The Use of Linguistic Terms in Database Models

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Abstract

Classical database systems have been introduced in the late 50's and have proved their usefulness in various domains. However, their incompetence to deal with vague and imprecise information, has lead to new data base designs. On the other hand the use of linguistic terms has also shown its usefulness. The assignment of linguistic terms to phenomena in order to describe the characteristics or properties of objects is very natural. People make such assignments every day. A drawback of most new database designs is that often the natural aspect of making assignment is lost. In this paper we introduce a new database model based on quasi-order relations (reflexive and symmetric). The proposed model describes the mathematical background of the assignment of values to database attributes, using the theory of evaluation problems and sets. The constructed model offers an interesting new approach to the theory of database design in combination with linguistic terms.

1 Introduction

The use of linguistic terms to describe phenomena or objects is a very natural process. People make such assignment every day. Linguistic terms are used in a lot of databases to attribute characteristics to all kind of objects. For example, in a criminal investigation database, a witness tries to describe a criminal using linguistic terms such as *young, tall and rather skinny*. Classical database systems have proved their usefulness in various domains. Their incompetence, however, to deal with vague and imprecise information has lead to new database models. The attributing of linguistic terms still plays a key role in these new designs. However, these new designs are sometimes still too limited to describe the real world (similarity-based framework [2]) or are sometimes too complex for those that have to make the proper assignments (possibility-based framework [18, 17, 4]). In the possibility-based framework, possibility distributions, i.e., mappings from the considered domain to the unit interval, are used as the attribute values. Imagine a witness of a murder telling a police-officer that the possibility that the criminal is *rather short* is 0.3, *short* 0.4, *average* 0.5, *tall* 0.9 and *very tall* 0.7. In this case the use of linguistic terms together with possibility distributions seems to mask their natural character for making assignments. In several other databases, such as large scale soil databases, where soil scientists have to take note of soil differences and all kind of soil characteristics, the use of partial degrees is not always very easy and natural. Another important aspect of the use of linguistic terms is that they are not always sharply defined and are already vague descriptors themselves. Moreover, their meaning may differ for different individuals. People from the Mediterranean will say more quickly that a person's haircolor is *blond* than someone from a Scandinavian country. For a basketball player someone from 1.80 m is probably *rather short*, but for several other sporters this is a perfectly *normal length*. The database should be capable of handling these diferent usages of the same linguistic terms, otherwise inconsistencies arise and possible queries from users will return irrelevant data. A new model, based on quasi-order relations (reflexive and symmetric) allows to better describe the mathematical background of the assignment of values to database attributes. The model uses the theory of evaluation problems [10] which seems a more natural way to deal with the attributing, and especially the attributing of linguistic terms. The individual user profile will be reflected using fuzzy relations on the different domains corresponding to the different attributes.

2 Data Model Based on Quasi-Order Relations

2.1 Evaluation Problems and Evaluation Sets

Considering a set X and an evaluation problem P in X, i.e., we try to compare any two elements on the basis of their satisfying a certain property P. We can define a relation R_P on X as follows :

$$\begin{aligned}(\forall(x,y)\in X^2)(xR_Py \quad &\Leftrightarrow \quad x \text{ is at most as } P \text{ as } y\\ &\Leftrightarrow \quad y \text{ fulfills the property } P\\ & \quad \text{at least as well as } x)\end{aligned}$$

Definition 1 (Quasi-Order Relation)
Let X be an arbitrary set, then we call a binary relation R on X a quasi-order relation if and only if this relation is reflexive and symmetric.

