

# **Model-Based Experimental Analysis**

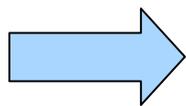
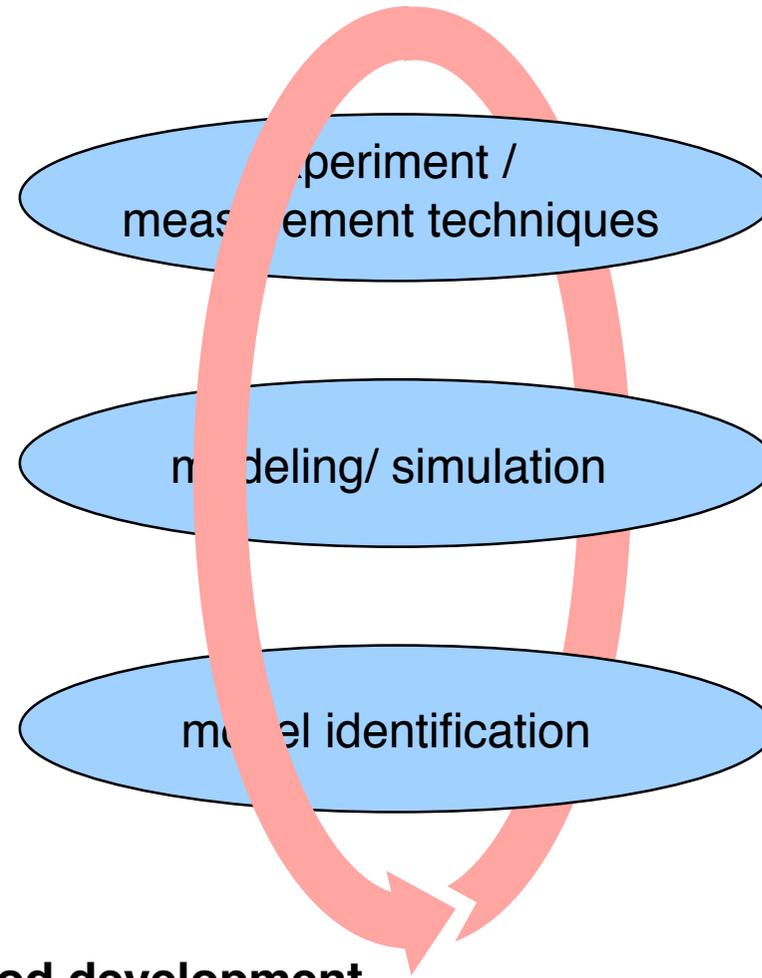
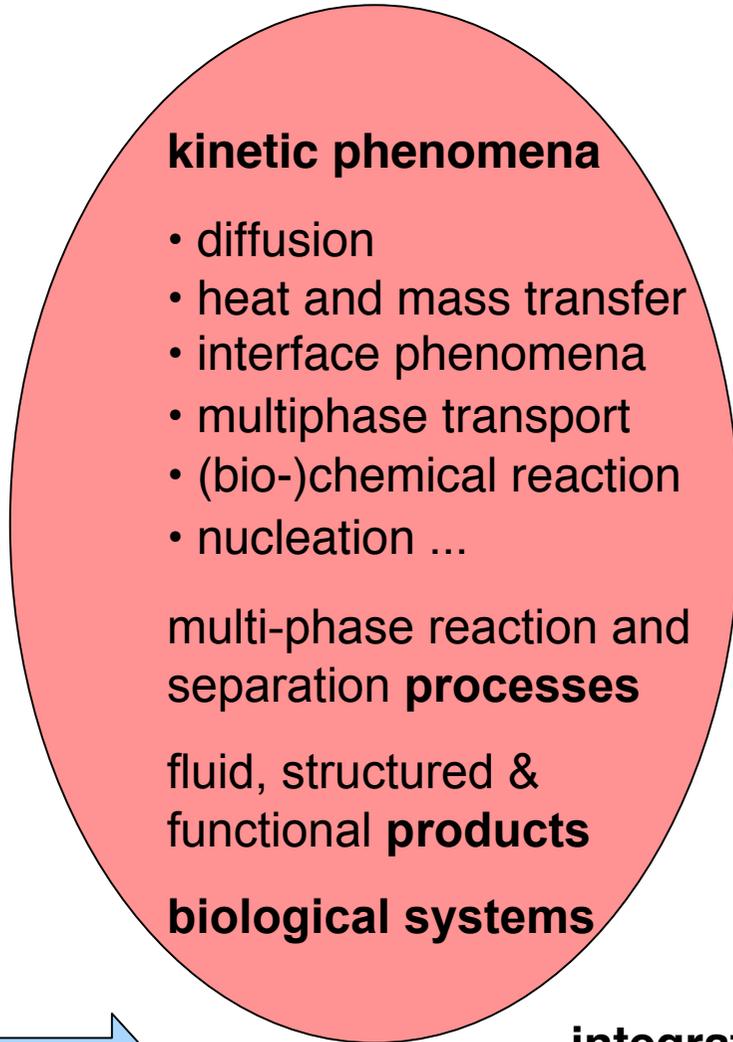
**A Systems Approach to Mechanistic  
Modeling of Reactive Systems**

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Lehrstuhl für Prozesstechnik  
RWTH Aachen**

**MaCKiE Annual Seminar, May 3, 2006, Gent, Belgium**

product & process systems

experimental and theoretical methods & tools

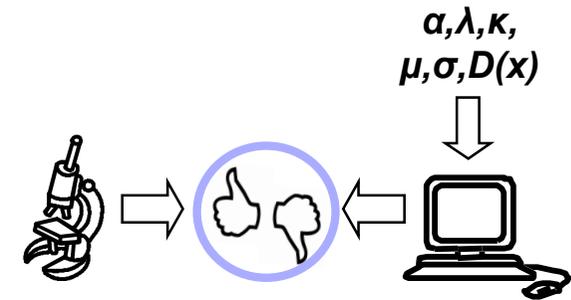


**integrated method development  
towards a work process of model-based experimental analysis**

cf. J.V. Beck, *Meas. Sci. Techn.* 9 (1998)

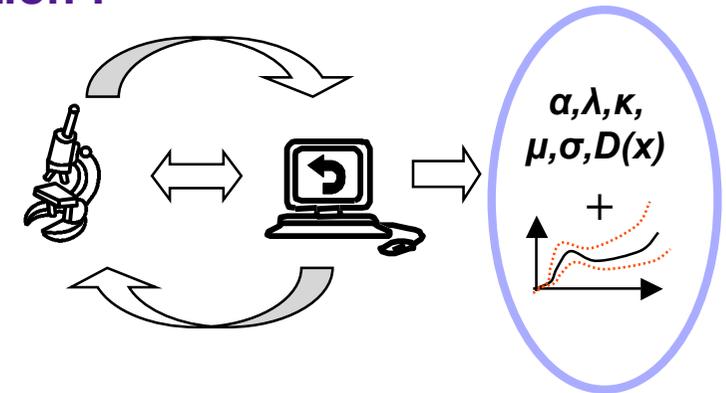
- **common approach in research and industrial practice**

- coupled phenomena
- detailed models, numerical case studies
- comparison of simulation and experimental results
- **evaluation of the model, but no model identification !**



- **suggested future approach**

- coordinated design of model and experiment
- model refinement based on experimental evidence
- accounting for inevitable measurement errors
- **identification of a valid (mechanistic) model (structure & parameters) !**

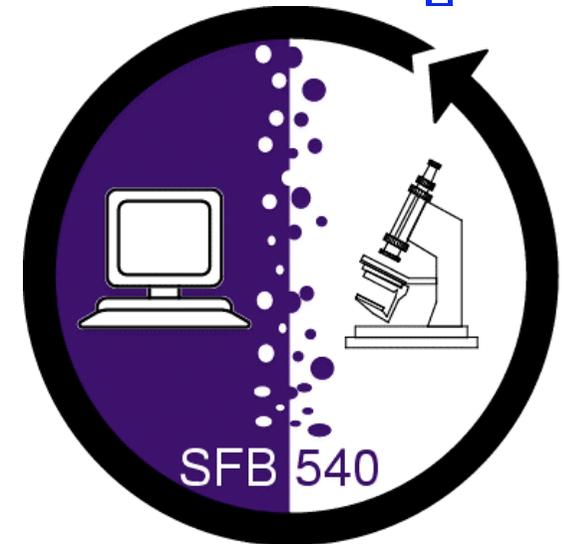


**model-based experimental analysis – MEXA:**  
valid models at minimal effort

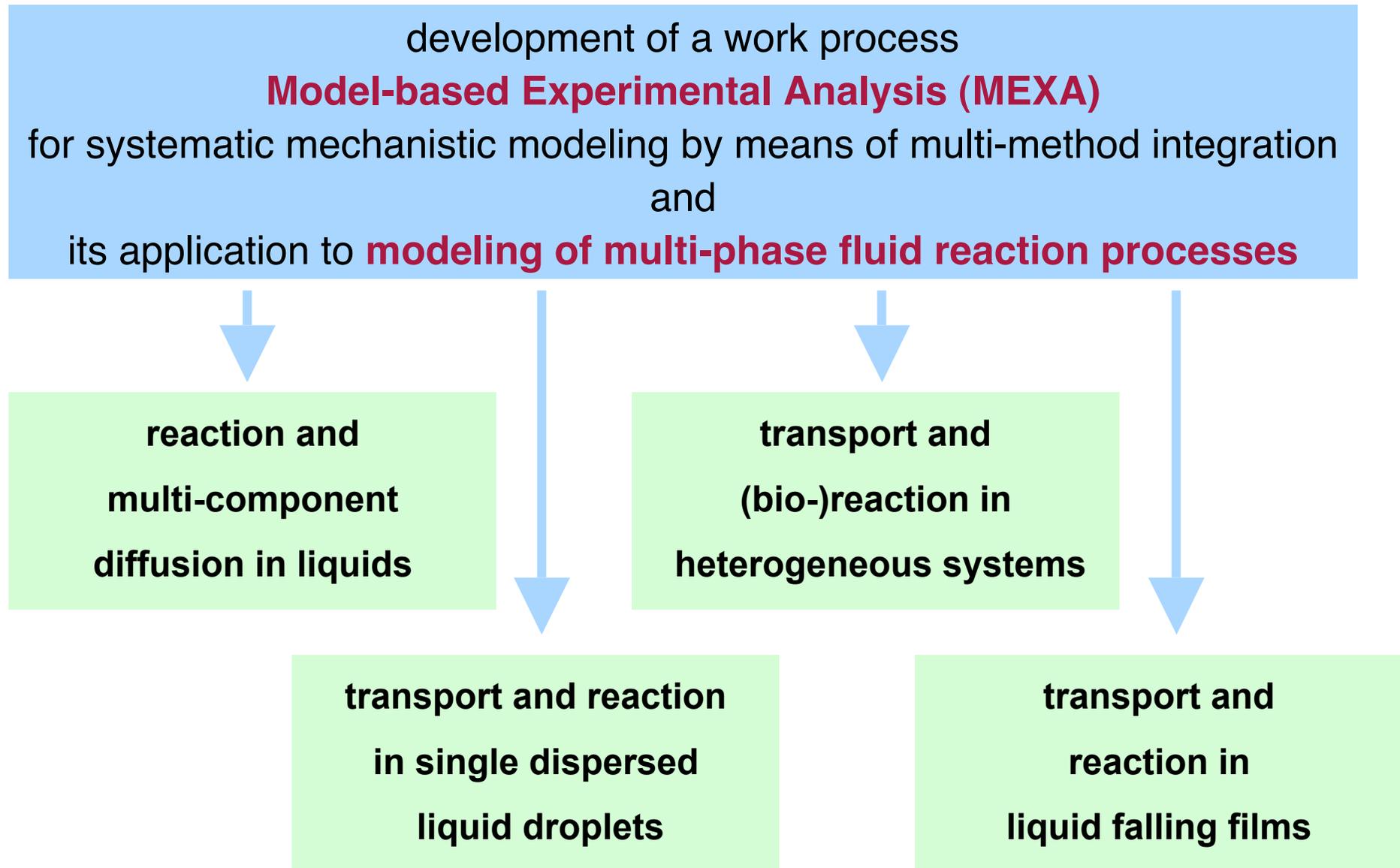
# Model-based Experimental Analysis of Kinetic Phenomena in Fluid Multi-phase Reactive Systems

