
THE LOGIC OF LAW

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Introduction

‘Law’, in the sense in which I shall use the word here, denotes an order of persons.¹ Within this general concept, we can distinguish between natural orders and artificial orders. Natural order, that is natural law, is the order of natural persons. Artificial order, often referred to as positive law, is an order of artificial persons. In the terminology of Rousseau, natural persons are physical persons (*personnes physiques*), while artificial persons are legal persons (*personnes morales*).² Artificial persons are positions, roles or functions in a system of rules, which defines a particular game, organization or society. The rules of the game or society tell us what those artificial persons are, and what they can and cannot do. Examples are White and Black in a game of chess as well as their subdivisions, King, Queen,

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¹ See F. van Dun, “The Lawful and the Legal”, *Journal des économistes et des études humaines*, VI, 4, 1996, 555–579. The present paper is a formal analysis of the figure on p. 575 of that article, and an elaboration of the argument first presented in a much older paper (in Dutch), “De logische structuur van het recht”, in B. Bouckaert, e.a. (eds), *Liber Amicorum Willy Calewaert: Recht en Criminaliteit* (Kluwer Rechtswetenschappen: Antwerpen, 1984), 87–122. For a comment on the fundamentally different concept of law as an order of actions, see the appendix to this paper.

² J.-J. Rousseau, *Du Contrat Social* (1762), Livre II, chapitre 7, where Rousseau discusses the need to transform human beings into citizens in these terms: “substituting a partial and moral existence for the physical and independent existence nature has conferred on us all.” From G.D.H. Cole’s translation, *The Social Contract* (J.M. Dent and Sons: London, 1923)

Knights, Pawns, etcetera. Other examples are Rector, Dean, Student, Faculty member, etcetera in a university, or King, Government, Parliament, Citizen, Chief of Staffs, Registered Alien, City Council, etcetera in the statutes and other legal rules of a nation state.

It is immaterial for understanding the concept of an artificial person that usually it takes a natural person (a human being) to play the role, occupy the position or perform the function of an artificial person. It is said that the Roman Emperor Caligula wanted to make his horse a Consul of Rome. Had he done that, it would not have changed the positive, legal rights and duties of a Roman Consul. What a King in chess is or can do is independent of whether the game is played by human chess players or by computer programs. Usually, one chess player moves all the pieces of a given color. However, in other rule-defined systems, two or more human individuals may occupy the same position or perform the same function (as when two part-timers occupy the position of Secretary). There is no direct connection between the order of human persons and an order of artificial persons, except for the fact that orders of the latter type are created or built up over time by human persons.

In this paper, I shall not discuss the methodological and ontological issues attending the distinction between social orders of artificial persons or personified social positions and the convivial order of natural persons.³ Our focus is on the general concept of an order of persons (whether natural or not). What follows is primarily a formal axiomatization of that concept.

A Formal Approach

Why formalize?

A formal approach is especially useful to immunize reasoning from the almost inevitable pitfalls of thinking when it uses a natural language. In a natural language, the same word may be used to express a variety of meanings, sometimes related, and sometimes unrelated. Likewise,

³ See my, "The Lawful and the Legal", op.cit.; also "Concepts of Order" in H. Bouillon & H. Kliemt (eds), *Ordered Anarchy, Jasey and His Surroundings* (Ashgate Publishing: Aldershot, 2007), p. 59-92.

different words can have more or less the same meaning. Beside their dictionary meanings, words also carry connotations that may vary from one language user to another and from context to context. A speaker or writer can hardly be aware of all the ways in which members of his audience might interpret his words.

Most natural languages have a variety of ways for dealing with issues of quantification, generality, and so on, but not always a systematic way. In ‘A dog is a mammal’ and in ‘A dog was lost during the trip’ the expression ‘a dog’ refers to different quantities of dogs. ‘You should talk to him’ neither implies nor excludes that you should talk *only* to him. ‘It is my property’ neither implies nor excludes that the property is exclusively mine. Similarly, ‘You are dealing with a person’ neither implies nor excludes that you are dealing with a natural, not an artificial person. ‘You are responsible for your property’ neither implies nor excludes that you are responsible for what another does with your property. In themselves, such utterances are ambiguous, but one who makes them will usually intend them to have a particular meaning. He may be wrong in assuming that all the people in his audience share or know about his habits of speech — and wrong in assuming that only his habits of speech make sense.

Consequently, the message sent is not always the message received — and for every message sent, there may be at least as many different messages received as there are people in the speaker’s audience. Both agreement and disagreement between a speaker and a person in his audience can be spurious as well as genuine, and it may take a long and in-depth exchange of questions and answers before the matter can be settled, if it can be settled at all. Two people may agree on the definition of the concept of man as “a rational animal”. It does not follow that they agree on the definitions of the concepts “rational” and “animal”, on the sorts of things that these concepts cover, or on the criteria and methods for determining whether this or that particular thing is of the requisite sort. It is not always easy — occasionally nearly impossible — to determine just at what level of discourse there is agreement or disagreement.

Natural languages are all-purpose languages. They serve as a common medium of communication among an indefinite number of persons in all sorts of situations. They are not particularly suited for highly specialized tasks, such as rigorous theory construction, analysis

and criticism. Moreover, not all theorists who present their theories in a natural language take the precaution of explicitly defining their terminology (and the concepts to which it refers) with the precision required to eliminate confusion or misinterpretation even in the mind of an attentive reader, full of goodwill, wholly intent on discovering what the author has to say.

In fields that are highly controversial and contested, goodwill may be a rare disposition. Discussants in such fields often use the same terminology with widely different meanings and theoretical associations. How is one to find out whether opponents are saying different things about the same subject, the same things about different subjects, or different things about different subjects? Not surprisingly, arguments couched in a natural language can be regarded as definitive proofs by one side in a discussion, while being dismissed as irrelevant or ignorant by the other side.

Formalization avoids such complications of the use of natural language by substituting symbols that have none of its distractions and that take their meaning exclusively from the definitions supplied in the formalization itself. Hence, a formalization makes it possible to get a clear view of the logical relations between the formal constructs defined in it. This helps us to determine whether disagreements in a discussion reflect differences in the logical basis of the arguments, the semantics or rules of correspondence that relate it to things and events in the real world, or merely the terminology in which it is expressed. Formalization can render this service by providing a common, exact language in which one can express the central tenets of all or most of the opposing positions under discussion.

What we can formalize is not the real world or any of its parts, but our ways of thinking and speaking about it. We should make sure that we know as precisely as possible what people are saying before we praise or criticize their saying it. As understood here, formalization is not the same as *formal modeling*. It does not pretend to simulate, let alone predict, how things behave under various conditions. Nevertheless, it is a valuable, sometimes indispensable tool in any serious intellectual undertaking, an aid to disciplining our thinking and speaking. An adequate formalization may show unambiguously (i.e., prove) that a given set of beliefs is consistent or inconsistent. It may show that the set is more (or less) complex than those who refer to it assume, or that it

answers fewer (or more) questions than some discussants allege it does. It may show that one's arguments rely on more or less subtle shifts in the definitions of concepts that are not reflected in the terminology, or that they presuppose a common logical basis for different beliefs where there is none; and so on.

The formalization in the next sections is intended to capture the common logical core of most material theories of law in the Western tradition — those of Locke, Hobbes, Rousseau, Kelsen, Rothbard, and others — to the extent that they are theories of the status in law of a person and the means (and actions) that belong to him. It does not deal with the semantics of such theories — i.e., with questions concerning the things that they consider persons or means, or the criteria they employ for ascertaining the truth of the statement that a means belongs to a person, etcetera. Differences between any two of them that are only in the semantics are beyond its scope, but differences in the formally distinguishable kinds of persons they admit as denizens of the order of law are not. Moreover, the formalization is not intended to, and cannot, answer questions such as “Does anything really belong to a person?” or “Is there anything out there that corresponds to the concept of a person?” Formalization does not render rational criticism, philosophy or science superfluous, but it can make them much more effective.

Separating logic and semantics

Of course, the relevance of a formalization appears only when its formal constructs can be and are interpreted in terms of real things and relations. If such an interpretation conflicts at any point with previously held beliefs (or prejudices) about such things or relations, one should consider whether to revise the formalization, or the beliefs themselves or maybe just the customary way of expressing them. The formalization should in any case be consistent, but it may be consistent and yet inadequate for a given purpose.

We can interpret the formal theory of law specified below by stipulating rules of correspondence that link its terms to various classes of things, events, actions, or relations in what the interpreter assumes to be the real world. Obviously, there may be an innumerable multitude of different, more or less plausible interpretations. I shall call such interpretations *material theories of law*, but apart from a few references

to Hobbes and Rousseau, I shall consider only certain *classes* of them towards the end of this paper. I assume that all or nearly all of the better known and historically influential material theories of law are or include interpretations of the formal theory that I shall set out below. To the extent that that assumption is true, the formal theory is a tool for identifying the logical basis of the differences among material theories as well as their similarities, regardless of the terminologies they use and the examples they give (often the only pointers to the interpretation intended by their authors).

Our interest in law obviously derives from our concerns about human beings and their relations. The same concerns motivate our attempt to develop a formal theory of law. However, the formal system does not and cannot fix a priori the semantics of ‘person’ or any other term of the formal language. The common understanding of the word ‘person’ is that it refers to a human being, but in some disciplines (for example theology, political, legal and social theory) many other things such as gods, demons, states, societies, social positions, communities, and the like, are personified or taken to be persons. Similarly, the common understanding of ‘natural person’ is that it too refers to a human being. Again, some disciplines or philosophies may assume that the formal characteristics of what I here call ‘a natural person’ apply to other sorts of things as well as to human beings. For example, a legal-positivistic theory of law may even stipulate that only sovereign states viewed as legal systems are persons *per se* — persons because of what they are (and at least in that sense, persons “by their own nature”) — and that human beings can be considered as persons only if the state’s legal system recognizes them as such. However, my aim here is not to judge the merits or demerits of particular material theories of law from a semantic (let alone a pragmatic) point of view. I merely want to elucidate their common logical forms and how these can be used to express particular conceptions of law as an order of persons.

Overview of the argument

I proceed from a formal analysis of the concept of *law as an order of persons and the means that belong to them* to an analysis of the concept of an order that includes natural persons and the means that naturally belong to them. From the notion of order among natural persons (natural law), I move on to the notion of order among human persons

(human law). Each step in this procession requires the introduction of a new concept, yet no addition requires a revision of the previously analyzed concept of order — it enriches but does not distort.

In the first section, I define a formal language using only the resources of first order predicate logic, extended with the logic of the identity of objects. Using that language, I then formulate a general theory or logic of law as a formal theory of order among persons and the means that belong to them. It is a general logical theory in the sense that it is very nearly a pure logic of the binary predicator 'x belongs to y'. Indeed, the general theory has only two axioms that restrict the possible interpretations of that predicator.⁴ By introducing new predicators — all of them defined in terms of the basic 'x belongs to y' — we can extend the vocabulary of the formal theory and rephrase its theorems in a concise manner that reveals its relevance for the philosophy of law. For example, I define the notions of 'a person' and 'a means' syntactically in terms of the relation 'x belongs to y'. The only presuppositions of the theory are that there may be things that are means but not persons, and that if 'A belongs to B' is true, then B must be a person while A must be a person or a means. A noteworthy result of this section is the partition of the class of persons in three mutually exclusive but collectively exhaustive classes, which we shall call 'sovereign', 'strictly autonomous,' and 'heteronomous'. In other words, the status in the law of a person is always given by his being a member of one and only one of these classes.⁵

⁴ A *primitive* expression of a formal system obviously cannot be defined explicitly *within the system*. It can only be defined *implicitly* by listing a number of 'axioms', i.e., formulas that turn into true propositions under any intended interpretation. I cannot vouch for all the connotations the phrase 'belongs to' evokes in a reader's mind. I only ask of the reader either that he bracket-out those connotations that do not fit the axioms or that he substitute another phrase that does fit the axioms and preserves the link with the class of intended interpretations (in this case, those that are organized as material theories of law).

⁵ Be careful not to substitute 'human being' for 'person' here. The formal theory is devoid of semantic relations to the world we know. A material theory of law need not recognize all or even any human beings as 'persons'. It may assign all or some human beings to the class of 'mere means' or even 'mere objects'.

Next, I consider an extension of the general theory to provide a formal framework for referring to actions. I introduce a couple of predicators that refer to relations between ‘actions’ and ‘means’ or ‘persons’. Again, there are just a few axioms that restrict the possible interpretations of those predicators. I conclude the first section with the formulation of a *general principle of justice*. Because the principle contains a predicator (‘is innocent’) that is not defined *in* the general theory, it is neither a theorem nor the negation of a theorem of that theory: its acceptance or rejection in no way affects the coherence of the general theory.

In the second section, I introduce another basic binary predicator, ‘x naturally belongs to y’, that I define implicitly by formulating a few axiomatic constraints. The main interest of that section is the formal definition of the predicator ‘is a natural person’. This gives us a formal perspective on the notion of natural law as an order of natural persons. Of course, the formal theory does not allow us to derive a theorem stating that there are natural persons. Neither does it have a theorem that determines the status in the law of all or some natural persons. From a formal point of view, there is no logical relation between ‘x belongs to y’ and ‘x belongs naturally to y’.

I then try to flesh out the natural-law perspective by introducing a number of propositions that together make up a naturalistic filter and a principle of natural justice. We may think of the former as *postulates of natural law*. Like the general principle of justice mentioned earlier, these postulates and the principle of natural justice are independent of the formal theory, but they formally link the general theory of order among persons to the theory of order among natural persons (the general theory or logic of natural law). Note that this theory of natural law is as formal as the general theory of law.

Because of the formal independence of the constitutive relations ‘x belongs to y’ and ‘x belongs naturally to y’, natural persons can be assigned the status of a sovereign, a strictly autonomous or a heteronomous person, or even the status of a non-person, either a mere means or a mere object. Leaving aside the last-mentioned status, theories

of law can be classified in 15 different types, according to their originary⁶ distribution of natural persons over the four types of status.

We can identify types of theories that conflict with the postulates of natural law or the principle of natural justice. We find that political and legal theories of natural law (which accommodate the notion of an originary right to unilateral rule or legislation) deny the principle of natural justice. Among the natural-law theories that do not deny that principle, we can identify those that fail to satisfy the condition that all natural persons have the same originary status in law — the condition of equality in the law. Like “natural justice”, “equality in the law” is neither implied nor denied by the formal theory; but no less than “justice”, it is often proclaimed a fundamental value of law. The main result of this exercise in filtering out various types of theories of natural law is that we can identify a single type that satisfies both conditions: “justice” and “equality in law”. This type encompasses all material theories that assign to every natural person the originary status of a sovereign self owner. Because every step in the derivation of this result was formalized, a critique of that type of theory can focus on the axioms, postulates and principles used in the derivation.