We assume that the relation R_P is a quasi-order relation. This relation R_P is the ordinal representation of the information about the solution of the evaluation problem considered. We stress that it is not necessary for any two elements x and y of X to be comparable w.r.t. P, i.e. $(x,y)\in X^2$ need not belong to R_P or its inverse relation R_P^{-1}, defined as :

$$(\forall(x,y)\in X^2)(xR_P^{-1}y \Leftrightarrow yR_Px)$$

The relation R_P also need not be antisymmetric, i.e. :

$$(\forall(x,y)\in X^2)([(x,y)\in R_P \text{ and } (x,y)\in R_P^{-1}] \not\Rightarrow x=y)$$

Some other interesting relations related to the evaluation problem are the indifference relation I_P, the incomparability relation O_P and the strict preference relation S_P, defined as :

$$\begin{aligned}I_P &= R_P\cap R_P^{-1}\\ &= \{(x,y)|(x,y)\in X^2 \text{ and } (x,y)\in R_P\\ &\quad \text{ and } (y,x)\in R_P\}\end{aligned}$$

and

$$\begin{aligned}O_P &= \text{co}\,(R_P\cup R_P^{-1})\\ &= \{(x,y)|(x,y)\in X^2 \text{ and } (x,y)\notin R_P\\ &\quad \text{ and } (y,x)\notin R_P\}\end{aligned}$$

and

$$\begin{aligned}S_P &= R_P\cap(\text{co}\,(R_P^{-1}))\\ &= \{(x,y)|(x,y)\in X^2 \text{ and } (x,y)\in R_P\\ &\quad \text{ and } (y,x)\notin R_P\}\end{aligned}$$

The relation I_P clearly defines an equivalence relation on X. A corresponding equivalence class contains those couples that are considered equal for the given property P, i.e. those couples (x,y) for which x satisfies P equally well as y. The relation O_P contains those couples that are not related to each other w.r.t. P, i.e. O_P contains those couples that are incomparable w.r.t. P. The relation S_P contains those couples (x,y) of X^2 for which x satisfies P strictly less well than y. One easily verifies that R_P on the one hand and the triple (S_P, I_P, O_P) on the other hand are equivalent ways of representing the same information about the evaluation problem [**?**].

2.2 Evaluation Sets and Database

The formalism of evaluation sets can be linked in a rather convenient way to relational database models. The assignment of values for a certain attribute A_i can be seen as an evaluation problem in D_i.

Example

The assignment of an attribute *land use* A_i to a a plot location can be translated to an evaluation problem on D_i. What the soil scientist will do is to assign the proper values for the considered plots, is compare any two elements w.r.t. to the property (attribute) *land use*. This will lead to a quasi-order relation R_{A_i} :

$$\begin{aligned}(\forall(x,y)\in(D_i)^2)(xR_{A_i}y \quad &\Leftrightarrow \quad x \text{ describes the land use}\\ &\quad \text{of the plot location at}\\ &\quad \text{most as well as } y)\\ &\Leftrightarrow \quad y \text{ describes the land use}\\ &\quad \text{of the plot location at}\\ &\quad \text{least as well as } x)\end{aligned}$$

Since a database contains multiple records (soil mapping units), analogous evaluation problem are to be considered for the different records.

$$\begin{aligned}(\forall(x,y)\in(D_i)^2)(xR^1_{A_i}y \quad &\Leftrightarrow \quad x \text{ describes the land use}\\ &\quad \text{of plot location 1 at}\\ &\quad \text{most as well as } y)\\ (\forall(x,y)\in(D_i)^2)(xR^2_{A_i}y \quad &\Leftrightarrow \quad x \text{ describes the land use}\\ &\quad \text{of plot location 2}\\ &\quad \text{at most as well as } y)\end{aligned}$$

Definition 2
Let X be an arbitrary set, then $\mathcal{Q}(X)$ is the set of all quasi-order relations on X, i.e.

$$(\forall R\in\mathcal{Q}(X))(\forall x\in X)((x,x)\in R) \qquad \textit{(reflexive)}$$
$$\begin{aligned}(\forall R\in\mathcal{Q}(X))(\forall(x,y,z)\in X^3)((x,y)\in R\\ \text{and } (y,z)\in R\Rightarrow(x,z)\in R) \qquad \textit{(transitive)}\end{aligned}$$

Definition 3 (Closeness Relation)
A closeness relation c_i on a domain D_i is a fuzzy relation on $D_i\times D_i$ to $[0,1]$ that is reflexive and symmetric.

