

Modeling Two-Phase-System Heat Conduction

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Heat conduction is the process of heat transfer from a high-temperature region to a low-temperature region through a body (gas, liquid, or solid) that is not in macroscopic relative motion. It is the macroscopic manifestation of particle activities at the molecular scale in the body. For single-phase systems, it is well-known that heat conduction obeys the Fourier law of heat conduction. Multiphase systems involve another scale, the microscale where different phases interplay with each other. By using the two-phase systems as an example, we show that this additional microscale activity shifts the Fourier heat conduction in the each phase into the dual-phase-lagging heat conduction in two-phase systems at the macroscale. This finding is significant because all results regarding dual-phase-lagging heat conduction can thus be applied to study the heat conduction in two-phase systems and the other way around. This finding also raises the question of reliability for existing thermal conductivity data that were obtained based on the *hypothesized* Fourier heat conduction at the macroscale.

Consider heat conduction in two-phase-systems denoted by β - and σ - phases, respectively. By the first law of thermodynamics and the Fourier law of heat conduction, we have a microscale model for heat conduction in two-phase-systems (Fig. 1)

$$(\rho c)_{\beta} \frac{\partial T_{\beta}}{\partial t} = \nabla \cdot (k_{\beta} \nabla T_{\beta}), \text{ in the } \beta\text{-phase} \quad (1)$$

$$(\rho c)_{\sigma} \frac{\partial T_{\sigma}}{\partial t} = \nabla \cdot (k_{\sigma} \nabla T_{\sigma}), \text{ in the } \sigma\text{-phase} \quad (2)$$

$$T_{\beta} = T_{\sigma}, \text{ at the } \beta - \sigma \text{ interface } A_{\beta\sigma} \quad (3)$$

$$\mathbf{n}_{\beta\sigma} \cdot k_{\beta} = \mathbf{n}_{\beta\sigma} \cdot k_{\sigma} \nabla T_{\sigma}, \text{ at the } \beta - \sigma \text{ interface } A_{\beta\sigma} \quad (4)$$

Here T is the temperature. ρ , c and k are the density, specific heat and thermal conductivity, respectively. Subscripts β and σ refer to the β - and σ -phases, respectively. $A_{\beta\sigma}$ represents the area of the $\beta - \sigma$ interface; $\mathbf{n}_{\beta\sigma}$ is the outward-directed surface normal from the β -phase toward the σ -phase, and $\mathbf{n}_{\sigma\beta} = -\mathbf{n}_{\beta\sigma}$ (Fig. 1).

Applying the volume averaging and multiscale theorems to scale-up the microscale model yields a macroscale model:

$$\frac{\partial \langle T_i \rangle^i}{\partial t} + \tau_q \frac{\partial^2 \langle T_i \rangle^i}{\partial t^2} = \alpha \Delta \langle T_i \rangle^i + \alpha \tau_T \frac{\partial}{\partial t} \left(\Delta \langle T_i \rangle^i \right) + \frac{\alpha}{k} \left[F(\mathbf{r}, t) + \tau_q \frac{\partial F(\mathbf{r}, t)}{\partial t} \right], \quad (5)$$

where the index i can take β or σ ; $\langle T_i \rangle^i$ is the i -phase intrinsic average temperature; γ_β and γ_σ are the β -phase and σ -phase effective thermal capacities, respectively; k_β and k_σ are the effective thermal conductivities of the β - and σ -phases, respectively; $k_{\beta\sigma} = k_{\sigma\beta}$ is the cross effective thermal conductivity of the two phases; h and a_v are the film heat transfer coefficient and the interfacial area per unit volume, respectively; and

$$\begin{aligned} \tau_q &= \frac{\gamma_\beta \gamma_\sigma}{h a_v (\gamma_\beta + \gamma_\sigma)}, & \tau_T &= \frac{\gamma_\beta k_\sigma + \gamma_\sigma k_\beta}{h a_v (k_\beta + k_\sigma + 2k_{\beta\sigma})}, \\ k &= k_\beta + k_\sigma + 2k_{\beta\sigma}, & \alpha &= \frac{k_\beta + k_\sigma + 2k_{\beta\sigma}}{\gamma_\beta + \gamma_\sigma}, \\ F(\mathbf{r}, t) + \tau_q \frac{\partial F(\mathbf{r}, t)}{\partial t} &= \frac{k_{\beta\sigma}^2 - k_\beta k_\sigma}{h a_v} \Delta^2 \langle T_i \rangle^i. \end{aligned} \quad (6)$$

This is the dual-phase-lagging heat-conduction equation with τ_q and τ_T as the phase lags of the heat flux and the temperature gradient, respectively. $F(\mathbf{r}, t)$ is the volumetric heat source. k , ρc and α are the effective thermal conductivity, capacity and diffusivity of two, respectively. Therefore, the interaction between the two phases shifts the Fourier heat conduction in each phase into the dual-phase-lagging heat conduction.

