An information-theoretic approach to build hypergraphs in psychometrics

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Abstract

Psychological network approaches propose to see symptoms or questionnaire items as interconnected nodes, with links between them reflecting pairwise statistical dependencies evaluated on cross-sectional, time-series, or panel data. These networks constitute an established methodology to visualize and conceptualize the interactions and relative importance of nodes/indicators, providing an important complement to other approaches such as factor analysis. However, limiting the representation to pairwise relationships can neglect potentially critical information shared by groups of three or more variables (higher order statistical interdependencies). To overcome this important limitation, here we propose an information-theoretic framework to assess these interdependencies and consequently to use hypergraphs as representations in psychometrics. As edges in hypergraphs are capable of encompassing several nodes together, this extension can thus provide a richer account on the interactions that may exist among sets of psychological variables. Our results show how psychometric hypergraphs can highlight meaningful redundant and synergistic interactions on either simulated or state-of-the-art, re-analyzed psychometric datasets. Overall, our framework extends current network approaches while leading to new ways of assessing the data that differ at their core from other methods, enriching the psychometrics toolbox, and opening promising avenues for future investigation.

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Introduction

The last decade has seen the growth of various network approaches that allow researchers to probe and exploit potential interdependencies of observable variables associated with a given construct, or between constructs themselves (Borsboom & Cramer, 2013; Borsboom et al., 2021; Golino & Epskamp, 2017; Marsman et al., 2018; Schmittmann et al., 2013; Van den Bergh et al., 2021). These approaches build networks where nodes represent measurable variables and links reflect relationships between them, with the purpose of focusing not on common underlying factors but on the structure of the inter-dependencies between the observables. Recently some approaches have been developed to include also factor nodes in the network (Haslbeck et al., 2021), see below for further details. Such *psychometric networks* are capable of reflecting dynamical processes (Schmittmann et al., 2013), and can consider lagged relationships that can reveal dynamics leading to the emergence of symptoms (Borsboom & Cramer, 2013). Crucially, these networks often reveal non-trivial features of the groups of variables they represent, which can be identified by leveraging tools of network science such as measures of centrality (Bringmann et al., 2019; Epskamp et al., 2018) or community detection (Van den Bergh et al., 2021). Furthermore, it has been argued that important outgoing effects or events can be identified from an examination of either the overall network or part of it — e.g. how interdependencies between certain symptoms may create a vicious circle triggering a series of negative events along the way (Schmittmann et al., 2013).

An illustrative case of the potential of network approaches can be found in the work on *comorbidity* (Fried et al., 2017), i.e. constructs that share multiple observables (e.g. depression and anxiety, which are sometimes considered as different facets of neuroticism (Lovibond & Lovibond, 1995; Van den Bergh et al., 2021), and can be studied in terms of common edges or nodes, without limitations coming from modelling these sets via multiple latent variables (Cramer et al., 2010; Jones et al., 2019).

While network properties such as centrality and modularity provide valuable information about the overall topology of a network (Borsboom et al., 2021), traditional network approaches are limited in a fundamental yet often unacknowledged way: as their construction is based solely on pairwise links, they cannot give account of potential higher order (i.e. beyond pairwise) interdependencies that involve three or more observables (James & Crutchfield, 2017; McGill, 1954; Rosas et al., 2016). In effect, the interdependencies between three or more variables are often nontrivial, poorly understood and, yet, can play fundamental roles in driving complex systems (Battiston et al., 2021). From a statistical perspective, a pairwise approach assessing the relation between two variables (conditioned on other variables or not) cannot inform on the existence of synergistic or redundant interdependencies, which by definition involve three or more variables (McGill, 1954; Williams & Beer, 2010). To sketch an example of redundancy in psychometrics, suppose one has three items that try to measure depression. All items measure the same thing, so they overlap substantially. Pairwise correlations will typically be high, but partial correlations will typically be low, because if you condition on another very similar item, there is not much (residual) information left, and the 'shared' residual information between two items (i.e. their partial correlation) will be small. As far as synergy is concerned, suppose that two uncorrelated items are influencing a third one. These two uncorrelated items would become correlated if, seeking for a mechanism, one would condition on the effect. An example of a synergistic triplet could be reading ability at age 9, IQ, and hours of practice per week. "Knowing" the values of two variables *simultaneously*, would tell us something about the third one. Importantly, in this paper we showed that the term "higher order" implies more than two variables, and it's not related to two-way interdependencies where the "third" variable is for example the product of the other two (i.e. moderation). Recent work is revealing how higher order interdependencies play key roles in diverse hallmark aspects of complex systems, including self-organisation (Rosas et al., 2018), emergence (Rosas, Mediano, et al., 2020), synchronisation (Skardal & Arenas, 2020; Tabar et al., 2024), percolation (Iacopini et al., 2019).

Hypergraphs are the natural extension of pairwise networks that can properly give an account of higher order interdependencies. In effect, analogously as graphs, hypergraphs are composed of nodes and *hyperedges* (or hyperlinks) connecting them, with the key difference that hyperedges can connect more than two nodes (Battiston et al., 2020; Bick et al., 2023; Boccaletti et al., 2023). Hyperedges can represent e.g. co-authors (Milojević, 2014), components of an ecosystem (Bairey et al., 2016), brain regions (Hindriks et al., 2024; Santoro et al., 2023),or financial entities, or individuals sharing a social or physical space in contagion models (Santoro et al., 2023; St-Onge et al., 2022). It is worth mentioning another relevant representation of interactions between three or more variables is the simplicial complex (Bianconi, 2021; Torres et al., 2021; Zhang et al., 2023), whose use we will not explore here. As an example, Figure 1 depicts a hypergraph where each node is a scholar, and each hyperlink (represented as a colored region encompassing the nodes) is a quite popular paper on network psychometrics. Translated to the approach presented here, the nodes would be the observables, and the hyperlinks would be a significant amount of joint information among all the observables. And as it happens with co-authorship, sometimes the paper contains more knowledge than the sum of the knowledge of the single authors (synergy), sometimes the overlap prevails (redundancy), and sometimes the individual contributions are simply independent. And a hyperlink (these three or more people co-authored a paper) can be more informative than pairwise links (these two people were co-authors in some paper). It is important to stress that this example is meant to show what an hypergraph is, but that the links in the example are directly quantifiable (co-authorship).

In the present work we put forward a framework that proposes hypergraphs to represent higher order *statistical* interdependencies. This approach is complementary to the pairwise one. Pairwise networks would still be obtained by (Pearson/Spearman) correlations and partial correlations, or by mutual information and conditional mutual information (Cover & Thomas, 2012) to stay in theme with the proposed approach, or by one of the many other methods proposed over the years to assess statistical dependencies. To put this more in context, the extension that we propose in terms of the first sentence of the abstract of the recent primer on network psychometrics (Borsboom et al., 2021):

> *In recent years, network analysis has been applied to identify and analyse patterns of statistical association in multivariate psychological data. In these approaches, network nodes represent variables in a data set, and edges represent pairwise conditional associations between variables in the data, while conditioning on the remaining variables.*

can be described as:

> *...network nodes represent variables in a data set, and hyperedges represent the joint informational contribution of groups of variables in the data to the typical patterns of the system, without the need for conditioning on the remaining variables.*

Crucially, we do not propose to *model mechanisms of* "higher order interdependencies" as typically done in Markov Random Field models familiar to network psychometrics, as described for example in Haslbeck et al. (2021). To illustrate how our approach is different, consider a simple linear regression $Y \sim X1 + X2$. It could be that the triplet $(Y, X1, X2)$ exhibits redundancy, synergy, or independence. According to Haslbeck et al. (2021), a *second-order* model would be $Y \sim X1 + X2 + X1 * X2$, involving the addition of more information/variables (product terms). Having as a goal to verify that the relationship between y and x1 changes depending on x2 this *second-order* model would need a 4x4 covariance matrix, while our approach would find a synergistic triplet containing Y, X1, and X2 with a 3x3 covariance matrix, and would do so regardless the model that resulted in that covariance matrix. In other words, the two perspectives are complementary, and they can be combined, but they refer to different phenomena. Rather than these "higher order mechanisms" we seek "higher order statistical dependencies", here by investigating the properties of the multivariate probability distributions. This point of view, aimed at disambiguating higher order mechanisms and higher order behaviors, is expressed in a recent correspondence (Rosas et al., 2022).

In the chosen formulation, the analysis of the marginal probability distributions leads to the fundamental distinction between predominantly redundant and predominantly synergistic dependencies, where the former would correspond to shared randomness being higher than collective constraints, and vice versa for the latter. As such, our proposed framework can be conceived as an extension of existing psychometric network approaches that, in turn, enable a systematic investigation of the role of interdependencies in psychometric scenarios, proposing to complement current representations based on pairwise interdependencies. In doing this, our approach is not necessarily overlapping, but rather complementary to the discovery of underlying structures such as latent factors and interdependencies used in psychometric models.

Defining and computing synergy is not a straightforward approach nor one on which all scholars agree. Among the different recipes, we choose the *O-information* (Rosas et al., 2019), an approach that in our opinion has the advantage of intuitively grasping the balance between the concepts of synergy and redundancy, and the ability to parsimoniously scale with the number of variables. The common information shared in groups of three or more variables computed via this metric is validated against null distributions, and against the possibility of this shared information resulting from lower-order interdependencies instead of containing new information. The proposed framework is illustrated on synthetic datasets where higher order behavior is generated according to one of several possible mechanisms is provided, and on standard datasets already analyzed in network psychometrics studies.

Methods

The state of the art: Exploratory Data Analysis and Networks

Let us consider n measurable variables $X_1, \ldots, X_n$, which could correspond to symptoms of a condition of interest or items in a questionnaire. An attractive way to describe the interdependencies between these variables is to represent them as a network (Vasiliauskaite & Rosas, 2020). In general, networks are defined by two elements: (a) a set of nodes/vertices denoted by V, and (b) a collection of links/edges between the nodes denoted by E. Individual edges are usually denoted as e_{ij}, where i and j represent the indices of two nodes. A network of observables typically represents each variable as a node and uses links to depict relationships between the variables.

Among the simplest and most used ways for establishing a relationship between two continuous random variables, one finds the partial Pearson correlation, conditioned against all other considered variables (Epskamp & Fried, 2018). Specifically, network analyses typically consider networks whose links are given by the *precision matrix* $C = \sum^{-1}$ with $\sum$ being the correlation matrix, which allows calculating partial correlations via $\rho_{ij} = -\frac{C_{ij}}{\sqrt{(C_{ii}\dot{C}_{jj})}}$. It is worth mentioning that accurate estimations of the precision matrix are non-trivial and prone to biases, but adequate techniques have been developed in recent years (Epskamp et al., 2018). In particular, it is common to use regularization techniques such as the LASSO (Foygel & Drton, 2010; Friedman et al., 2008; Meinshausen & Bühlmann, 2006) to restrict the number of non-zero entries in the precision matrix.

Information theory

Entropy and Mutual Information

Random variables are a mathematical tool to model scenarios that include randomness and uncertainty. If a model considers a variable X that can display different values when measured according to factors outside of our control, we usually model the frequency of occurrence of each possible outcome via a probability distribution $p(x)$. The amount of information associated with each possible outcome is given by another function

$h(x)$ which has the following properties: (i) it is a monotonic function of the probability $p(x)$ (less likely events are more informative), and (ii) two statistically independent events $\{X = x\}$ and $\{Y = y\}$ together provide the sum of the information given by each, i.e. $h(x, y) = h(x) + h(y)$. From such considerations, one can shown that $h(x) = -\log_2 p(x)$, where the negative sign ensures that information is positive or zero (Shannon, 1948).

The mean value of information $h(x)$, denoted by $H(X) = \mathbb{E}\Big\{h(X)\Big\}$, is called *entropy*, and measures the average amount of information that one would gain when measuring X. Similarly, the entropy of (a vector of) two variables can be calculated via $H(X, Y) = \mathbb{E}\Big\{h(X, Y)\Big\}$. In general, the entropy of two variables is less than the sum of their entropies when measured separately — or, equivalently, the information provided by two variables together is less than the sum of the information provided by each separately. Hence, the difference $I(X; Y) := H(X) + H(Y) - H(X, Y)$, known as Shannon's *mutual information* (Shannon, 1948), is precisely how much information is shared by both variables. An alternative formula is given in terms of conditional entropies by $I(X; Y) = H(X, Y) - H(X|Y) - H(Y|X)$.

Higher order interdependencies via Total Correlation and Dual Total Correlation

While Shannon's mutual information provides an encompassing account of the interdependencies between two (sets of) variables, it is unable to fully explore the rich interplay that can take place within triple or higher order interdependencies (James & Crutchfield, 2017; McGill, 1954; Rosas et al., 2016). Two popular multivariate extensions of the Shannon mutual information are the *Total Correlation* (TC) (Watanabe, 1960) and the *Dual Total Correlation* (DTC) (Te Sun, 1978), which can be calculated as

$$\mathrm{TC}(X_1, \dots, X_n) := \sum_{i=1}^{n} H(X_i) - H(X_1, \dots, X_n), \tag{1}$$

$$\mathrm{DTC}(X_1, \dots, X_n) := H(X_1, \dots, X_n) - \sum_{j=1}^{n} H(X_j | X_1, \dots, X_{j-1}, X_{j+1}, \dots, X_n). \tag{2}$$

On the one hand, the TC follows the idea of calculating the difference between the sum of the information provided by each variable separately and the information provided by them together, extending the formula $I(X_1; X_2) = H(X_1) + H(X_2) - H(X_1, X_2)$. On the other hand, the DTC captures the information in each variable that cannot be extracted from other variables (captured by the terms in the form of $H(X_j|X_1, \ldots, X_{j-1}, X_{j+1}, \ldots, X_n)$) and subtract it to the total information $H(X_1, \ldots, X_n)$, generalising the formula $I(X_1; X_2) = H(X_1, X_2) - H(X_1|X_2) - H(X_2|X_1)$. Both the TC and DTC are non-negative quantities that are zero if and only if all variables $X_1, \ldots, X_N$ are jointly statistically independent — i.e. if $p(X_1, \ldots, X_n) = \prod_{i=1}^{n} p(X_i)$. However, while they are both equal to the mutual information for $n = 2$, they are in general different when $n \geq 3$. One way to understand their difference is by noting that while the DTC measures the amount of *shared randomness* between the variables (i.e. the information that can be collected in one variable that also refers to the activity of another), the TC accounts for the effect of *collective constraints* (i.e. joint patterns that the system is precluded to explore). For example, if X_1, X_2, X_3 are three binary variables satisfying $X_1 = X_2 = X_3$, this means that they hold relatively little shared randomness (just the entropy in one of them) while they have strong shared constraints, as there are many configurations that are prohibited to the system (namely, all in which some of the variables are not equal); therefore, for this example TC > DTC. For more details about this interpretation, we refer the reader to Rosas et al. (2019, Sec. 2).