As mentioned above, we acknowledged the fact that linguistic terms are not always sharply defined and their meanings may sometimes overlap. Therefore a closeness relation is introduced to show the relationship between the domains elements. Keeping all this in mind, we propose the following definition.

Definition 4 (DBMS - Quasi-Order Relations)

Let R(U) be a relational scheme on the domains $D_1, D_2, \ldots, D_n$, where U is the set of all attributes $A_1, A_2, \ldots, A_n$ on $D_1, D_2, \ldots, D_n$ with domain(A_i) = $D_i, i = 1, 2, \ldots, n$. With every domain D_i we associate a closeness relation c_i. A n-ary relation R of $R(U)$ is then a subset of the Cartesian product $\mathcal{Q}(D_1) \times \mathcal{Q}(D_2) \times \ldots \times \mathcal{Q}(D_n)$, i.e. $R \subseteq \mathcal{Q}(D_1) \times \mathcal{Q}(D_2) \times \ldots \times \mathcal{Q}(D_n)$. An n-tuple t of R is then of the form : $t = (R_{A_1}, R_{A_2}, \ldots, R_{A_n})$ with $R_{A_i} \in \mathcal{Q}(D_i), i = 1, 2, \ldots, n$.

In this model we can have tuples $t = (R_{A_1}, R_{A_2}, \ldots, R_{A_6})$ where for instance :

R_{A_3}	well	mod.	poor	very poor
well	1	1	1	1
mod.	0	1	1	1
poor	0	0	1	0
very poor	0	0	0	1

2.3 Tuple Equality

As mentioned above it is very important to be able to compare individual records. Once tuple equality is properly defined, topics related to data redundancy can be tackled.

Since a record, in the database model, is a tuple of quasi-order relations, first of all a formula for expressing equality of two quasi-order relations $R^{\alpha}_{A_i}$ and $R^{\beta}_{A_i}$ on the domain D_i must be constructed. Because a quasi-order relation on D_i is in fact an ordinary subset of $D_i \times D_i$ the characteristic maping of the equality relation of the two quasi-order relations could be readily defined as :

$$
\begin{aligned}
E(R^{\alpha}_{A_i}, R^{\beta}_{A_i}) = 1 \quad &\Leftrightarrow \quad (\forall (x,y) \in (D_i)^2) \\
&\qquad ((x,y) \in R^{\alpha}_{A_i} \Leftrightarrow (x,y) \in R^{\beta}_{A_i}) \\
&\Leftrightarrow \quad (\forall (x,y) \in (D_i)^2) \\
&\qquad (x R^{\alpha}_{A_i} y \Leftrightarrow x R^{\beta}_{A_i} y) \\
E(R^{\alpha}_{A_i}, R^{\beta}_{A_i}) = 0 \quad &\Leftrightarrow \quad \neg(\forall (x,y) \in (D_i)^2) \\
&\qquad ((x,y) \in R^{\alpha}_{A_i} \Leftrightarrow (x,y) \in R^{\beta}_{A_i})
\end{aligned}
$$

Reminding that with every domain D_i there is associated a closeness relation c_i and taking into account that we are working with vague, imprecise information, this approach seems to harsh. Therefore, below we present a general formula to express equality of two relations between domains X and Y, keeping in mind that with each domain a closeness relation is associated. Once this is defined, the equality of two tuples can be readily derived.

The general idea, used to define the equality of relations R^{α} and R^{β} between X to Y is that a relationship R^{α} in a couple (x, y) should be preserved in R^{β} and vice versa. When closeness relations are involved there should exist a relationship R^{β} between close enough or comparable elements (x', y') and vice versa, e.g. a relation R^{α} between (x, y) can be partially matched by a relation R^{β} between (x', y') if the couples (x, y) and (x', y') are comparable (closeness relation). In what follows we shall make this idea more precise.