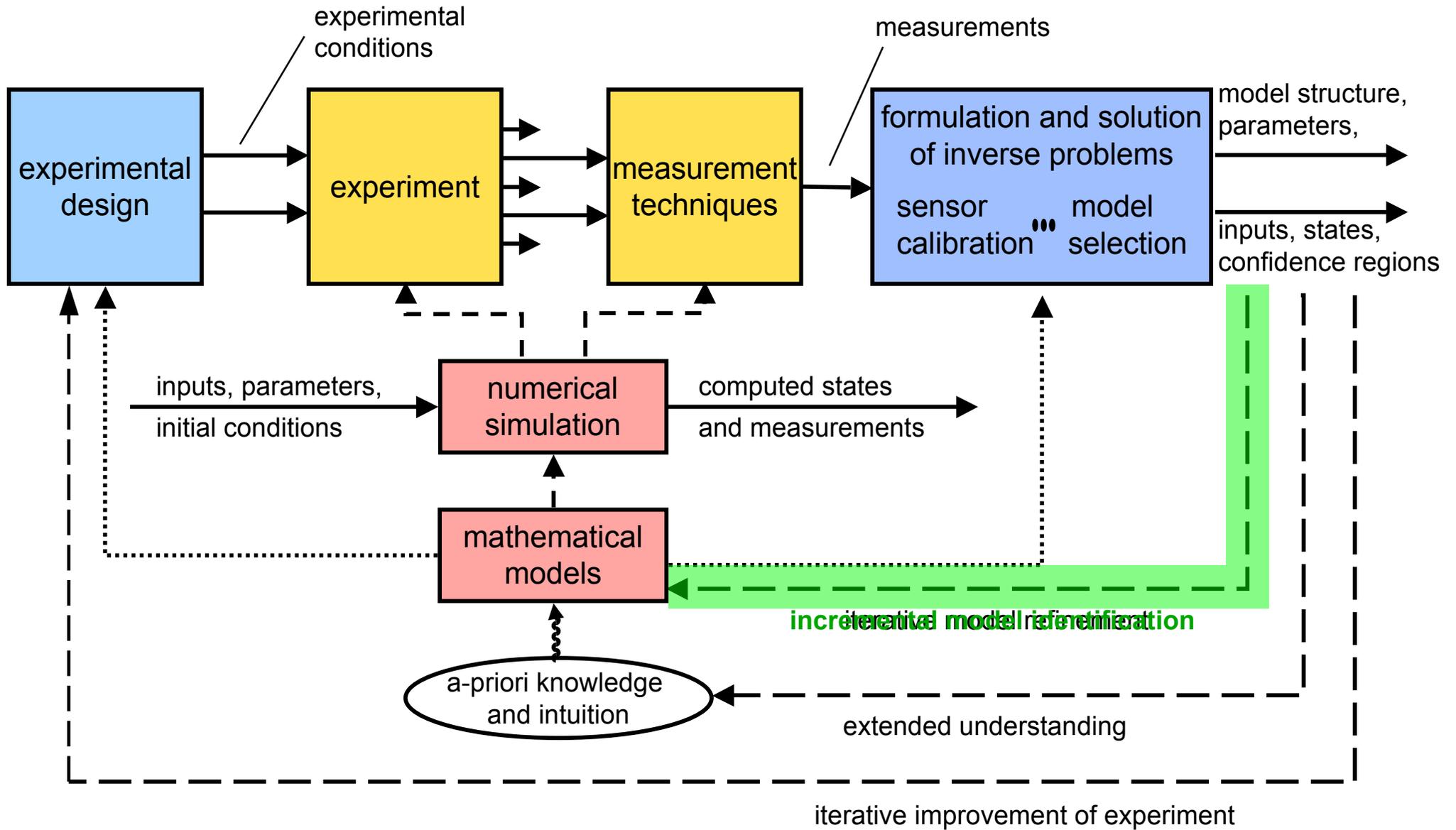
12 research groups with cross-disciplinary expertise

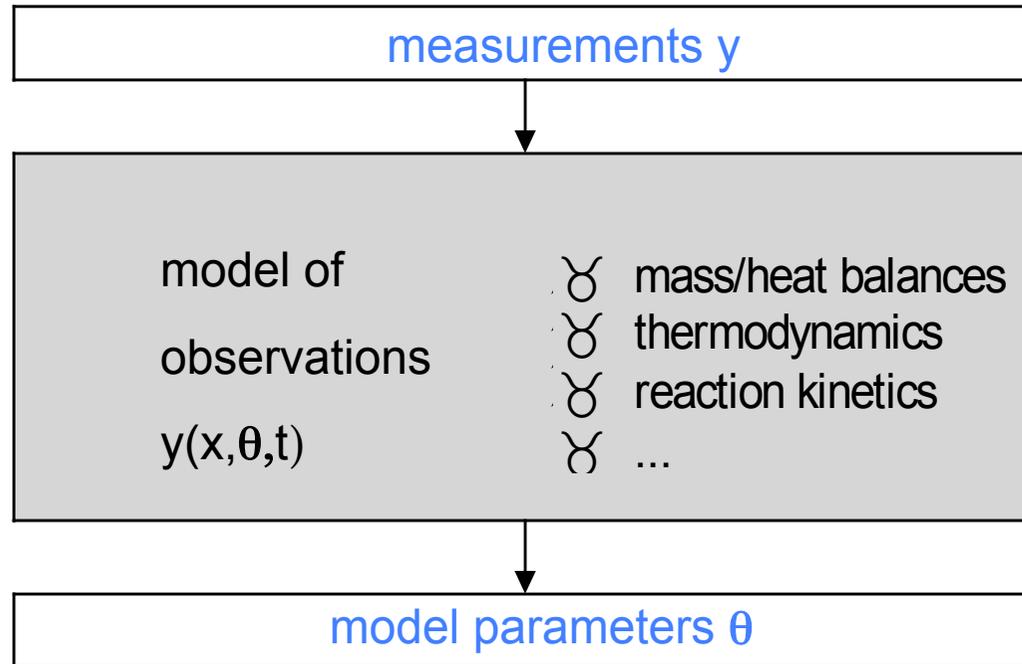
- biotechnology (Ansorge-Schuhmacher)
- biochemical engineering (Büchs)
- reaction engineering (Greiner, Leitner)
- thermal separations (Pfennig)
- transport phenomena (Kneer)
- multiphase fluid dynamics (Modigell)
- computational engineering science (Behr)
- process systems engineering (Bardow, Marquardt)
- numerical mathematics (Reusken)
- scientific computing (Bischof, Bucker)
- NMR imaging (Blümich, Stapf)
- optical spectroscopy (Koß, Lucas, Poprawe)



**Funded by DFG  
(Deutsche Forschungs-  
gemeinschaft) since 1999  
Director: W. Marquardt**





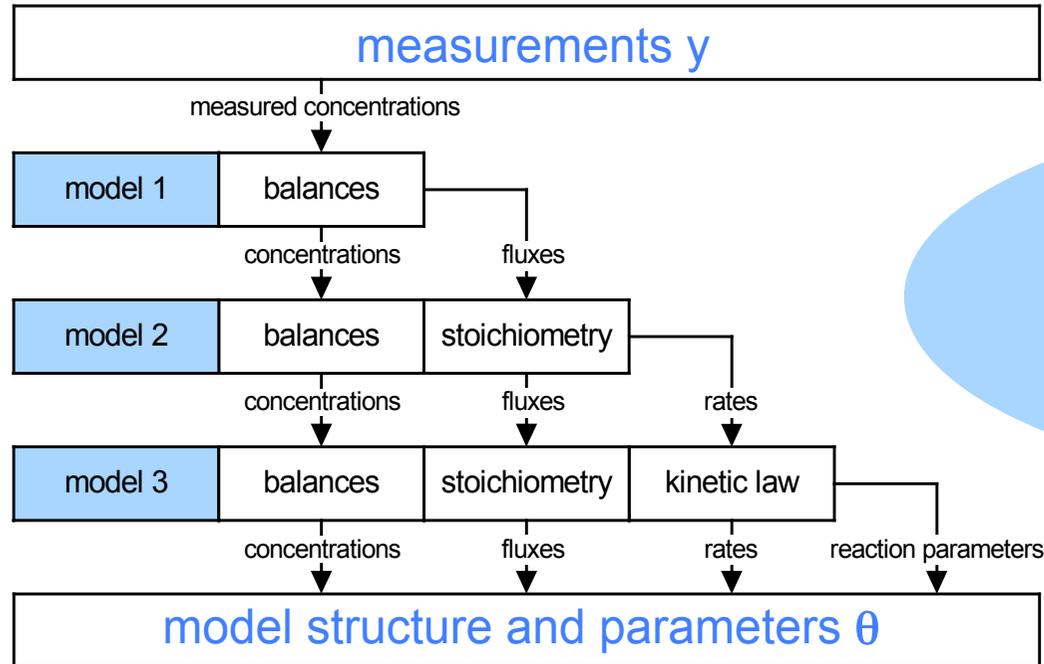


➡ Overall process model  $y(\mathbf{x}, \theta, t)$  is fitted to experimental data:

$$\min_{\hat{\mathbf{e}}} \frac{1}{2} \sum_{i=1}^n w_i \sum_{j=1}^m \left( y(\mathbf{x}, \hat{\mathbf{e}}, t_j) - \tilde{y}(t_j) \right)^2$$

s.t. dynamic model & constraints

- What if we do not know any candidate model structure ?
- How to select a suitable model structure ?
- Is bias due to model structure defects or a lack of information content in data ?
- How to deal with very few or very many observations ?
- How to deal with convergence & robustness problems of estimation algorithm?



**Decompose  
the model identification and  
selection problem into fully  
transparent steps !**

- **computationally efficient (minutes rather than days)**
- **numerically robust and fully transparent**
- **a-priori knowledge can be integrated into the identification process**
- **complex and interacting kinetic phenomena can be identified**

- **differential methods in reaction kinetics**, e.g. Connors (1990)  
→ estimate reaction rate by FD, then estimate kinetic parameters
- **hybrid modeling**, e.g. Psychogios & Ungar (1992), Tholudor & Ramirez (1999)  
→ combine first-principles models with neural nets
- **inverse problems in population balances**, Mahoney, Doyle & Ramkrishna (2002)  
→ calculate growth rate as model-based data, correlate with states

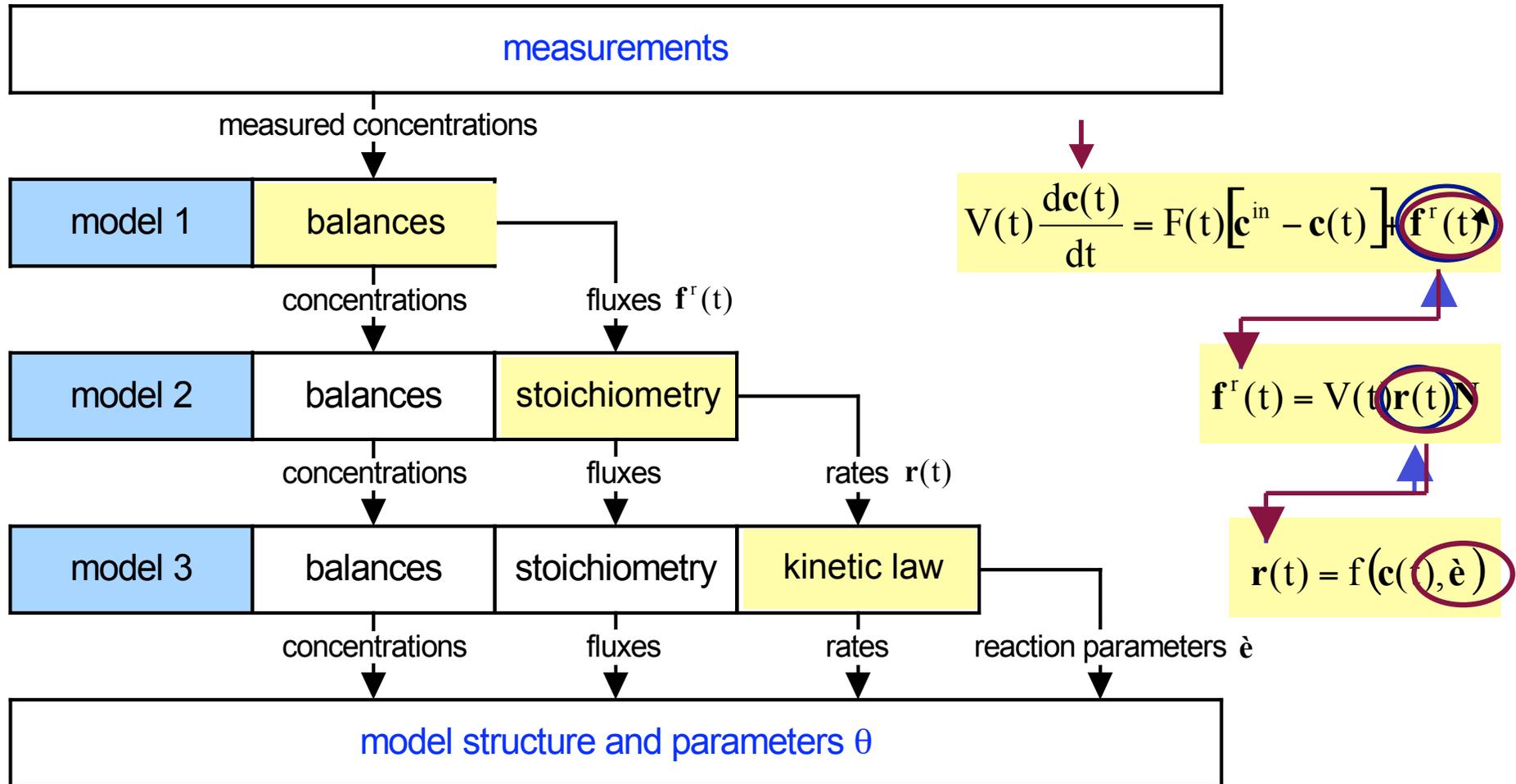


- **derived from intuition and physical insight**
- **often ad-hoc methods**
- **problem-specific solutions**