A noteworthy case is the three-variable setting. Let's consider the expression of DTC in this case:

$$\mathrm{DTC}_{N=3} = I(X_1; X_2|X_3) + I(X_2; X_3|X_1) + I(X_3; X_1|X_2) + I(X_1; X_2; X_3). \tag{3}$$

The last term is known as interaction information McGill, 1954) and can be defined in term of entropies as:

$$\begin{aligned} I(X_1; X_2; X_3) \coloneqq{}& H(X_1) + H(X_2) + H(X_3) - H(X_1, X_2) - H(X_2, X_3) \\ & - H(X_1, X_3) + H(X_1, X_2, X_3) \end{aligned} \tag{4}$$

$$= I(X_1; X_2) - I(X_1; X_2|X_3) \ . \quad (5)$$

This quantity indicates the amount of information shared by three variables (Ince et al., 2017). The relation between DTC, (conditioned) mutual information, and interaction information for three variables is depicted in figure 2.

Let us illustrate how these measures differ in some simple examples. First, let's consider three fair coins that are always equal, i.e. $X_1 = X_2 = X_3$ with X_1 distributed as a Bernoulli variable with parameter $p = 0.5$. Then, a simple calculation shows that $\mathrm{TC} = 2$ while $\mathrm{DTC} = 1$, which means that while the system obeys many collective constraints (i.e. there are many things the system cannot do) while having little shared randomness (only one bit). To contrast, let us consider a system made of two independent fair coins X_1 and X_2, and make $X_3 = X_1 + X_2 (mod 2)$ be an exclusive-OR (XOR) of them. Then, a direct calculation shows that $\mathrm{TC} = 1$ while $\mathrm{DTC} = 2$, showing that this system has relatively fewer collective constraints while having more shared randomness. In fact, the first system can be better described via its shared randomness (the three variables share the same bit), while the second is better described by its collective constraint (the third variable has to be an XOR of the other two).

Measuring higher order effects via the O-information

An attractive way to exploit these complementary views is then by considering their difference,

$$\Omega_n(X_1, \ldots, X_n) = \mathrm{TC}(X_1, \ldots, X_n) - \mathrm{DTC}(X_1, \ldots, X_n) \ . \quad (6)$$

which is known as the *O-information* (Rosas et al., 2019). Put simply, the O-information compares how hard is to describe the interdependencies of a system as collective constraints or as shared randomness — being positive if the former would lead to a more parsimonious description, and negative if the latter would be preferable. Importantly, it has been shown that this leads to an effective way to account for the balance between high- and low-order statistical constraints in a system: Low-order constraints impose strong restrictions on the system and allow a little amount of shared information between variables, while higher order constraints impose collective restrictions that enable large amounts of shared randomness. By construction, $\Omega(X_1, \ldots, X_n) < 0$ implies a predominance of higher order constraints within the system of interest, a condition that is usually referred to as *statistical synergy*. Conversely, $\Omega(X_1, \ldots, X_n) > 0$ implies that the system is dominated by low-order constraints, which imply *redundancy* of information. From the example in the previous paragraph, it is evident that O-information and interaction information are equivalent for three variables, although they generally differ for four or more variables. For more details related to the O-information and its interpretation, we refer the reader to Rosas et al. (2019). It is important to stress that the entropy, the mutual information, and related techniques can be calculated using different kinds of estimators, the classical ones being based on binning of the data or measuring their distance in the space of all possible realizations (embedding), while other ones are often preferred due to their convenient tradeoff between loss of accuracy and simplicity of calculation. These methods are reviewed in Ince et al. (2017). Here we perform rank-based normalization of the data using the inverse complementary error function and use an estimator based on the covariance matrix, in line with what is still the most used estimator for pairwise networks in psychometrics (Epskamp & Fried, 2018). Furthermore, this estimator is particularly parsimonious, analytically tractable, and robust for distributions that moderately deviate from gaussianity.

Measuring higher order effects on data

Consider a scenario where we have data from n variables of interest, denoted by $X_1, \ldots, X_n$. It is important to note that the O-information of the whole, $\Omega(X_1, \ldots, X_n)$, only provides information about the *overall balance* of synergy and redundancy, and hence it is also interesting to calculate the O-information of *multiplets*, i.e. subsets of variables, as they can show a different balance that may be buried among other patterns.

Following these considerations, we employ the following analytical pipeline:

1. Calculate the O-information for multiplets of sizes from 3 to m (with $m \leq n$; the

multiplet with n=2 represents pairwise mutual information).

2. Partition the multiplets depending on the sign of the O-information into positive (redundant) and negative (synergistic).

3. Evaluate the bias-corrected and accelerated confidence intervals of the O-information values via nonparametric bootstrap with replacement (Davison & Hinkley, 1997; DiCiccio & Efron, 1996; Efron & Tibshirani, 1994).

4. Compare the bootstrap confidence interval of each multiplet with subsets of its components one order below. Multiplets are discarded if the confidence intervals overlap, as this indicates that they do not display additional synergy or redundancy.

5. Build two hypergraphs with the significant multiplets: one with hyperedges of positive O-information related to dominantly redundant interdependencies, and another with hyperedges of negative O-information associated with synergy.

The maximum size of the multiplet can be chosen according to the scientific question, and be limited by computational constraints. When considering and statistically testing all possible multiplets, as we do here, the computational complexity is of the order of $(n_{boot} + 1) \times n^m$. Another possibility is to choose a greedy approach, building the multiplets each time adding the variables providing a significant increase in information, which scales as $(n_{boot} + 1) \times n \times m$. The outputs of this pipeline are *redundancy* and *synergy hypergraphs*, which give an overview of the structure of interdependencies in the system by encoding the multiplets whose interdependencies are dominantly redundant or synergistic, respectively. These two hypergraphs can then be used to perform further analysis related to their topology, including centrality or clustering analyses.

In this paper, step 1 is carried out by transforming the data to obtain standard normal samples with the same empirical cumulative distribution function value as the input as described in Ince et al. (2017), providing the advantages of Gaussian parametric estimation for variables with different marginal distributions. Here we choose a covariance based estimator for a clearer comparison with previous estimators on pairwise networks in psychometrics, and because it allows a parsimonious yet robust estimation of the O-information, but it is important to stress that this choice does not require the variables to be normally distributed and that the framework itself is independent of the type of estimator used. The loss in accuracy for datasets which deviate from gaussianity is in our experience, for the dimensionality and distribution of the data most often encountered in network psychometrics, less severe than the problems encountered by other estimators. The bootstrap replicates the data structure by building covariance matrices obtained shuffling the observables, and the distribution of bootstrapped data allows for a meaningful definition of the confidence interval. The significant multiplets are those with a bias-corrected and accelerated bootstrap confidence interval not containing zero and have an associated p-value smaller than a fixed significance level, e.g. 0.01), corrected for multiple comparisons using the Bonferroni-Holm false discovery rate (FDR). In larger datasets, where the number of possible multiplets can become very high, one can choose to focus on those with a higher value and thus validate via bootstrap only the n most informative ones. Concerning step 4, an alternative approach is to compare O-information values estimated on the same cardinality, by means of surrogates of the samples of the variables which is added to the rest of the system, as described in Scagliarini, Marinazzo, et al. (2022). After step 4 is completed, the output will be a collection C of multiplets of different lengths, $X_1, \dots, X_m$ with $m < n$, featuring any of the n variables under analysis. This collection C can then be interpreted as a list of hyperlinks indicating non-overlapping higher order interdependencies between the considered variables. For illustration, the analyses presented in this paper focus on multiplets of order 3, 4, and 5.

Generating higher order behaviors on synthetic data

In order to evaluate our approach, this section provides a simple possible procedure to generate synthetic data with specific levels of higher order *information*. It is crucial to stress once more that the approach that we propose here is aimed to find higher order behavior (or higher order effects), and that these might originate from different mechanisms (or causes), which can be themselves higher order, or not. We appreciate that for colleagues more used to generative and causal models this might seem dissonant, but we think that this is an important complementary mindset. We thus employ a general approach based on three steps, which are described as follows. First, we construct triplets of variables generically named *x,y,z* with a pre-specified level of higher order information — specifically, we focus on multivariate Gaussian systems and build 3×3 covariance matrices that exhibit the desired level of higher order information. These covariance matrices are then stacked on the diagonal of a $n \times n$ covariance matrix, where n is the total number of nodes in the system. As the second step, we build connections between some of these triplets (if desired) by setting the corresponding elements of the off-diagonal covariance matrix to non-zero values. As the final step, we use the resulting covariance matrix to generate multivariate Gaussian data. To reduce the corresponding degrees of freedom, we set the means to zero. Moreover, as the scale is not important we standardize each of the variables — so that the covariance matrix of the resulting data is in fact a correlation matrix. The most challenging step of this process is the construction of 3×3 correlation matrices with a pre-specified level of higher order information, and is described below.

From Eqs. (6), (1), and (2), the O-information between three variables (X, Y, Z) (coinciding with the interaction information) can be calculated as

$$\Omega(X; Y; Z) = I(X; Y) - I(X; Y|Z), \tag{7}$$

Moreover, if we assume (X, Y, Z) follow a multivariate Gaussian distribution, then their O-information can be calculated from their correlation matrix as follows:

$$\Omega(X; Y; Z) = \frac{1}{2} \log \frac{1 - \rho(X, Y|Z)^2}{1 - \rho(X, Y)^2} = \frac{1}{2} \log \frac{|\Sigma|}{|\Sigma_{xy}\Sigma_{yz}\Sigma_{xz}|}, \tag{8}$$

The first equality in equation 8 uses the fact that for multivariate Gaussian variables $I(X; Y) = -\frac{1}{2} \log(1 - \rho(X, Y)^2)$ and $I(X; Y|Z) = -\frac{1}{2} \log(1 - \rho(X, Y|Z)^2)$, where $\rho(X, Y)$ is the Pearson correlation between X and Y and $\rho(X, Y|Z)$ is the conditional

Pearson correlation of X and Y given Z, which can be calculated as

$$\rho(X,Y|Z) = \frac{\rho(X,Y) - \rho(X,Z)\rho(Y,Z)}{\sqrt{1-\rho(X,Z)^2}\sqrt{1-\rho(Y,Z)^2}}. \tag{9}$$

The second equality can be directly verified, and $|\Sigma|$ denotes the determinant of the correlation matrix of (X, Y, Z) and $|\Sigma_{ij}|$ the determinant of the submatrix of the corresponding variables.

Overall, the second expression shows that the O-information is transformation invariant (i.e. it does not depend on the mean values or variances of its arguments but only on the three correlations ρ_{xy}, ρ_{yz}, and ρ_{xz}), and the latter that it is symmetric on its three arguments.

While finding the O-information that corresponds to three correlation values is straightforward, the reverse operation is highly non-trivial. To facilitate this process, we will use a parameterization based on factor analysis. The factor model with a single factor and n indicators y_k, $k = 1, 2, \ldots, n$, can we written as

$$y_k = \lambda_k \eta + \epsilon_k \tag{10}$$

where η is a latent random variable, $\lambda_k \in \mathbb{R}$ is the factor loading for indicator k, and ϵ is additive Gaussian noise with covariance matrix Θ with components $\theta_{i,j}$. For simplicity we set $\mathrm{Var}(\eta) = 1$, $\mathrm{E}(\epsilon_k) = 0$, and $\mathrm{Cov}(\eta, \epsilon_k) = 0$ for all $k = 1, \ldots, n$. Then it follows that the covariance matrix of $y_1, \ldots, y_n$, denoted by Σ, is given by

$$\Sigma = \lambda\lambda^T + \mathbf{\Theta}, \tag{11}$$

where $\lambda = (\lambda_1, \ldots, \lambda_n)$.

If we have a triplet of three variables X, Y and Z, the factor model can be expressed by the equations:

$$X = \lambda_x \eta + \epsilon_x$$

$$Y = \lambda_y \eta + \epsilon_y$$

$$Z = \lambda_z \eta + \epsilon_z.$$

In order to create a correlation matrix that either exhibits redundancy, independence, or synergy, we will first fix the factor loadings to apriori chosen constants. Several choices are possible, but we have used $\lambda_x = \sqrt{0.99}$, $\lambda_y = \sqrt{0.70}$ and $\lambda_z = \sqrt{0.30}$. To ensure that the resulting covariance matrix has a unit diagonal, we set the value of θ_x, θ_y and θ_z to $(1 - 0.99)$, $(1 - 0.70)$ and $(1 - 0.30)$ respectively. The only parameter that we will vary is the error covariance (ecov) $\mathrm{Cov}(\epsilon_y, \epsilon_z)$. All other off-diagonal elements of $\boldsymbol{\Theta}$ are fixed to zero. The possible range of this ecov parameter is about $[-0.458, +0.458]$ as determined by $\sqrt{1 - 0.70} \times \sqrt{1 - 0.30}$. Otherwise, the correlation matrix would no longer be positive definite. Changing this ecov parameter allows us to set the level of O-information. For example, if we let ecov to be -0.14849, the level of O-information is smaller than 1e-07, implying independence. If we let ecov to be -0.39 or 0.22, the level of information is about -0.58 (indicating synergy) and 0.175 (indicating redundancy) respectively.