Definition 5 (Implication Operator) *An implication operator $\rightarrow$ is a mapping from $[0,1] \times [0,1]$ to $[0,1]$ which corresponds to the classical implication in the corner points of its domain, i.e. $0 \rightarrow 0 = 1$, $0 \rightarrow 1 = 1$, $1 \rightarrow 0 = 0$ and $1 \rightarrow 1 = 1$.*

Let R^{α}, R^{β} be relations between the finite domains X and Y. We implicitly assume $c_X = \mathrm{Id}_X$ and $c_Y = \mathrm{Id}_Y$, i.e., the closeness relations on X, respectively Y are the identity relations. Then we define the implication between relations R^{α} and R^{β} as :

$$
R^{\alpha} \rightsquigarrow R^{\beta} = \begin{cases} 1 & \text{if } (\forall (x,y) \in X \times Y)((x,y) \in R^{\alpha} \Rightarrow (x,y) \in R^{\beta}) \\ 0 & \text{otherwise} \end{cases}
$$

what can be rewritten as :

$$
\begin{aligned}
R^{\alpha} \rightsquigarrow R^{\beta} &= \min_{(x,y) \in X \times Y} \max[R^{\beta}(x,y), (\mathrm{co}\, R^{\alpha})(x,y)] \\
&= \min_{(x,y) \in X \times Y} \max[1 - R^{\alpha}(x,y), R^{\beta}(x,y)]
\end{aligned}
$$

Using an implication operator $\rightarrow$ we may write down the more general expression :

$$
R^{\alpha} \rightsquigarrow R^{\beta} = \min_{(x,y) \in X \times Y} [R^{\alpha}(x,y) \rightarrow R^{\beta}(x,y)]
$$

Definition 6 (Triangular Norm) *A triangular norm τ is mapping from $[0,1] \times [0,1]$ satisfying commutativity, associativity, non-decreasingness and the boundary condition $(\forall x \in [0,1])(\tau(x,1) = x)$ [19].*

Definition 7 (Comparability of Ordered Pairs)

Let X and Y be arbitrary sets, c_X and c_Y closeness relations on X, respectively Y. Furthermore, let τ be *a T-norm, then we can define the comparability of the two couples (x,y) and (x',y') as :*

$$c_{X\times Y}((x,y),(x',y')) = \tau[c_X(x,x'), c_Y(y,y')]$$

One readily verifies that the relation $c_{X\times Y}$ is a closeness relation on $X \times Y$.

Let us now take $(c_X, c_Y) \neq (\mathrm{Id}_X, \mathrm{Id}_Y)$ and follow the general idea that a relationship R^α in (x,y) should imply a relationship R^β between comparable elements (x',y').

The degree to which there is a relationship R^β between (x,y) or comparable elements (x',y') can readily be defined as :

$$R^\beta_{c_{X\times Y}}(x,y) = \max_{(x',y')\in X\times Y} \tau[R^\beta(x',y'), c_{X\times Y}((x,y),(x',y'))]$$

or equivalently

$$R^\beta_{c_{X\times Y}}(x,y) = \max_{(x',y')\in X\times Y} \tau[R^\beta(x',y'), c_X(x,x'), c_Y(y,y')]$$

Definition 8 (Envelope of a relation)
Let R^β be a relation between X and Y, c_X and c_Y be closeness relation on X, respectively Y, then we call the relations $R^\beta_{c_{X\times Y}}$ the envelope of relation R^β according to $c_{X\times Y}$.

Note that, if we denote $R^\beta_{c_{X\times Y}}$ as $c_{X\times Y}(R^\beta)$, $c_{X\times Y}$ can be interpreted as an operator which maps any *relation* R^β between X and Y to the *fuzzy relation* $R^\beta_{c_{X\times Y}}$ between X and Y. Moreover, there is nothing that prevent us from considering the image $c_{X\times Y}(S)$ of a fuzzy relation S between X and Y, as follows. For any x in X and y in Y :

$$c_{X\times Y}(S)(x,y) = \max_{(x',y')\in X\times Y} \tau(c_X(x,x'), S(x',y'), c_Y(y,y'))$$

