the fact that the *Great Inflation* saw highly volatile technology shocks, while during the *Great Moderation* aggregate demand shocks were relatively more important. Moreover, it is the relative importance of aggregate shocks that explain output fluctuations, not changes in monetary policy, that is a crucial driver of the observed changes in wage indexation in the US.

This paper partially responds to concerns about the lack of endogenous channels explaining price and wage inflation persistence (see for instance Benati, 2008). We suggest several directions for future research based on elements not addressed in our paper. First, as our framework uses rational expectations, implementing learning rules by agents (e.g., to learn about trend inflation) could be of interest. Second, instead of using a time-dependent model, a state-dependent model could be used for wage (and price) determination. Third, rather than assuming state-contingent securities, Heterogeneous Agent New Keynesian (HANK) models could be considered to address cases where income risk is not fully insurable.

CRediT authorship contribution statement

Julio A. Carrillo: Conceptualization, Methodology, Software, Writing – original draft, Writing – review & editing. **Gert Peersman:** Conceptualization, Writing – review & editing, Supervision. **Joris Wauters:** Conceptualization, Software, Writing – original draft, Writing – review & editing.

Appendix A. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.jmacro.2022.103417.

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