In the third section, I introduce a formal placeholder for human beings into the framework of the theory of natural law, so that we can make explicit the possible relations between the class of natural persons and the class of human beings. Obviously, not all material theories of law posit the same relation between those classes. I conclude the analysis with a brief look at the different ways in which the theory of human law can be integrated formally into the general theory. Note that theories that satisfy the requirements of “natural justice” and “equality” fit the individualist-libertarian paradigm of law, if their semantic rules

⁶ ‘Originary’ refers to the status that is presumed to be a person’s normal definitive status. Obviously, certain events may defeat the presumption, such as, in the case of a human person, his committing a crime, becoming insane, being manumitted or adopted by persons of another status. (Because of biological constraints, babies and children are not considered persons in the full sense. Yet, most material theories of law assume that being born or even conceived sets new human beings on a path that, *in the normal course of events*, will lead them into a status that can be predicted from their earliest moments. Hence, people can claim to be *born free*, *born a citizen* of this or that state, or *born a slave*.)

validate the proposition that any human being is normally a natural person.

SECTION I

The General Theory

The system L_0 .

The binary predicator $\mathbf{Bo:oj}$ denotes the basic or primitive relation of the formal system L_0 we are about to construct. In view of the intended interpretations, we read it as ‘ o_i belongs to o_j ’.⁷ Another primitive predicator is \mathbf{Io} , which we read as ‘ o is innocent’. It will be some time before we find a use for it, but we might as well introduce it here. In addition, the standard expression for the identity of objects, $\mathbf{o_i=o_j}$, is a primitive expression of L_0 .

⁷ Occasionally other readings, such as ‘ o_i is property of o_j ’, ‘ o_j has a property right in o_i ’, ‘ o_j is responsible or answerable for o_i ’ or some other appropriate expression, may be used. *Such variants merely serve a stylistic purpose.* By stipulation, all of them are used here as interchangeable ways of reading the formula ‘ Bxy ’ of the formal theory. I am not suggesting that those expressions cannot or should not be used to refer to different relationships in any material theory of law that uses them. Note, however, that I do *not* use ‘ y owns x ’ to express Bxy (see note 10 below).

In suggesting readings for the formulas, I have tried to find terms that are in rough conformity with accepted usage. Nevertheless, there are some cases where no common term is available. In other cases, the formal approach reveals distinctions that are not commonly made in ordinary language. For those cases, we can do little more than make up distinguishing labels as the need arises. Remember that we are concerned with the formal structures we use in our thinking, not with the idiosyncrasies of a natural language or any of its users. The argument is in the formalization, not in the suggested readings.

To avoid a clutter of quotation marks, I shall simply write *person* rather than ‘*person*’ (with quotation marks to indicate that what is meant is: ‘person’ or whichever noun you prefer for what you believe is the appropriate referent of y in ‘ x belongs to y ’); *mutatis mutandis* for ‘means’ and other terms.

The well-formed formulas (henceforth wff's) of L_0 are defined recursively as follows. $Bo_i o_j$, Io_i and $o_i = o_j$ are wff's if each of o_i and o_j stands either for a variable or a constant name referring to an object in the domain $D(L_0)$. If F is a wff then so is the negation of F [written as ' $\sim F$ ', read as 'not F ']. Also, if F is a wff and ' v ' a variable name referring to an object in $D(L_0)$, the universal quantification of F [written as ' $(\Delta v)F$ ', read as 'for all v , F '] and the particular quantification [written as ' $(\nabla v)F$ ', read as 'For at least one v , F ' or 'There is at least one v for which F is true'⁸] are wff's. If F and G are wff's then the conjunction of F and G [written as ' $F \& G$ ', read as 'F and G'] and the adjunction of F and G [written as ' $F \vee G$ ', read as 'F or G'] are wff's. Also, the implication of G by F [written as ' $F \rightarrow G$ ', read as 'F only if G' or 'if F then G'] and the equivalence of F and G [written as ' $F \leftrightarrow G$ ', read as 'F if and only if G'] are wff's. Finally, we shall use round and occasionally square brackets to group wff's and so indicate the priority of the logical operators.⁹

A formula is closed if there are no free variables in it. A variable is free unless a quantifier binds it. For example, suppose that F is a closed formula of L_0 and that ' o ' is a constant name referring to a particular object in $D(L_0)$; then the variable v is free in ' $F \& Bo v$ ' but not in ' $(\Delta v)(F \& Bo v)$ ' or ' $F \& (\nabla v)(Bo v)$ '. A closed formula of L_0 is called a proposition.

The variables o and o' in $Bo o'$, $o = o'$ or Io range over a domain $D(L_0)$ of objects. Within that domain, we distinguish between mere objects and objects of a special kind. Among the latter, we make a

⁸ In logic, 'there is' is to be distinguished from 'there exists'. Thus, 'A is' says that A has being, not necessarily that it has existence (in the sense that it can be located in space or time). For example, there are numbers (even if, unlike trees or clouds, they are not existents).

⁹ To save on the use of brackets, we adopt the following conventions. 1) We leave out the outermost brackets: we write ' F ' instead of ' (F) '. 2) No brackets are used for elementary formulas: for example, we write ' $Bo_i o_j$ ' instead of ' $(Bo_i o_j)$ ' and ' $\sim Bo_i o_j$ ' instead of ' $\sim(Bo_i o_j)$ '. 3) By default, the implicator ' \rightarrow ' is the first operator to consider for parsing a formula. Thus, we should read ' $A \& B \rightarrow C \vee D$ ' as if it were written ' $(A \& B) \rightarrow (C \vee D)$ '. 4) By default, the negator ' \sim ' is the last operator to consider in parsing a formula. Thus, we should interpret ' $\sim A \& B$ ' as if it were written ' $(\sim A) \& B$ '.

distinction between ‘means’ and ‘persons’. They are defined in terms of the relation Boo' :

$$\text{(SD1)} \quad \text{Po}' =: (\nabla o) \text{Boo}'$$

O' is a person =: there is at least one object O such that O belongs to O'.¹⁰

$$\text{(SD2)} \quad \text{Mo}' =: (\nabla o) \text{Bo}'o$$

O' is a means =: there is at least one object O such that O' belongs to O.

An object that belongs to no person and for which there is no means that belongs to it, is a *mere object*. In terms of the intended interpretation, it is outside the law — like the sun or a particular cell or molecule in one’s body.

In the following presentation, we use the variables p, q, \dots to refer to persons. We use the variables x, y, \dots to refer to means. Subscripts or superscripts may be used to expand the number of symbols for variables — for example p_1, q', x_1, y' . If ever we need to refer indiscriminately to objects in $\mathbf{D}(\mathbf{L}_0)$, we use the variables o, o', o'', \dots (as we did in SD1 and SD2).

By using distinct sets of variables to refer to the class of persons, respectively to the class of means, we can simplify considerably the presentation of the system by having recourse to the technique of many-sorted quantification. Thus $(\blacktriangle p)\mathbf{F}$ and $(\nabla q)\mathbf{G}$ are to be read respectively as ‘for all *persons*, \mathbf{F} [is the case]’ and as ‘for at least one *person*, \mathbf{G} [is the case]’. On the other hand we read $(\blacktriangle x)\mathbf{F}$ and $(\nabla y)\mathbf{G}$ respectively as ‘for all *means*, \mathbf{F} [is the case]’ and as ‘for at least one *means*, \mathbf{G} [is the case]’.

Given the syntactical definitions SD1 and SD2, we can immediately derive two theorems: $(\blacktriangle p)(\nabla x) \text{Bxp}$ and $(\blacktriangle x)(\nabla p) \text{Bxp}$ — for every person, there is at least one means that belongs to him; and for every means, there is at least one person to whom it belongs. The proofs

¹⁰ In this general theory, we do not distinguish between natural and artificial persons. Note that ‘ x belongs to y ’ is not intended to express ownership. Thus, the apartment that I rent belongs to me, but it also belongs to the person who rented it to me and who will normally regain full ownership of it the moment my lease terminates. A pasture may belong as a “commons” to each villager, although none of them is its owner.

of these theorems are too simple to spell out. In any case, the formal presentation should make it easy to check their validity. The same remarks apply to all the other theorems. While I shall occasionally provide short sketches of proofs, I shall list most theorems without proving them.

The logical use of Bxp is constrained by two axioms.

(A₀1) $(\blacktriangle p)(\nabla q)(Bpq)$

Every person belongs to at least one person.¹¹

(A₀2) $(\blacktriangle x)(\blacktriangle p)(\blacktriangle q)(Bxp \ \& \ Bpq \rightarrow Bxq)$

If person P belongs to Q, P's property also belongs to Q.

In view of SD2, the first axiom implies that every person is a means. In other words, the set of persons is a subset of the set of means. Consequently, the name of any person is the name of a means and the variables x, y, \dots can be instantiated with the names of persons. The first axiom ensures that if something is identified as a person, it is meaningful to ask to whom he belongs. The answer may be "to himself alone", "to himself and one or more other persons" or "to one or more other persons". The axiom only excludes the possibility that a person (in the sense of the formal theory) belongs to no one.

The second axiom specifies that B_{oo} is a transitive relation if the middle term (which is 'p' in the formulation of A₀2 above) refers to a person. It makes persons the central elements of law. Means of action 'follow' the persons to whom they belong. Thus, what lawfully belongs to a corporate person also belongs to those persons to whom the corporation belongs.¹² A slave's property also belongs to his master (assuming that there is such a thing as lawful slavery).

¹¹ Using one of the other suggested readings for Bxy (see note 7 above), we may read A₀1 as "every person is the property of some person" or "there is no person for whom no person is responsible or answerable."

¹² Complications arise if a corporation is considered an artificial person distinct from its shareholders, yet legally entitled to *own* property in its corporate capacity. For more on the curious concept of corporate collectives, see the subsection *Collectives* below.

Persons

The notion of law that is relevant here is that of an order of persons. Our first task, therefore, is to define different sorts of persons that we can distinguish in terms of the theoretical apparatus at our disposal and to spell out the relations that obtain between persons of the same sort or of different sorts. We begin with the concept ‘a real person’.

(DP1) $P_{rp} =: B_{pp}$.

P is a real person =: P belongs to himself.

A person that does not belong to himself (hence, by A01, belongs to one or more *other* persons) we shall call an *imaginary* person. According to an alternate reading of DP1, such a person is not answerable or responsible for himself — another person has to answer for him.¹³ Obviously, every person is a real or an imaginary person.

(DP2) $P_{fp} =: B_{pp} \ \& \ (\Delta q)(B_{pq} \rightarrow p=q)$

P is a free person =: P belongs to himself and only to himself.

A person is free if and only if nobody else has a lawful claim on his person. He is a “self owner”. A material theory of law should be able to say which of the persons it recognizes are free (that is, solely responsible for themselves). Obviously,

TP1) $(\Delta p) (P_{fp} \rightarrow P_{rp})$

Free persons are real persons.

TP2) $(\Delta p) (\sim P_{rp} \rightarrow \sim P_{fp})$

Imaginary persons are not free.

¹³ Think of an artificial person, such as a corporation or “the President of the Republic” (not the man or woman but the constitutional position or office): it does not have the capacity of speech, which is the pre-eminent characteristic of *natural* personhood. In that sense, it is a person only in the imagination of other and natural persons. Of course, that is a natural-law perspective. In the perspective of legal positivism, “legal personality” is a quality bestowed by the personified legal system (e.g., the state), regardless of any naturalistic characteristic such as speech or the ability to reason and imagine. In the latter perspective, human beings might be assigned the status of an imaginary person (or even no personal status whatsoever).

Note that a person that is not free may be real and therefore responsible for himself, though not, of course, solely responsible for himself. Others share that responsibility.

We now define three general relational concepts in terms of a person's relation to others: sovereignty, autonomy and heteronomy.

$$(DP3) \quad Sp =: (\blacktriangle q)(Bpq \rightarrow p=q)$$

P is a sovereign person =: P belongs to no person but himself.

It follows immediately that

$$TP3) (\blacktriangle p) (P_{fp} \rightarrow Sp)$$

Free persons are sovereign.

Given **Ao1**, we derive

$$TP4) (\blacktriangle p)(Sp \rightarrow P_{fp})$$

Only free persons can be sovereign.

$$TP5) (\blacktriangle p) (Sp \leftrightarrow P_{fp})$$

A person is free if and only if he is sovereign.

$$TP6) (\blacktriangle p) (Sp \rightarrow P_{rp})$$

A sovereign person is a real person.

Although their definitions are different, the concepts of a free person and a sovereign person are logically equivalent in the general theory of law. Both apply only to real persons. Next, we define autonomy.

$$(DP4) \quad Ap =: (\blacktriangle q)(Bpq \rightarrow Bqp)$$

P is an autonomous person =: if P belongs to some Q then Q belongs to P.

From **Ao1** and **Ao2**, we deduce

$$TP5) (\blacktriangle p) (Ap \rightarrow P_{rp})$$

An autonomous person is a real person.

Proof: Let P be an autonomous person. From **Ao1** it follows that P belongs to some person Q. Given that P is autonomous, it follows that any such Q belongs to P. Now, if P belongs to Q and Q to P, then according to **Ao2**, P belongs to P. That is to say, P is a real person. Q.E.D.

TP6) ($\blacktriangle p$) ($S_p \rightarrow A_p$)
Every sovereign person is an autonomous person.

Note that it does not follow that every autonomous person is sovereign. We define the concept of heteronomy simply as the negation of autonomy.

(DP5) $H_p =: \sim A_p$
P is a heteronomous person =: P is not an autonomous person.

Because $\sim A_p \leftrightarrow (\nabla q) (B_{pq} \& \sim B_{qp})$, a heteronomous person P belongs to some person Q who does not belong to P. Thus, there necessarily is another person who is responsible for a heteronomous person. Obviously,

TP7) ($\blacktriangle p$)($A_p \vee H_p$)
Every person is either an autonomous person or a heteronomous person.

TP8) ($\blacktriangle p$)($\sim P_r p \rightarrow H_p$)
An imaginary person is heteronomous.

Proof: Suppose that A is an imaginary person. Then, A does not belong to himself. Now suppose that A is *not* heteronomous. Then, A must belong to some person, B, who must belong to A. By A_o2, A must then belong to himself. However, this contradicts the supposition. Q.E.D. Intuitively, since an imaginary person does not belong to himself, and is therefore not answerable for himself, someone else must be responsible for him if he is to be part of the order of law. Obviously, because an heteronomous person belongs to another:

TP9) ($\blacktriangle p$)($H_p \rightarrow \sim P_r p$)
Heteronomous persons are not free persons.