Clearly, $c_{X\times Y}$ can then be interpreted as an operator on the set of the fuzzy relations between X and Y. Note that, since c_X and c_Y are reflexive,

$$S \sqsubseteq c_{X\times Y}(S) \tag{1}$$

where we write $S_1 \sqsubseteq S_2$ iff $(\forall (x,y) \in X \times Y)(S_1(x,y) \leq S_2(x,y))$. This justifies our calling $c_{X\times Y}(S)$ an *envelope* of S. Moreover, if S_1 and S_2 are fuzzy relations between X and Y then clearly

$$S_1 \sqsubseteq S_2 \Rightarrow c_{X\times Y}(S_1) \sqsubseteq c_{X\times Y}(S_2) \tag{2}$$

since both τ and max are monotone operators. Also, it is interesting to note that if τ is distributive w.r.t. max

$$c_{X\times Y}(S_1 \cup S_2) = c_{X\times Y}(S_1) \cup c_{X\times Y}(S_2)$$

where $S_1 \cup S_2$ is the union of the fuzzy relations S_1 and S_2 between X and Y, i.e., for any x in X and y in Y

$$(S_1 \cup S_2)(x,y) = \max(S_1(x,y), S_2(x,y))$$

Finally, if c_X and c_Y are in particular τ-transitive (and therefore τ-equivalence relations, since they are already reflexive and symmetric) and τ is distributive w.r.t. max, then

$$c_{X\times Y}(c_{X\times Y}(S)) = c_{X\times Y}(S) \tag{3}$$

(1), (2) and (3) imply that $c_{X\times Y}$ is a *fuzzy closure* operator [14].

Theorem 1
Let R^β be a relation between X and Y, where c_X and c_Y are closeness relations on X, respectively Y, Let τ be a t-norm distributive w.r.t. max*, then the envelope of R^β according to $c_{X\times Y}$ is equal to $c_X \circ R^\beta \circ c_Y$, where $\circ$ is the round composition (max-τ) of fuzzy relations.*

Considering again an implication operator $\rightarrow$, we can define the degree to which R^α implies R^β as :

$$R^\alpha \rightsquigarrow_{C_{X\times Y}} R^\beta = \tau_{(x,y)\in X\times Y}[R^\alpha(x,y) \rightarrow R^\beta_{c_{X\times Y}}(x,y)]$$

Note that $\rightsquigarrow_{C_{X\times Y}}$ can be considered as a binary fuzzy relation on the domain of the (fuzzy) relations between X and Y.

Theorem 2
Let c_X and c_Y be similarity relations on X, respectively Y. Furthermore, consider an implication operator satisfying $(\forall b \in [0,1])(0 \rightarrow b = 1 \text{ and } 1 \rightarrow b = b)$ and take minimum as t-norm. The fuzzy relation $\rightsquigarrow_{C_{X\times Y}}$ is then min-transitive.

We note that some of the popular implication operators, such as the Gaines 43 implication operator, the Lukasiewicz implication operator and the Kleene-Dienes implication operator satisfy the desired property.

The equality of the relations R^α and R^β can be defined as the following fuzzy relation :

$$E_{c_{X\times Y}}(R^\alpha, R^\beta) = \tau(R^\alpha \rightsquigarrow_{C_{X\times Y}} R^\beta, R^\beta \rightsquigarrow_{C_{X\times Y}} R^\alpha)$$

Theorem 3 *The equality relation $E_{c_{X\times Y}}$ as defined above is a fuzzy relation on the domain of the relations between X and Y which is both reflexive and symmetric.*

Theorem 4
Let again c_X and c_Y be similarity relations on X, respectively Y, $\rightarrow$ an implication operator satisfying

$(\forall b \in [0,1])(0 \rightarrow b = 1 \text{ and } 1 \rightarrow b = b)$ *and take minimum as t-norm, then the operator* $E_{c_{X\times Y}}$ *is an equivalence relation.*