Table 2, Table 3 and Table 1 report the correlations, the partial correlations, and the difference of their squared values for the cases above, where we use $A.x$, $A.y$ and $A.z$ to refer to the three variables that make up the triplet A.

As a straightforward consequence of equation (8), the difference between the squared correlation and the squared partial correlation will have positive nondiagonal entries for redundancy, negative for synergy, and zero for independent interaction (zero O-information).

Obviously, the level of O-information will be reflected also in the partial correlations. As a result, if we apply the graphical lasso to these triplets and visualize them using the *qgraph* package, they result in different patterns as can be seen in Figure 3.

A final model can combine several triplets, each with its own value for the ecov parameter. The full covariance matrix is then constructed by placing the model-implied covariance (correlation) matrices of the triplets on the main diagonal. In addition, we can allow for variables to be connected by setting the corresponding off-diagonal elements of the covariance matrix to a non-zero value. An example covariance matrix for three connected triplets is shown in Table 4.

	A.x	A.y	A.z
A.x	1		
A.y	0.832	1	
A.z	0.545	**0.678**	1

	A.x	A.y	A.z
A.x	1		
A.y	0.751	1	
A.z	-0.048	0.483	1

	A.x	A.y	A.z
A.x	0		
A.y	0.129	0	
A.z	0.295	0.226	0

Table 1

Redundancy, $ecov = 0.22$*.*

Top: correlation.

Middle: Partial correlation.

Bottom: squared correlation minus squared partial correlation.

	A.x	A.y	A.z
A.x	1		
A.y	0.832	1	
A.z	0.545	**0.310**	1

	A.x	A.y	A.z
A.x	1		
A.y	0.832	1	
A.z	0.545	-0.310	1

	A.x	A.y	A.z
A.x	0		
A.y	0	0	
A.z	0	0	0

Table 2

Zero II, $ecov = -0.14849$*.*

Top: correlation.

Middle: Partial correlation.

Bottom: squared covariance minus squared partial correlation.

	A.x	A.y	A.z
A.x	1		
A.y	0.832	1	
A.z	0.545	**0.068**	1

	A.x	A.y	A.z
A.x	1		
A.y	0.951	1	
A.z	0.883	-0.823	1

	A.x	A.y	A.z
A.x	0		
A.y	-0.211	0	
A.z	-0.483	-0.684	0

Table 3

Synergy, $ecov = -0.39$*.*

Top: correlation.

Middle: Partial correlation.

Bottom: squared covariance minus squared partial correlation.

A visualization of the pairwise and partial correlation networks estimated from this covariance matrix via the *EBICglasso* function with default parameters from the *qgraph* package is presented in Figure 4.

	A.x	A.y	A.z	B.x	B.y	B.z	C.x	C.y	C.z
A.x	1								
A.y	*0.832*	1							
A.z	*0.545*	*0.678*	1						
B.x	0	0	0	1					
B.y	0	0	0	*0.832*	1				
B.z	0	0	**0.150**	*0.545*	*0.308*	1			
C.x	0	0	0	0	0	0	1		
C.y	0	0	0	0	0	0	*0.832*	1	
C.z	0	0	**0.150**	0	0	**0.150**	*0.545*	*0.068*	1

Table 4

Covariance matrix representation of the example displayed in Figure 4 including off-diagonal covariances in bold.

Results

Synthetic datasets

The simulations described above where implemented in *lavaan* (Rosseel, 2012). Having discussed the basic elements of our procedure, we now present several potential model variations, changing the level of O-information for one or more triplets. For this purpose, each of the suggested models has been scaled up to include between nine and twelve triplets. All of the following layouts will use one particular set of residual correlations, meaning that only residual correlations are used between z-variables. These layout designs are depicted in figure 5 and show how each node from a triplet can be connected to two, three or four nodes belonging to other triplets. For the results below we simulated 2000 observables. We first analysed a system composed of 27 variables arranged into nine triplets with various covariance structures, which were — as triplets — independent of each other. We considered three covariance structures, which give zero, positive, and negative O-information, respectively (we informally refer to the O-information of these triplets as “triplet information”). Our analysis pipeline identifies redundant and synergistic interdependencies in multiplets of order 3, but nothing in higher orders, see figure 6. The areas representation of the hypergraphs are obtained with a modified version of the HyperNetX package (Praggastis et al., 2019).

As a second step, we analysed scenarios where three groups of three triplets each were linked through their z variables (layout 2 in Figure 5). These links through the z variables in each triplet — weaker than the inter-triplet links — resulted in the appearance of redundant multiplets in the independent case, see figure 7. For the case of positive triplet information, these links result in new redundant triplets connecting the z variables of different triplets, generating a more connected hypernetwork. This is a clear example of a higher order behavior emerging from pairwise links. Even if the triplets have zero O-information in their covariance matrices, the coupling between them gives rise to redundant triplets. The synergistic triplets are not much disturbed by these links, while one multiplet of order 4 appears. Interestingly, while the resulting hypergraph of redundant hyperedges follows the pairwise structure, it is much less straightforward to derive the resulting synergistic hyperedges with the current approaches. When two triplets of variables are coupled (i.e. six variables that are strongly correlated within two disjoint groups of two, and two variables each from a different triplet have a weak residual correlation), then one can expect to observe interdependencies involving multiplets of cardinality 4 or more. For example, colliders to a given node then become colliders also for variables in the other triplet. Hence, it is not surprising to find synergistic circuits of 4 variables in these cases. On the other hand, depending on the sample size and on the details of the internal correlations of triplets, just some of the synergistic circuits of order >3 will be detected. This is completely different from LASSO, because the latter aims to get a description of the system "in economy", hence it will discard the weak residual correlation between triplets and it detects only the triplet’s dependencies. By contrast, the information-based approach can instead also detect subtle dependencies of order higher than three.

Finally, we analysed scenarios with three groups of four triplets linked through the z variables, see layout 3 in Figure 5. The results are in figure 8. The between-triplets links result in synergistic multiplets of order 3 also in the cases when the covariance matrices are designed to give zero or positive O-information. In the synergistic case, synergy extends to order 4 maintaining the same integration of linked multiplets. Overall, the inclusion of larger cliques produces a richer array of higher order interdependencies, resulting in redundant and synergistic hypergraphs that emerge more clearly compared to the corresponding pairwise network.

Influence of sample size

We retrieved the informational multiplets from the simulated data (layouts 1 and 2 described above) as we varied the sample size from 50 to 2000, randomly selecting a subset of the original dataset constituted by 2000 samples, repeated 100 times, and then averaging the number of retrieved multiplets for each type and size. Figure 9 reports the numbers of multiplets retrieved after analyzing data generated according to layout 1 (left column) and layout 2 (right column). The types of interdependencies explicitly sought by structuring the covariance matrix to have a certain amount of O-information (zero for neither synergy nor redundancy, positive for redundancy, negative for synergy) are all retrieved starting at less than 200 observables. This is the case for both models. Additionally, we see how more subtle effects arising from inter-triplet coupling (redundancy with negative or zero intra-triplet O-information, synergy with positive intra-triplet O-information compatible with zero) need more observables to become evident. For example, in the case of variables coupled in a group of three with zero O-information, the first redundant multiplets to become detectable are the internal ones, followed by the ones corresponding to the z variables connected by black lines in panel 2 of Figure 5.

Again, there is no "ground truth", even if some lower and upper limits to the number of multiplets of variables with higher order behavior exist (e.g. zero for independent triplets with no O-information, 12 for the case described above).

Reanalysis of the Empathy dataset — Briganti et al. (2018)

We re-analysed the psychometric data presented in Briganti et al., 2018 around the construct of empathy ($N = 1973$). In this case we focused on a maximum of 50 most informative multiplets for each type and order. We find (figure 10) that the O-information can identify significant synergistic and redundant interdependencies co-existing within the hypergraph representation of the construct. Interestingly, multiplets of orders 3 and 4 identify the same major factors obtained with the pairwise psychometric network construction — namely fantasy, perspective taking, personal distress, and empathic concern Briganti et al. (2018). Since network embeddings tend to place at the center items engaging in more multiplets at once, our hypergraph visualisations contribute to conveying visually Briganti and colleagues' findings that empathic concern represents a pivotal dimension of empathy, engaging with several other nodes/items. However, this happens only for redundant interdependencies up to order 3 and 4: at higher multiplet orders, fantasy becomes a more relevant major factor. This highlights higher order interdependencies between cognitive-related projection abilities into fictional characters, measuring fantasy, and more emotionally-focused items relative to distress, empathy, and anticipation of the future. Please note that these distinct trends could not have been retrieved by pairwise psychometric networks.

These results relate with recent independent studies indicating a stronger-than-expected interplay between fantasy and emotions in regulating empathy (Brown et al., 2017; Nomura & Akai, 2012; Stotland et al., 1978). Interestingly, synergistic interdependencies highlight items 3 and 24 as pivotal. Whereas item 3 relates to the ability to see things from the perspective of other individuals, and is thus evidently crucial for synergistic interdependencies relative to empathy, item 24 ("I tend to lose control during emergencies") is relative to emotional regulation and distress. These results suggest that emotional regulation — the ability to exert control over emotional responses — provides information over the whole construct but only when combined with items of all other four factors, thanks to hyperedges.

The retrieved informational hyperedges could be interpreted also as quantitative indications that emotional regulation and empathy greatly overlap with each other, mainly in terms of understanding emotions, as also pointed out in independent studies (Stotland et al., 1978; Thompson et al., 2019; Zaki, 2020). Although the causal directionality between empathy and emotion regulation has not been clearly assessed, exploratory studies have shown how a lack of empathy can trigger emotional dysregulation in young adults (Schipper & Petermann, 2013), further underlining the relevance of the synergistic centrality of emotion regulation found in our hypergraph representation.

The issue of visualization

One of the reasons why networks are used is that they offer an appealing visual representation of the system under study, complementing our knowledge of it. In some cases (proteomics, large social networks, etc.) networks involve hundred or thousands of nodes, a moment arrives (sooner or later depending on the sparsity of the network) when the nodes and the links are no longer all visible. Even a "hairball" network can arguably be visually appealing, and more importantly, its local and global network properties remain well-defined, even if they cannot be easily eyeballed from a figure. In hypergraphs, this "visual curse of dimensionality" necessarily arrives earlier, as it is also evident from the figure chosen as an example in the paper describing XGI (Landry et al., 2023), a recent Python package for the analysis and display of higher order networks. It is also well represented in this paper by the "crop circles" appearance of some of the figures shown so far.

We don't think this is necessarily a problem, even more so for the approach presented here, which is by construction non-parsimonious: again what matters is to find groups of variables sharing information.

Possible mitigation strategies could be interactive figures or notebooks in which users can toggle to evidence or hide certain nodes, labels, or hyperlinks. On the other hand we do not recommend thresholding, since the output of the algorithm contains multiplets that have all been already statistically validated.

Complementary to the network visualization, one could visualize for each variable the number of multiplets it is involved in, and the (averaged) informational value across these, as exemplified in figure 11.

Also for this dataset, we studied the effect of sample size, observing that redundant multiplets emerge earlier, and the maximum value is reached with a few hundred observables or less. More subtle effects (synergy) become more evident with an increasing number of observables (figure 12).

Reanalysis of the PTSD dataset — (Armour et al., 2017)

As a second re-analysis, we studied the PTSD dataset (Armour et al., 2017) which includes information about $N = 221$ U.S. veterans. Also in this case we focused on a maximum of 50 most informative multiplets for each type and order. Our analyses did not identify significant synergistic interdependencies between variables, see Figure (13), which reflects a lack of higher order interdependencies across items in the four major factors — which include elements as distinct as intrusions, avoidance, cognitive and mood alterations, and arousal and reactivity alterations. In contrast, several redundant interdependencies were found among items in multiplets of orders 3 and 4. This finding suggests that the above factors may be driven by similar underlying components. The redundant O-information hyperedges identify items about nightmares, flashbacks, physiological cue reactivity, and avoidance of reminders as being highly central. This combination of behavioral, emotional, and memory-related patterns underlines the complexity of PTSD as a condition affecting not only the emotional and cognitive spheres of individuals, but also their behavior and physiology, as pointed out by recent reviews (Armour et al., 2020; Bisson et al., 2021).

The lack of a clearer internal structure and of synergistic interdependencies might be due not only to the intrinsic complexity of the construct but also to sample size issues, or other methodological issues. To better understand this, we re-analysed the same dataset while using a larger number of variables (Armour et al., 2017) to include covariates, see Figure (14). In this second hypergraph, clinical covariates like anxiety and depression levels were identified as being pivotal for redundant interdependencies across multiplet orders 3, 4, and 5. This indicates that anxiety and depression levels overlap significantly with behavioral, emotional, and physiological symptoms of PSTD, in agreement with independent studies (Armour et al., 2020; Bisson et al., 2021). The addition of clinical covariates polarised the hypergraph structure and led to a more pronounced separation between intrusion variables and others. Furthermore, intrusive thoughts were found to be an item bridging multiplets at order 4 in terms of redundant information. Hence, our quantitative analysis indicates that intrusive thoughts play a key role in PSTD disorders since they possess richly redundant information and are thus relevant for understanding the whole construct.

Our findings relate with other studies highlighting intrusive thoughts as key signs of PTSD (Brooks et al., 2019; Ressler et al., 2022), potentially rising from a maladaptive thought suppression, i.e. a conscious, imperfect, cognitive-driven suppression of a negative thought can enhance the prominence and recurrence of the thought itself, causing worry and rumination. The persistence of intrusive thoughts has documented consequences over behavior and emotional wellbeing (Armour et al., 2020; Brooks et al., 2019; Ressler et al., 2022), which support the redundancy higher order patterns found in our work. Interestingly, synergistic interdependencies are missing even in this expanded set of items. Future research should investigate whether this pattern is due to sample size issues or other methodological aspects of PTSD measurements.