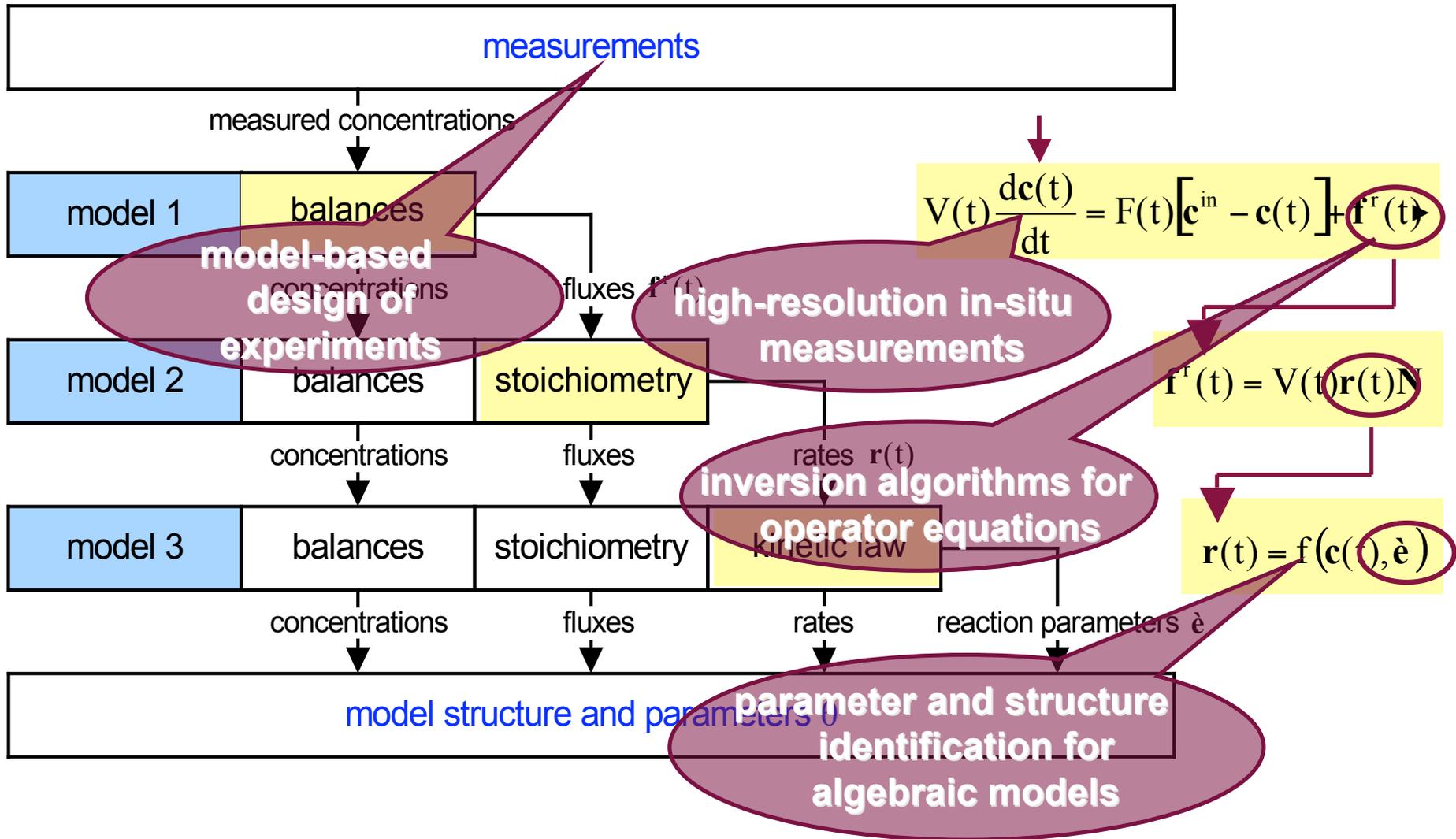
**a generic principle ?**

Illustration with reaction with a CSTR



(Marquardt, 1998)

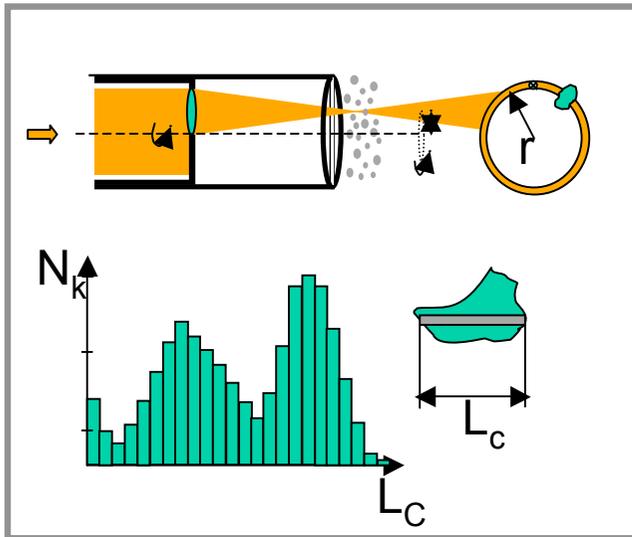
What are the ingredients for implementation ?



**non-invasive, in-situ measurements of field data**

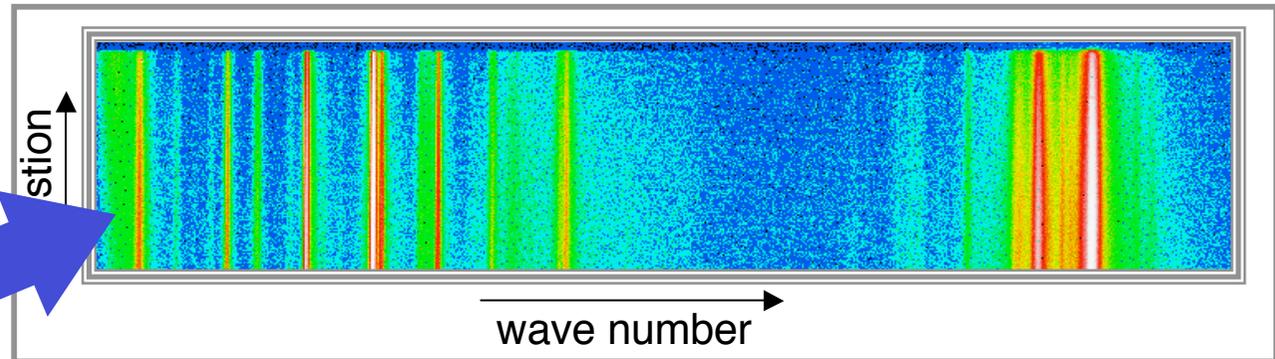
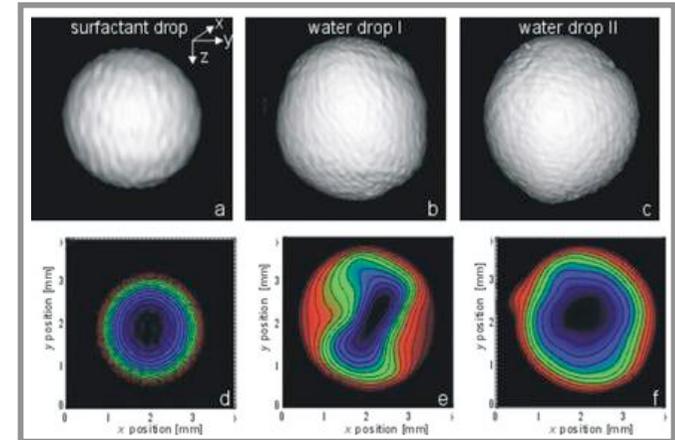
- observation of qualitative behavior
- quantitative characterisation of kinetic phenomena

particle size distribution (FBRM)  
Kail, Briesen, Marquardt et al., LPT



concentrations on a line,  
Raman spectroscopy  
Koß, Lucas et al., CRC 540

velocity profiles in a levitated droplet, NMR imaging, Blümich et al., CRC 540



**interpretation of the primary measurement data,  
calibration, quantification of measurement errors !**

Most established spectral analysis methods such as

- PCA, PLS or
- classical least squares

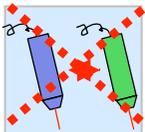
are **linear** and **cannot model all nonlinear effects that occur in real mixtures:**



Non-linear effects due to molecular interaction



Reactive mixtures: restricted extrapolability



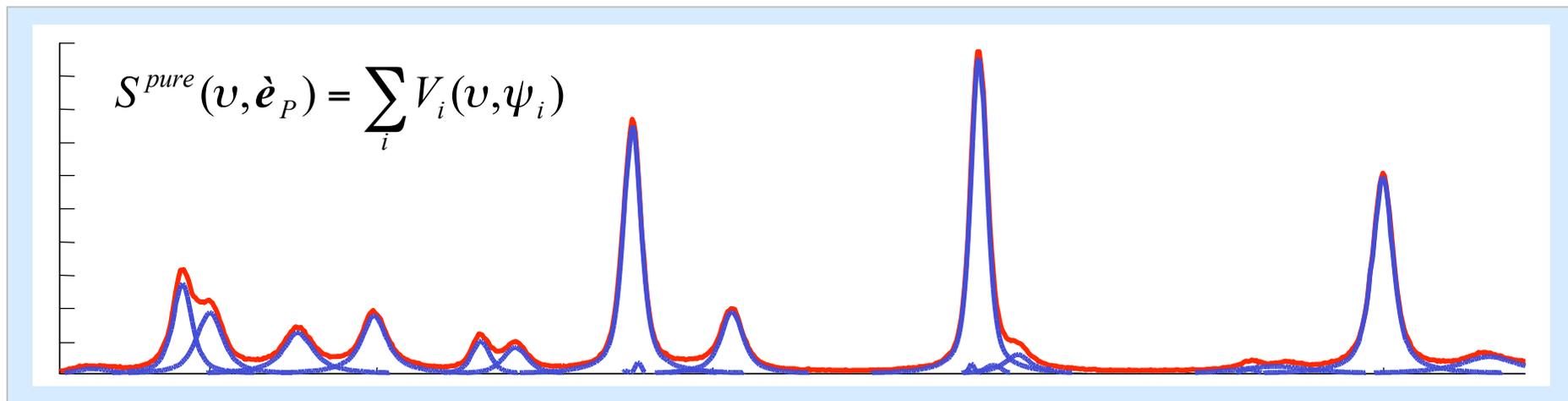
Replacement  
of measurement device or varying temperatures

## Indirect Hard Modeling: a nonlinear spectral analysis approach

### Development of a rigorous mathematical model of the spectrum

Consideration of physical effects of the spectrum through phenomenological modeling

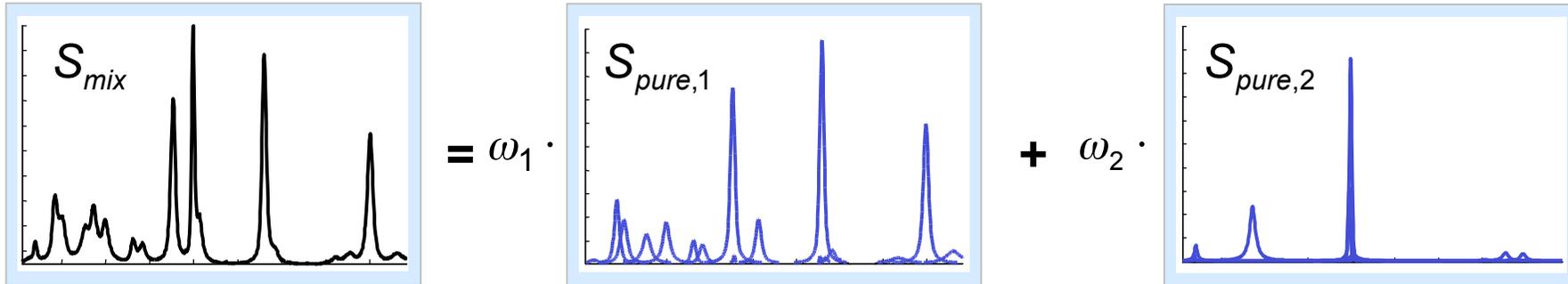
#### Step 1: Modeling of pure component spectra (during calibration)



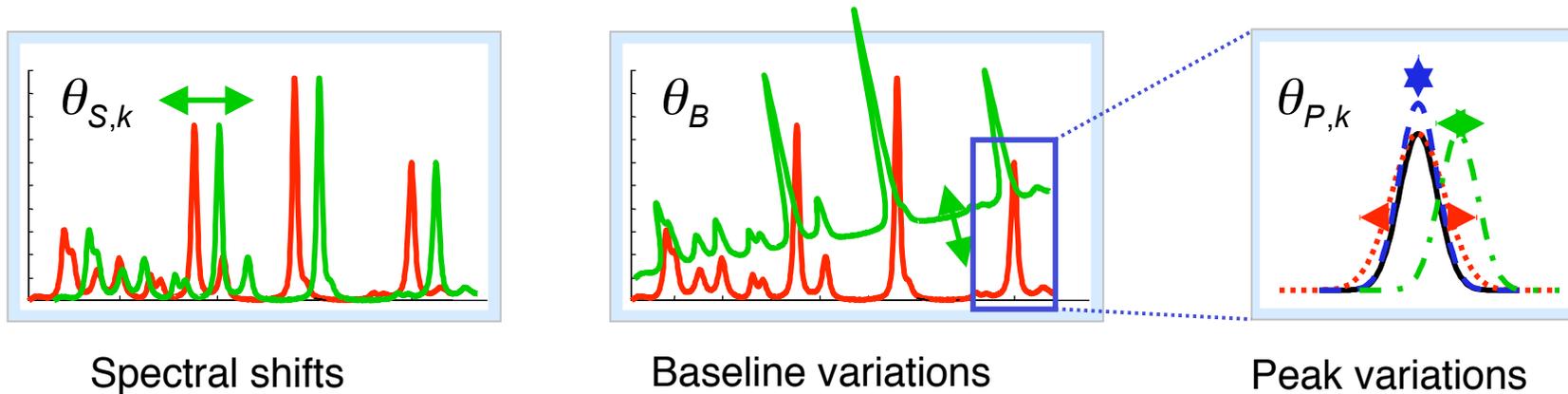
Generation of **pure component models** using **automatic peak fitting algorithm**

(Alsmeyer et al., *Applied Spectroscopy*, 58 (8), 2004)