Finally, let $R^{\alpha}_{A_i}$ and $R^{\beta}_{A_i}$ be two quasi-order relations on the domain D_i, i.e. $R^{\alpha}_{A_i}$, $R^{\beta}_{A_i} \in \mathcal{Q}(D_i)$. Furthermore, let c_i be a closeness relation on D_i. The degree to which these two quasi-order relations are equal is then :

$$E_{c_i}(R^{\alpha}_{A_i}, R^{\beta}_{A_i}) = \tau(R^{\alpha}_{A_i} \leadsto_{c_i} R^{\beta}_{A_i}, R^{\beta}_{A_i} \leadsto_{c_i} R^{\alpha}_{A_i})$$

The envelope of the quasi-order relation $R^{\beta}_{A_i}$ according to closeness relation c_i becomes a sort of convolution of $R^{\beta}_{A_i}$ with c_i, i.e. $(R^{\beta}_{A_i})_{c_i} = c_i \circ R^{\beta}_{A_i} \circ c_i$.

Definition 9 (Tuple Equality)
Given two tuples $t^{\alpha} = (R^{\alpha}_{A_1}, R^{\alpha}_{A_2}, \ldots, R^{\alpha}_{A_n})$ *and* $t^{\beta} = (R^{\beta}_{A_1}, R^{\beta}_{A_2}, \ldots, R^{\beta}_{A_n})$ *and closeness relations* $c = (c_1, c_2, \ldots, c_n)$ *where* $c_i \in \mathcal{Q}(D_i)$ *then the degree wherein the tuples* t^{α} *and* t^{β} *are equal is defined as :*

$$\mathcal{E}_c(t^{\alpha}, t^{\beta}) = \tau(E_{c_1}(R^{alpha}_{A_1}, E_{c_2}(R^{\beta}_{A_1}), \ldots, E_{c_n}(R^{\alpha}_{A_n}, R^{\beta}_{A_n}))$$

3 Application : Individual User Profile

The above defined equality operator can also be used to reflect an individual understanding of the used linguistic terms. When somebody launches a query to a database, where this query contain certain linguistic components, the user's personal interpretation of these components should be clear for the retrieval mechanism, otherwise the user will get information that does not match *his* selection criteria. In this case the *closeness* relations, associated with the different domain elements, should somehow reflect the user's interpretation of the linguistic terms. Therefore, the relation c_i from D_i to D_i should show the relationship between the interpretation of the domain elements for a specific user and their *intrinsic interpretation* as used in the stored data :

$c_i(x, y)$ = the degree to which the use of x by the user corresponds to the intrinsic meaning of y as used in the stored data

Considering again the example of a person's haircolor in the introduction it can be easily seen that such a relation c_i will in general not be a closeness relation. The proposed equality operator, with some modification, can still be used to match the descriptions of an individual with those descriptions stored in the databases. The necessary relations, i.e., the relations $c_i, i = 1, \ldots, n$, can be easily obtained from the assignments of indiviual user(s) made for a number of test objects.

However, further research has to be done to fully highlight the possibilities of the above defined equality operator.

4 Conclusion

Evaluation problems and evaluation sets offer a nice starting point for a mathematical description of the process of attribute assignment. This approach seems to be especially useful when the domains of the corresponding attributes consist of linguistic terms. The associated closeness relations reflect the relationship between the linguistic terms. Furthermore, closeness relations offer the possibility to overcome inconsistencies in large scale databases and to flatten minor differences when comparing tuples. Since the defined equality operator for database tuples $\mathcal{E}_c$ is both reflexive and symmetrical, it should be possible to extend the definition, properties and theorems related to redundancy [5, 6]. However, further research is needed, e.g. about the properties of the defined equality operator. Finally, we remark that it is possible to extend the proposed data model to a more general one using fuzzy quasi-order relations. The given definitions and operators can be readily extended, leading to a more general framework for describing vague information. The design of yet another model for describing vague information should make us realize that somehow a more general framework should be developed. Different kinds of imprecise information often ask for different models. These models are fit to describe and handle this specific kind of information. However, an important issue will be to combine it all.

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