Discussion

We proposed a novel approach to capture higher order interdependencies, in the sense of effects, or behavior, rather than in the sense of mechanisms, between observables in psychometrics and described them in the form of hypergraphs. In particular, we introduced an analysis pipeline that builds two hypergraphs reflecting redundant and synergistic interdependencies. This approach capitalizes on recent advances in multivariate information theory, which have provided data-efficient metrics to capture these phenomena in multiplets of different sizes, and has the scope to build up and extend existent network approaches to psychometrics. Current network approaches to psychometrics are based on a key assumption: that relationships between variables can most of the time be represented as pairwise networks — either in cases of having a common cause or effect, or being linked by a directed flow of influences (i.e. as a directed acyclic graph) or an Ising-like structure (Borsboom et al., 2021). This assumption, in turn, leads to the adoption of pairwise random Markov fields as a natural modelling choice for identifying the interdependencies between variables. However, recent work (Grasso et al., 2021; Mediano et al., 2021; Williams, 2011) has convincingly shown the relevance of higher order interdependencies, in which relationships often take place between groups of variables in a way that cannot be reduced to a set of pairwise links. Building on these insights, our proposed framework aims to relax these assumptions along two axes, extending and complementing, not substituting, the pairwise approach.

On the first axis, we propose to disentangle the analysis of higher order mechanisms (and structures) from the higher order behaviors that can emerge in complex systems. As described in Rosas et al. (2022) in the context of physical systems, there is a fundamental distinction between mechanisms that address how the system is structured and behaviors that correspond to emergent properties related to what the system "does." Furthermore, the existence of higher order mechanisms do not imply higher order behaviors or *vice-versa*. Consequently, our proposed approach focuses on the analysis of higher order patterns in the data, while explicitly acknowledging that this does not bring straightforward implications on how the data was generated in the first place.

The second axis relates to possible confounding effects and their nature. Variables sharing information, yet considered as individual or pairwise interacting units, can pose problems for proper network reconstruction and in assigning a given centrality score to an individual variable. The presence of a variable could either enhance or deteriorate statistical dependencies between other variables, possibly influencing results for network estimation when using established centrality indices of strength, betweenness, and closeness (Hallquist et al., 2021). Analogous issues of deteriorated model validity would hold also for other approaches like factor analysis (Tuccitto et al., 2010). Another possibility could be the existence of a statistical dependency between two variables which is considered insignificant in an exploratory analysis (Epskamp et al., 2018; Golino & Epskamp, 2017). This issue increases the concern of detecting false positives regarding statistical dependencies during estimation. While some efforts have been done in this direction, e.g. Haslbeck et al. (2021), correlations can persist even after conditioning on a latent variable, a phenomenon known as local dependency (McDonald, 2013). Our work contributes quantitatively to this direction while exploiting the powerful statistical framework of higher order interdependencies in complex networks (Rosas et al., 2022). Considering higher order interdependencies involving the observables only, and thus transferring these directly to hyperedges may let network approaches to be even more complementary to classical factor analysis. Crucially this approach could reconcile two competing points of view on shared variance and unique covariance between variables (Forbes et al., 2021; Waldorp & Marsman, 2022). The instructive debate in these two papers clearly shows how a representation in mere terms of pairwise networks entails difficulties of interpretation that are instead naturally addressed considering the functional hyperlinks. Indeed the covariance matrix is obviously pairwise, and neglects the higher order correlations; the partial covariance matrix is a pairwise network where there is a peculiar (and rigid) way of stripping all the rest of the system while evaluating the interdependence between two variables. The correspondence cited above shows that even in the simplest cases several clarifications are in order to get to the right interpretations of the results, and that one would probably need to look at both covariance and partial covariance matrix to draw conclusions. Using our formalism (functional hyperlinks) situations like those described in the papers (unidimensional latent variable model) are handled in a natural way, recognizing for example that in figure 1 of Waldorp and Marsman (2022) that the three variables form a redundant triplet. In addition to redundancy, our approach would similarly and with the same efficiency address the possibility of synergy, incorporating the notion of suppressors.

The approach presented here may also be helpful for detecting latent factors and correctly attribute observables to them, which is also the goal of recent work aimed at modelling local dependency as a trace for *redundancy* (Christensen et al., 2020; Golino, Christensen, et al., 2021; Golino, Moulder, et al., 2021; Vijayakumar et al., 2022). Redundancy comes from the fact that within a latent variable model, locally dependent variables contain less information than the one predicted by the (inflated) model parameters. Christensen and colleagues (Golino, Christensen, et al., 2021) measured redundancy and local dependency by considering psychometric networks of pairwise relationships enriched with weighted topological overlap, i.e. a metric estimating how similarly connected any two nodes are in a network. Their innovative approach showed how considering “higher order” interactions through network overlap provided superior factor estimation compared to IRT modelling. Our approach adopts a different perspective, substituting the pairwise psychometric network with a hypergraph structure in which higher order interdependencies are naturally captured by information-theoretic hyperlinks. Thus, it is worth noting that the terms “higher order,” “redundancy” or “3-way interactions” can have a different flavor or even some different definitions than the ones used so far. An important difference is that the structures simulated in the papers cited above do indeed generate redundancy (in the sense of higher order informational content), but not synergy (see Appendix A in the Supplementary Material). Whereas redundancy can be captured by local dependency between variables, synergy must be interpreted in different terms (Stramaglia et al., 2016), i.e. as the amount of information available only when variables are jointly considered. Our approach enables the quantification of synergistic interdependencies also within psychometric networks, extending to psychological representations a measure originally devised for brain data and networks.

A note here is in order on the choice of Gaussian variables in the simulations. Readers more familiar with the random Markov field (RMF) approach could object that Gaussian distributions by definition cannot encode higher interdependencies than two-way interdependencies, as the presence of an interaction with three or more variables would break Gaussianity. On the other hand, it has been shown that Gaussian multivariate processes may show nontrivial properties of marginal distributions, i.e. higher order dependencies, like synergy and redundancy (Barrett, 2015). The fact that higher order dependencies can take place in the absence of higher order interaction in the RMF sense is highly non-trivial. As a matter of fact, the same synergistic triplets of the covariance-Gaussian-based layout 1 in Figure 5, reported in the bottom right part of figure 6 would be obtained by explicitly modelling a variable with a multiplicative relationship between the other two.

The presented framework uses the O-information in its basic form, but there is a number of variations of it that can be used for future work. In effect, recent work has introduced multiple extensions of the O-information, including dynamical versions (Stramaglia et al., 2021) that provide an expansion of Transfer Entropy (or Granger Causality in the Gaussian case Barnett et al. (2009)), point-wise extensions which provide O-information values for each individual pattern (Scagliarini, Marinazzo, et al., 2022), and a spectral decomposition that resolve the O-information into different frequency bands (Faes et al., 2022). These tools could be used to build hypergraphs extending temporal psychological networks, as that presented in Fried et al. (2022). Please note that the metrics introduced in Stramaglia et al. (2021) can be used to calculate conditional O-information metrics, which provide a close simile to the conditional correlation that is often used in the network psychometric literature. Other interesting avenues for future work include leveraging recent algorithms to identify hypergraph centrality (Tudisco & Higham, 2021) and modularity (Chodrow et al., 2021; Kamiński et al., 2020), and also harmonic analysis and dimensionality reduction (Medina-Mardones et al., 2021).

On top of the ubiquitous yet elusive nature of the concept of synergy, different approaches to compute it have been developed, each one with its challenges, and without a true consensus among the different scholars working on the topic. For a sample of the proposed approaches and debates see Lizier et al. (2018), Olbrich et al. (2015), Quax et al. (2017), Rosas, Mediano, et al. (2020), and Williams and Beer (2010). One limitation of the O-information (and any other coarse-grained metric over an information decomposition, see references above) is that a multiplet is labelled synergistic or redundant depending on its sign, and in this sense, it reflects the net balance between the two quantities and only acknowledges the dominant one. Note that redundancy might originate from underlying psychological correlations, for instance, two or more symptoms might overlap rather than cause one another, thus bringing more redundancy within the analysis (Epskamp et al., 2018). For these reasons redundancy is prevalent in real datasets — where it is most likely to be found by experimental design or construction, and hence this could cover synergistic interdependencies making them harder to find. One solution could be to first reduce the redundancy in the sense of common factors (Golino, Christensen, et al., 2021), or consider implementing a full partial information decomposition e.g. as proposed in Bertschinger et al. (2014), James et al. (2018), and Rosas, Mediano, et al. (2020)) where redundancy and synergy can be considered as distinct information atoms that can be assessed entirely separately.

In the introduction we mentioned simplicial complexes (Bianconi, 2021) as an alternative representation of higher order networks. In simplicial complexes, if interdependencies exist at a higher order, they must exist at all lower orders. In this sense we think that simplicial complexes are not the most appropriate way to describe higher order statistical dependencies as we see them here, since adjacency matrices of statistical dependencies are full by definition. With our approach we separate different orders by explicitly checking that a higher order multiplet contains more new information with respect to the lower order ones.

Regarding the (visual) representation of hypergraphs, in addition to the dedicated subsection of the results section, from the more conceptual point of view, a recent study (Scagliarini, Nuzzi, et al., 2022) has proposed lower order descriptors of higher order dependencies, quantifying the variation of the O-information when a smaller (as low as one or two) number of variables are added to the rest of the system to form these groups, allowing a mapping from higher order interactions to lists and pairwise networks.

To conclude, by taking inspiration from complex systems whose behavior is not equal to the one resulting from the sum of its parts, here we asked ourselves whether the interdependencies observed in psychometric data could be described in a way that could account for beyond-pairwise (albeit conditioned), higher order interdependencies. Our proposed framework builds hyperlinks directly from the multivariate distribution of the data, identifying multiplets with a dominantly redundant or synergistic role. These analyses may ultimately inform on *which kind* of information is being shared by different behavioral variables or symptoms, and how these informational multiplets change in time. This higher order perspective complements current network approaches, leading to ways of assessing the data that greatly differ from what can be attained by other methods such as factor analyses. By doing this, the proposed approach enriches the toolbox of psychometricians and opens promising avenues for future investigation.

Declarations

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Conflicts of interest/Competing interests

None

Ethics approval

Not applicable

Consent to participate

Not applicable

Consent for publication

Not applicable

Availability of data and materials

The empirical data, and code to reproduce the original pairwise networks also reported here are found at these repositories https://osf.io/73c4q/ and https://osf.io/mj5wa/.

Code availability

The code for simulations, analyses, and plotting is described in this github repository https://github.com/danielemarinazzo/O_info_psycnet.

Authors' contributions

DM: conception of the study, writing, editing figures, coding; **JVR**: writing, coding; **F.E.R**: conception of the study, writing, editing figures; **M.S.**: conception of the study, writing; **R.C.**: coding, editing figures; **N.C.**: coding; **S.S.**: conception of the study,

writing, coding; **Y.R.**: conception of the study, writing, coding.

References

Armour, C., Fried, E. I., Deserno, M. K., Tsai, J., & Pietrzak, R. H. (2017). A network analysis of dsm-5 posttraumatic stress disorder symptoms and correlates in u.s. military veterans. Journal of Anxiety Disorders, 45, 49–59.

Armour, C., Greene, T., Contractor, A. A., Weiss, N., Dixon-Gordon, K., & Ross, J. (2020). Posttraumatic stress disorder symptoms and reckless behaviors: A network analysis approach. Journal of Traumatic Stress, 33(1), 29–40.

Bairey, E., Kelsic, E. D., & Kishony, R. (2016). High-order species interactions shape ecosystem diversity. Nature communications, 7(1), 1–7.

Barnett, L., Barrett, A. B., & Seth, A. K. (2009). Granger causality and transfer entropy are equivalent for gaussian variables. Physical review letters, 103(23), 238701.

Barrett, A. B. (2015). Exploration of synergistic and redundant information sharing in static and dynamical gaussian systems. Physical Review E, 91(5), 052802.

Battiston, F., Amico, E., Barrat, A., Bianconi, G., Ferraz de Arruda, G., Franceschiello, B., Iacopini, I., Kéfi, S., Latora, V., Moreno, Y., et al. (2021). The physics of higher-order interactions in complex systems. Nature Physics, 17(10), 1093–1098.

Battiston, F., Cencetti, G., Iacopini, I., Latora, V., Lucas, M., Patania, A., Young, J.-G., & Petri, G. (2020). Networks beyond pairwise interactions: Structure and dynamics. Physics Reports, 874, 1–92.

Bertschinger, N., Rauh, J., Olbrich, E., Jost, J., & Ay, N. (2014). Quantifying unique information. Entropy, 16(4), 2161–2183.

Bianconi, G. (2021). Higher-order networks. Cambridge University Press.

Bick, C., Gross, E., Harrington, H. A., & Schaub, M. T. (2023). What are higher-order networks? SIAM Review, 65(3), 686–731.

Bisson, J. I., Wright, L. A., Jones, K. A., Lewis, C., Phelps, A. J., Sijbrandij, M., Varker, T., & Roberts, N. P. (2021). Preventing the onset of post traumatic stress disorder. Clinical psychology review, 86, 102004.

Boccaletti, S., De Lellis, P., Del Genio, C., Alfaro-Bittner, K., Criado, R., Jalan, S., & Romance, M. (2023). The structure and dynamics of networks with higher order interactions. Physics Reports, 1018, 1–64.

Borsboom, D., & Cramer, A. O. (2013). Network analysis: An integrative approach to the structure of psychopathology. Annual review of clinical psychology, 9, 91–121.

Borsboom, D., Deserno, M. K., Rhemtulla, M., Epskamp, S., Fried, E. I., McNally, R. J., Robinaugh, D. J., Perugini, M., Dalege, J., Costantini, G., et al. (2021). Network analysis of multivariate data in psychological science. Nature Reviews Methods Primers, 1(1), 1–18.

Briganti, G., Kempenaers, C., Braun, S., Fried, E. I., & Linkowski, P. (2018). Network analysis of empathy items from the interpersonal reactivity index in 1973 young adults. Psychiatry Research, 265, 87–92.