**Step 2: Mixture spectrum is modeled as:  
Linear combination of parameterized non-linear pure component models**

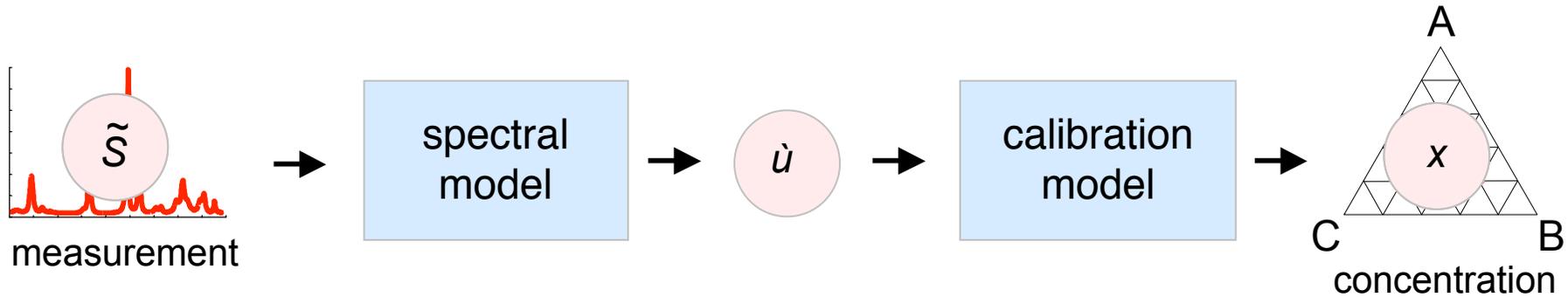


**All non-linear effects are modeled phenomenologically:**



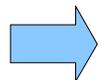
**Full spectral model:**

$$S(\nu) = B(\theta_B) + \sum_k \omega_k \cdot S_k^{pure}(\nu, \theta_{P,k}, \theta_{S,k})$$



**Linear, physically motivated calibration model**  
 (Non-linear effects are corrected by the spectral model)

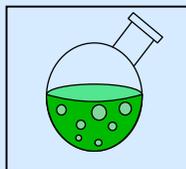
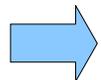
$$\frac{\omega_i}{\omega_j} = K_{i,j} \frac{x_i}{x_j} \Rightarrow x_i = \frac{1}{\sum_k \frac{\omega_k}{\omega_i} K_{k,i}}$$



**Highly reduced amount of calibration measurements**

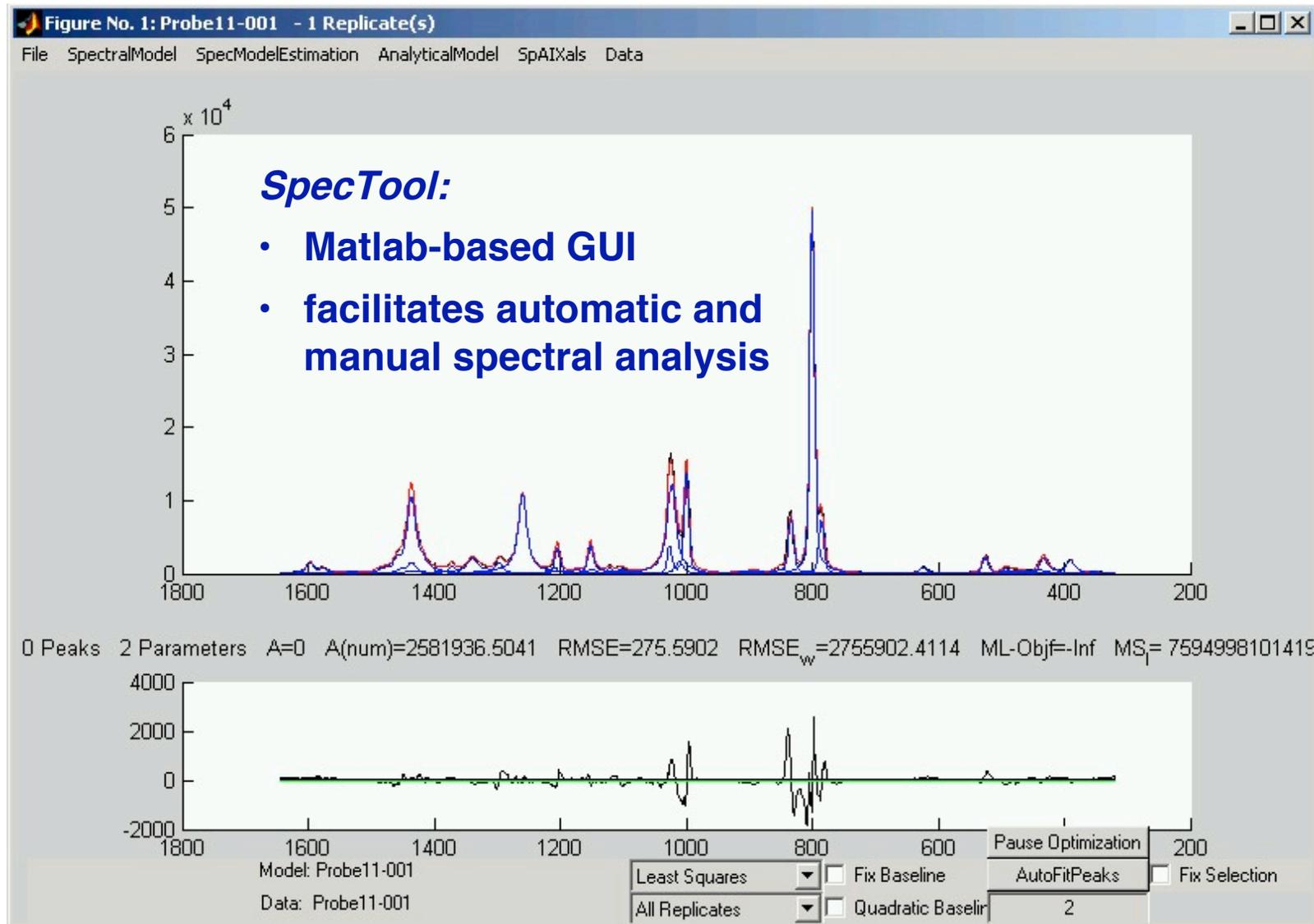
**Theory:** One calibration measurement

**Practice:** Few measurements

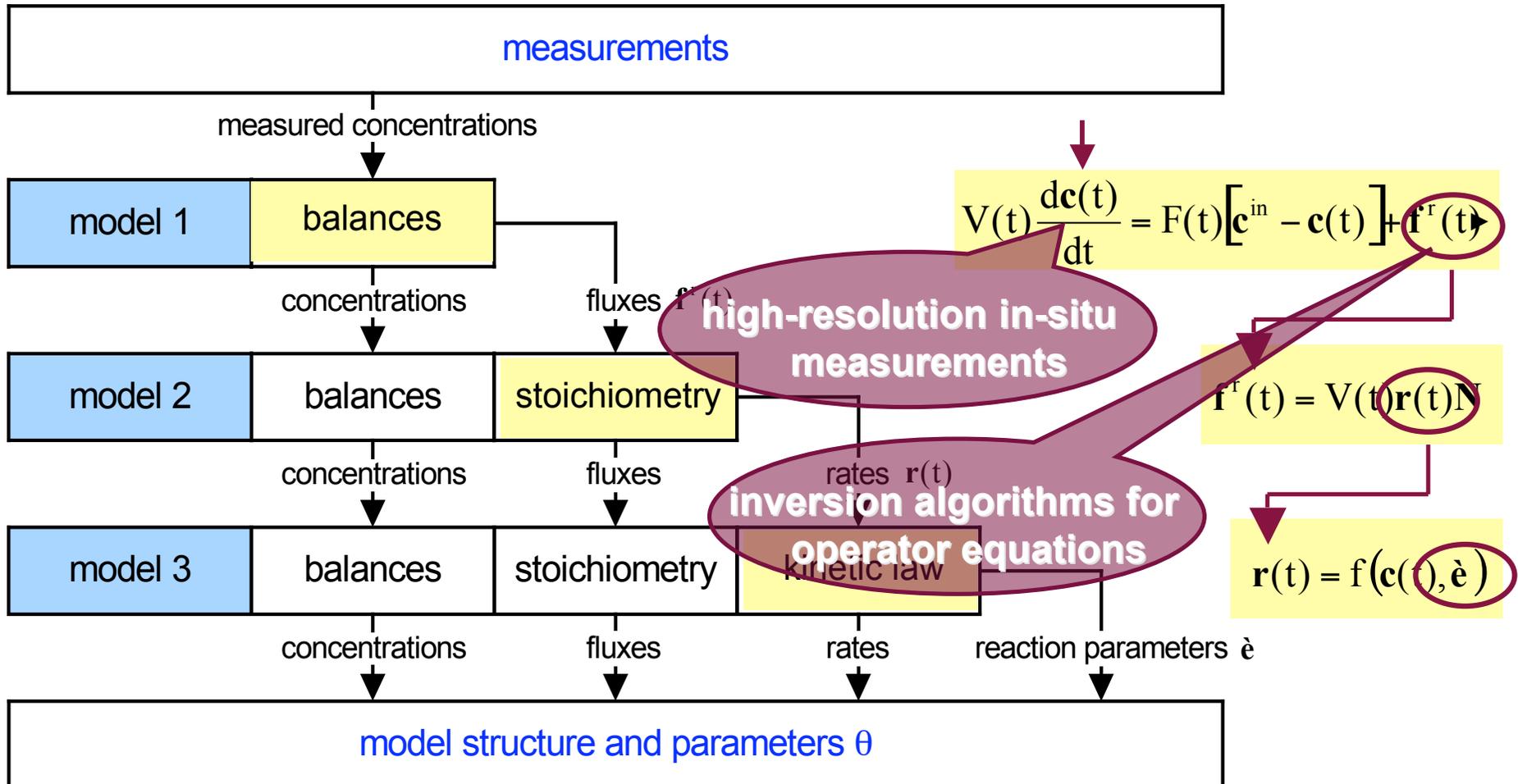


**Good extrapolability:**

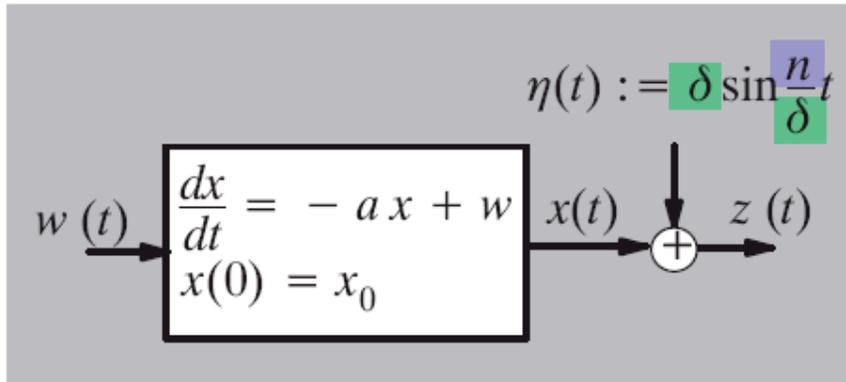
**Calibration in concentration subspace is possible**



What are the ingredients for implementation ?



illustrating linear example (scalar):



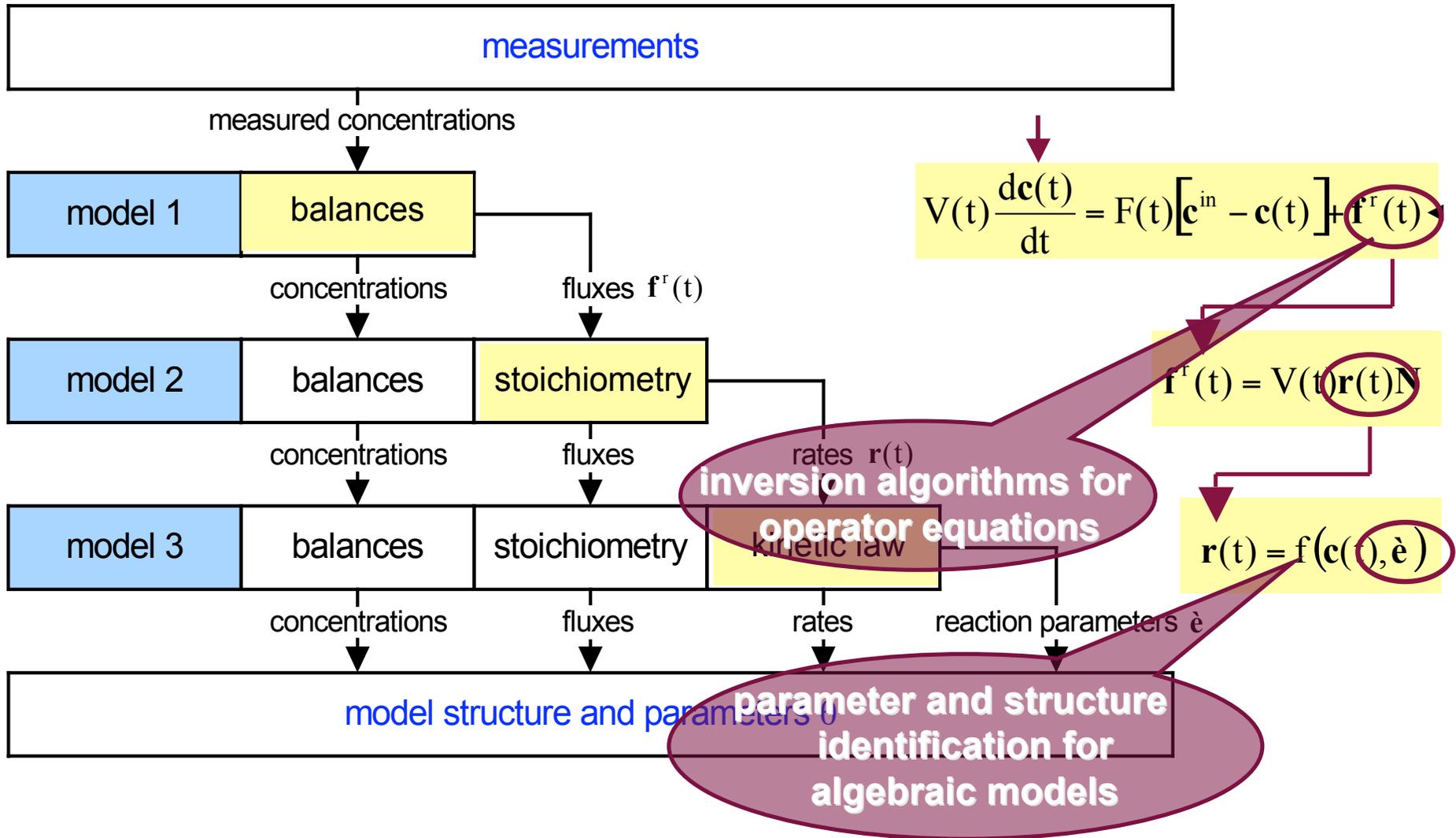
a problem is well-posed if

- a solution exists
- the solution is unique
- small errors in the data lead to small errors in the solution