We shall say that B is a *master* of A and A is a *serf* of B, if A is a heteronomous person and belongs to B who does not belong to A. Thus, we may use the term 'serf' as a synonym for 'heteronomous person'. Obviously, according to the definitions and axioms, a serf may have more than one master, and a master may have more than one serf. Note that the definition does not imply that a master is autonomous. B, the master of A, may be a serf of C.

We shall make a distinction between the master-serf relationship and the ruler-subject relationship. If A belongs to B and B is an autonomous person, then we shall say B is a *ruler* and A is his *subject*. Clearly, a master need not be a ruler, because the concept ‘master’ does not imply autonomy. On the other hand, a subject is not necessarily a serf, nor is a ruler necessarily a master. For example, A may belong to autonomous B (and therefore be a subject of B); yet, A may be autonomous himself (in which case B belongs to A and A is a ruler of B). Indeed, it is logically possible for two persons to be at once rulers and subjects of one another.¹⁴

Because not every autonomous person is sovereign, we have use for the following definition.

(DP6) $A!p =: Ap \ \& \ \sim Sp$

P is a strictly autonomous person =: P is an autonomous person who is not sovereign.

Obviously,

TP10) $(\blacktriangle p)(Ap \rightarrow Sp \vee A!p)$

Every autonomous person is either sovereign or strictly autonomous.

TP11) $(\blacktriangle p)(A!p \rightarrow (\nabla q)(Bpq \ \& \ \sim p=q))$

A strictly autonomous person belongs to another person.

TP12) $(\blacktriangle p)(\blacktriangle q)(A!p \ \& \ Bpq \rightarrow A!q \ \& \ Bqp)$

If P is a strictly autonomous person who belongs to Q, then Q is a strictly autonomous person who belongs to P.

TP13) $(\blacktriangle p)(A!p \rightarrow \sim P!p)$

Strictly autonomous persons are not free persons.

TP14) $(\blacktriangle p)(Sp \vee A!p \vee Hp)$

Every person is sovereign, strictly autonomous or heteronomous.

TP15) $\sim(\nabla p)(Sp \ \& \ A!p)$

No person is at once sovereign and strictly autonomous.

TP16) $\sim(\nabla p)(A!p \ \& \ Hp)$

No person is at once strictly autonomous and heteronomous.

¹⁴ This possibility lies at the heart of Rousseau’s notion of citizenship, which implies that each citizen should rule and at the same time should be under the rule of every other citizen of the state. We shall discuss Rousseau’s conception of citizenship briefly in the text.

TP17) $\sim(\nabla p)(Sp \ \& \ Hp)$

No person is at once sovereign and heteronomous.

TP14–17 tell us that the set of persons is partitioned in three jointly exhaustive but mutually exclusive sets of respectively sovereign, strictly autonomous or heteronomous persons. About the number of persons (if any) in any of those sets, the formal theory has little to say. However, some quantitative results are implied:

TP18) $(\blacktriangle p)(\sim Sp \rightarrow (\nabla q)(Bpq \ \& \ \sim p=q))$

Every non-sovereign person belongs to at least one other person.

TP19) $(\blacktriangle p)(Ap \ \& \ (\blacktriangle q)(Aq \rightarrow p=q) \rightarrow Sp)$

If P is the only autonomous person, then P is sovereign.

TP20) $(\blacktriangle p)((\blacktriangle q) p=q \rightarrow Sp)$

If P is the only person, then P is sovereign.

Thus, if there is only one person, that person must be sovereign. Consequently, if there is a non-sovereign person then there must be at least two persons. A sovereign person may be the only person in the world, but a strictly autonomous or a heteronomous person must have personal company.

By inductive generalization, we can derive the following propositions. (The expression ‘finite world’ refers to a world with a finite number of persons in it; ‘infinite world’ accordingly refers to a world with an infinite number.)

MT1: Only in an infinite world can all persons be heteronomous.

MT2: Only in an infinite world can there be serfs who are not subjects.

MT3: In a finite world there must be at least one autonomous person.

MT4: If there is a serf in a finite world, he must be the subject of some ruler(s).

Thus, in a finite world, there must be at least one sovereign person or at least one community of strictly autonomous persons. Whether or not there also are heteronomous persons, the formal theory does not specify. However, if there are, then we are dealing with a world in which there are rulers and subjects (and maybe also masters and serfs).

Collectives

A strictly autonomous person is always ‘in community’ with at least one other strictly autonomous person. If A is a strictly autonomous person and belongs to B then B must be a strictly autonomous person and belong to A. We shall refer to A and B as ‘*members of the same autonomous collective*’.

(DP7) $SAC_{pq} =: A!p \ \& \ Bpq$

P and Q are members of the same autonomous collective =: P is a strictly autonomous person and belongs to Q.

Obviously,

TP21) $(\blacktriangle p) (A!p \rightarrow (\nabla q) (\sim p=q \ \& \ SAC_{pq}))$

TP22) $(\blacktriangle p) ((\nabla q) SAC_{pq} \rightarrow A!p)$

In short, strictly autonomous persons *are* members of one or another autonomous collective. Therefore, we may treat the statement ‘P is a strictly autonomous person’ as synonymous with ‘P is a member of an autonomous collective’. From DP7, TP12, and axiom **A_o2** it follows that ‘being a member of the same autonomous collective’ is a reflexive, symmetrical and transitive relation for strictly autonomous persons.

TP23) $(\blacktriangle p) (A!p \rightarrow SAC_{pp})$

TP24) $(\blacktriangle p)(\blacktriangle q) (SAC_{pq} \rightarrow SAC_{qp})$

TP25) $(\blacktriangle p)(\blacktriangle q)(\blacktriangle r) (SAC_{pq} \ \& \ SAC_{qr} \rightarrow SAC_{pr})$

An autonomous collective has at least two members, but it may well have many more. Of course, there may be any number of autonomous collectives. From the mere fact that A and B are both strictly autonomous it does not follow that they belong to the same autonomous collective (i.e., to one another).

Obviously, no person can be a member of more than one autonomous collective. If C1 and C2 are different autonomous collectives (there being at least one person who is a member of one but not of the other), then none of the members of C1 is a member of C2, and vice versa. Of particular relevance is the following theorem:

TP26) $(\Delta p)(\Delta q)(\Delta x)(SAC_{pq} \rightarrow (Bxp = Bxq))$

Whatever belongs to one member of an autonomous collective also belongs to every other member.

Autonomous collectives are perfect communities, exhibiting a perfect communism, every member of the collective sharing the person and the means of all other members.

The members of an autonomous collective may be masters and rulers of other persons, who would then be serfs and subjects of every one of the members of the collective. However, while the members of an autonomous collective necessarily are rulers and subjects of each other, they cannot be serfs of any master (for then they would not be autonomous persons). Nor can they be subjects of any person who is not a member of the collective, since that too would conflict with their status as autonomous persons.¹⁵

We should distinguish an autonomous collective from a *hegemonic* collective. The latter necessarily comprises a class of serfs, all of whom are subjects of the same rulers (the same masters). A sovereign master (ruler) with his serfs forms a hegemonic collective. If several sovereign persons have a number of serfs in common then they and their common serfs form a hegemonic collective. The rulers in a hegemonic collective need not be sovereign. They may form an autonomous collective among themselves. Think for example of an autonomous republic of citizens (in the manner of Rousseau) that rules over a subject population that is denied 'political rights' or membership in the community of citizens. The structure of such a hegemonic collective is obviously different from one in which the rulers are sovereign persons. In the latter case, all of the subjects belong to the same group of rulers, but the rulers themselves do not belong to one another.

Clearly, sovereign persons have no master. Although several sovereign persons may have many means (including serfs or subjects) 'in

¹⁵ However, their ruler may instruct them to treat each other *as if* they were members of the same autonomous collective. If that is the case, they are still heteronomous in the order (*law*) of persons, even though the *legal system* imposed by their ruler requires them to deal with one another (but not with the ruler) as if each of them belonged to every other. Autonomy in law is not the same as legal autonomy.

common', they are always independent of one another. They cannot be part of an autonomous collective; and if they are part of a hegemonic collective, their position in it must be that of a ruler. Sovereign persons cannot be subjects and need not be rulers, unlike strictly autonomous persons, who are necessarily both. Thus, sovereign persons need not be members of any collective and they do not constitute a collective of any sort.

A digression on autonomous collectives

Autonomous collectives are well known in the history of the political philosophy of law and rights. For example, we may represent Hobbes's natural condition of mankind as an autonomous collective. In the natural condition, Hobbes wrote, there is no distinction between 'mine' and 'thine' as every person has a right to everything, including 'one another's body'. In that sense, every person belongs to every other. Consequently, there is no distinction between justice and injustice.¹⁶ Hobbes's argument was that the natural condition was an impractical, indeed life threatening, state of affairs. For him it was a dictate of reason that men should abandon the condition of the autonomous collective and should reorganize in one or more separate hegemonic collectives. Each of those would be defined by the relationship between a free person (ruler-master) and a multitude of subjects (who are also serfs). For Hobbes, then, the sacrifice of the 'equality' of the original autonomous collective was a necessary condition for survival, peace and comfort.

Perhaps the most significant aspect of Hobbes's theory was that it suggested an alternative reading for the constitutive relation of the 'commonwealth'. Next to the reading 'subject belongs to sovereign', which represents the political or power relation, Hobbes introduced the legal reading 'subject authorizes sovereign'. He took his inspiration for this from the Stoics. They interpreted the wise man's 'I accept the will of the Gods' as 'It is my will that the Gods do whatever they do' and therefore as 'The Gods do what I want them to do.' In a similar way, for Hobbes, the submission of the weak to the strongman meant that they

¹⁶ Thomas Hobbes, *Leviathan*; Book I, Chapter 13, "Of the Naturall Condition of Mankind, as Concerning Their Felicity, and Misery."

accepted his actions as if they were mere executions of their own will.¹⁷ With that move, Hobbes gave formal expression to the modern conception of the state as an agency relationship between a multitude of ‘principals’ (the subjects or citizens) and a single ‘agent’ (the sovereign). Thus, the legal fiction that the State, in exercising its sovereign rule over its subjects, merely acts according to the will of its citizens was put in place.

Rousseau’s theory of the social contract requires every person who enters into the State to give all his possessions, all his rights, indeed himself, to all the other parties to the contract. Thus, the social contract founds an autonomous collective. However, Rousseau set out to prove to his own satisfaction — and against Hobbes — that an autonomous collective could be a viable option, at least in theory, if certain conditions were met. The essential condition was that human nature should be changed, because an autonomous collective made up of natural human persons would inevitably be a Hobbesian ‘war of all against all’. To meet that condition, it was necessary that a political genius (Rousseau’s ‘legislator’) should succeed in turning (“educating”) natural men and women into artificial citizens of the right kind. Rousseau’s argument was that if it were possible to make citizens out of human beings then the autonomous collective would become a self-governing, free *real unity* of equal citizens. The social contract, therefore, must be interpreted as creating a new entity, ‘the People’ or ‘the Republic’ (the State), which is the personified autonomous collective itself. That artificial person, however, can come to life only if the living human material that constitutes it takes the form of the Citizen — if it can be made to identify fully with the State.

Like Hobbes’s subjects, Rousseau’s citizens interpret their subjection to the State as their authorization of the State. However, in the autonomous collective of the State, Rousseau’s citizens do not act according to their particular ‘natural will’ (their given human nature) but according to the statutory ‘general will’ of the collective itself (which is to become second nature to them). The general will is the same for all citizens *qua* citizens, because by definition a citizen *qua* citizen is

¹⁷ On the neo-Stoic and neo-skeptical origin of this ‘moral alchemy of power,’ see Frank van Dun, ‘Philosophical Statism and the Illusions of Citizenship’, in B. Bouckaert & A. Godart-Van der Kroon (eds.), *Hayek Revisited* (Edward Elgar, Cheltenham, 2000), p.95–96.

animated by nothing else than the statutory purpose of the association. Rousseau's citizens, therefore, are committed to act according to the legal rules that express the determinations of the 'general will' in particular circumstances.

Rousseau and Hobbes, then, agreed on the thesis that the principle of freedom among likes (natural persons of the same kind) — the principle of natural law — had to be replaced by positive legislation. Rousseau, however, thought that it was theoretically possible to reproduce 'the freedom among likes' of the natural law as 'liberty and equality' for the members of an autonomous collective.¹⁸ That was the basis of his claim to have *squared the political circle*, i.e. to have solved the problem of proving the legitimacy of the fact that man, who is born free, is everywhere kept in chains.¹⁹ Rousseau thought he had proven that the state could be legitimate, in accordance with the formal requirements of justice. Formally, his solution requires that we distinguish between natural persons and citizens (artificial persons). We have to suppose that for every Jean and Jacques, members of the same autonomous collective, there is a person that is different from both, a *citizen Jean* and a *citizen Jacques*. We also have to suppose that *as citizens* Jean and Jacques are merely numerically different manifestations or aspects of the same person, the People. We can express those suppositions formally as follows:

(▲p) [A!p → (▼q) (∼q=p & q=c(p))]

For every member of an autonomous collective there is another person who is his civic persona.

(▲p)(▲q) (SACpq → c(p) = c(q))

The civic personae of any two members of the same autonomous collective are identical.

We should represent the relation between a natural person and his legal or civic personality (in Rousseau's theory) as follows:

¹⁸ On the difference between 'freedom and likeness' and 'liberty and equality', see Frank van Dun, "The Lawful and the Legal", *op. cit.*, especially sections 3.1 and 3.2.

¹⁹ Cf. the first paragraph of *Du Contrat Social*.

$(\blacktriangle p)[A!p \rightarrow B_{Lp}c(p) \ \& \ \sim B_{Lc(p)}p]$

A member of an autonomous collective *legally* belongs to his own civic persona but the latter does not *legally* belong to him.

$(\blacktriangle x)(\blacktriangle p) (A!p \ \& \ Bxp \rightarrow B_{Lxc}(p))$

Whatever belongs to a member of an autonomous collective *legally* belongs to his civic persona.

Thus, as natural persons, A and B may be members of the same autonomous collective, and then they are strictly autonomous in their dealings with one another. On the other hand, as natural persons they also legally and heteronomously belong to their own civic persona, the People. They are subjects and serfs of the People, who is a single sovereign person.²⁰ Hence, in the State, the People may use force against them to free them from their own human nature and to make them into what they have committed themselves to be by entering into the social contract: citizens. That, of course, is Rousseau's "paradox of liberty".²¹ It is not really a paradox within his system. Free natural men have no place in the state, as they would immediately destroy the unity that is the necessary condition of the sovereignty (hence the liberty) of the People and therefore of the citizens who are the People. Enforcement of the legal rules ("laws") liberates the citizen by coercing the human beings whose natural drives and personal interests would otherwise keep them from acting as real citizens.