Bringmann, L. F., Elmer, T., Epskamp, S., Krause, R. W., Schoch, D., Wichers, M., Wigman, J. T., & Snippe, E. (2019). What do centrality measures measure in psychological networks? Journal of Abnormal Psychology, 128(8), 892.

Brooks, M., Graham-Kevan, N., Robinson, S., & Lowe, M. (2019). Trauma characteristics and posttraumatic growth: The mediating role of avoidance coping, intrusive thoughts, and social support. Psychological Trauma: Theory, Research, Practice, and Policy, 11(2), 232.

Brown, M. M., Thibodeau, R. B., Pierucci, J. M., & Gilpin, A. T. (2017). Supporting the development of empathy: The role of theory of mind and fantasy orientation. Social Development, 26(4), 951–964.

Chodrow, P. S., Veldt, N., & Benson, A. R. (2021). Generative hypergraph clustering: From blockmodels to modularity. Science Advances, 7(28), eabh1303.

Christensen, A. P., Garrido, L. E., & Golino, H. F. (2020). Unique variable analysis: A novel approach for detecting redundant variables in multivariate data.

Cover, T. M., & Thomas, J. A. (2012). Elements of information theory. John Wiley & Sons.

Cramer, A. O., Waldorp, L. J., Van Der Maas, H. L., & Borsboom, D. (2010). Comorbidity: A network perspective. Behavioral and brain sciences, 33(2-3), 137–150.

Davison, A. C., & Hinkley, D. V. (1997). Bootstrap methods and their application. Cambridge university press.

DiCiccio, T. J., & Efron, B. (1996). Bootstrap confidence intervals. Statistical science, 11(3), 189–228.

Efron, B., & Tibshirani, R. J. (1994). An introduction to the bootstrap. CRC press.

Epskamp, S., Borsboom, D., & Fried, E. I. (2018). Estimating psychological networks and their accuracy: A tutorial paper. Behavior Research Methods, 50(1), 195–212.

Epskamp, S., Cramer, A. O., Waldorp, L. J., Schmittmann, V. D., Borsboom, D., et al. (2012). Qgraph: Network visualizations of relationships in psychometric data. Journal of Statistical Software, 48(4), 1–18.

Epskamp, S., & Fried, E. I. (2018). A tutorial on regularized partial correlation networks. Psychological methods, 23(4), 617.

Faes, L., Mijatovic, G., Antonacci, Y., Pernice, R., Barà, C., Sparacino, L., Sammartino, M., Porta, A., Marinazzo, D., & Stramaglia, S. (2022). A framework for the time-and frequency-domain assessment of high-order interactions in brain and physiological networks. arXiv preprint arXiv:2202.04179.

Forbes, M. K., Wright, A. G., Markon, K. E., & Krueger, R. F. (2021). Quantifying the reliability and replicability of psychopathology network characteristics. Multivariate behavioral research, 56(2), 224–242.

Foygel, R., & Drton, M. (2010). Extended bayesian information criteria for gaussian graphical models. Advances in neural information processing systems, 23.

Fried, E. I., Papanikolaou, F., & Epskamp, S. (2022). Mental health and social contact during the covid-19 pandemic: An ecological momentary assessment study. Clinical Psychological Science, 10(2), 340–354.

Fried, E. I., van Borkulo, C. D., Cramer, A. O., Boschloo, L., Schoevers, R. A., & Borsboom, D. (2017). Mental disorders as networks of problems: A review of recent insights. Social Psychiatry and Psychiatric Epidemiology, 52(1), 1–10.

Friedman, J., Hastie, T., & Tibshirani, R. (2008). Sparse inverse covariance estimation with the graphical lasso. Biostatistics, 9(3), 432–441.

Golino, H. F., Christensen, A. P., Moulder, R., Kim, S., & Boker, S. M. (2021). Modeling latent topics in social media using dynamic exploratory graph analysis: The case of the right-wing and left-wing trolls in the 2016 us elections. Psychometrika, 1–32.

Golino, H. F., & Epskamp, S. (2017). Exploratory graph analysis: A new approach for estimating the number of dimensions in psychological research. PloS One, 12(6), e0174035.

Golino, H. F., Moulder, R., Shi, D., Christensen, A. P., Garrido, L. E., Nieto, M. D., Nesselroade, J., Sadana, R., Thiyagarajan, J. A., & Boker, S. M. (2021). Entropy fit indices: New fit measures for assessing the structure and dimensionality of multiple latent variables. Multivariate Behavioral Research, 56(6), 874–902.

Grasso, M., Albantakis, L., Lang, J. P., & Tononi, G. (2021). Causal reductionism and causal structures. Nature neuroscience, 24(10), 1348–1355.

Hallquist, M. N., Wright, A. G., & Molenaar, P. C. (2021). Problems with centrality measures in psychopathology symptom networks: Why network psychometrics cannot escape psychometric theory. Multivariate Behavioral Research, 56(2), 199–223.

Haslbeck, J. M., Borsboom, D., & Waldorp, L. J. (2021). Moderated network models. Multivariate behavioral research, 56(2), 256–287.

Hindriks, R., Broeders, T. A., Schoonheim, M. M., Douw, L., Santos, F., van Wieringen, W., & Tewarie, P. K. (2024). Higher-order functional connectivity analysis of resting-state functional magnetic resonance imaging data using multivariate cumulants. Human Brain Mapping, 45(5), e26663.

Iacopini, I., Petri, G., Barrat, A., & Latora, V. (2019). Simplicial models of social contagion. Nature communications, 10(1), 1–9.

Ince, R. A., Giordano, B. L., Kayser, C., Rousselet, G. A., Gross, J., & Schyns, P. G. (2017). A statistical framework for neuroimaging data analysis based on mutual information estimated via a gaussian copula. Human brain mapping, 38(3), 1541–1573.

James, R. G., & Crutchfield, J. P. (2017). Multivariate dependence beyond shannon information. Entropy, 19(10), 531.

James, R. G., Emenheiser, J., & Crutchfield, J. P. (2018). Unique information via dependency constraints. Journal of Physics A: Mathematical and Theoretical, 52(1), 014002.

Jones, P. J., Ma, R., & McNally, R. J. (2019). Bridge centrality: A network approach to understanding comorbidity. Multivariate behavioral research, 1–15.

Kamiński, B., Prałat, P., & Théberge, F. (2020). Community detection algorithm using hypergraph modularity. International Conference on Complex Networks and Their Applications, 152–163.

Landry, N. W., Lucas, M., Iacopini, I., Petri, G., Schwarze, A., Patania, A., & Torres, L. (2023). Xgi: A python package for higher-order interaction networks. Journal of Open Source Software, 8(85), 5162.

Lizier, J. T., Bertschinger, N., Jost, J., & Wibral, M. (2018). Information decomposition of target effects from multi-source interactions: Perspectives on previous, current and future work.

Lovibond, P. F., & Lovibond, S. H. (1995). The structure of negative emotional states: Comparison of the depression anxiety stress scales (dass) with the beck depression and anxiety inventories. Behaviour research and therapy, 33(3), 335–343.

Marsman, M., Borsboom, D., Kruis, J., Epskamp, S., van Bork, R. v., Waldorp, L., Maas, H. v. d., & Maris, G. (2018). An introduction to network psychometrics:

Relating ising network models to item response theory models. Multivariate behavioral research, 53(1), 15–35.

McDonald, R. P. (2013). Test theory: A unified treatment. psychology press.

McGill, W. (1954). Multivariate information transmission. Transactions of the IRE Professional Group on Information Theory, 4(4), 93–111.

Mediano, P. A., Rosas, F. E., Luppi, A. I., Carhart-Harris, R. L., Bor, D., Seth, A. K., & Barrett, A. B. (2021). Towards an extended taxonomy of information dynamics via integrated information decomposition. arXiv preprint arXiv:2109.13186.

Medina-Mardones, A. M., Rosas, F. E., Rodriguez, S. E., & Cofre, R. (2021). Hyperharmonic analysis for the study of high-order information-theoretic signals. Journal of Physics: Complexity, 2(3), 035009.

Meinshausen, N., & Bühlmann, P. (2006). High-dimensional graphs and variable selection with the lasso. The annals of statistics, 34(3), 1436–1462.

Milojević, S. (2014). Principles of scientific research team formation and evolution. Proceedings of the National Academy of Sciences, 111(11), 3984–3989.

Nomura, K., & Akai, S. (2012). Empathy with fictional stories: Reconsideration of the fantasy scale of the interpersonal reactivity index. Psychological Reports, 110(1), 304–314.

Olbrich, E., Bertschinger, N., & Rauh, J. (2015). Information decomposition and synergy. Entropy, 17(5), 3501–3517.

Praggastis, B., Arendt, D., Joslyn, C., Purvine, E., Aksoy, S., & Monson, K. (2019). Hypernetx.

Quax, R., Har-Shemesh, O., & Sloot, P. M. (2017). Quantifying synergistic information using intermediate stochastic variables. Entropy, 19(2), 85.

Ressler, K., Berretta, S., Bolshakov, V. Y., Rosso, I. M., Meloni, E. G., Rauch, S. L., Carlezon, W. A., et al. (2022). Post-traumatic stress disorder: Clinical and translational neuroscience from cells to circuits. Nature Reviews Neurology, 18(5), 273–288.

Rosas, F. E., Mediano, P., Jensen, H. J., Seth, A. K., Barrett, A. B., Carhart-Harris, R. L., & Bor, D. (2020). Reconciling emergences: An information-theoretic approach to identify causal emergence in multivariate data. PLoS computational biology, 16(12), e1008289.

Rosas, F. E., Mediano, P., Ugarte, M., & Jensen, H. J. (2018). An information-theoretic approach to self-organisation: Emergence of complex interdependencies in coupled dynamical systems. Entropy, 20(10), 793.

Rosas, F. E., Mediano, P. A. M., Gastpar, M., & Jensen, H. J. (2019). Quantifying high-order interdependencies via multivariate extensions of the mutual information. Phys. Rev. E, 100, 032305.

Rosas, F. E., Mediano, P. A., Luppi, A. I., Varley, T. F., Lizier, J. T., Stramaglia, S., Jensen, H. J., & Marinazzo, D. (2022). Disentangling high-order mechanisms and high-order behaviours in complex systems. Nature Physics, 1–2.

Rosas, F. E., Mediano, P. A., Rassouli, B., & Barrett, A. B. (2020). An operational information decomposition via synergistic disclosure. Journal of Physics A: Mathematical and Theoretical, 53(48), 485001.

Rosas, F. E., Ntranos, V., Ellison, C. J., Pollin, S., & Verhelst, M. (2016). Understanding interdependency through complex information sharing. Entropy, 18(2), 38.

Rosseel, Y. (2012). Lavaan: An r package for structural equation modeling and more. version 0.5–12 (beta). Journal of statistical software, 48(2), 1–36.

Santoro, A., Battiston, F., Petri, G., & Amico, E. (2023). Higher-order organization of multivariate time series. Nature Physics, 19(2), 221–229.

Scagliarini, T., Marinazzo, D., Guo, Y., Stramaglia, S., & Rosas, F. E. (2022). Quantifying high-order interdependencies on individual patterns via the local o-information: Theory and applications to music analysis. Physical Review Research, 4(1), 013184.

Scagliarini, T., Nuzzi, D., Antonacci, Y., Faes, L., Rosas, F. E., Marinazzo, D., & Stramaglia, S. (2022). Gradients of o-information: Low-order descriptors of high-order dependencies. arXiv preprint arXiv:2207.03581.

Schipper, M., & Petermann, F. (2013). Relating empathy and emotion regulation: Do deficits in empathy trigger emotion dysregulation? Social neuroscience, 8(1), 101–107.

Schmittmann, V. D., Cramer, A. O., Waldorp, L. J., Epskamp, S., Kievit, R. A., & Borsboom, D. (2013). Deconstructing the construct: A network perspective on psychological phenomena. New ideas in psychology, 31(1), 43–53.

Shannon, C. E. (1948). A mathematical theory of communication. Bell system technical journal, 27(3), 379–423.

Skardal, P. S., & Arenas, A. (2020). Higher order interactions in complex networks of phase oscillators promote abrupt synchronization switching. Communications Physics, 3(1), 1–6.

St-Onge, G., Iacopini, I., Latora, V., Barrat, A., Petri, G., Allard, A., & Hébert-Dufresne, L. (2022). Influential groups for seeding and sustaining nonlinear contagion in heterogeneous hypergraphs. Communications Physics, 5(1), 1–16.

Stotland, E., et al. (1978). Empathy, fantasy and helping. Sage.

Stramaglia, S., Angelini, L., Wu, G., Cortes, J. M., Faes, L., & Marinazzo, D. (2016). Synergetic and redundant information flow detected by unnormalized granger causality: Application to resting state fmri. IEEE Transactions on Biomedical Engineering, 63(12), 2518–2524.

Stramaglia, S., Scagliarini, T., Daniels, B. C., & Marinazzo, D. (2021). Quantifying dynamical high-order interdependencies from the o-information: An application to neural spiking dynamics. Frontiers in Physiology, 1784.

Tabar, M. R. R., Nikakhtar, F., Parkavousi, L., Akhshi, A., Feudel, U., & Lehnertz, K. (2024). Revealing higher-order interactions in high-dimensional complex systems: A data-driven approach. Physical Review X, 14(1), 011050.

Te Sun, H. (1978). Nonnegative entropy measures of multivariate symmetric correlations [Publisher: Elsevier]. Information and Control, 36, 133–156.

Thompson, N. M., Uusberg, A., Gross, J. J., & Chakrabarti, B. (2019). Empathy and emotion regulation: An integrative account. Progress in brain research, 247, 273–304.

Torres, L., Blevins, A. S., Bassett, D., & Eliassi-Rad, T. (2021). The why, how, and when of representations for complex systems. SIAM Review, 63(3), 435–485.

Tuccitto, D. E., Giacobbi Jr, P. R., & Leite, W. L. (2010). The internal structure of positive and negative affect: A confirmatory factor analysis of the panas. Educational and psychological measurement, 70(1), 125–141.