(Hadamard, 1923)

- decide on a set of measurements for best identifiability
  - at least as many measurements as unknown fluxes (Hirschorn, 1979)
  - methods for quantification of identifiability (e.g. Asprey, 2003)
- balance information content of measurements and resolution of the flux parameterization
  - choose spatial and temporal resolution of flux function
  - adaptive discretization methods (e.g. Binder et al. 2000)
- compromise between bias and variance in estimates
  - balanced choice of discretization, early stopping and regularization
  - systematic methods for the selection of regularization operators and multiple regularization parameters (e.g. Ascher, Haber, 2001, Engl et al., 1996, Belge et al. 2002)

## What are the ingredients for implementation ?



**find the most appropriate functional representation  
for the correlation of fluxes and states: structure and parameters**

**generate candidate model structures**

- experience-based, qualitative reasoning, kinetic power laws (Schaich et al., 2001)
- molecular scale modeling (Barrett & Prausnitz, 1975, Liu et al., 1998)
- multivariate regression & data mining (Bates & Watts, 1988, Hastie et al. 2001)

**parameter estimation and model structure selection**

- error-in-variables formulation (Britt & Lücke, 1978, Boggs et al. 1992)
- inference approach, decision tree & statistical tests (Verheijen, 2003)
- Bayesian a-posteriori probability tests (Stewart et al., 1998)
- combinatorial search (McKay et al., 1997, Skrifvars et al., 1998)

Given: noisy data set  $S$

$$S = \{(\mathbf{x}_i, \tilde{y}_i) \in \mathbb{R}^d \times \mathbb{R}\}_{i=1}^M \quad \tilde{y}_i = f(\mathbf{x}_i) + \varepsilon$$

Recover unknown function  $f \in V$  from data  $S$  "as good as possible"

$$\min_{f_N \in V_N} \frac{1}{M} \sum_{i=1}^N (f_N(\mathbf{x}_i) - \tilde{y}_i)^2$$

Restriction to finite dimensional subspace  $V_N$

$$+ \lambda \Phi(f)$$

Tikhonov regularization term enforcing smoothness of  $f_N$

Binder *et al.* (2000)

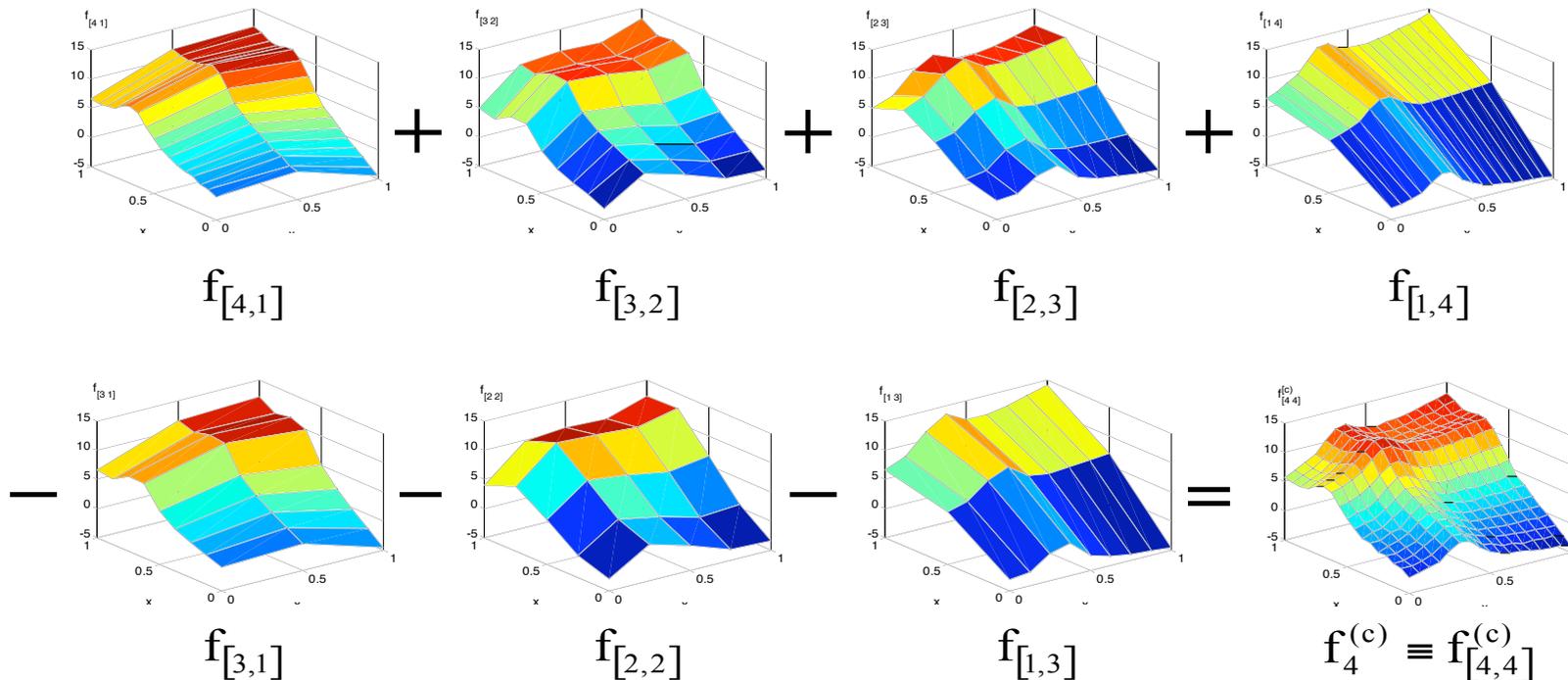
Ascher and Haber (2001)

Desirable properties

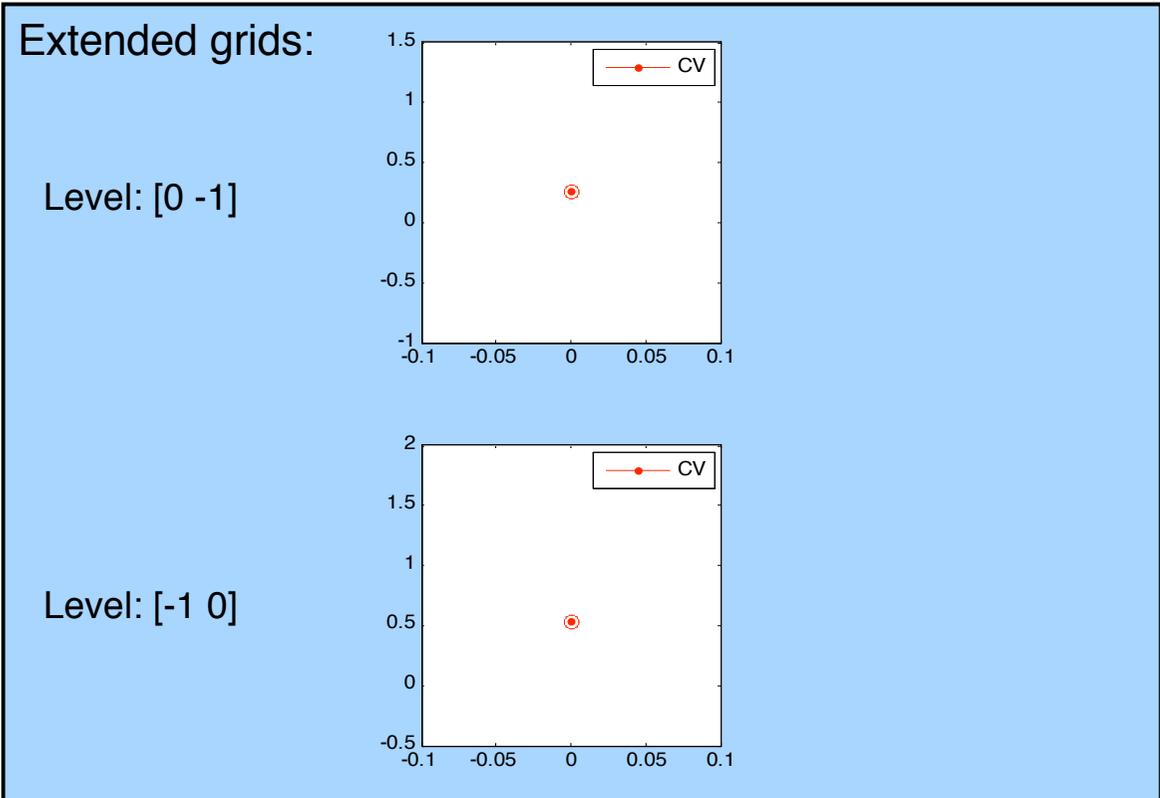
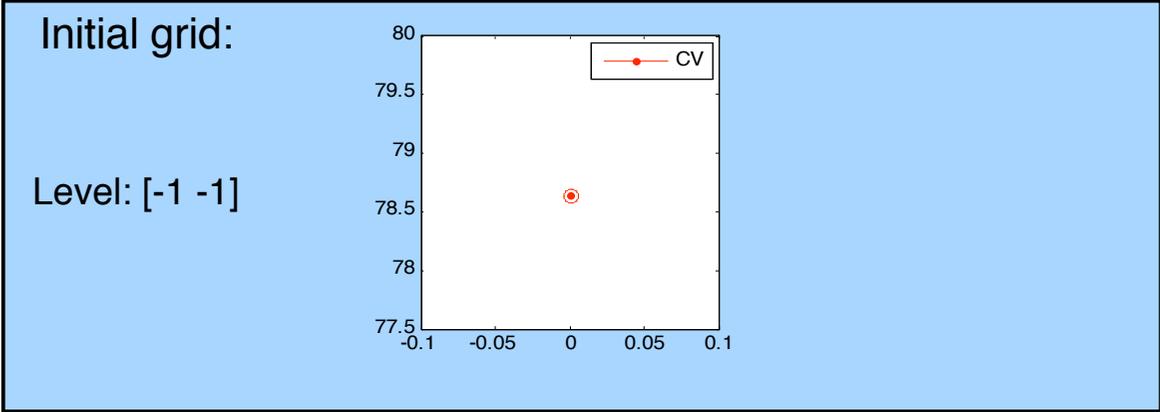
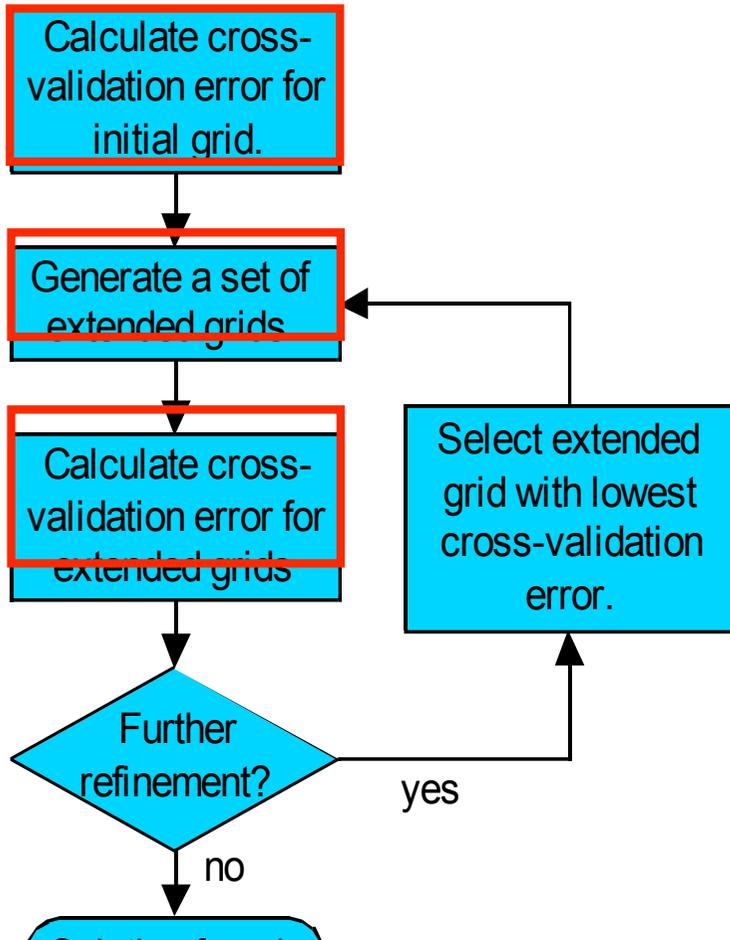
- linear scaling with number of data points (avoid „curse of dimensionality“)
- properly exploit information content in data (avoid under-/overfitting)