Note that we had to introduce a modal notion 'belongs legally' (B_{Lxp}) to make sense of the theory. Obviously, the same relation cannot express that one natural person belongs to another natural person and that one such person belongs to some artificial persona.²² Note also that

²⁰ *The Social Contract*, II, 4: "Each man alienates, I admit, by the social compact, only such part of his powers, goods and liberty as it is important for the community to control; but it must also be granted that the Sovereign is sole judge of what is important."

²¹ *The Social Contract*, I,7: "In order then that the social compact may not be an empty formula, it tacitly includes the undertaking, which alone can give force to the rest, that whoever refuses to obey the general will shall be compelled to do so by the whole body. This means nothing less than that he will be forced to be free...."

²² Indeed, if A is a natural member of an autonomous collective and A belongs to $c(A)$ in the same way in which he belongs to the other natural members of the collective, then $c(A)$ would be just another member of the

Rousseau's solution of the problem of the legitimacy of the State rests crucially on his inversion of the natural order of things. While the aspect person (the citizen) is the product of the human imagination, the theory elevates him to the status of an autonomous person for whom his creators are merely subjects and serfs. It does so by way of redefining the perspective on order among persons in terms of a modal notion $\text{B} \Box c(p)$ that requires a reference to the aspect person $c(p)$ instead of the natural person p to whom things ordinarily belong. If it were not for that inversion of the natural order of things, the notion of an aspect person would be unobjectionable. For example, suppose we accept:

$(\blacktriangle p) (\text{B}c(p)p \ \& \ \sim \text{B}p(c(p)))$

Aspect persons are the serfs of the persons from whom they were abstracted.

Aspect persons would be simply heteronomous (artificial or imaginary) persons under the responsibility of their human masters. Then, Jacques's rights-as-a-citizen could never supersede the personal rights he has as a natural person.

In the final analysis, Rousseau's theory ends up in the same corner as Hobbes's. For both, the social contract establishes a sovereign who rules his subjects, who are also his serfs. In Rousseau's approach, the social contract does so in two steps. First, it ostensibly establishes only an autonomous collective in which every person belongs to every other, but then this collective is personified and turned into a sovereign ruler ('the People') in its own right. That second step is accomplished by legally interpreting the autonomous collective of human persons as an autonomous collective of their identical civic personae. The latter collective, therefore, legally has only one member, the People, who necessarily is autonomous and indeed sovereign. The human persons ultimately drop out of the picture. However, as in reality the transformation of men into citizens takes time, they must be given a

collective — a strictly autonomous person. Rousseau's theory of the state would then simply be Hobbes's theory of the natural condition of mankind with an additional number of ghostly fictions participating in the war of all against all. Rousseau avoided this consequence by identifying the civic personae of all members of the autonomous collective and reinterpreting the collective as a single artificial sovereign person composed not of natural persons but of citizens (who only have "a partial and moral existence").

place in the State until the transformation is complete. Their place is that of a heteronomous person, a serf. From the citizen's point of view, then, the State is not a hegemonic collective and every citizen is autonomous and sovereign. The political circle is squared: citizens are 'free and equal'. From the human point of view, however, the State is a hegemonic collective, with the citizens constituting the ruling class and the (not yet fully socialized) human persons constituting the class of serfs. From that point of view, the political circle is not squared at all.²³ Nevertheless, Rousseau's legal pyrotechnics, masquerading as political philosophy, had a great future. It spawned a numerous offspring of ideologies of 'the republican state' and its fulfillment of the requirements of liberty and equality. In the legal-political theory of the State, fiction trumps reality.

Rights

In this section we introduce 'rights talk', without adding anything to the theoretical apparatus we have used so far. In other words, we reduce the notion of rights fully to the notion of 'belonging'.²⁴ First, we define the notion of a right to deny a person the use of some means.

$$(DR1) \quad Dpxq =: (Bxp \vee Bqp) \ \& \ \sim Bpq$$

P has a right to deny Q the use of X =: either X or Q belongs to P and P does not belong to Q.

Thus, to refute the claim that P has a right to deny Q the use of X, one may point out that neither X nor Q belongs to P, or that P is a serf or subject of Q. As immediate consequences we have

$$TR1a) \ (\blacktriangle p) \ \sim Dppp$$

No person has a right to deny himself the use of himself.

²³ Rousseau later regarded his squaring of the political circle as no more than an act of imagination, perhaps a reverie. Letter to Mirabeau, 26 July, 1767, Rousseau, *Correspondance complète*, XXXIII, no. 5991.

²⁴ Thus, if there are several mutually irreducible modal forms of the underlying relation '*x* belongs to *p*' then we should have several mutually irreducible modal forms of rights as well. For example, we should have 'moral rights' as well as 'lawful rights' if things may morally as well as lawfully belong to a person.

TR1b) ($\blacktriangle p$)($\blacktriangle q$) ($S_p \rightarrow (\sim q = p \rightarrow D_{ppq})$)

A sovereign person has a right to deny use of himself to every other person.

TR2a) ($\blacktriangle p$)($\blacktriangle x$) ($D_{pxp} \rightarrow B_{xp}$)

A person has a right to deny himself the use of X only if X belongs to him.

TR2b) ($\blacktriangle p$)($\blacktriangle x$) ($D_{pxp} \rightarrow \sim B_{pp}$)

Only an imaginary person has a right to deny himself the use of any means.

The proof of TR1a is trivial: substitution of p for x and q in DR1 gives $B_{pp} \& \sim B_{pp}$, which is a contradiction. Interpreted as referring to human persons, the theorem suggests that no one has a personal right to commit suicide or to fully incapacitate himself in some other way. An act of such a sort, while physically possible, would remove him (at least as a person) from the order of law. Would it not appear self-contradictory to maintain that there is a lawful right for any person to place himself outside the law or reduce himself to a mere means or object? Note, however, that, according to TR1b, the act of killing oneself might be considered rightful, e.g., when a free man prefers death as the only way to avoid being taken into slavery. The description under which a particular act should be subsumed under the logic of law ('denying oneself the use of oneself' *versus* 'denying others the use of oneself') is of course a question of interpretation and practical application that cannot be answered on formal-logical grounds alone. Theorems TR2a,b merely explicate necessary (though not sufficient) conditions of the "right to deny".

We use the concept D_{pxq} to define the much more common notions of being free to use, and having the right to the use of, some means without the consent of some person. Thus

(DR2) $F_{pxq} =: \sim D_{qxp}$

P is free to use X without the consent of Q =: Q has no right to deny P the use of X.

(DR3) $R_{pxq} =: B_{xp} \& \sim D_{qxp}$

P has a right to the use of X without the consent of Q =: X belongs to P and Q has no right to deny P the use of X.

(DR4) $R^*_{px} =: (\blacktriangle q)R_{pxq}$
P has an absolute right to the use of X =: P has a right to the use of X without the consent of any person.

Obviously,

TR3) $(\blacktriangle p)(\blacktriangle q)(\blacktriangle x) (R_{pxq} \rightarrow F_{pxq})$
If P has a right to the use of X without the consent of Q, then P is free to use X without Q's consent.

The definitions, in conjunction with the axioms of L_0 , imply that real, and only real, persons have the right to the use of themselves without their own consent:

TR4) $(\blacktriangle p) (B_{pp} \rightarrow R_{ppp})$
 TR5) $(\blacktriangle p) (R_{ppp} \leftrightarrow P_{ip})$

Not surprisingly, all autonomous, and in particular sovereign, persons have the absolute right to the use of themselves:

TR6) $(\blacktriangle p) (A_p \rightarrow R^*_{pp})$
 TR7) $(\blacktriangle p) (S_p \rightarrow R^*_{pp})$

No person has a right to the use of what belongs to an independent person (a fortiori, a sovereign) person without the latter's consent:

TR8) $(\blacktriangle p)(\blacktriangle q)(\blacktriangle x) (B_{xq} \ \& \ \sim B_{qp} \rightarrow \sim R_{pxq})$
 TR9) $(\blacktriangle p)(\blacktriangle q)(\blacktriangle x) (S_q \ \& \ B_{xq} \ \& \ \sim p=q \rightarrow \sim R_{pxq})$

These theorems bring into focus the concept of consent as a necessary condition for the use of other independent persons.

Of particular interest are the following theorems:

TR10) $(\blacktriangle p)(\blacktriangle q) (B_{pq} \rightarrow R_{qpp})$
If P belongs to Q then Q has a right to the use of P without the latter's consent.²⁵

²⁵ There is a difference between a material theory of law that interprets an employment contract as a ground for saying that the employee belongs to the employer (and therefore, loses his autonomy for the duration of the contract) and a theory that denies that such a contract has any effect on the personal status of the parties. Similarly, a material theory that says that committing a

TR11) $(\blacktriangle x)(\blacktriangle p)(\blacktriangle q)(\blacktriangle r) (R_{pxq} \& B_{rq} \rightarrow R_{pxr})$

If P has a right to the use of X without consent of Q then P has a right to the use of X without the consent of any person R that belongs to Q.

Together with the following theorems about strictly autonomous and heteronomous persons, they help to clarify what was said earlier about autonomous and hegemonic collectives:

TR12) $(\blacktriangle p)(\blacktriangle q)(\blacktriangle x) (SAC_{pq} \& B_{xp} \rightarrow R_{qxp})$

A member of an autonomous collective has a right to the use of any other member's means without the latter's consent.

TR13) $(\blacktriangle p)(\blacktriangle q) (SAC_{pq} \rightarrow R_{pqq})$

Members of the same autonomous collective have right to the use of each other without consent.

TR14) $(\blacktriangle p)(\blacktriangle x) (H_p \& B_{xp} \rightarrow (\blacktriangledown q) R_{qxp})$

For every heteronomous person P there is another person Q who has a right to the use of P's means without the latter's consent.

TR15) $(\blacktriangle p)(\blacktriangle x) (H_p \& B_{xp} \rightarrow (\blacktriangledown q) \sim R_{pxq})$

For every means X that belongs to a heteronomous person P there is a person Q without whose consent P has no right to the use of X.

TR16) $(\blacktriangle p) (H_p \rightarrow (\blacktriangledown q) R_{qpp})$

For every heteronomous person P there is a person Q who has a right to the use of P without the latter's consent.

Applied to human persons, TR12 and TR13 lead us back to the Hobbesian “natural condition of mankind” and the implied right to everything and everybody.

Property

We can easily define some concepts or forms of property, such as common property, communal property and ownership, in terms of the primitive ‘x belongs to y’.

crime may result in the criminal coming to belong to his victims for as long as he is not willing to provide full restitution is different from one that denies the victims any rights to the *person* of the criminal. However, the conditions that determine when “P belongs to Q” remains or ceases to be true may vary from one material theory of law to another. None of these variations is specifically accepted or rejected in the formalization.

(DY1) $O_{xp} =: B_{xp} \ \& \ (\blacktriangle q)(B_{xq} \rightarrow B_{qp})$
P owns X =: X belongs to P and to no person that does not belong to P.²⁶

It follows that an imaginary person cannot own what belongs to it. To put this differently, an owner must be a real person:

TY1) $(\blacktriangle x)(\blacktriangle p)(O_{xp} \rightarrow P_{rp})$
Only real persons own anything.
 TY2) $(\blacktriangle x)(\blacktriangle p)(O_{xp} \rightarrow A_p)$
Only autonomous persons own anything.
 TY3) $(\blacktriangle x)(\blacktriangle p)(H_p \rightarrow \sim O_{xp})$
No heteronomous person owns anything.
 TY4) $(\blacktriangle p)(S_p \rightarrow O_{pp})$
Only self owners are sovereign.
 TY5) $(\blacktriangle p)(A_p \rightarrow O_{pp})$
Every autonomous person is a self owner.
 TY6) $(\blacktriangle p)(O_{pp} \rightarrow A_p)$
Every self owner is an autonomous person.

With respect to autonomous collectives, we have

TY7) $(\blacktriangle x)(\blacktriangle p)(\blacktriangle q)(A!_p \ \& \ O_{xp} \ \& \ B_{pq} \rightarrow O_{xq})$
If a member of an autonomous collective owns X then every member of that collective owns X.

Thus, the members of an autonomous collective have collective ownership of every means that is owned by any member.

(DY2) $C_{n\ x\ p\ q} =: B_{xp} \ \& \ B_{xq}$
X is common property of P and Q =: X belongs to P and to Q.
 (DY3) $C_{i\ x\ p\ q} =: R_{pxq} \ \& \ R_{qxp}$
X is communal property of P and Q =: P and Q have right to the use of X without one another's consent.

Hence, the theorems:

²⁶ Under this definition, a ruler can own what belongs to his subjects. On the other hand, if a house belongs also to the renter (who is not a serf of the owner) then the owner does not have full ownership of the place for the duration of the lease.

TY8) $(\Delta x)(\Delta p)(\Delta q)(C_{ixpq} \rightarrow C_{ixpq})$

Only common property can be communal property.

TY9) $(\Delta x)(\Delta p)(\Delta q)(C_{ixpq} \rightarrow B_{pq} \& B_{qp})$

If P and Q have communal property then they belong to one another.

TY10) $(\Delta x)(\Delta p)(\Delta q)(B_{pq} \& \sim B_{qp} \rightarrow \sim C_{ixpq})$

There is no communal property between master and serf.

TY11) $(\Delta x)(\Delta p)(\Delta q)(C_{ixpq} \rightarrow \sim S_p \& \sim S_q)$

If X is communal property of P and Q then neither P nor Q is a sovereign person.

TY12) $(\Delta p)(\Delta q)(\Delta x)(SAC_{pq} \& B_{xp} \rightarrow C_{ixpq})$

Whatever belongs to any member of an autonomous collective is communal property of all its members.

Obviously, in an autonomous collective, collective ownership implies communal property. However, something may be the communal property of the members yet no one of them may have right to its use without the consent of some person who is not a member of the collective. A sovereign person, in contrast, may have common property with others, but no communal property.