Tudisco, F., & Higham, D. J. (2021). Node and edge nonlinear eigenvector centrality for hypergraphs. Communications Physics, 4(1), 1–10.

Van den Bergh, N., Marchetti, I., & Koster, E. H. (2021). Bridges over troubled waters: Mapping the interplay between anxiety, depression and stress through network analysis of the dass-21. Cognitive Therapy and Research, 45(1), 46–60.

Vasiliauskaite, V., & Rosas, F. E. (2020). Understanding complexity via network theory: A gentle introduction. arXiv preprint arXiv:2004.14845.

Vijayakumar, R., Choi, J. Y., & Jung, E. H. (2022). A Unified Neural Network Framework for Extended Redundancy Analysis. Psychometrika.

Waldorp, L., & Marsman, M. (2022). Relations between networks, regression, partial correlation, and the latent variable model. Multivariate Behavioral Research, 57(6), 994–1006.

Watanabe, S. (1960). Information theoretical analysis of multivariate correlation [Publisher: IBM]. IBM Journal of research and development, 4(1), 66–82.

Williams, P. L. (2011). Information dynamics: Its theory and application to embodied cognitive systems [Doctoral dissertation, Indiana University].

Williams, P. L., & Beer, R. D. (2010). Nonnegative decomposition of multivariate information. arXiv preprint arXiv:1004.2515.

Zaki, J. (2020). Integrating empathy and interpersonal emotion regulation. Annual review of psychology, 71, 517–540.

Zhang, Y., Lucas, M., & Battiston, F. (2023). Higher-order interactions shape collective dynamics differently in hypergraphs and simplicial complexes. Nature communications, 14(1), 1605.

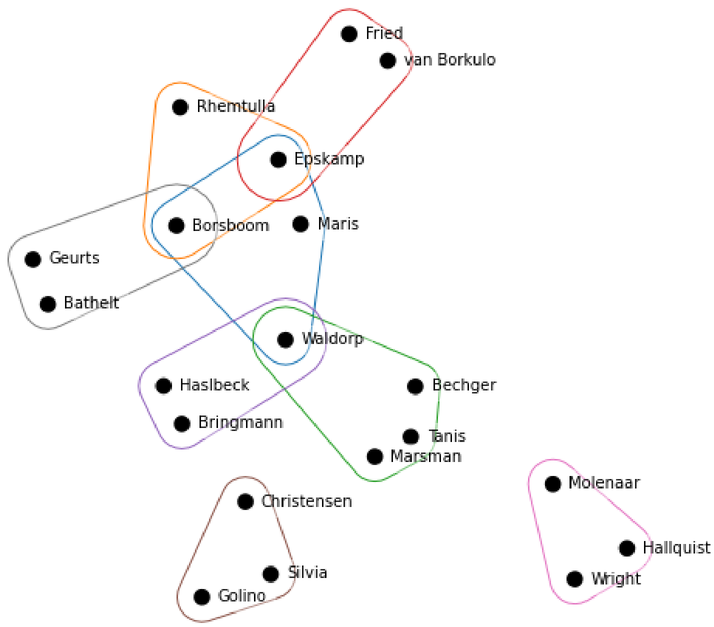

Figure 1

Illustrative example of an hypergraph, where each node is a scholar, and each hyperlink (represented as a colored string encompassing the nodes) is a quite popular paper on network psychometrics co-authored by the nodes.

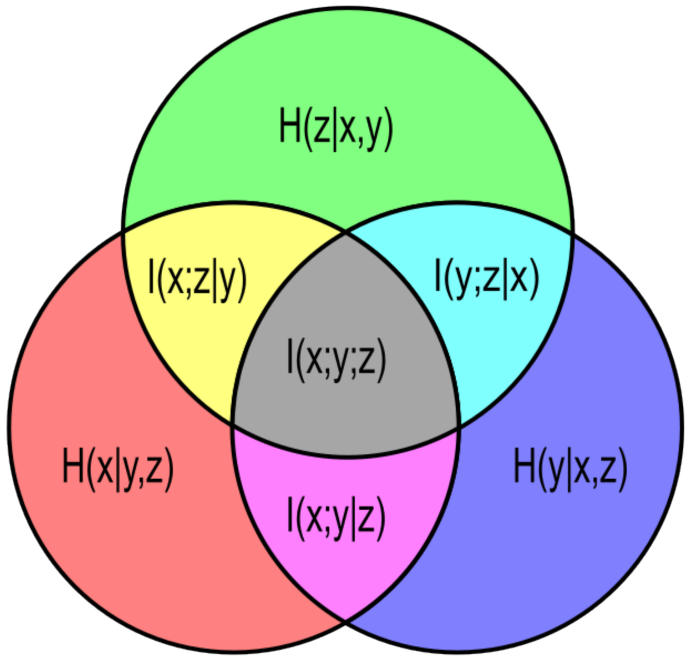

Figure 2

Venn diagram of information theoretic measures for three variables x, y, and z. The dual total correlation is represented by the union of the three mutual information terms and is shown in the diagram by the yellow, magenta, cyan, and gray regions. The gray region represents the interaction information. From https://en.wikipedia.org/wiki/Dual_total_correlation.

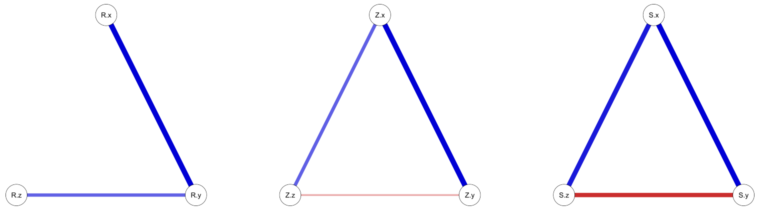

Figure 3

qgraph visualizations of the triplet. Both color intensity and edge thickness represent edge weight. Red indicates negative edge weight, blue indicates positive edge weight. Left: redundancy; Middle: Zero interaction information; Right: synergy.

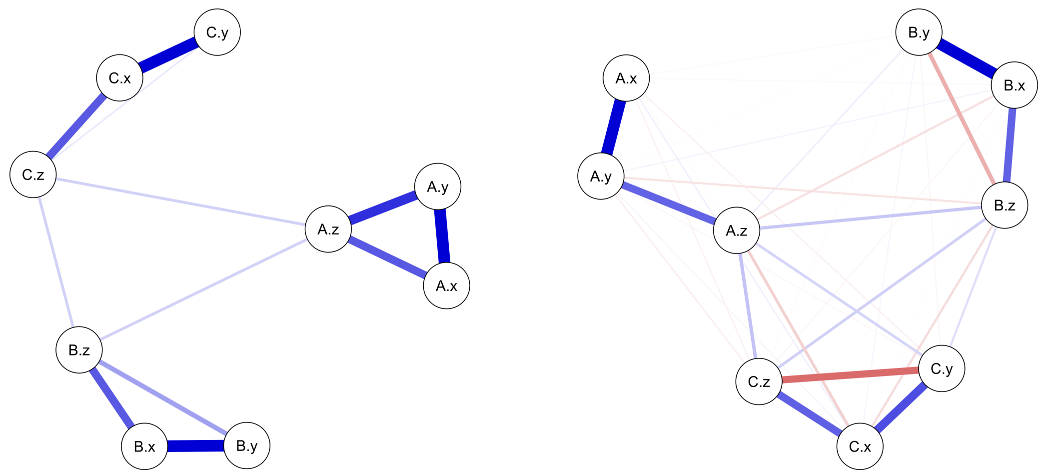

Figure 4

qgraph network visualizations of three triplets with residual correlations between triplets. Triplet A includes redundancy; triplet B includes zero O-information; triplet C includes synergy. The primary link between the triplets are done via the variables A.z, B.z and C.z. Residual correlations were set at 0.15. Left: pairwise correlations; Right: partial correlations

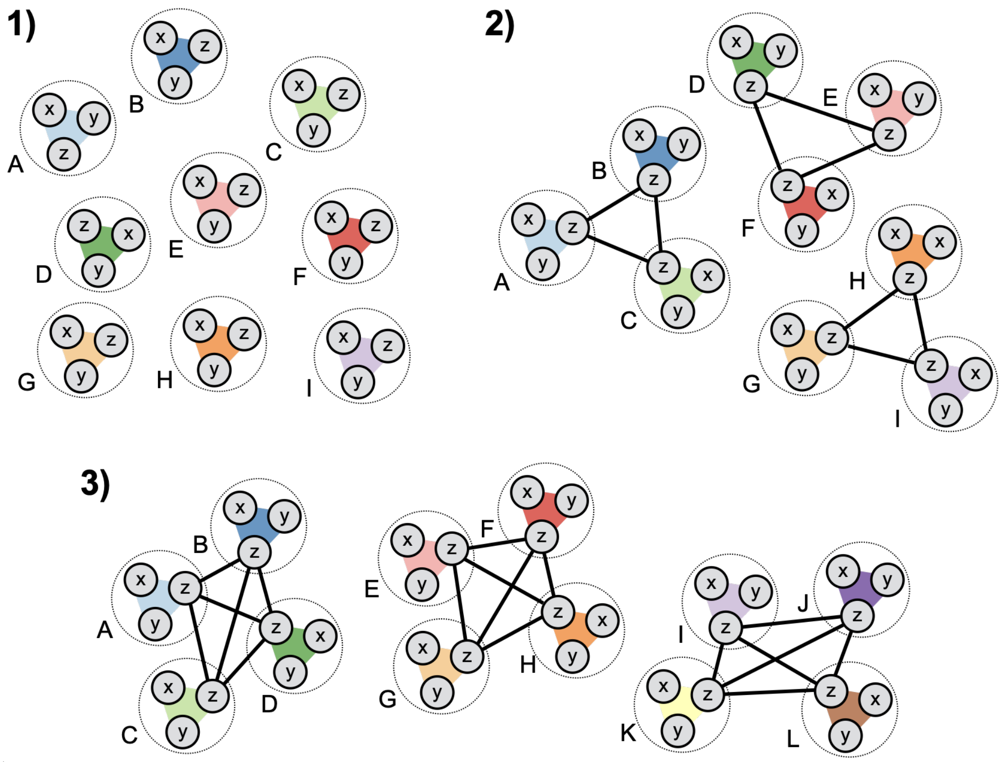

Figure 5

Visual overview of the presented model layouts. Each triplet, indicated by a capital letter and a circle, is a set of three variables originating via the same latent variable. Black lines represent residual correlation linking different triplets. The colors are used as a further way of differentiating the variables within each triplet in the following figures. ***1)*** *Nine independent triplets.* ***2)*** *Three clusters of three triplets. All triplets within a cluster are connected to two others via their z-variables.* ***3)*** *Expansion of Model Layout 2. Three clusters of four triplets. Each triplet within a cluster is connected to the three other related triplets via the z-variables.*

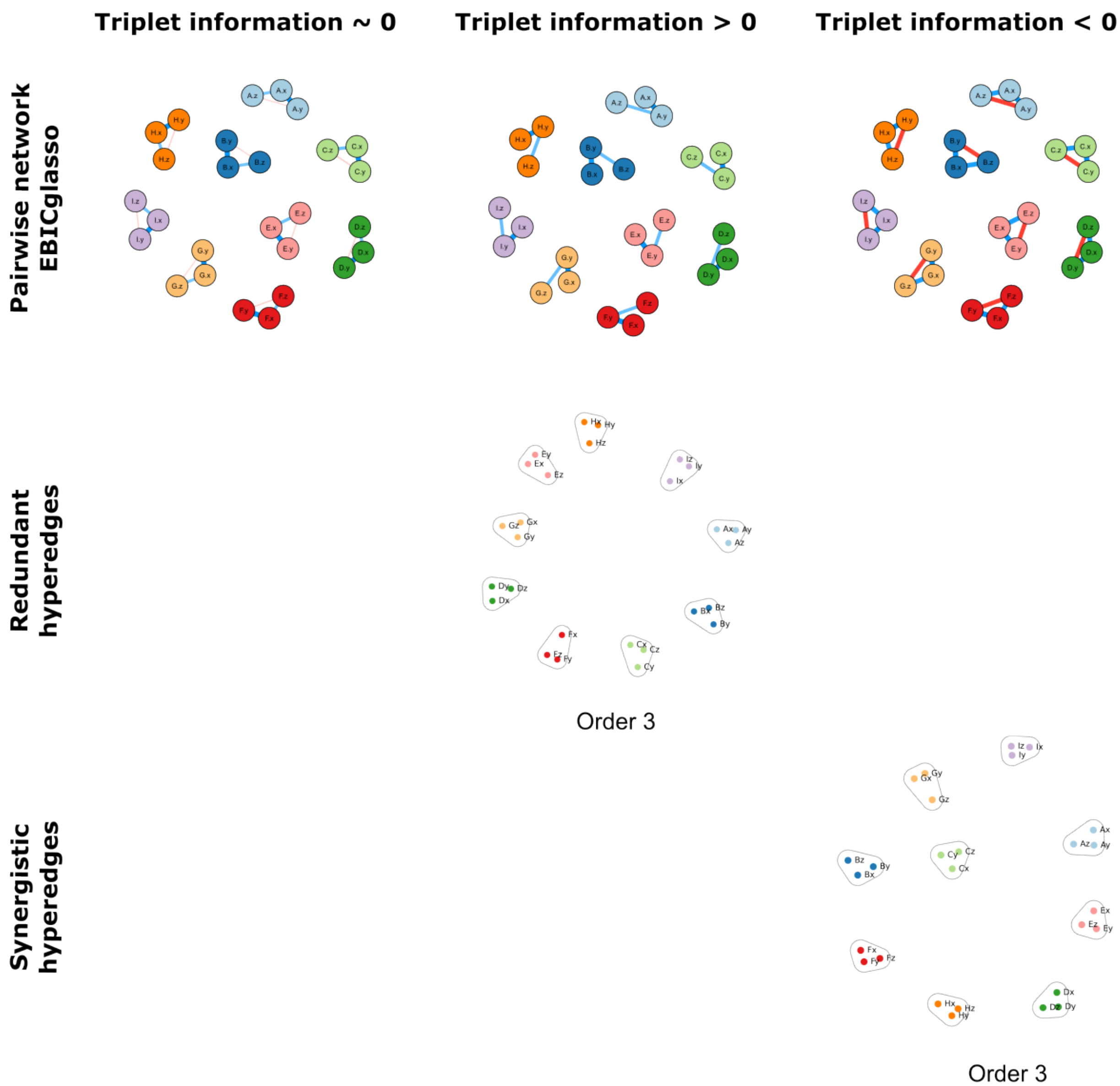

Figure 6

Results obtained from layout 1 in Figure 5 when implemented with zero (i.e. independent), positive (i.e. redundant), and negative (i.e. synergistic) O-information in each triplet. ***Top row:*** *network obtained with EBICglasso (Epskamp et al., 2018) and represented with qgraph (Epskamp et al., 2012), where both color intensity and edge thickness represent edge weight. Red indicates negative edge weight, blue indicates positive edge weight. Left: Zero interaction information; Middle: redundancy; Right: synergy.* ***Middle row:*** *significant multiplets of redundant O-information.* ***Bottom row:*** *significant multiplets of synergistic O-information.*