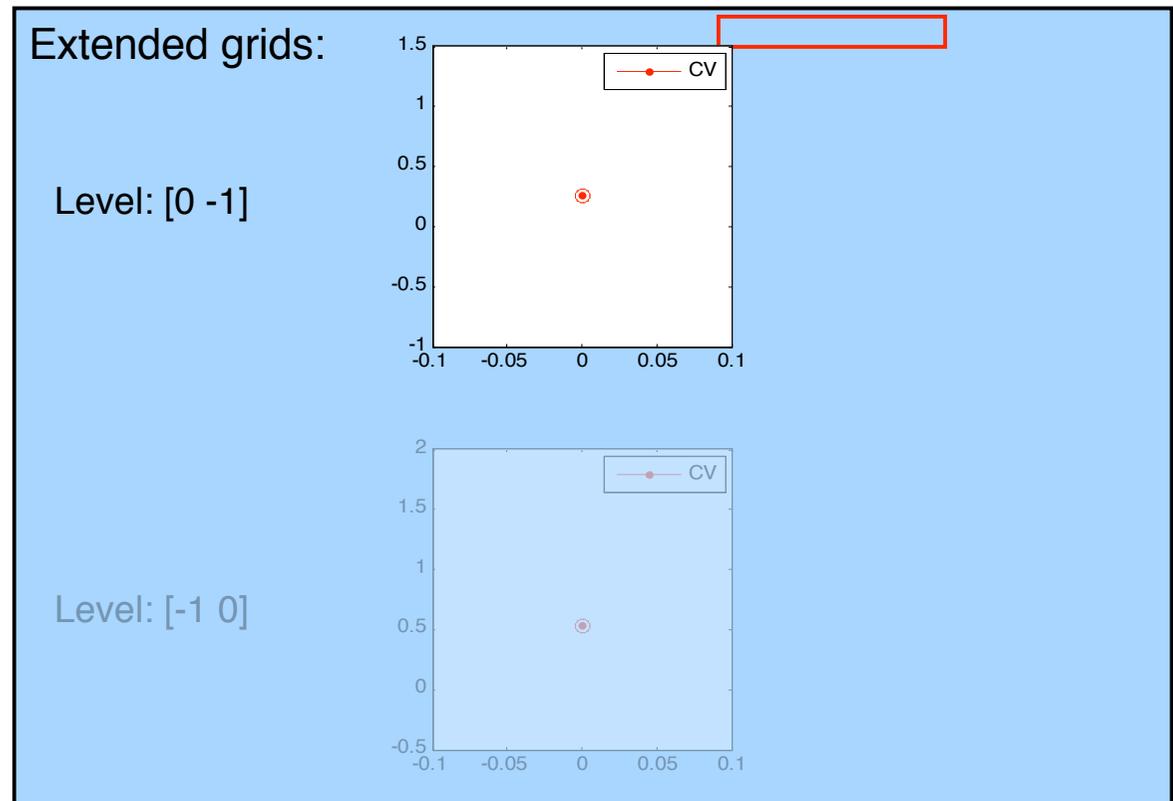
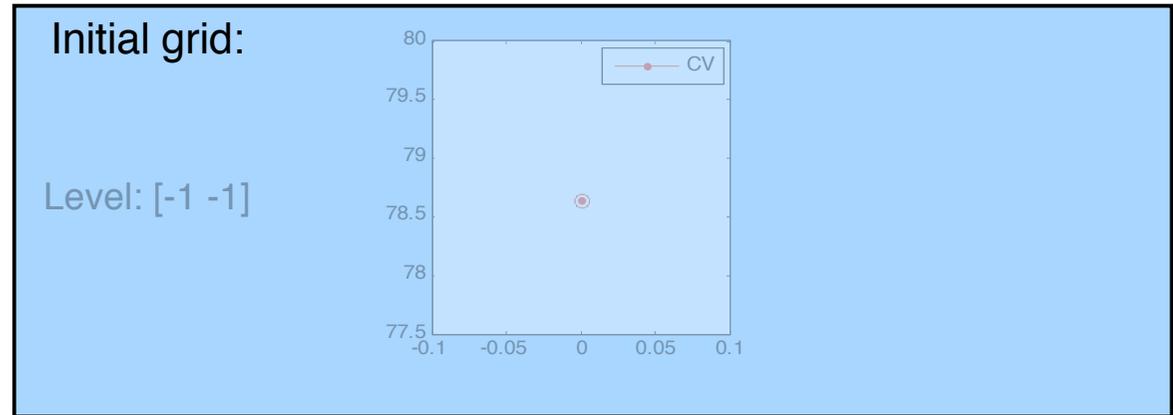
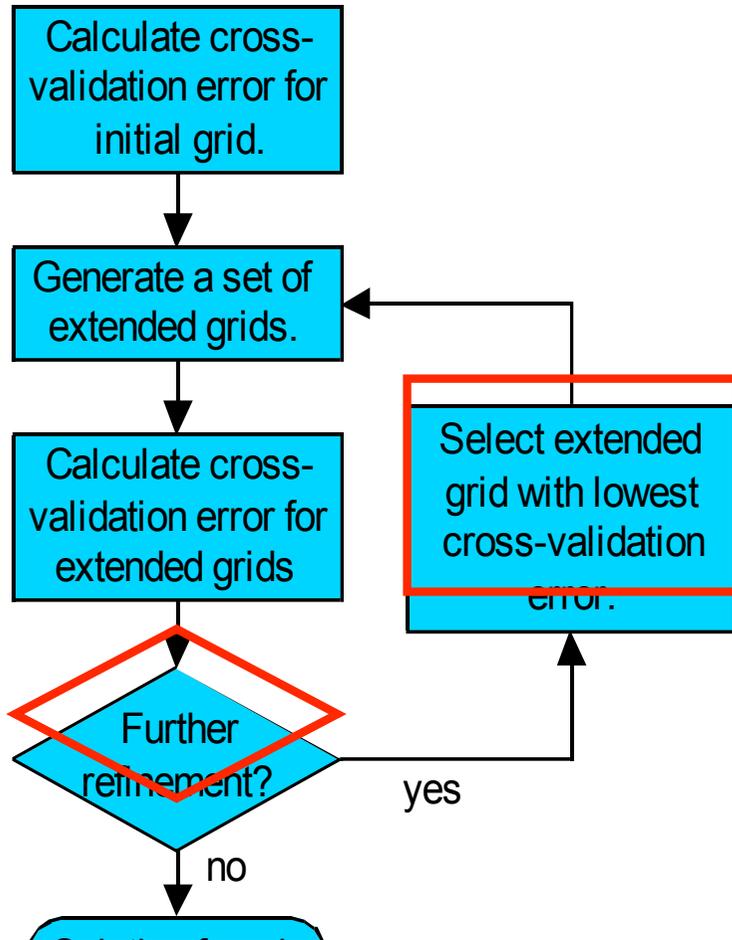
(Garcke et al., 2001; Brendel & Marquardt, 2003)

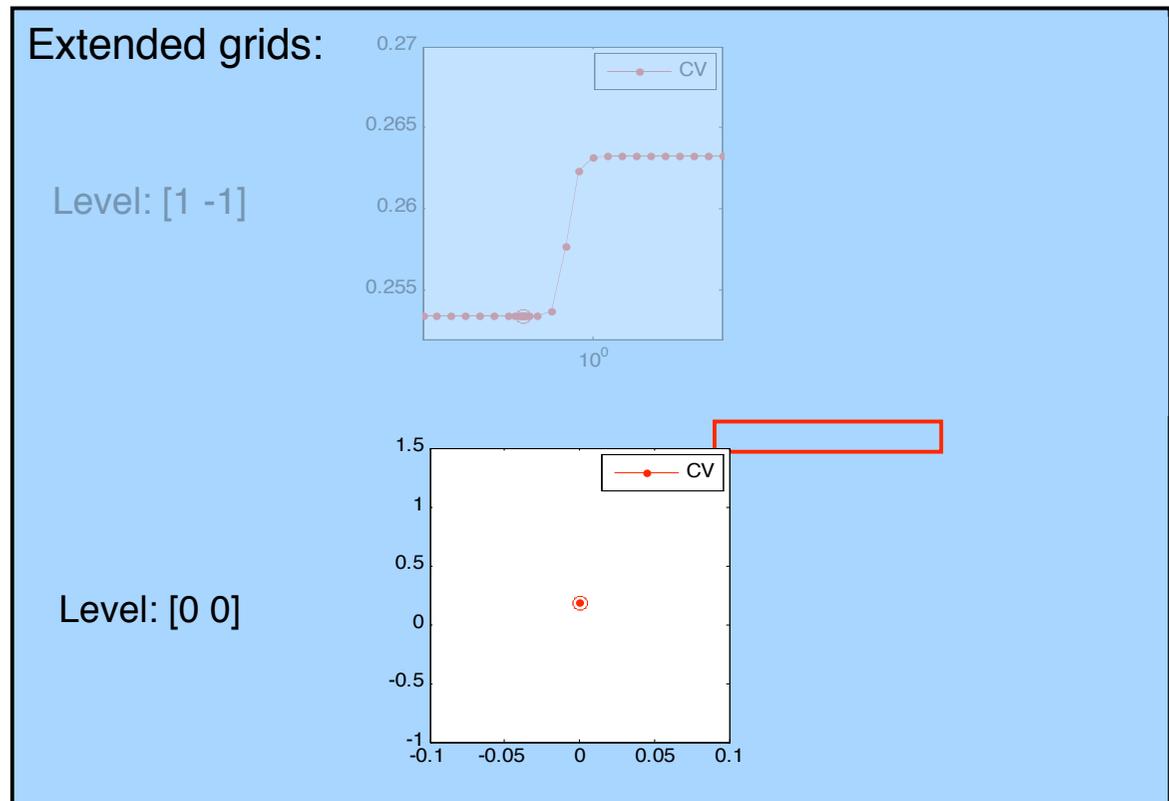
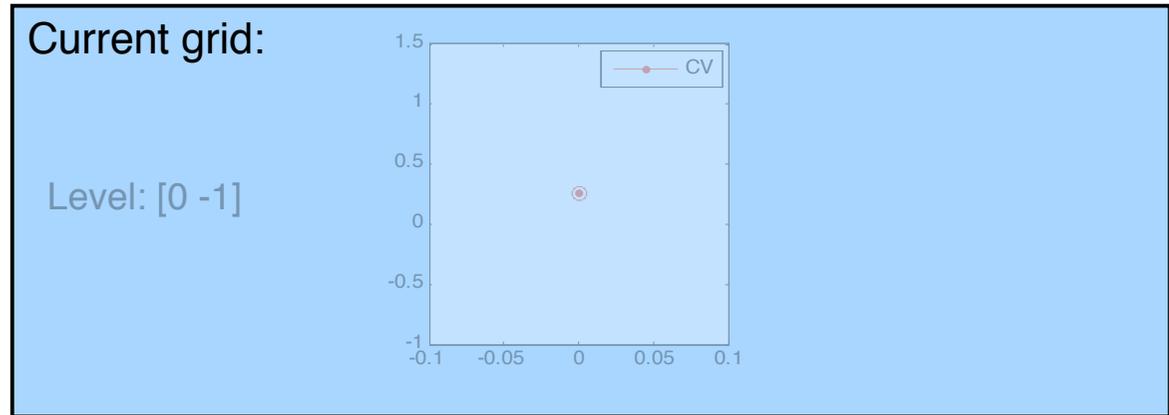
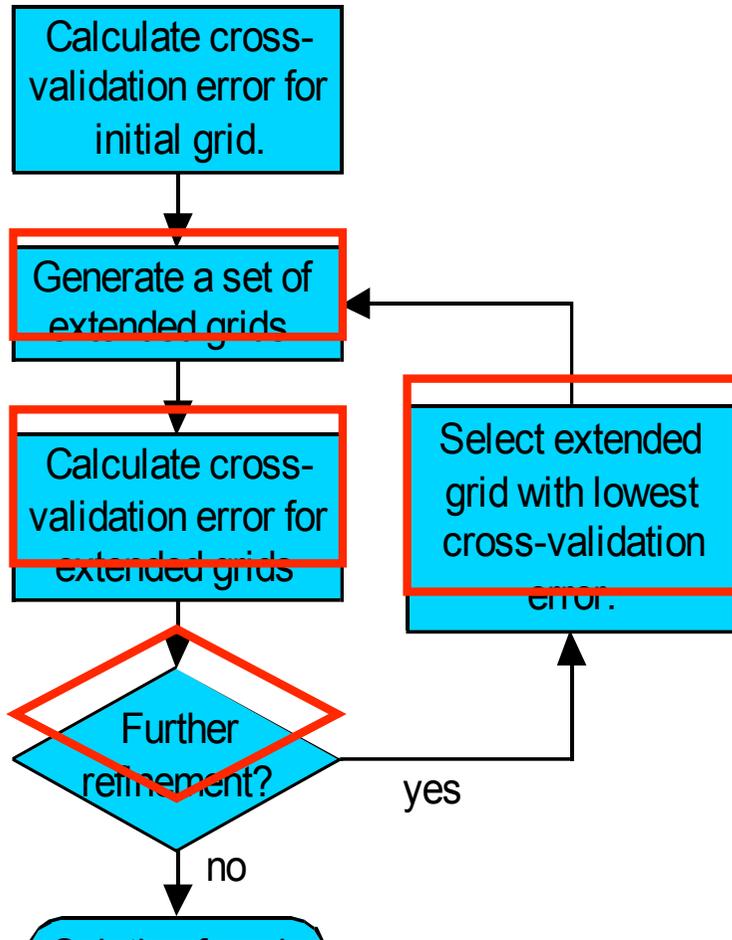
- hierarchical d-dimensional finite-element discretization of unknown function with d-linear hat functions, Yserentant (1992)
- significant reduction of number of parameters by successive approximation on subgrids and subsequent linear combination to a sparse grid approximation