So far, the formal theory involves only the relation ‘x belongs to y’, where x may be a ‘means’ but also a ‘person’ and y must be a ‘person’. Yet, we have been able to identify quite a number of structures that lend themselves easily to interpretations of the intended class, viz., propositions of the kind one would expect to find in a material theory of law. Obviously, many more such structures could be defined. One might then enquire which structure is most suited for a particular concept as used in a particular material of law or by a particular commentator on such a theory. We can then discover whether a controversy about, say, ownership is rooted in different semantic interpretations of the same formal structure or in the fact that one party in the discussion defines “ownership” in terms of definition DY1 and another party refers to another formal structure. For example, if “ownership” were defined as ‘ $B_{xp} \& (\Delta q)(B_{xq} \rightarrow p=q)$ ’ then it would always be *exclusive* ownership. Members of an autonomous collective could not, then, own anything: under this definition, TY7 would not be a theorem. A collective would have to be considered an autonomous person in its own right (a “self owner”) if one were to hold the view that it may own things. Moreover, it would be illogical to say that a renter

has a right to deny the use of the house to its owner, even while the lease is still running. The renter can have that right only if the house also belongs to him, but then the “owner”, not having exclusive ownership, would not be the owner (in the intended strong sense). Both the weaker and the stronger definitions may coexist within the same theory, but they should then be distinguished by suitably different terms and special care should be taken in argumentation to avoid inadvertent shifts from one definition to another.

In the next paragraphs, it will become even clearer how easy it is to lose one’s way in arguing about law. We shall add ‘actions’ to the domain of the formal theory in order to introduce concepts such as “freedom to do”, “right to do”, and “obligation”. Because of their relative complexity, they are probably far more controversial, even at the purely syntactical level, than the concepts defined so far. (Readers who are not interested in those complexities may jump to the paragraph “General Principle of Justice”. While action concepts are ubiquitous in discussions about law, they are not directly relevant to the central theme of this paper, viz., the analysis of the concept of law as an order of persons and the means that belong to them.)

Actions

The system L_1 .

We now expand the domain $\mathbf{D}(L_0)$ of the theory and add a separate category of objects that we call ‘actions’.²⁷ Let $\mathbf{D}(L_1)$ be the name of the expanded domain. As variables for actions, we use i, j, \dots (possibly with subscripts or superscripts).

The actions domain is linked to the original objects domain by means of two primitive predicators Uio and Vio . We read them as ‘action i uses o ’ (Uio) and ‘action i affects o ’ (Vio). Obviously, we need

²⁷ One may well have misgivings about treating actions (or, more generally, events) as ‘objects’; however, they are not pertinent here since our treatment is purely formal and thus does not involve us in any ontological argument. The only relevant requirement is that it be possible, at least in principle, to identify actions (not only to differentiate an action from a means or a person but also to differentiate one action from another).

to add the formulas ‘Uio’ and ‘Vio’ to the primitives of L_0 to generate an expanded language L_1 to speak about $D(L_1)$. As most material theories of law make the distinction between ‘use’ and ‘affect’ in one way or another, it is not amiss to make room for it in our formal theory.²⁸ Obviously, whether and how to distinguish cases where an action uses or affects some means or person, are not matters that we can decide with the formal apparatus.

The use of the added primitive predicators is constrained by four axioms:

$$(A_{11}) \quad (\blacktriangle i)(\blacktriangle x)(U_{ix} \rightarrow V_{ix})$$

Every action that uses a means affects it.

$$(A_{12}) \quad (\blacktriangle i)(\blacktriangledown x) U_{ix}$$

Every action uses at least one means.

$$(A_{13}) \quad (\blacktriangle x)(\blacktriangledown i) U_{ix}$$

For every means there is an action that uses it.

$$(A_{14}) \quad (\blacktriangle i)(\blacktriangle x)(\blacktriangle p)(V_{ix} \& B_{xp} \rightarrow V_{ip})$$

An action that affects a means that belongs to a person, affects that person.

The first axiom requires us to interpret U_{ix} and V_{ix} in such a way that whenever U holds for some pair (i,x) V holds for the same pair. However, it allows that with respect to some such pair V holds but U does not: an action may ‘affect’ a means without ‘using’ it. The other axioms ensure that we can always ask about the connection of an action to the other objects (means, persons) in the domain.

From the perspective of a theory of law, the primary purpose of introducing the concept of action is to answer questions about what sort of things a person has, or does not have, the right to do. To achieve that purpose, we first define with respect to actions some concepts that are analogous to those that we introduced earlier:

$$(DA1) \quad D^a p i q =: (\blacktriangledown x)(V_{ix} \& D_{pxq})$$

P has a right to deny Q to do I =: P has a right to deny Q the use of some means that I affects.

²⁸ Obviously, as with ‘belong to’ (see note 24 above), particular theories of law may postulate the co-existence of several irreducible modal forms of ‘use’ and ‘affect’.

(DA2) $R^a p i q =: (\blacktriangle x)(\forall i x \rightarrow R p x q)$

P has a right to do I without the consent of Q =: P has a right to the use, without the consent of Q, of all means that are affected by I.²⁹

The following definitions extend concepts of property and ownership to actions:

(DA3) $B^a i p =: (\blacktriangle x)(\forall i x \rightarrow B x p)$

Action I belongs to P =: All the means that I affects belong to P.

(DA4) $O^a i p =: (\blacktriangle x)(\forall i x \rightarrow O x p)$

P owns action I =: P owns all the means that I affects.

Among the theorems that we can prove, we note the following:

TA1) $(\blacktriangle i)(\blacktriangle p)(\blacktriangle q)(\blacktriangle x)(\forall i x \& B x q \rightarrow (R^a p i q \rightarrow B q p))$

If action I affects property of Q then P has a right to do I without the consent of Q only if Q belongs to P.

Concerning the property of actions, we may note these theorems:

TA2) $(\blacktriangle i)(\blacktriangle p)(\blacktriangle q)(B^a i p \& S p \& R^a q i p \rightarrow p = q)$

If action I belongs to sovereign person P then no other person has a right to do I without the consent of P.

TA3) $(\blacktriangle i)(\blacktriangle p)(\blacktriangle q)(O^a i p \rightarrow R^a p i q)$

If P owns I then P has the right to do I without the consent of any Q.

TA4) $(\blacktriangle i)(\blacktriangle p)(H p \rightarrow \sim O^a i p)$

No heteronomous person owns any action.

Again, we have theorems that illustrate the perfect communism of autonomous collectives

²⁹ A weaker notion of 'P having the right to do I without the consent of Q' would be $(\blacktriangle x)(\forall i x \rightarrow \sim D q x p)$. We can define similar but slightly different concepts by substituting 'Uix' for 'Vix' in the defining formulas of DA1 and DA2. However, in this presentation we shall not consider such variants. The interested reader is invited to do the exercise himself.

TA5) $(\blacktriangle i)(\blacktriangle p)(\blacktriangle q)(\blacktriangle r) (SAC_{pq} \& R^a_{pir} \rightarrow R^a_{qir})$

If one member of an autonomous collective has the right to do an action without the consent of a person R, then so does every other member.

TA6) $(\blacktriangle i)(\blacktriangle p)(\blacktriangle q) (SAC_{pq} \& O^a_{ip} \rightarrow O^a_{iq})$

If any member of an autonomous collective owns an action, then so does every other member.

TA7) $(\blacktriangle i)(\blacktriangle p)(\blacktriangle q) (SAC_{pq} \rightarrow \sim D^a_{piq})$

No member of an autonomous collective has a right to deny any action to any other member.

By means of the fourth action axiom we can prove the equivalence

TA8) $(\blacktriangle i)(\blacktriangle p)(\blacktriangle q) (R^a_{piq} \leftrightarrow B^a_{ip} \& (B^a_{iq} \vee B_{pq} \rightarrow B_{qp}))$

P has the right to do I without consent of Q if and only if action I belongs to P and if either I or P belongs to Q then Q belongs to P.

Thus, R^a_{piq} turns out to be formally analogous to R_{pxq} , which was defined as $B_{xp} \& (B_{xq} \vee B_{pq} \rightarrow B_{qp})$.

Generic actions

We should think of the objects ('actions') that we introduced to define the expanded domain $\mathbf{D}(\mathbf{L}_1)$ as concrete individual actions. They are specified action events except that the specification does not mention who performs the action or which means it uses or affects.³⁰ Normally, of course, we refer only to kinds of actions, such as going to the hospital, reading a book, etc. Many theoretical discussions are exclusively in terms of generic actions.

Generic actions typically can be instantiated in many ways, each of which may be different from other instantiations with respect to the means it uses or affects. To accommodate references to generic actions, we can use action predicates such as '[action I] is of kind Z'. One advantage of introducing general action predicates is that we can negate and logically combine them: 'I is of kind Z_1 but not of kind Z_2 '. The

³⁰ This is analogous to the treatments of 'means' in the earlier parts of the paper: we have assumed that we can identify individual means without specifying whose means they are (or what purpose they serve).

basic structure of the logic of rights to do actions of some kind or other then comes into view.

Rights, obligations, freedoms.

We should now be able to define such concepts as having right to do some kind of action, or having right to deny some kind of action to a person, having obligations, being free, and so on. However, because of the high level of abstraction of such definitions, there may be no intuitively straightforward way to do that. For example, with respect to ‘having the right to do some generic action’, we could define

$$(DA5) \quad W^a_p Z_q =: (\nabla i) (Z_i \ \& \ R^a_{piq})$$

P has a *weak* right to Z without the consent of Q =: There is at least one action I of kind Z such that P has a right to do I without consent of Q.

Alternatively, we could define

$$(DA6) \quad S^a_p Z_q =: (\blacktriangle i) (Z_i \rightarrow R^a_{piq})$$

P has a *strong* right to Z without the consent of Q =: P has a right to do any action of kind Z without consent of Q

In their use of the expression ‘has a right to do some kind of action’ some material theories may exhibit a preference for one of those notions over the other. Perhaps the same theory uses that expression now in one sense, then in another. There is nothing intrinsically wrong with that, but the indiscriminate use of the same expression to convey different logical structures may be utterly confusing.

Because the DA6 concept is ‘stronger’ than the concept defined in DA5, its negation is ‘weaker’ than the negation of the DA5 concept. Thus, if “freedom of speech” were a weak right, saying that a person does *not* have the right to speak his mind would imply that he does not have the right to do anything that comes under the concept of speaking one’s mind. If, on the contrary, “freedom of speech” were defined as a strong right, saying that a person does *not* have the right of free speech would imply only that there are ways of speaking his mind that are not within his right (although there may very well be other ways that are covered by it). Demagogues often appeal to such a strong definition, while at the same time fallaciously accusing those who oppose their

claims of denying the weak definition: “You oppose free speech (in the strong sense of DA6)? Then you deny free speech (in the weak sense of DA5). Therefore, you assert that there is not a single act of free speech such that one has a right to do it without the consent of this, that, or any other person.”

We note that

$$(\blacktriangle p) ((\blacktriangle q) W^a p Z q \rightarrow A p)$$

If P has the weak right to Z without anybody’s consent, then P is an autonomous person.

We have already used a few action predicates: Vix and Uix . Thus, under the conventions adopted in this section, we can consider expressions such as $W^a p Vxq$ and $S^a p Uxq$. We can prove

$$(\blacktriangle x)(\blacktriangle p)(\blacktriangle q) (W^a p Uxq \leftrightarrow Rpxq)$$

P has a weak right to use X without consent of Q if and only if P has a right to the use of X without Q’s consent.

$$(\blacktriangle x)(\blacktriangle p) ((\blacktriangle q) W^a p Uxq \rightarrow Oxp)$$

If P has a weak right to use X without anybody’s consent then P owns X.

In other words, ownership is a necessary (but it is not a sufficient) condition for the right to use a means without anybody’s consent. It is not a sufficient condition because there may be no action that uses the means one owns that does not have significant side effects on other means (and other persons).

Consider now the theorem

$$(\blacktriangle p)(\blacktriangle q) (W^a p \sim Zq \leftrightarrow (\nabla i) (R^a piq \& \sim Zi))$$

P has the weak right to do something else than Z without consent of Q, if and only if P has a right to do some action that is not of the kind Z without consent of Q. Equivalently:

$$(\blacktriangle p)(\blacktriangle q) (\sim W^a p \sim Zq \leftrightarrow (\blacktriangle i) (R^a piq \rightarrow Zi))$$

P does not have the weak right to do something else than Z without consent of Q, if and only if P has a right to do any action without consent of Q only if it is of kind Z.

This is an immediate consequence of DA5. The right part of the second formulation of the theorem captures at least one sense of ‘P is

under an obligation to Q not to Z'. However, the same is true for the right part of the following theorem.

$$(\blacktriangle p)(\blacktriangle q) (\sim S^a p \sim Z q \leftrightarrow (\nabla i) (\sim Z i \& \sim R^a p i q))$$

P does not have the strong right to not-Z without consent of Q, if and only if there is an action of kind not-Z that P does not have the right to do without consent of Q.

Clearly, the ambiguity of the natural language formula 'P is under an obligation to Q to Z' reflects the ambiguity of 'P has a right to Z without consent of Q'. Thus, we should at least distinguish a 'strong' and a 'weak' form of obligation, the first defined in terms of 'weak right' and the latter in terms of 'strong right'. Moreover, in some instances, 'P is under an obligation to Q to Z' may mean something along the lines of 'P is not free not to Z without the consent of Q'. Obviously, even with the rather simple formalization used so far, we can identify numerous logical structures that arguably are hard to keep apart in natural languages, or even in the technical (but not formalized) languages of lawyers, moralists and philosophers.

Similar complications attend the interpretation of 'freedom to do some generic action'. We could trace it back to definition DR2 (one's freedom to use some means without the consent of another person). Then, we should note at least the following possibilities for defining "P's freedom to Z":

$$(\nabla i) [Z i \& (\blacktriangle q) (\sim p=q \rightarrow \sim D^a q i p)]$$

There is some action of kind Z, such that no other person has a right to deny P to do it.

$$(\blacktriangle q) [\sim p=q \rightarrow (\nabla i) (Z i \& \sim D^a q i p)]$$

For every other person Q, there is an action of kind Z such that Q has no right to deny P to do it.

$$(\blacktriangle i)(\blacktriangle q) [Z i \& \sim p=q \rightarrow \sim D^a q i p]$$

No other person has a right to deny P to do any action of kind Z.

Alternatively, we may have in mind the sort of freedom that one would positively call a right rather than the mere absence of another's right. Here, too, we have several formulations of different strength:

$$(\nabla i) (Z i \& (\blacktriangle q) (\sim p=q \rightarrow R^a p i q))$$

There is an action I of kind Z, such that P has a right to do I without the consent of any other person.

$(\blacktriangle q) [\sim p=q \rightarrow (\nabla i) (Zi \& R^a piq)]$

For every other person Q, there is an action of kind Z such that P has a right to do it without consent of Q.

$(\blacktriangle i)(\blacktriangle q) [Zi \& \sim p=q \rightarrow R^a piq]$

For every action of kind Z, P has a right to do it without any other person's consent.

Relevant harms and wrongs.