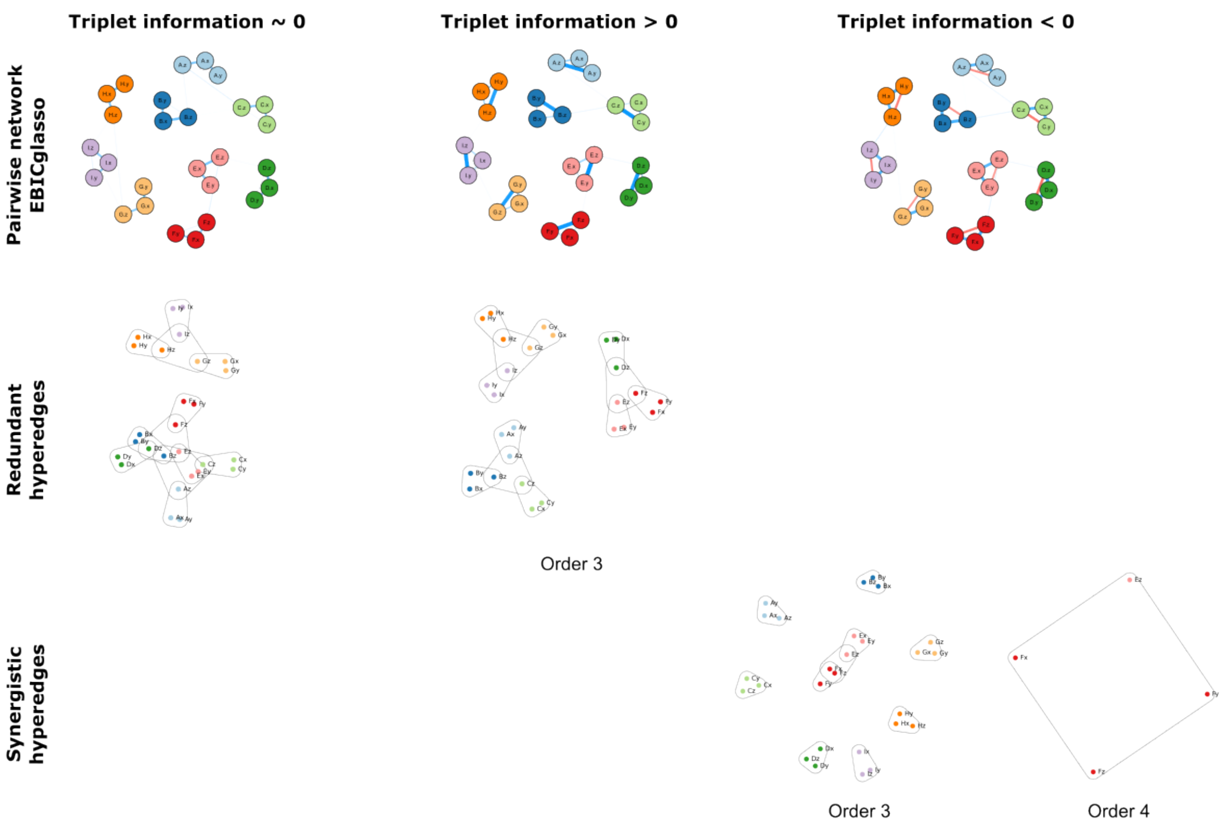

Figure 7

Analogous results as shown in Figure 6 but for Model 2 in Figure 5.

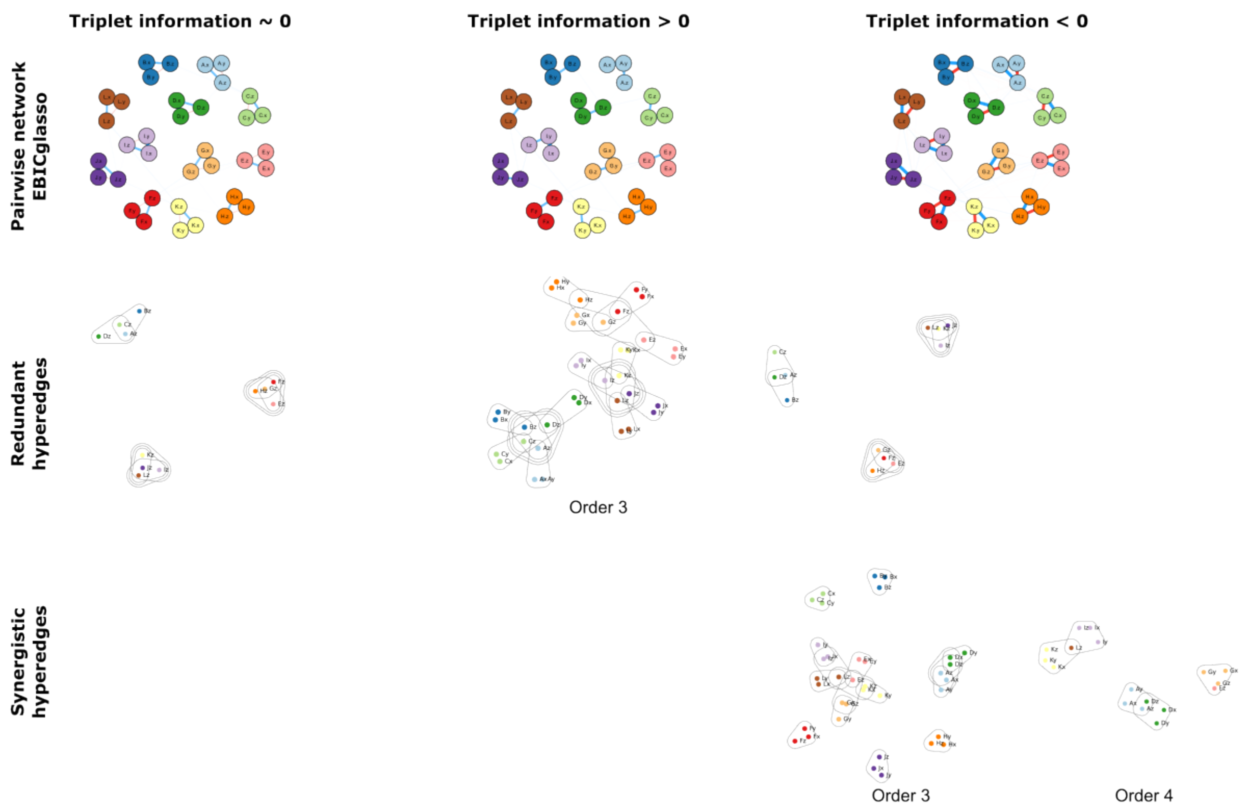

Figure 8

Analogous results as shown in Figure 6 but for Model 3 in Figure 5.

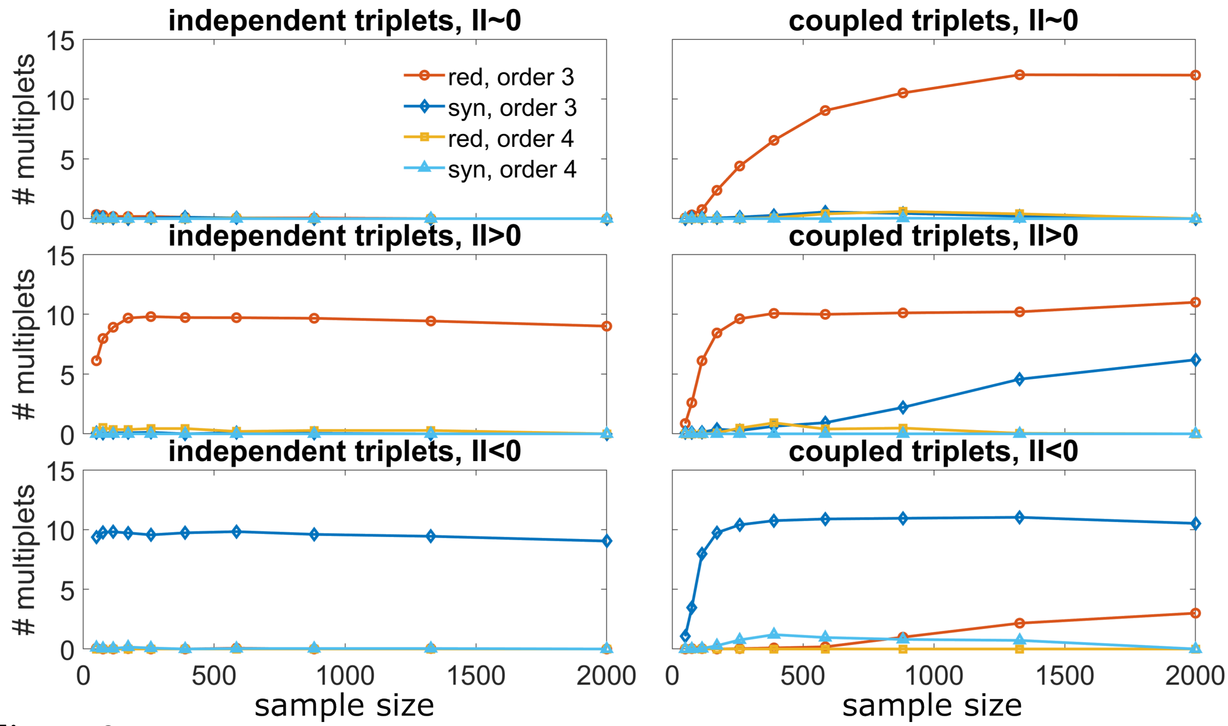

Figure 9

Average number of redundant and synergistic informational multiplets of size 3 and 4, obtained by computing the O-information on datasets simulated according to layout 1 and layout 2, varying the number of observables.

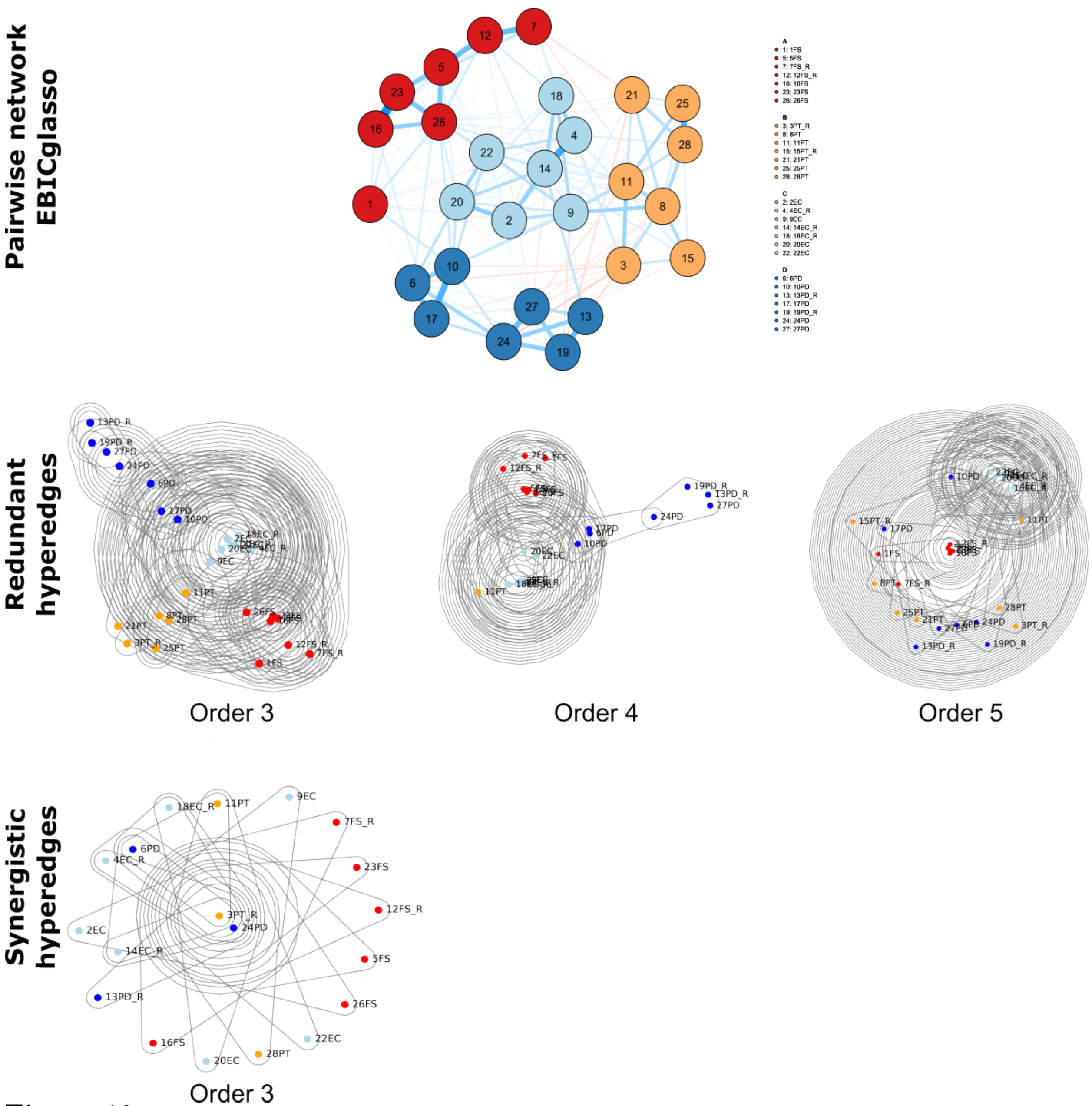

Figure 10

Empathy dataset from Briganti et al. (2018). Colors indicate membership of a component of items, as in the reference paper. Top row: network obtained with EBICglasso. Middle row: significant multiplets of redundant O-information. Bottom row: significant multiplets of synergistic O-information.