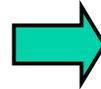
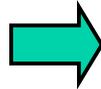
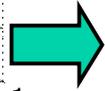
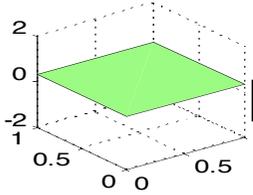
- approximation quality close to full grid approximation







Initial grid



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- number of tested grids scales linearly with number of dimensions instead of exponentially
- curse of dimensionality in selecting the discretization is avoided
- no guarantee to find optimal discretization

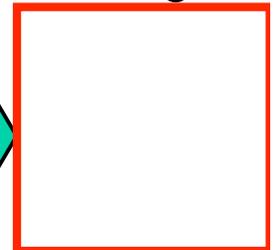
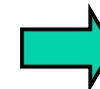
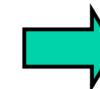
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Final grid

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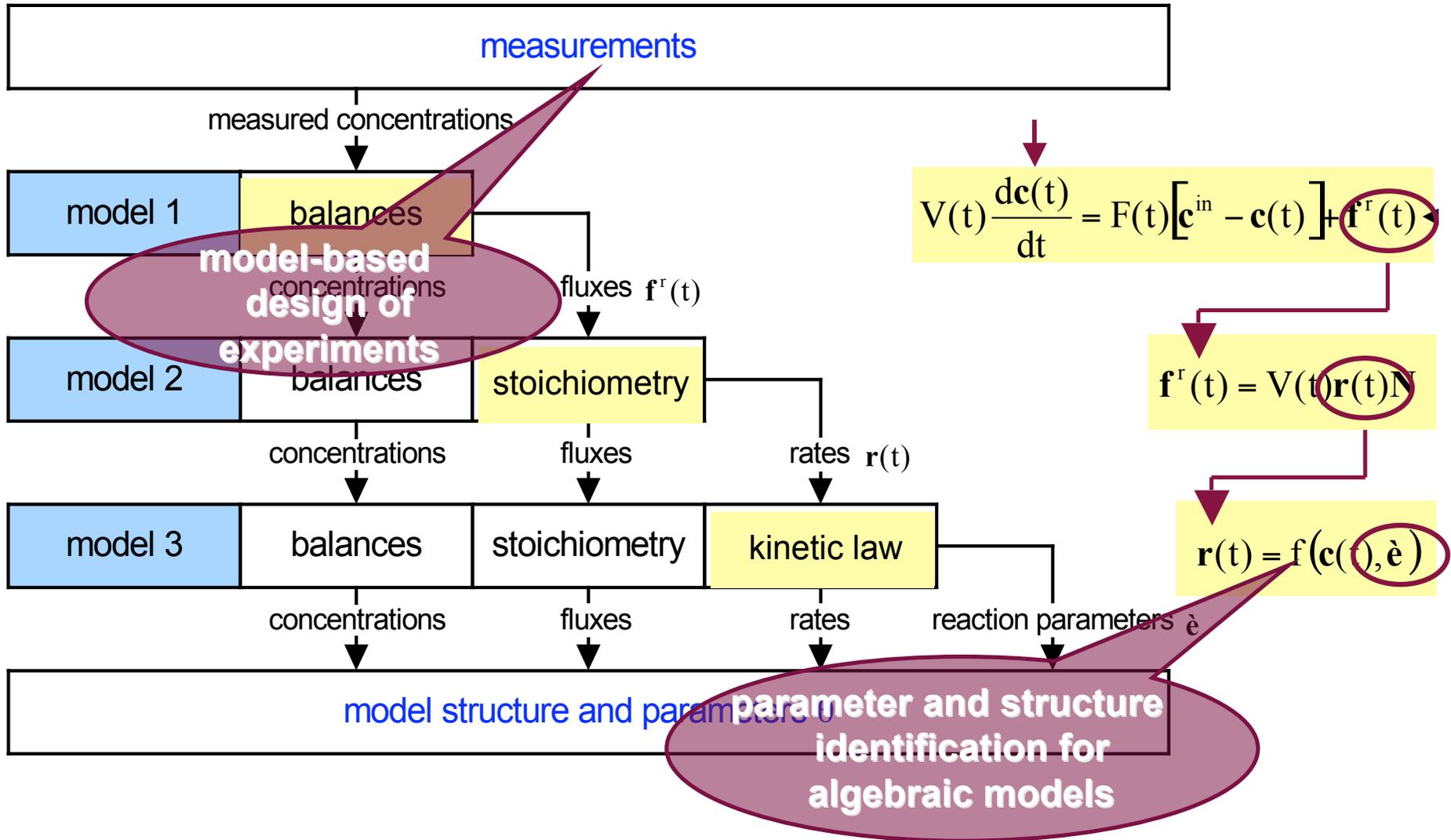
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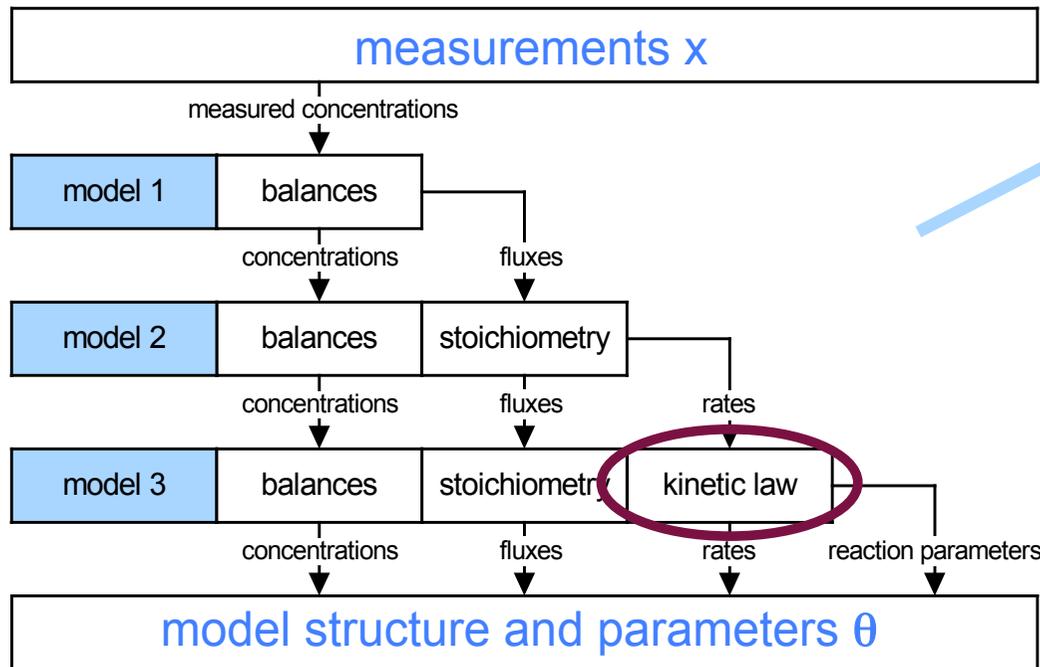
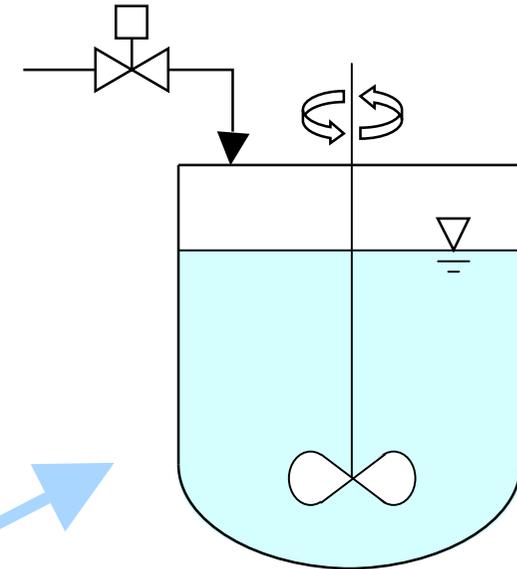
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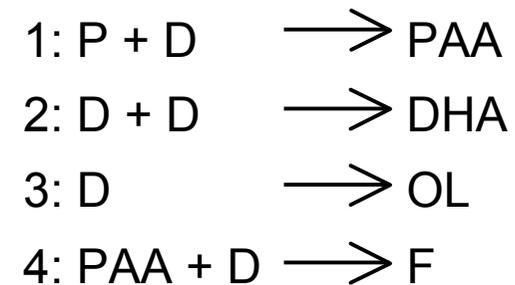
## What are the ingredients for implementation ?



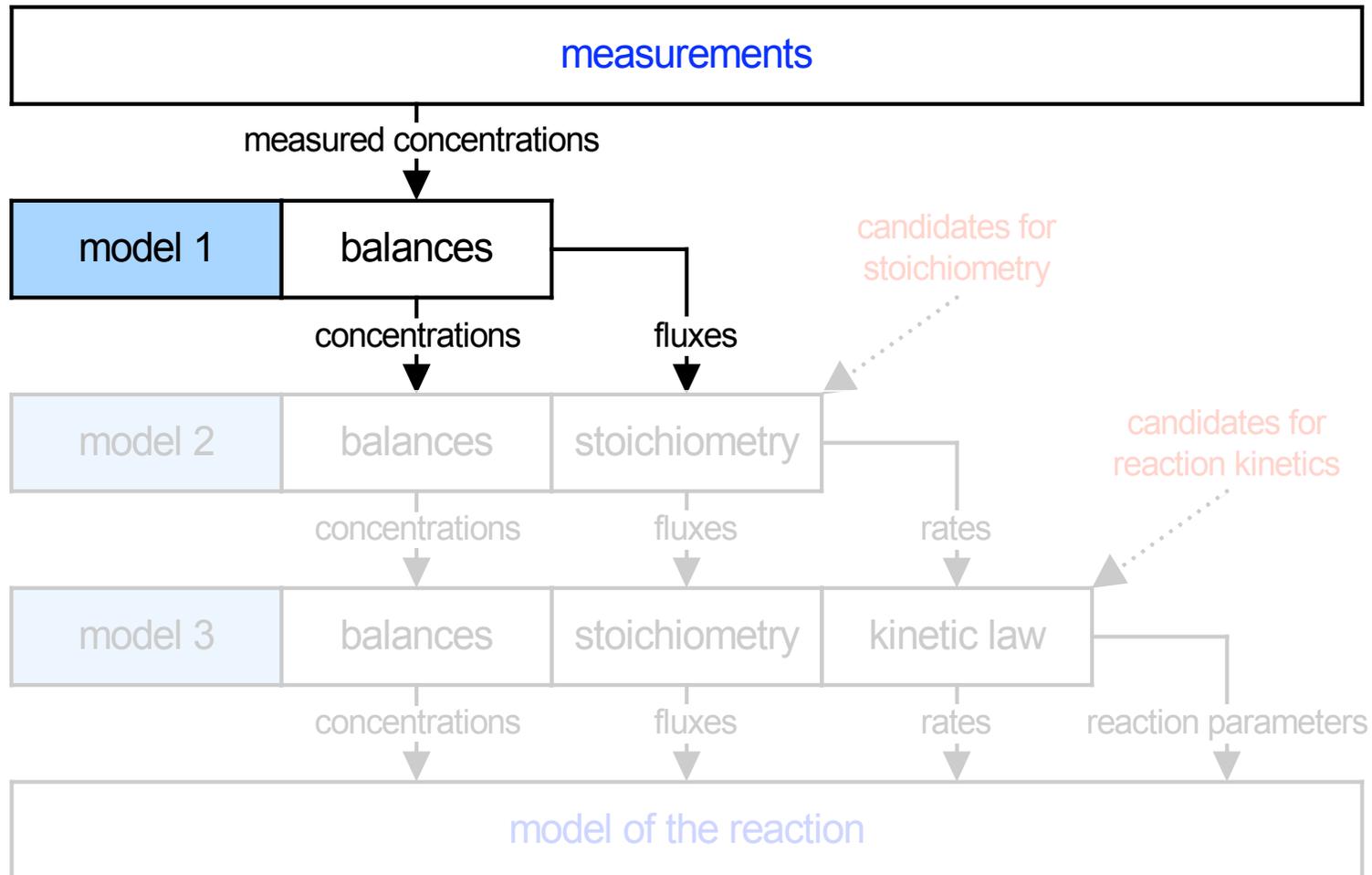
- no. of reactions, stoichiometry and kinetics unknown
- isothermal semi-batch CSTR experiments
- concentration measurements (ex-situ, e.g. GC; in-situ, e.g. Raman/IR spectroscopy)
- a number of simulated semi-batch reactor experiments, 60 min (cases: noise, sampling ...)