Kinds of actions are often identified in terms of the effects of actions. Let us introduce binary predicates of the form 'action I produces state of affairs S' and represent them by expressions of the form 'i>S'. Then we can read ' $W^a p \gg S q$ ' as 'P has a weak right to produce S without consent of Q' and interpret it as ' $(\nabla i) (i \gg S \& R^a piq)$ '.

Consider a state of affairs Φ such that any action that puts a means (or a person) in that state harmfully affects that means (or person) in a way that is significant or relevant from the point of view of the law: $(\blacktriangle i) (i \gg \Phi x \rightarrow \nabla ix)$. If that condition holds for a means x then we shall write $F\Phi x$ — which we may read as: Φ is an F-state for x (think "forbidden state"). An example could be 'X is destroyed'. We can easily prove

$(\blacktriangle x)(\blacktriangle p)(\blacktriangle q) (F\Phi x \& Bxp \& \sim Bpq \rightarrow \sim W^a q \gg \Phi xp)$

No person has a right to put what belongs to an independent person in F-state Φ without that person's consent.

$(\blacktriangle p)(\blacktriangle q) (F\Phi p \& \sim Bpq \rightarrow \sim W^a q \gg \Phi pp)$

No one has a right to put an independent person in F-state Φ without his consent.

Suppose, for example, that $\sim Ip$ (i.e. 'P is not innocent') is deemed an F-state for P — every action that puts P in a condition where P is not innocent relevantly affects P. Then, with the possible exception of P's masters or rulers, no one has liberty to make it happen that P loses his innocence without his consent. No one can lawfully make an independent person lose his innocence without his consent. Also

$(\blacktriangle p) (F\Phi p \& P_{fp} \rightarrow (\blacktriangle q) (\sim p=q \rightarrow \sim W^a q \gg \Phi pp))$

No other person has a right to put a free person in F-state Φ without his consent.

For example, if $\sim P_i q$ ('Q is not a free person') is deemed an **F**-state for a free person Q, then every other person is under a strong obligation to Q not make him lose his freedom. Consequently, an action that makes Q lose his freedom must be undertaken with the consent of Q himself. No free person can lose his freedom against his will.³¹ Of course, whether a particular theory of law does or should consider any state of affairs an **F**-state, is not a matter that can be decided on formal grounds.

Next, consider a state of affairs Ψ such that any action that puts a means (or person) in that state is one that no person has a right to do except possibly with the consent of every person: $(\blacktriangle i)(\blacktriangle x)(\blacktriangle p)(\blacktriangle q)$ ($i \gg \Psi x \rightarrow \sim R^a p i q$). To put this differently: if action I produces Ψ , then there is no person Q without whose consent P has a right to do I. We may call Ψx an excluded or **X**-state: $X\Psi x$. Obviously, no person has a right to put any means or person in an **X**-state unless, perhaps, he does so with the consent of every person:

$$(\blacktriangle x)(\blacktriangle p)(\blacktriangle q) (X\Psi x \rightarrow \sim W^a p \gg \Psi x q).$$

Let us assume that 'no innocence' is an **X**-state for any person. Then,

$$(\blacktriangle p)(\blacktriangle q)(\blacktriangle r) \sim W^a p \gg \sim I q r$$

There is no person R without whose consent P has a right to deprive a person Q of his innocence.

$$(\blacktriangle p)(\blacktriangle q) \sim W^a p \gg \sim I p q$$

There is no person Q without whose consent P has a right to deprive himself of his innocence.

If we assume further that every action that puts a free person in a condition where he is no longer free makes him lose his innocence, then

³¹ An anonymous referee asked, "Does it make sense to say someone lost his freedom with his consent? How was it a loss of freedom?" The answer is that, in this formalization, we are concerned only with freedom as a status in law, not with freedom in, say, a physical or psychological sense. He who consents to commit a crime (cf. the *mens rea* requirement) willingly gives up his status as a free person in the order of law, if that is the lawful consequence of his crime. (See below, the section on the principle of natural justice. Note that not all material theories of law accept that principle.)

— given that ‘not innocent’ is considered an **X**-state for any person — it follows that

$$(\blacktriangle p)(\blacktriangle q) (P_{fp} \rightarrow \sim W^a p \gg \sim P_{fpq})$$

There is no person Q without whose consent a free person P has a right to deprive himself of his freedom.

The same result follows immediately from the alternative assumption that ‘no freedom’ is an **X**-state for a free person. Presumably, the assumption captures the essence of the thesis that freedom is an absolute, inalienable right. By analogy, the assumption that ‘no innocence’ is an **X**-state for any person would represent the thesis that ‘innocence’ is an absolute, inalienable right. Again, whether a particular material theory of law does or should consider $\sim I_p$ (or any other condition) an **X**-state, is not a matter that can be decided on formal grounds.

Clearly, our formalization can give plausible definitions of concepts such as freedom and obligation in terms of the primitive relations Bxp , Uix and Vix . It also highlights the more or less subtle ambiguities of words such as ‘freedom’ and ‘obligation’ in natural languages and discourses about law. We can easily add more definitions and derive more theorems but we shall not do so. In any case, it should be clear that L_0 and L_1 are useful tools for formalizing significant parts of our thinking about law.

General Principle of Justice

It is time to turn our attention to the predicator I_{oI} , which is a primitive of L_0 . Its use is constrained by the axiom

$$(A_03) \quad (o)(I_o \rightarrow (\nabla p)(p=o))$$

A lawfully innocent object is a person.

In other words, only persons can be innocent in law — which is not to say, of course, that only innocents can be persons. A given material theory of law might postulate that the concept of innocence does not apply to some persons or classes of persons (for example, imaginary persons). Nor is it logically necessary for such a theory to assume that, for a person, the loss of innocence entails the loss of his

status as a person in law. We use the concept of an innocent person to formulate a general principle of personal justice.

General principle of justice (GJ): Only innocent persons are free.

$[(\blacktriangle p)(P!p \rightarrow Ip)]$. In other words, a person who is not lawfully innocent cannot be considered in justice to be lawfully free — and therefore to belong only to himself. To have lost his innocence (in the relevant law-related sense), he must have done something or something must have happened that gave some other person a lawful claim to his person. A non-innocent person always belongs to some other person. While this is compatible with his being a member of an autonomous collective, it does rule out that he is a sovereign person.

Notice that GJ does not specify that in justice all innocent persons are free persons. Such a specification would not make sense in the formal theory of law, which does not specify the material conditions that are necessary or sufficient for ascertaining that a particular person is innocent or not. A material theory of law might permit us to say that a slave, serf or subject is innocent without compelling us to say that he is a free person. GJ only rules out that we consider a person lawfully free but not lawfully innocent.

In the formal theory, it follows from GJ that

TG1) $(\blacktriangle p)((\blacktriangle q) p=q \rightarrow Ip)$

If someone is the only person in the world, he is innocent.

If there is a non-innocent person, there must be at least two persons.

TG2) $(\blacktriangle p) \sim Ip \rightarrow (\blacktriangle q) \sim Sq$

If no person is innocent, then no person is sovereign.

Consequently, if no person is innocent, then no person is free; every person belongs to some other person. If we are dealing with a finite world then we also have

TG3) $(\blacktriangle p) \sim Ip \rightarrow (\nabla q) A!q$

If there are no innocent persons in a finite world, then some persons must be members of one or more autonomous collectives.³²

³² As formalized, the theorem is incomplete: the condition that the number of persons is finite is not made explicit.

In a finite world without innocent persons, there are some strictly autonomous persons and maybe some heteronomous persons, but there are no sovereign persons.

SECTION II

Natural Persons and Natural Law

The system L₂.

Let us define another primitive relation between persons and means. We refer to it with the binary predicator B_n . We read the formula $B_n o_1 o_2$ as ' o_1 naturally belongs to o_2 ' or as ' o_1 belongs to o_2 by nature'. From L_1 , we get to the system L_2 by expanding the language with the new predicator and adding a few axioms that constrain its use. First, however, we give a definition of the concept 'natural person'.

(DN1) $P_n p =: B_n p p$

P is a natural person =: P belongs to himself by nature.

A person who is not natural we shall call an artificial person. The relevant axioms for the relation $B_n x p$ are:

(A₂₁) $(\blacktriangle p)(\blacktriangle x) (B_n x p \rightarrow P_n p)$

Only to a natural person does any means belong naturally.

(A₂₂) $(\blacktriangle p)(\blacktriangle q) (B_n p q \rightarrow p = q)$

No person belongs naturally to any other person.

(A₂₃) $(\blacktriangle p)(\blacktriangle q)(\blacktriangle x) (B_n x p \ \& \ B_n x q \rightarrow p = q)$

No means belongs naturally to more than one person.

The axioms make good intuitive sense if we think of human beings as natural persons. Some means belong by nature to a human person. They are his somatic means, which are embodied in his physical being and which are under his control in a way in which no other (extra-somatic) means is or can be. With respect to, say, a corporation, the concept of a means that belongs to it by nature does not make sense. As for the second axiom, no human person belongs by nature to any other person, whether human or not. For example, no person has control over

a human person or his body in the same natural, immediate way in which he has control over himself or his body and its parts. Axiom 3 captures the separate existence of natural human persons, at least in the sense that those parts of the world that naturally belong to one such person (his body, its limbs and other somatic means) do not and cannot in the same way belong to another.³³ Clearly,

$$\text{TN1) } (\blacktriangle p) (P_{np} \rightarrow (\nabla x) B_{nxp})$$

For every natural person, some means naturally belongs to him.

$$\text{TN2) } (\blacktriangle p)(\blacktriangle q) (P_{np} \& P_{nq} \& \sim p=q \rightarrow (\nabla x) (B_{nxp} \& \sim B_{nxq}))$$

For every pair of natural persons, there is a means that naturally belongs to one of the pair but not to the other.

The axioms exclude the possibility of one person being by nature the serf or subject of another. Consequently, we cannot define, in terms of B_{nxp} , a concept analogous to that of an autonomous collective.

A note on positivism

Legal positivists might object to the use of the term ‘natural’ for qualifying persons. However, the term, while it is suggestive, is not logically important. Another term, say ‘necessary’, might do as well. What is important is that we have at our disposal a concept of a person that is independent of the concept of a person as defined in the general theory of law, yet sufficiently similar to be subsumed under the latter theory. Although we may believe that human persons are natural persons, and perhaps the only natural persons, we cannot charge a purely formal theory with these assumptions.

Natural law theorists focus on natural persons (in an ordinary sense of the word ‘natural’) as the persons whose existence is necessary to make sense of law as an order of persons. From their point of view, all other orders of persons — orders of artificial persons such as

³³ Viable Siamese twins do not appear to be exceptions (but I am not an expert on Siamese twins). If viable, even craniothoracopagus twins, joined at the head and chest, with only one brain, one heart and combined gastrointestinal tracts, presumably would be one person with more controllable [lower] limbs than an ordinary person would have. Viable Siamese twins usually, it seems, are two persons whose bodies happen to be linked in a particular way.

corporations or states — are ‘law’ only by analogy to the natural order of human persons.

In contrast with the natural law theorists, positivists deal primarily with what they call ‘legal persons’. Some of those persons, for example states (or in more abstract language, legal systems), they treat as theoretical representations of pre-existing data from which any legal analysis must start. These systems are the ‘legally necessary persons’ that serve to anchor the positivists’ theoretical constructions in some reality that is not itself one of those constructions. The existence of those particular legal persons is a necessary supposition of any (positivistic) theory of law. Other legal persons have no axiomatic or ‘legally necessary’ existence. Hence, there is nothing that belongs to them as a matter of legal necessity and nothing to which they belong as a matter of legal necessity. They are legal persons only if and because they stand in some legally relevant relation to one of the axiomatically acknowledged legally necessary anchorpersons. That relation — and not any material condition per se — determines their status as legal persons in some legal system. Even to a legal positivist, something must be ‘given’ if his theory is to have any relevance. He may refuse to talk about human beings as natural persons (in the common sense of the word ‘natural’), or about their natural rights, but he too must acknowledge that his theories are about things to which the formal concept of law applies.

A positivist, therefore, need not object to the axioms **A1–3**, if he reads B_{xp} as ‘x belongs as a matter of legal necessity to p’ and P_{xp} as ‘p is a legally necessary person’, or in some similar fashion. If a reader so wishes, he may substitute such readings wherever the text has ‘natural’, ‘naturally’ or ‘by nature’. However, he will find that the axioms more readily yield an intuitively convincing interpretation when we apply them to human beings than when we apply them to abstract constructions such as states or social systems. That is no defect of the axioms but a consequence of the fact that we can understand a positivistic theory of artificial persons only through the analogy with real human persons. In any case, a positivistic theory of ‘legal necessity’ is formally equivalent to a theory of natural law, no matter how much its material interpretations differ from it. The difference between natural law theory (as the theory of law of natural persons) and legal positivism cannot, and should not be made, at the level of our formal theory.

However, we can introduce it in that theory by means of special differentiating postulates.

The Postulates of Natural Law

With the system L_2 we are in a position to begin to make sense of natural law as an order of natural persons (as defined in the previous section). To do that, we need to introduce some postulates of natural law. They are intended to capture the distinctive convictions that make up the idea of a natural order or law of persons, as far as we can express them in our formal system. They also provide a logical link between, on the one hand, the concepts of a natural person and what naturally belongs to him and, on the other hand, the general theory of law as an order of persons. Please note that I write ‘natural person’ and not ‘human person’. We are still concerned about formal structures without forcing a particular semantic interpretation on them.

Postulate of Finitism (PF): The number of natural persons is finite.

No matter what a material theory of law may say about other sorts of persons, it cannot be a theory of natural law unless it denies that there is at any time an actual infinity of natural persons.

Postulate of Naturalism (PN): Every means belongs to at least one natural person.

$[(\blacktriangle x)(\blacktriangledown q)(Bxq \ \& \ P_nq)]$. Note that the postulate says ‘belongs’, not ‘belongs by nature’. According to Naturalism, the responsibility for any means or person — and therefore for any action — ultimately always rests with a natural person. Adding this postulate to our formal apparatus, we can deduce a number of interesting theorems.

NL1) $(\blacktriangle p)(\blacktriangledown q)(Bpq \ \& \ P_nq)$

Every person belongs to at least one natural person.

NL2) $(\blacktriangle p)(P_{fp} \rightarrow P_{np})$

Only natural persons are free.

NL3) $(\blacktriangle p)(S_p \rightarrow P_{np})$

Only natural persons are sovereign.