By Eq. (6), we can readily obtain that, in two-phase-system heat conduction

$$\frac{\tau_T}{\tau_q} = 1 + \frac{\gamma_\beta^2 k_\sigma + \gamma_\sigma^2 k_\beta - 2\gamma_\beta \gamma_\sigma k_{\beta\sigma}}{\gamma_\beta \gamma_\sigma (k_\beta + k_\sigma + 2k_{\beta\sigma})}. \quad (7)$$

It can be large, equal or smaller than 1 depending on the sign of $\gamma_\beta^2 k_\sigma + \gamma_\sigma^2 k_\beta - 2\gamma_\beta \gamma_\sigma k_{\beta\sigma}$. Therefore, by the condition for the existence of thermal waves that requires $\tau_T / \tau_q < 1$, we may have thermal waves in two-phase-system heat conduction when

$$\gamma_\beta^2 k_\sigma + \gamma_\sigma^2 k_\beta - 2\gamma_\beta \gamma_\sigma k_{\beta\sigma} < 0. \quad (8)$$

Note also that for heat conduction in two-phase-systems there is a time-dependent source term $\frac{k_{\beta\sigma}^2 - k_\beta k_\sigma}{ha_\nu} \Delta^2 \langle T_i \rangle^i$ in the dual-phase-lagging heat conduction [Eqs. (5) and (6)].

Therefore, the resonance can also occur. These thermal waves and possibly resonance are believed to be the driving force for the conductivity enhancement.

Thermal waves have been observed in casting sand experiments by two independent groups. Substantial increases in thermal conductivity have also been confirmed experimentally for the porous-media fluids and nanofluids. However, the reported data of effective thermal conductivity are all in between those of two phases so that the k_β -enhancement appears only at $k_\sigma > k_\beta$. On the other hand, our theory shows that the k_β -enhancement can occur for all cases with $k_{\beta\sigma} > -k_\sigma/2$ [Eq. (6)]. Therefore, it is possible to have some thermal-wave fluids that can support very strong thermal waves and resonance such that their conductivities are higher than those of two phases. We present the first experimental evidence of substantial increases in fluid thermal conductivity (up to 139%) by adding some fluid even with lower conductivity (Figs. 2 and 3).

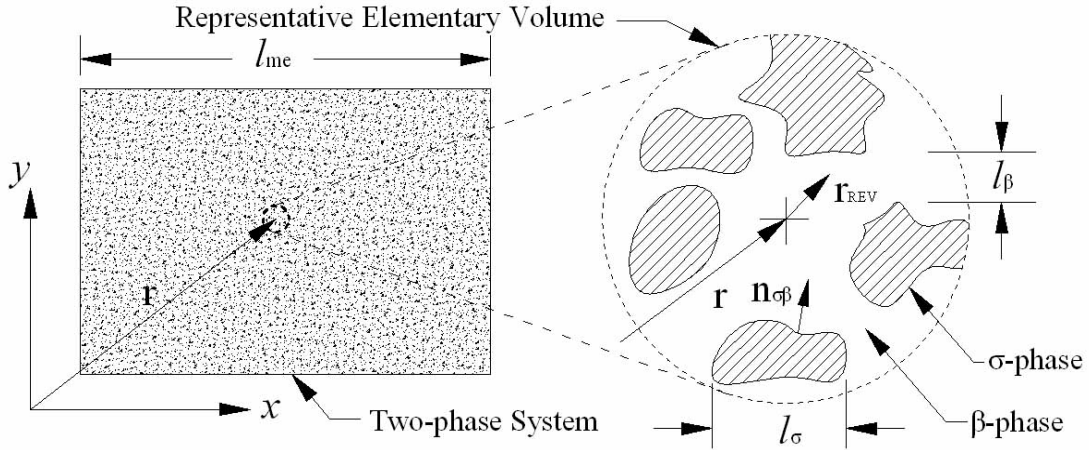


Figure 1: Two phase systems

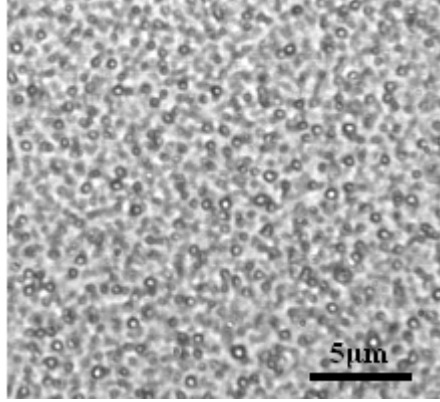


Figure 2: Oil/water emulsion under microscope [oil volume fraction=10%; temperature=23 °C; oil droplet mean diameter of 383.9nm with a coefficient of variation (CV) of 4.02%].