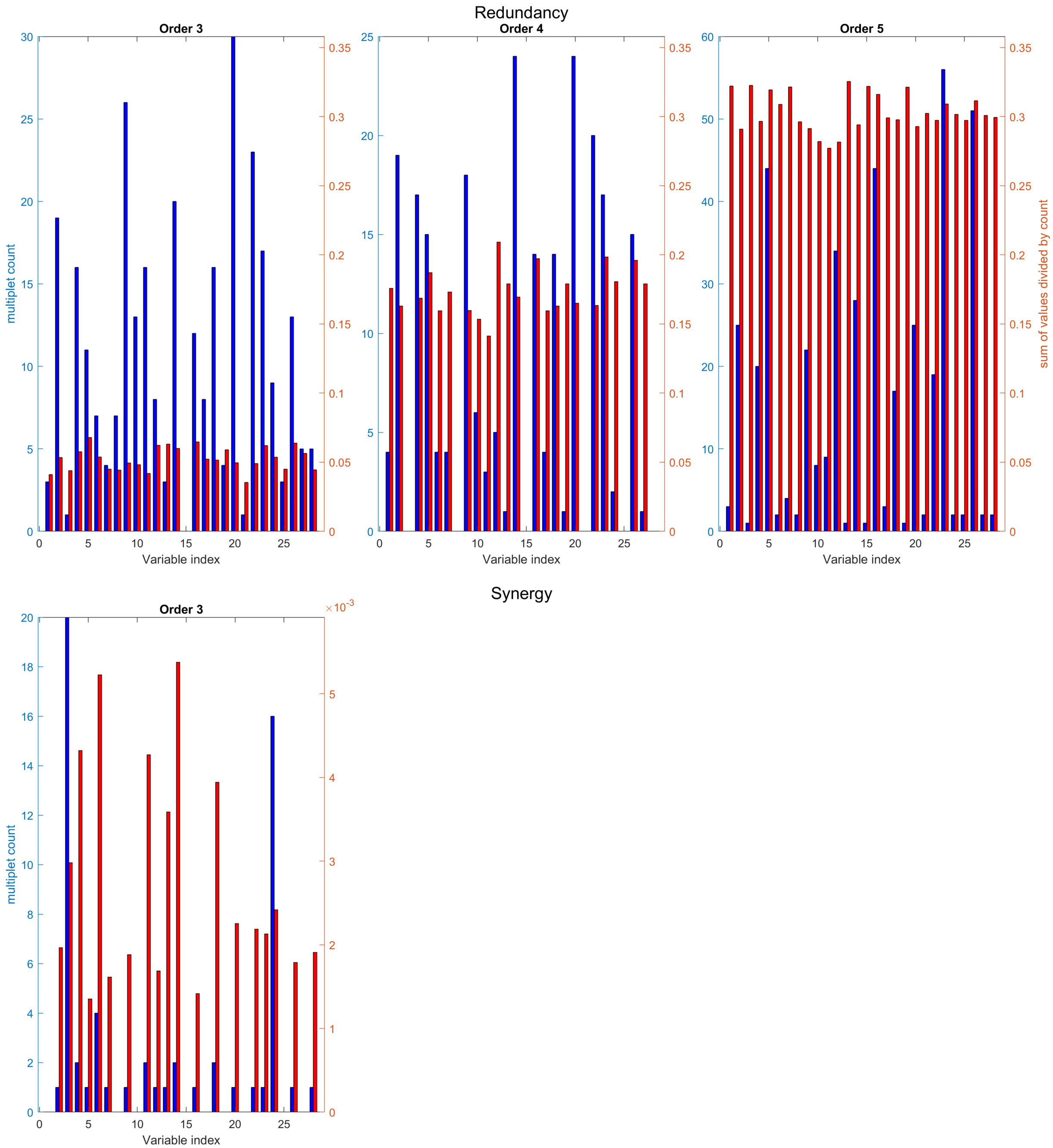

Figure 11

From the Empathy dataset from Briganti et al. (2018). Bar plots depicting the number of times each variables is involved in a multiplet (blue), and the average informational value (red)

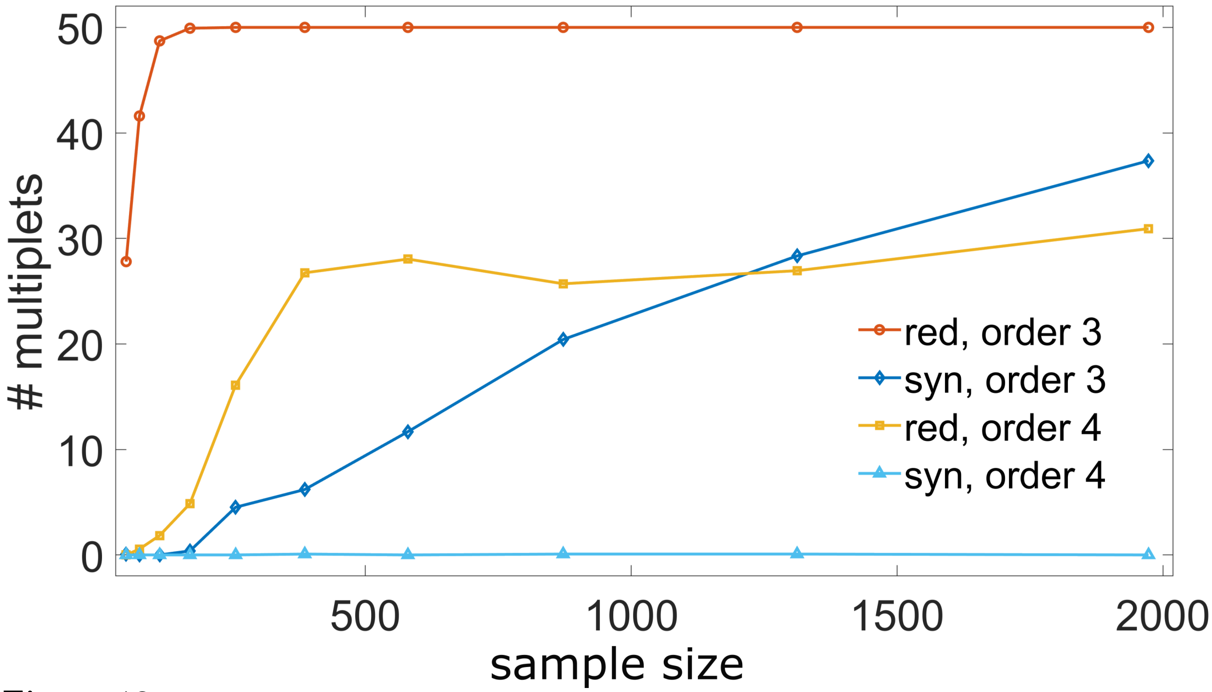

Figure 12

Average number of redundant and synergistic informational multiplets of size 3 and 4, obtained by computing the O-information on the Empathy dataset (Briganti et al., 2018).

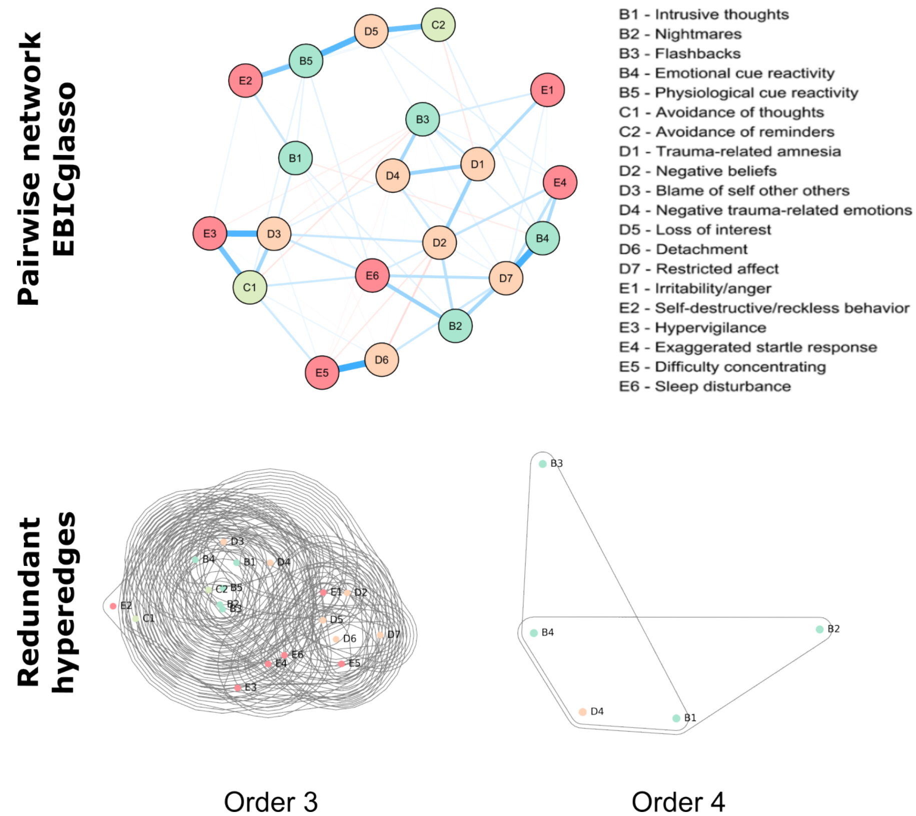

Figure 13

PTSD dataset from Armour et al. (2017). Colors indicate membership in a class of items. Top row: network obtained with EBICglasso. Bottom row: significant multiplets of redundant O-information.

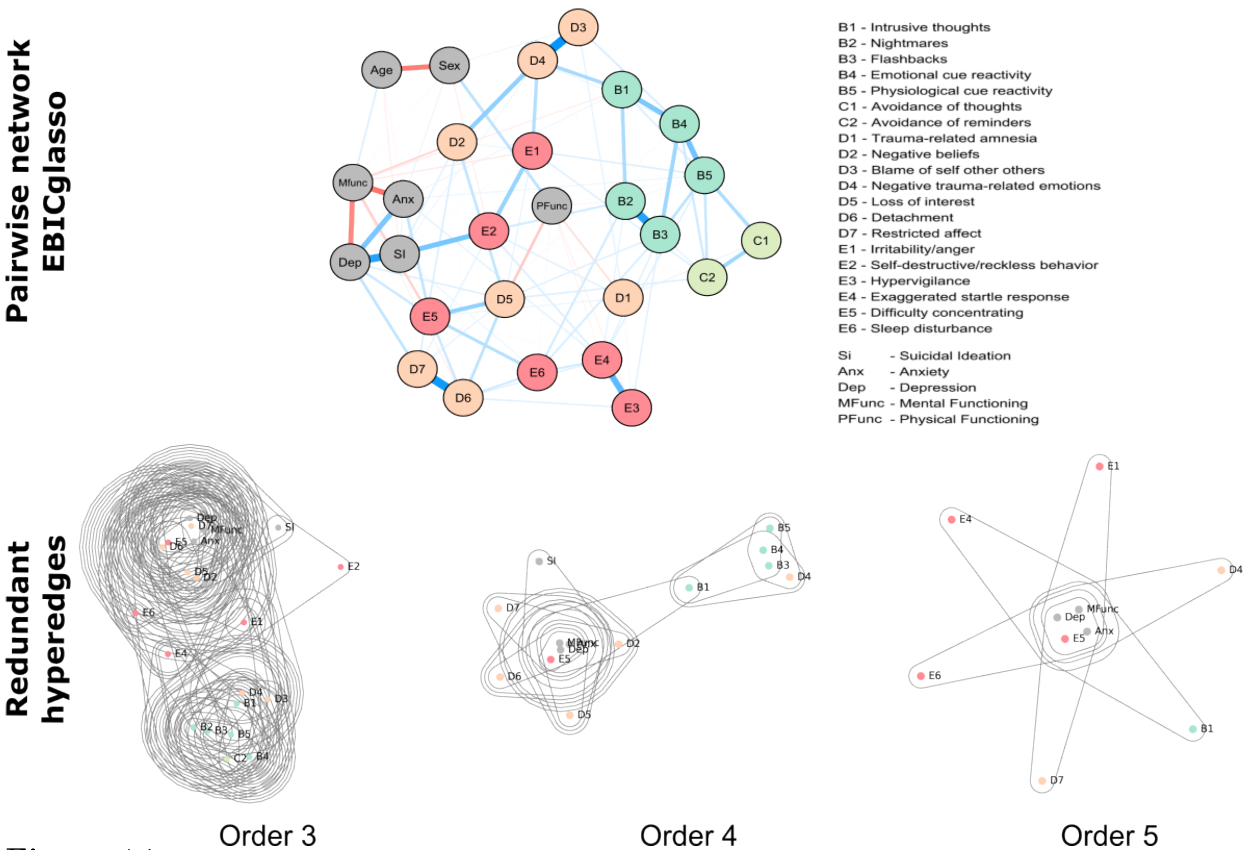

Figure 14

PTSD dataset from Armour et al. (2017). Colors indicate membership in a class of items. Grey items are the covariates. Top row: network obtained with EBICglasso. Bottom row: significant multiplets of redundant O-information.