Thus, Naturalism forces any natural law theory that assigns sovereignty to a person of whatever kind to classify such a person as a ‘natural’ one. In conjunction with the postulate of finitism, Naturalism implies that not every natural person can be heteronomous, i.e. that some natural persons must be autonomous. Indeed, according to PN every person belongs to some natural person. Consequently, a heteronomous natural person must belong to some other natural person who does not belong to him. However, if *every* natural person is heteronomous, there must be an infinite supply of such persons — which contradicts the postulate of finitism.

NL4) (∇p) ($P_{np} \ \& \ Ap$)

There is at least one autonomous natural person.

Thus, given the postulates of finitism and naturalism, we can deduce that either some natural persons are sovereign or some of them are members of one or more autonomous collectives.

In addition to the postulates of finitism and naturalism, which determine the basic structure of natural law, we have two postulates that determine the relations between B_{nxp} and B_{xp} , i.e. between what naturally belongs to a person and what lawfully belongs to him.

Postulate of Consistency (PC): What belongs naturally to a person lawfully belongs to him.

$[(\blacktriangle p)(\blacktriangle x) (B_{nxp} \rightarrow B_{xp})]$. A natural law theory holds that whenever it is established that something belongs naturally to a person, that fact is enough to say that the thing in question is the lawful property of that person. From the postulate of consistency and **Ao2**, we deduce

NL5) ($\blacktriangle p$) ($P_{np} \rightarrow P_{rp}$)

Only real persons are natural persons.

NL6) ($\blacktriangle p$)($\blacktriangle q$)($\blacktriangle x$) ($B_{nxp} \ \& \ B_{pq} \rightarrow B_{xq}$)

What belongs naturally to a person belongs to whomever he belongs to.

Postulate of Individualism (PI): What belongs naturally to a person belongs only to those persons to whom he belongs

$[(\blacktriangle p)(\blacktriangle q)(\blacktriangle x) (B_{nxp} \& B_{xq} \rightarrow B_{pq})]$. There can be no claim to a person's natural property that is separate from a claim to that person himself. In natural law, the natural property of a person is inseparable from the person whose natural property it is.

From the postulates of individualism and consistency, it follows that

NL7) $(\blacktriangle p)(\blacktriangle q)(\blacktriangle x) (B_{nxp} \rightarrow (B_{xq} \leftrightarrow B_{pq}))$

What belongs naturally to a person P belongs to another person Q if and only if P belongs to Q.

NL8) $(\blacktriangle p)(\blacktriangle q)(\blacktriangle x) (B_{nxp} \& D_{qxp} \rightarrow B_{pq})$

Q has a right to deny P the use of what naturally belongs to P only if P belongs to Q.

NL9) $(\blacktriangle p)(\blacktriangle q) (P_{np} \& D_{qpp} \rightarrow B_{pq})$

Q has a right to deny a natural person P the use of himself only if P belongs to Q.

The Principle of Natural Justice

Earlier I stated a principle of personal justice: only innocent persons are free. Here I should add what I take to be the principle of personal justice in natural law. To simplify formulas, we will use the shorthand I_n ('is an innocent natural person'): $I_{np} =: P_{np} \& I_p$.

Principle of natural justice (NJ): Innocent natural persons are free.

$[(\blacktriangle p) (I_{np} \rightarrow P_{ip})]$. In natural law, a person who is not free is either an artificial person or else not an innocent person. Together with the general principle of justice, this gives us

NJ1) $(\blacktriangle p) (P_{np} \rightarrow (P_{ip} = I_p))$

A natural person is free if and only if he is innocent.

'Natural justice' and 'consistency' entail

NJ2) $(\blacktriangle p) (I_{np} \rightarrow A_p)$

An innocent natural person is autonomous.

No innocent natural person is heteronomous.

NJ3) $(\blacktriangle p) (I_{np} \rightarrow \sim A!_p)$

No innocent natural person is strictly autonomous.

NJ4) $(\blacktriangle p) (P_{np} \rightarrow (S_p \leftrightarrow I_p))$

A natural person is sovereign if and only if he is innocent.

Thus, there is no innocent way in which a natural person can justly deprive himself of his freedom, sovereignty or autonomy by making another person responsible for him — either as his master in a hegemonic relationship or as his ruler-and-subject in an autonomous collective.³⁴ Obviously, it is not a matter of formal logic to identify or distinguish the real-world conditions of innocence or criminality. Moreover, injustice happens: innocent persons are sometimes deprived of their freedom — but that is not an argument against the principle of justice.

Other consequences of the principles of natural justice are

NJ5) $(\blacktriangle p)(\blacktriangle q) (I_{np} \& I_{nq} \& \sim p=q \rightarrow (\nabla x)(B_{xp} \& \sim B_{xq}))$

For every pair of innocent natural persons, some means belong(s) to only one of them.

NJ6) $(\blacktriangle p) (I_{np} \rightarrow (\nabla x)(\blacktriangle q)(\sim p=q \rightarrow B_{xp} \& \sim B_{xq}))$

For every innocent natural person, there is a means that belongs exclusively to him.

NJ7) $(\blacktriangle p)(\blacktriangle q)(\blacktriangle x) (B_{nxp} \& B_{nxq} \& I_p \rightarrow p=q)$

What belongs naturally to an innocent person belongs to him exclusively.

NJ8) $(\blacktriangle p)(\blacktriangle x) (B_{nxp} \& I_p \rightarrow O_{xp})$

An innocent person owns what naturally belongs to him.

Theories of Natural Law

Because the class of persons can be partitioned in three jointly exhaustive but mutually exclusive subclasses of sovereign, heteronomous and strictly autonomous persons, the status of a person in law is ‘sovereignty’, ‘strict autonomy’ or ‘heteronomy’. Therefore, we can make an exhaustive list of all logically possible types of theories of order among natural persons (theories of natural law), subject to two straightforward conditions. First, we consider only theories concerning

³⁴ Thus, as a matter of natural justice (formally considered!), only non-innocent (e.g., criminal) natural persons can lawfully be serfs or members of an autonomous collective.

the *originary*³⁵ status in law of natural persons — in other words, the natural rights the theories assign to innocent natural persons.

Second, we consider only theories that refer to natural persons as such. We have seen in our discussion of Rousseau that we can consider natural persons under a certain aspect, e.g. as citizens, and assume that they accordingly have rights not as natural persons but as citizens. However, the aspects under which we can consider natural persons are innumerable and do not form a closed set. Therefore it is pointless to try to list all possible ‘aspect persons’ $a(P)$, $b(P)$, $c(P)$, ... that we might associate with any particular natural person. A theory of law that took aspect persons as its starting point would be indeterminate. It would allow us to say that P is one person but also that, from the point of view of law, $w(P)$, e.g. P -as-a-woman, is a different person with a different set of rights. Similar constructions are possible, as the case may be, for P 's rights as a member of some minority or other, a worker, a child, a pensioner, a veteran, an obese person, and so on and so forth. The multiplication of persons would apply to every natural person P . It would then be all too tempting to dismiss P altogether and simply add P -as-a-human-being, say $b(P)$, to the list of aspect persons. As soon as we admit aspect persons as persons in their own right (not simply as heteronomous serfs of a natural person), we can assign a different status in law to each aspect. Consequently, a natural person P , considered under one aspect, say $a(P)$, might be sovereign and at the same time, considered under another aspect, say $b(P)$, heteronomous or a member of this or that autonomous collective — yet P himself need not have a status in law. In short, P might not be deemed a person and would have no rights unless some “authority” classified him as a member of some relevant group or category. Arguably, that is very nearly the ruling conception of persons and rights in fashionable opinion today. However, it is indicative of an almost complete dissociation of the concepts of ‘person’ and ‘rights’ from any reality. With the suggestion that a natural person is simply a ‘theoretical construct’, the result of assembling different aspect persons, it is also a denial of the proposition that a natural person *is* indivisibly a person — in short, an individual.

³⁵ See note 6 above.

Leaving aside aspect persons, we see that there are only so many logically different types of theories of the natural rights of natural persons. We have listed them in the following table.

TType	S	A!	H	M	Originary status in law of natural persons
<i>Equal originary status for all</i>					
0					None has a status in law, all are mere objects
1	*				All sovereign
2		*			All strictly autonomous
3			*		All heteronomous
4				*	None is a person, all are mere means
<i>Unequal originary status</i>					
5	*	*			All autonomous but only some sovereign
6	*		*		Some sovereign, the rest heteronomous
7	*			*	Some sovereign, the rest mere means
8		*	*		Some strictly autonomous, the rest heteronomous
9		*		*	Some strictly autonomous, the rest mere means
10			*	*	Some heteronomous, the rest mere means
11	*	*	*		Some autonomous, the rest heteronomous
12	*	*		*	Some autonomous, the rest mere means
13	*		*	*	Some sovereign, the rest heteronomous or mere means
14		*	*	*	Some strictly autonomous, some heteronomous, etc.
15	*	*	*	*	Some of every kind

The column 'S' has an entry '*' for theories that assign at least one natural person the originary status of a sovereign person. Similarly, column 'A!', respectively 'H,' has an '*' for theories that assign at least one natural person the status of a member of an autonomous collective, respectively the status of a heteronomous person. An asterix in column 'M' identifies theories that deny personal standing to at least one natural person (giving him the status of a mere means). Theories of type 0 assign no natural person any status in law, neither as a person nor as a means. Such theories consider natural persons as mere objects. (The table does not list the type of theory that assigns only some natural persons the status of a mere object).

Equal versus unequal originary status

The information that a theory assigns an equal status to all natural persons does not tell what that status is. However, the ‘equal status’ theories are, philosophically speaking, considerably less demanding than the ‘unequal status’ types. In particular, they need no justifying argument for discriminating among innocent natural persons. An argument for assigning to such persons one status rather than another is all they need to provide. Note, however, that a theory that assigns the originary status of a member of an autonomous collective to some or all innocent natural persons need not assign all of them to the same collective. Similarly, theories that originally assign an heteronomous status to some or all innocent natural persons need not assign them all to the same masters. Finally, theories that assign the status of a mere means to some or all innocent natural persons need not make them the property of the same non-natural or non-innocent person. Theories of types 2, 3 and 4, then, require not only an argument for justifying their pick of the originary status in law of any natural person, but also an argument justifying a particular distribution of natural persons among an untold number of autonomous collectives, hegemonic collectives or non-natural persons. Only theories of type 1, which assert that every natural person originally (in his state of innocence) is a sovereign person, avoid those complications of discrimination and distribution. In fact, formally speaking, there is only one such theory, although there may still be any number of schemes for interpreting it in terms of real things and relations.

Compatibility with postulates of natural law and natural justice

Let us check which types of theory are compatible with the postulates of natural law and the principle of natural justice. The results are summarized in the next table. Because we are interested only in originary rights, we assume a condition in which all natural persons are innocent: $(\blacktriangle p) (P_{np} \rightarrow Ip)$.

The postulates of natural law (Finitism and Naturalism — PNL in the table below) imply that all means and all persons (including all natural persons) belong to a finite number of natural persons. Therefore, at least some natural persons must be persons in the sense of the general theory of law. This consequence rules out TT0 and TT4. Moreover, the

same postulates imply that there should be at least one autonomous natural person. Therefore, the postulates of natural law rule out TT3 and TT10.

TT	S	A!	H	M	PNL	PC	NJ	
0					No	No	No	
1	*				yes	yes	yes	
2		*			yes	yes	No	√
3			*		No	yes	No	
4				*	No	No	No	
5	*	*			yes	yes	No	√
6	*		*		yes	yes	No	√
7	*			*	yes	No	No	
8		*	*		yes	yes	No	√
9		*		*	yes	No	No	
10			*	*	No	No	No	
11	*	*	*		yes	yes	No	√
12	*	*		*	yes	No	No	
13	*		*	*	yes	No	No	
14		*	*	*	yes	No	No	
15	*	*	*	*	yes	No	No	

According to the postulate of consistency, every natural person is a real person and therefore a person in the sense of the general theory of law. This rules out any type of theory that holds that some natural persons are not persons but mere objects or mere means. Thus, the postulate of consistency — PC in the table — eliminates TT0, TT4, TT7, TT9–10, and TT12–15.

The principle of natural personal justice states that all innocent natural persons are free and therefore sovereign. It rules out all types of theories except TT1. Thus, we see that only TT1 is compatible with the postulates of natural law and the principle of natural justice.

Natural law without natural justice

In the last table of the previous section, we have marked with a ‘√’ all types that satisfy the postulates of natural law but not the principle of natural justice. They may be called types of *political or legal theory of law*, which separate law from justice.

Ttype	S	A!	H	M	Originary status in law of natural persons
<i>Equal originary status for all</i>					
2		*			All strictly autonomous
<i>Unequal originary status</i>					
5	*	*			All autonomous but only some sovereign
6	*		*		Some sovereign, the rest heteronomous
8		*	*		Some strictly autonomous, the rest heteronomous
11	*	*	*		Some autonomous, the rest heteronomous

Each one of those theories implies that at least some innocent natural persons belong to another person. Moreover (because of PN, the postulate of naturalism), they imply that some innocent natural persons belong to at least one other natural person. Consequently, some natural person has a right to the use of another innocent natural person without the latter’s consent. In other words, some natural persons have the right to rule other innocent natural persons without their consent — that is, to legislate for or to impose their ‘will’ on others. TT2- and TT5-theories restrict this right to situations where the right to rule is mutual: it exists only within autonomous collectives. TT6-theories imply that at least some natural persons are sovereign and that at least some of those have the right to rule other innocent natural persons without their consent. TT8-theories imply that some natural persons are members (and therefore rulers and subjects) of autonomous collectives and rulers of other innocent natural persons who are merely subjects. Finally, TT11-theories stipulate that some innocent natural persons are subjects of others (sovereigns or members of autonomous collectives).

The common element of those theories is the idea of one or more natural persons ruling innocent others — and that idea, disguised as the power of legislation, is the centerpiece of most political or legal theories of law. Clearly, all attempts to justify legislation (as distinct from

contractual obligation) must reject the principle of natural justice, which is that innocent natural persons are free.

SECTION III

Human Beings and Human Law

The system L_3 .

We return once more to $\mathbf{D}(L_0)$, this time to introduce the concept of a human being. The concept is referred to with the unitary predicator μ_0 , where o represents an object in $\mathbf{D}(L_0)$. Adding this predicator to the language of L_2 , and making the appropriate changes to the definition of well-formed formulas, we get L_3 . We do not add any axioms for μ : we do not specify aprioristically any constraints on what a human being is supposed to be in the context of a discussion of law. Thus, we can accommodate the postulate of anti-humanism:

Postulate of Anti-humanism: No human being is a natural person.

In the language of legal positivism; no human being is a person by “legal necessity”. Obviously, anti-humanism has no use for the principle of natural justice in its consideration of human beings. It may acknowledge that only innocent humans can be free persons, but it does not hold that in justice an innocent human being is entitled to freedom.