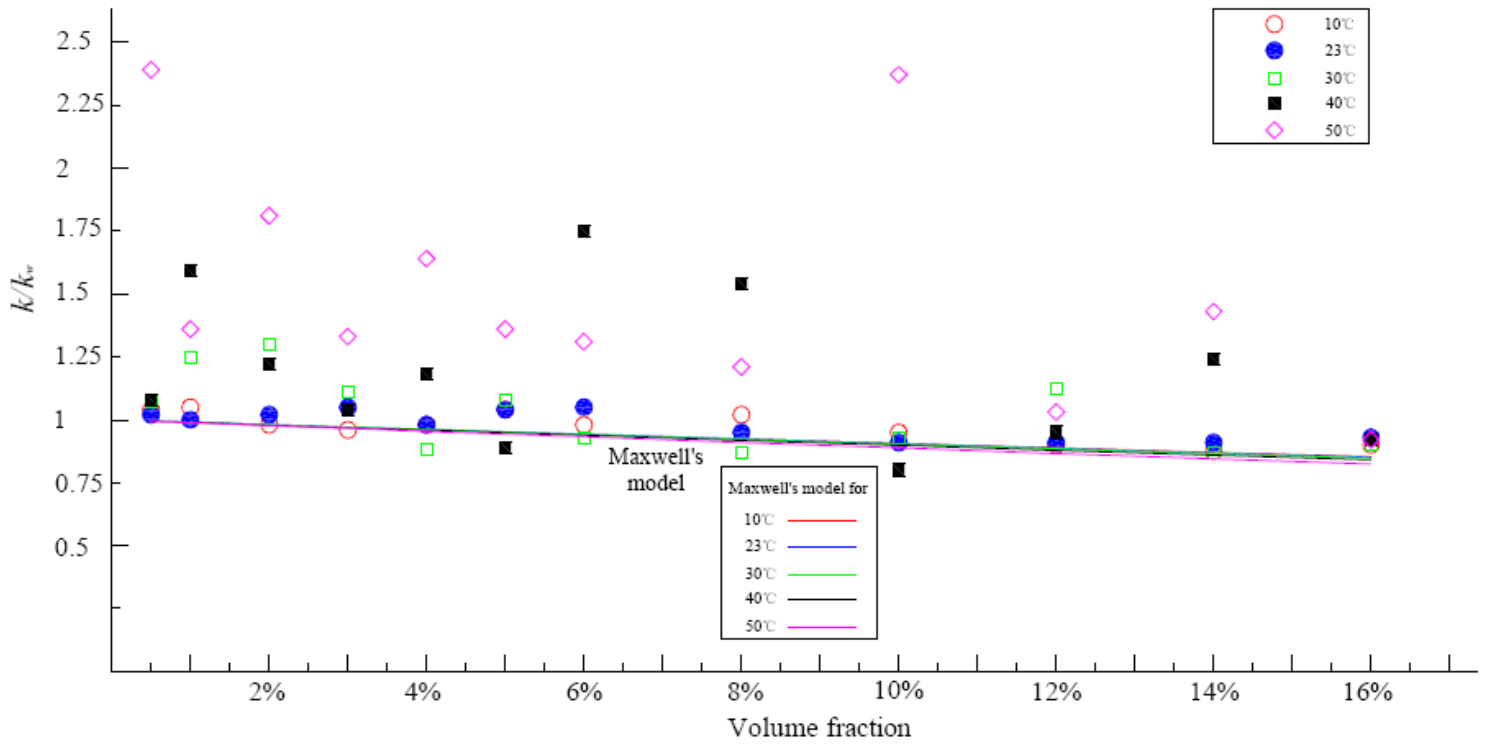


Figure 3: Variation of k/k_w with oil volume fraction and emulsion temperature (k : emulsion thermal conductivity; k_w : water thermal conductivity).