Weaker versions of anti-humanism imply that only some humans are not natural persons. An anti-humanism of this sort could ride in on the back of the postulate of humanist naturalism.

Postulate of humanist naturalism (PHN): Every natural person is a human being.

PHN leaves open the possibility that some human beings are not natural persons. Because it implies that only humans are natural persons, it is unacceptable to those who believe the natural law comprises non-human yet natural persons (gods, demons, personified historical or sociological phenomena like tribes, nations, states or whatever).

To simplify formulas, we define the shorthand I_μ ('innocent human person') as follows

$$I_\mu p =: \mu p \ \& \ I p$$

In conjunction with the postulate of naturalism (PN) and the general principle of justice (GJ), the postulate of humanist naturalism implies

$$(\blacktriangle p) (P_{fp} \rightarrow I_\mu p)$$

All free persons are innocent human beings.

Radically opposed to anti-humanism is the postulate of naturalist humanism:

Postulate of naturalist humanism (PNH): Every human being is a natural person.

The postulate of naturalist humanism leaves open the possibility that there are natural persons other than human beings. It is bound to raise controversies about what non-human natural persons there could be. In conjunction with the principle of natural justice (NJ), naturalist humanism implies

$$(\blacktriangle p) (I_\mu p \rightarrow P_{fp})$$

All innocent human beings are free persons.

PNH is probably too strong: it seems that some human beings (human in a biological sense, e.g., "human vegetables") are not and cannot act as persons. A weaker version avoids this complication, albeit at the cost of inviting controversy about the semantics of 'human person':

Weak postulate of naturalist humanism (PNH'): Every human person is a natural person.

The conjunction of the two postulates PHN and PNH' gives us a general postulate of humanism.

Postulate of humanism (PH): **All human persons are natural persons; nothing else is a natural person.**³⁶

In conjunction with the postulates of natural law and the principles of general and natural justice (GJ and NJ), PH implies

(\blacktriangle p) ($P_{fp} \leftrightarrow I_{\mu}p$)

All and only innocent human persons are free.

Human Law

If we accept the postulate of humanism then the concept of natural human law is formally unambiguous (cf. TT1). However, it does not leave room for an originary unilateral right of legislation, only for contractual obligation. In that sense, it has decidedly anarchistic implications, as indeed we should expect from any theory that treats all human persons alike and assigns them the rights of a sovereign, i.e., free person in their originary, innocent state. Not surprisingly, at all times major political and social thinkers have attempted to deny this conception of natural human law by attacking either the thesis that natural human persons are free or the thesis that they all have an equal originary status in the order of persons.

Under the postulate of naturalist humanism, all human beings are natural persons, but there may be other non-human natural persons. Assuming there are such persons, theories that assign an unequal originary status to natural persons nevertheless can assign an equal status to all innocent human beings (i.e., an equal originary status or equal natural rights to all human beings). This opens the door for theories asserting the existence of non-human legislators that lawfully rule human beings. For example, TT6-, TT8- and TT11-theories may envisage that at least some non-human persons (gods, nations, states) are “naturally” autonomous while human persons are “naturally” heteronomous. The affirmation of equality among humans is then coupled with a denial of natural justice (freedom) for all humans. Similarly, TT2- and TT5-theories allow us to envisage a situation in

³⁶ For a philosophical defense of that proposition, see F. van Dun, *Het Fundamenteel Rechtsbeginsel* (Kluwer-Rechtswetenschappen: Antwerpen, 1983).

which all human beings are members of some autonomous collective or other. This too is consistent with an equal status for all human persons and with a denial of freedom for all human persons.

Under the postulate of humanist naturalism, all natural persons are humans but there may be human beings that are not natural but artificial persons or even mere means or objects. That gives us a possibility for asserting the right of legislation, this time for human beings that are natural persons over those that are not. Here the denial of natural justice (freedom) for some humans is a consequence of the denial of equality among humans.

It would appear that up to the middle of the eighteenth century denials of human natural law and justice generally took the form of a denial of the equality in law of all human beings. Plato argued that while equality in natural law is a fact, it nevertheless must be denied by ‘the noble lie’³⁷ if politics is ever to rise above the institutionalization of war. Equal human nature must be doctored by political education to make it fit the requirements of inequality that the political order imposes. Aristotle, in contrast, asserted that apart from a small elite of well-born, educated male Greeks, human beings are persons only in an imperfect sense, naturally fit to be ruled but not to rule.³⁸ Equality was not given in nature, and therefore not a requirement of natural law. Much later, Hobbes rested his case for the state as the source of sovereign positive legislation on the supposition that natural equality means a natural autonomous collective, which is a state of universal war. Survival requires the institutionalization of inequality. Thus, whether inequality was seen as a natural fact (Aristotle) or a necessary condition of political existence and survival (Plato, Hobbes), equality was shunted aside so that at least some humans could be free. All of those views are compatible with humanist naturalism (“Every natural person is a human being”). Note, however, that the argument was that *natural* equality (if it existed at all) had to be sacrificed and replaced by *social* and *political* inequality. Indeed, the argument was part of the larger argument that

³⁷ Plato, *The Republic*, Book 3, 413c-415c.

³⁸ The doctrine that there are ‘slaves by nature’ (in book I of *Politics*) is perhaps the most telling illustration of Aristotle’s attempt to justify social inequality as being ‘according to nature.’

man could only survive as a social being — that is to say, a member of a particular society or social organization.³⁹

Medieval political theology came to rest on the postulate of naturalist humanism: human beings are natural persons but God also is a person — or, as Trinitarian theology would say, a personal being — by his very nature. To this postulate, the theologians later added the idea that only God is a sovereign personal being. Perhaps to maintain their ground against the strident claims on behalf of royal absolutism, the later scholastics presented God as the supreme legislator. Accordingly, regardless of their political or social status, all human beings are equally his servants and subjects. This meant human equality (as a matter of natural law) but also no freedom for any human person. The biblical notion of a covenant was thereby abandoned. It had implied the separation of created human nature from the divine nature of the Creator and allowed for the coexistence of God and human beings, each of them sovereign in their own domains, yet bound by their covenants.

With God as the supreme legislator, equality without freedom was sanctified. However, it could still be maintained without contradiction that the laws of God decreed that his servants should respect one another as free persons. Thus, they might be ordered to treat one another *as if* they were free and sovereign persons, leaving the exercise of unilateral rule over all human beings to God and to God only. That position would have implied an affirmation of natural human equality coupled with freedom as a legal right under divine law.

From the mid-eighteenth century onwards, the natural equality of human beings was sanctified. At the same time, divine law was virtually nullified. Thus, natural equality was taken for granted as the pre-eminent social and political norm — the “just society” had to be a society of equals. In contrast, freedom, as a natural or divinely ordained human right, rapidly lost ground.⁴⁰ The arguments of “enlightened”, progressive philosophy against natural law and justice began to focus on

³⁹ In the terminology of “The Lawful and The Legal”, *op. cit.*, section 4.2, we should say ‘member of an exclusive society’. An exclusive society or social order (Dutch: ‘maatschappij’) is to be contrasted with the inclusive society or convivial order (Dutch: ‘samenleving’).

⁴⁰ The separation of “natural justice” and “freedom” annuls the principle of natural justice.

human freedom as their primary target. Rousseau, for example, turned Plato's argument concerning the dangers of equality against freedom. He argued that while human beings are in fact "born free", they would have to trade in their human freedom for 'civic liberty' if man is to become truly social⁴¹ and politics is to rise above the institutionalization of war. Civic liberty, of course, was not a natural right of human persons, but a political right of the citizen. While it implied that every citizen was at once the ruler and the subject of every other citizen, it also implied that natural human persons as such have no right in the state. Since citizens qua citizens were by definition essentially identical and therefore equal, only civic liberty was compatible with equality and therefore 'just'.

Marx, for his part, turned the Aristotelian argument that there is no such thing as natural equality against freedom. A true Gnostic,⁴² he argued that natural human freedom was an illusion. Claims based on natural law were therefore simply false. Only man-as-Man (or rather man-as-Everything) — Marx's *universal individual* — could be truly free, but that Man was decidedly not the natural, historical human being — Marx's *particular individual* — that we know from experience. Again, the logic was that since universal individuals are essentially identical and therefore equal, only they could be truly 'free' without jeopardizing equality. Of course, the Marxist notion of 'true freedom' was not that of natural freedom. Its basic formulation was that Man is free only to the extent that he can control the natural and social conditions of his existence.⁴³ Translated into terms that take account of the existence of many individuals, it held that an individual is free only to the extent that he can control or rule others. Thus, all individuals can be free only in an autonomous collective, where everybody rules everybody. The apparent

⁴¹ I.e., an inseparable and dependent part of a particular society. 'Social' derives from the Latin *socius*, partner, companion, but originally follower (from *sequi*, to follow).

⁴² See Frank van Dun, Natural Law, Liberalism, and Christianity, *The Journal of Libertarian Studies*, XV, n°3, Summer 2001, p.1-36. Also, of course, Eric Voegelin, *Modernity Without Restraint*, Edited with an Introduction by Manfred Henningsen (University of Missouri Press, 2000), passim. This is volume 5 of *The Collected Works of Eric Voegelin*: it contains his *The Political Religions*; *The New Science of Politics*; and *Science, Politics, and Gnosticism*.

⁴³ M.C. Howard & J.E. King, *The Economics of Marx*, (Penguin, Harmondsworth: 1976), p.15.

paradox of that statement is ‘resolved’ by shifting the focus from ‘particular individuals’ to ‘universal Man’, the common, indeed identical, aspect person which alone has standing in the final communist society. That ‘solution’ is formally the same as Rousseau’s ‘squaring of the political circle’ by banishing natural human beings from the State and redefining politics as the affair of the Citizen.⁴⁴

Rousseau’s theory, with its hypostatization of the abstract ‘man-as-citizen’, inaugurated the fashion of appealing to mystical aspect persons that plagues positivist legal theory to this day. While it dispenses with pre-modern forms of the belief in non-human natural persons, it opens the door for a myriad of other abstractions to replace the gods of yore. Those are the superstitions we now invoke to justify political rule and legislation and to avoid the requirement of natural justice for human persons.

CONCLUSION

Starting from the concepts “person” and “means” and the relation “belongs to”, and having added a few others relating to actions — all of them defined implicitly and sparsely by means of a few axioms — we have been able to derive a set of formal propositions that lay out the basic structure of an order (or law) of persons. With the addition of, again, a few principles and postulates relating to justice and natural persons, we were able to get the outline of natural law as the order of natural persons. Thus, we have not only a derivation of a set of theorems about law but also a demonstration of the fact that, contrary to the claims of most legal positivists, the concept of natural law does not depend on questionable, “unscientific” metaphysics or theology. Admittedly, concepts such as “person”, “means” and “belongs to” are not strictly empirical. They are not mere representations of “raw data” that could be gathered by counters, photosensitive instruments, thermometers or other such devices — but then a strictly empiricist

⁴⁴ Just as Rousseau’s State, if run by natural human beings, would be the epitome of injustice, so would Marx’s communist society, if inhabited by particular individuals, be no more than “raw communism” — “merely a *manifestation* of the vileness of private property.” K. Marx, ‘Private Property and Communism’, in his *Economic and Philosophical Manuscripts*, 1844.

scientific canon is singularly inappropriate for the study of anything other than phenomena that can *only* be observed.

Obviously, to get from here to a material theory of law, we need to have a relevant scheme of interpretation for linking the various elements of the formal theory to what is out there in the real world in which we human persons live. The search for such a scheme lies outside the scope of this paper. The same goes for answering the question, “So what?” Indeed, a mere description of the human condition in terms of a reasonably and carefully interpreted theory of law does not tell us why we ought to respect the law it describes.⁴⁵

Appendix

Order of persons versus order of actions

The concept of law as an order of persons should be distinguished not only from the concept of law as a system of rules or norms (cf., for example, Kelsen and Hart) but also from the concept of law as an order of actions. The latter underlies the approach of F.A. Hayek, *Law, Legislation and Liberty*, Vol. 1: *Rules and Order* (Routledge & Kegan Paul: London, 1973), and to some extent Bruno Leoni, in the essays appended to *Freedom and the Law* (Liberty Fund: Indianapolis, 1991) and in *Law, Liberty, and the Competitive Market*, (Transaction Publishers: New Brunswick, 2009). These authors view law as an order of actions analogous to the orderly nature of free-market processes, even though the concept of a market already implies a reference to an order of ‘persons’ that includes recognition of a market participant’s rights and obligations under a regime of private (or, in any case, “several”) property and freedom of contract. Consequently, they end up arguing that competition yields an order of actions that is superior to central planning in the field of law no less than in the field of economics. However, that does not tell us anything about the entities involved in these processes or about what these processes mean in terms of human

⁴⁵ On this, see Frank van Dun, “Argumentation Ethics and The Philosophy of Freedom”, in *Libertarian Papers*, I, 19 (2009); and *Het Fundamenteel Rechtsbeginsel*.

freedom. As far as law and economics (as orders of actions) are concerned, a world organized into, say, five hundred slaveholding plantations that engage in “market competition” may be superior, from the slaveholders’ point of view, to one in which they all take directions from the same central planning bureau. That does not make it a world in which one would find much evidence of the kind of freedom or liberty that classical liberals such as Hayek and Leoni cherish. A dictator who allows his subjects to compete for his favors may be better served than one who tries to control their every move — but that too is hardly an argument in support of the classical liberal view of law and economics. (Besides, a dictator’s values being as subjective as anybody else’s, it is by no means certain that he would not prefer the enjoyment of seeing people jump at his command to the enjoyment of the gifts they bring.) In a system in which there is only one or a handful of consumers, while everybody else is merely a “human resource” or “human capital”, *consumer sovereignty* and *human freedom* are neither logically nor causally related. Hayek and Leoni were certainly sympathetic to the idea of human persons having natural rights, but they did not incorporate it in their theories of orders of action. Hence, they put the cart before the horse by trying to vindicate their classical liberal or libertarian views on human liberty with arguments derived from the formal properties of competition versus central direction as “processes of discovery”. However, such arguments work just as well for any order of persons — for a bunch of slaveholders or dictators no less than for a world in which all people treat one another as free and equal. After all, the fact that the slaves or the dictator’s subjects do not “like” their condition does not count for more than the fact that the chickens in an industrial chicken farm would escape at the earliest opportunity. Unless we have independent arguments against keeping other people (as distinct from chickens) as slaves, there is no reason to take account of the satisfaction of slaves except to the extent that it may affect the costs of managing them. It is the classical liberal or libertarian views on the right order (“law”) of persons that vindicates advocacy of the competitive order of actions, not the other way around.