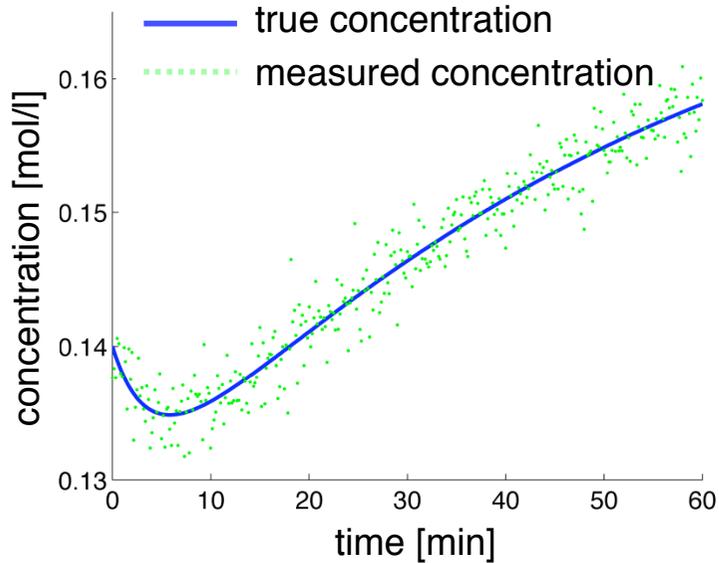
**Acetoacetylation of Pyrrole with Diketene**



(Brendel, Bonvin, Marquardt, 2006)



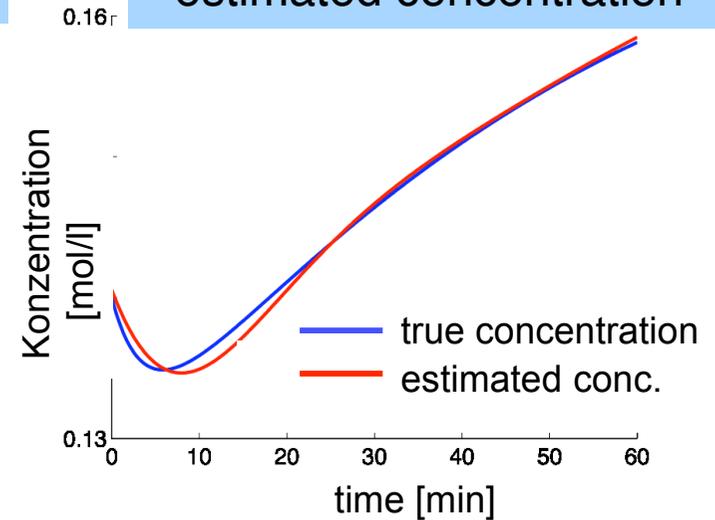
concentration measurements



regularisation



estimated concentration



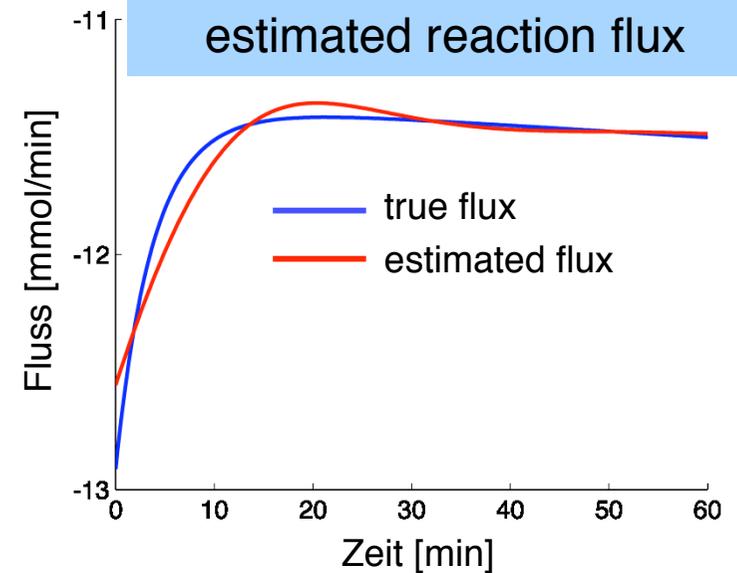
inverse problem:

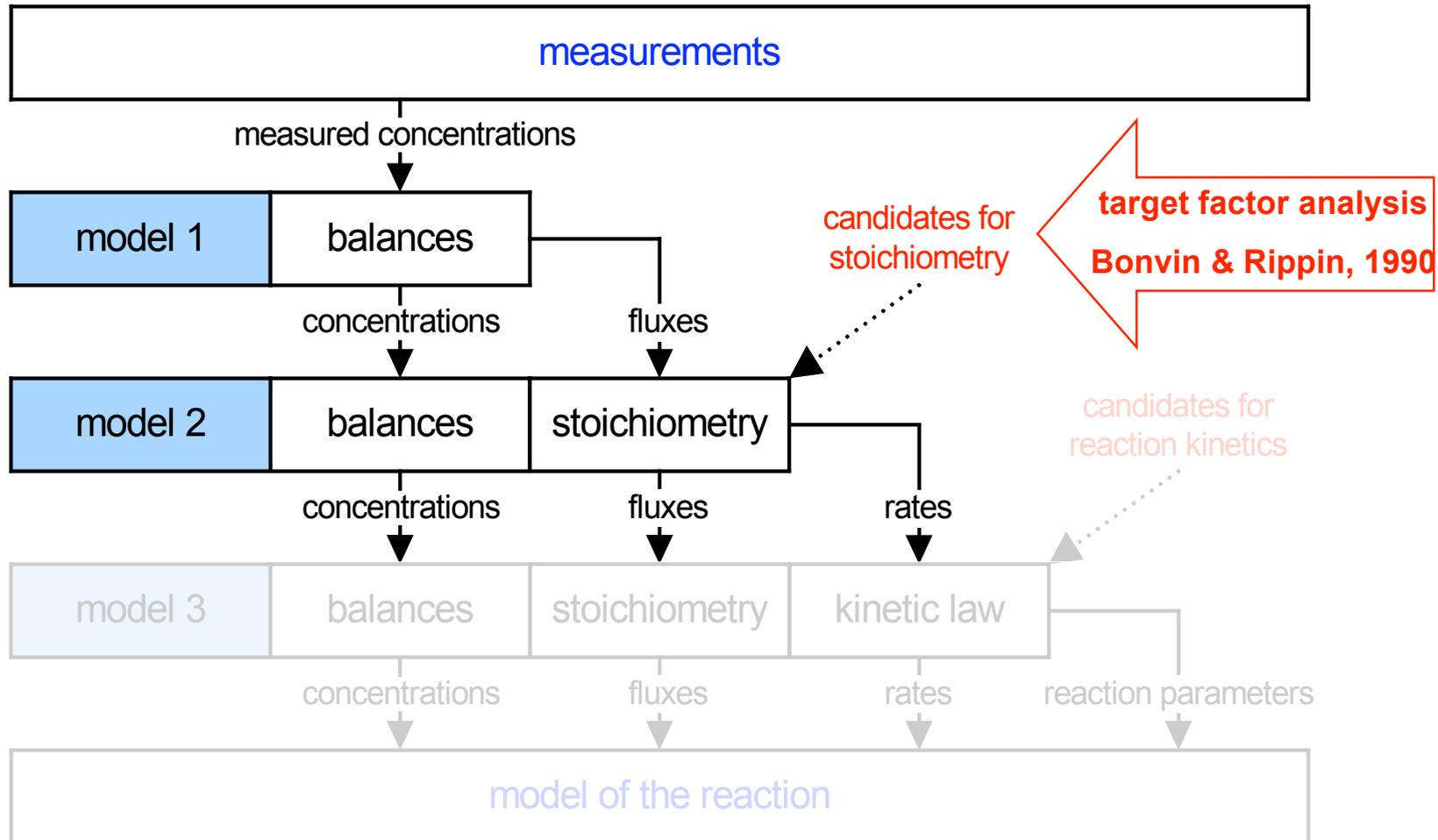
$$f_i^r(t) = V(t) \frac{dc_i(t)}{dt} - F[c_i^{in} - c_i(t)]$$

mass balances (semi-batch)



estimated reaction flux





Propose an  $R \times S$  dimensional stoichiometric matrix  $\mathbf{N}_p$  with

$R$  = number of reactions

$S$  = number of involved species

Set up a  $B \times S$  dimensional data matrix  $\mathbf{D}$

$B$  = Number of observations

$\mathbf{D}$  = can be expressed as  $\mathbf{X} * \mathbf{N}$ , where  $\mathbf{X}$  is a matrix indicating the extents of the  $R$  reactions and  $\mathbf{N}$  is the correct stoichiometric matrix

$$\begin{aligned} \text{Singular Value Decomposition} \rightarrow \mathbf{D} &= \underbrace{\mathbf{U} * \mathbf{S}} * \underbrace{\mathbf{V}^T} \\ &= \mathbf{X}_a * \mathbf{N}_a, \end{aligned}$$

where the rows of  $\mathbf{N}_a$  represent a basis for the observed stoichiometric space

Discard those stoichiometries of  $\mathbf{N}_p$  that cannot be approximately described as linear combination of the rows of  $\mathbf{N}_a$

## Stoichiometry

$$\mathbf{f}_D^r = V(-r_1 - 2r_2 - r_3 - r_4)$$

$$\mathbf{f}_P^r = V(-r_1)$$

$$\mathbf{f}_{PAA}^r = V(r_1 - r_4)$$

$$\mathbf{f}_{DHA}^r = V(r_2)$$

$$\mathbf{f}^r = V\mathbf{r}\mathbf{N}$$

## Determination of reaction rates

- BLS problem

$$\mathbf{r} = \arg \min \left\| \mathbf{N}\mathbf{r} - \frac{1}{V}\mathbf{f}^r \right\|$$

$$\text{s.t. } \mathbf{r}_l \leq \mathbf{r} \leq \mathbf{r}_u$$

- $\mathbf{f}^r$ : reaction fluxes
- $\mathbf{r}$ : reaction rates
- $\mathbf{N}$ : stoichiometric matrix
- $V$ : volume

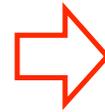
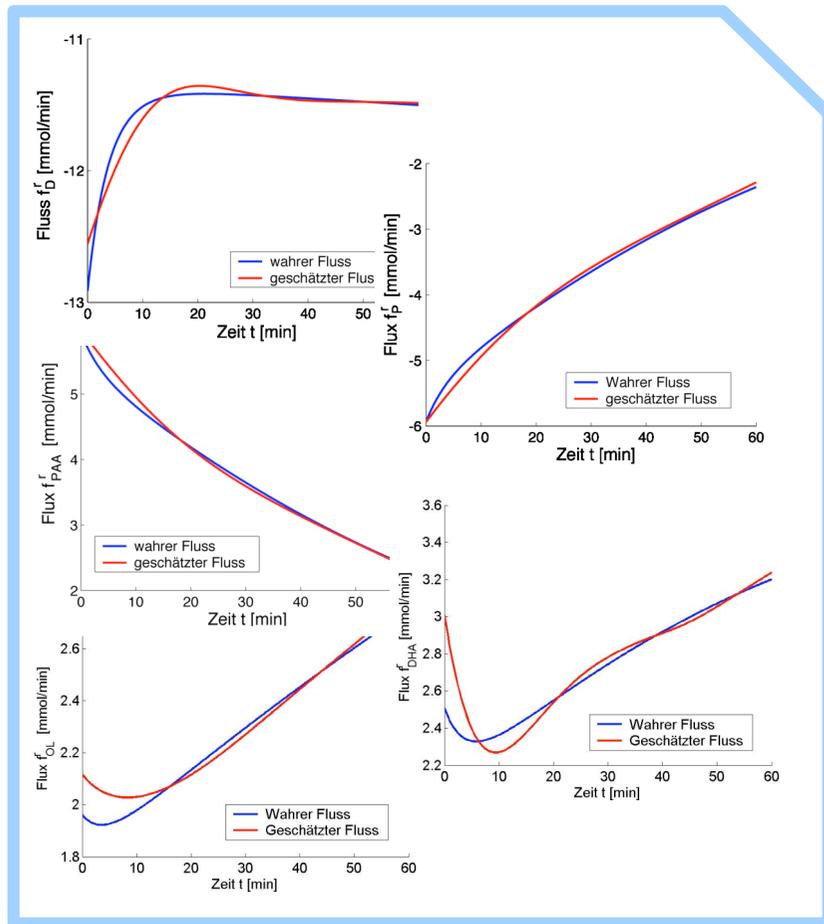
estimated  
reaction fluxes



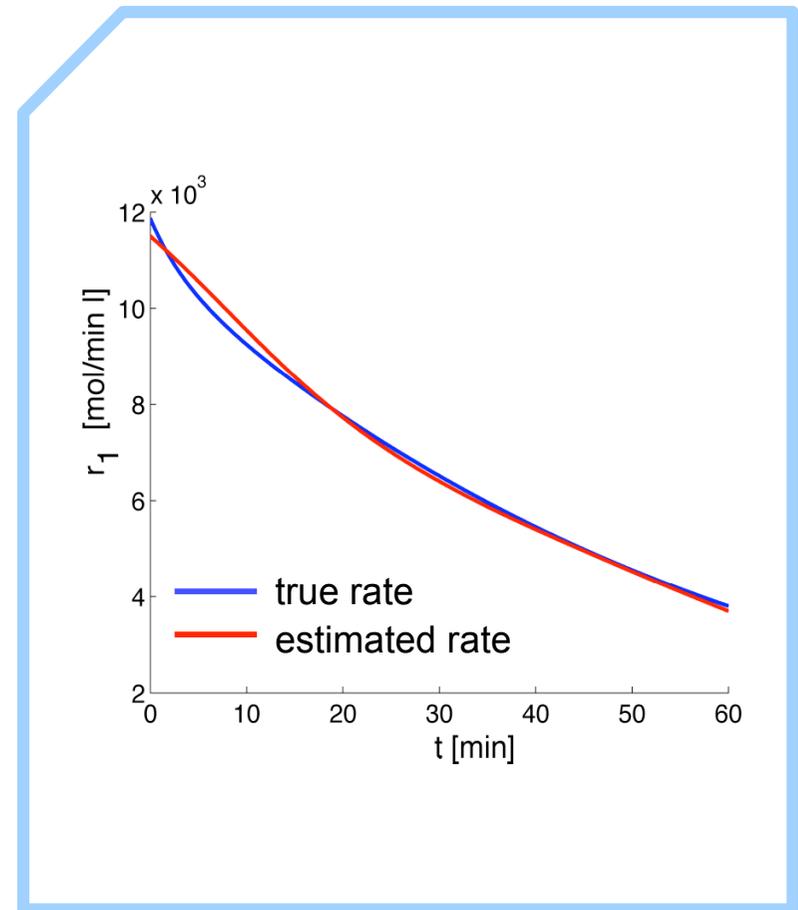
stoichiometry

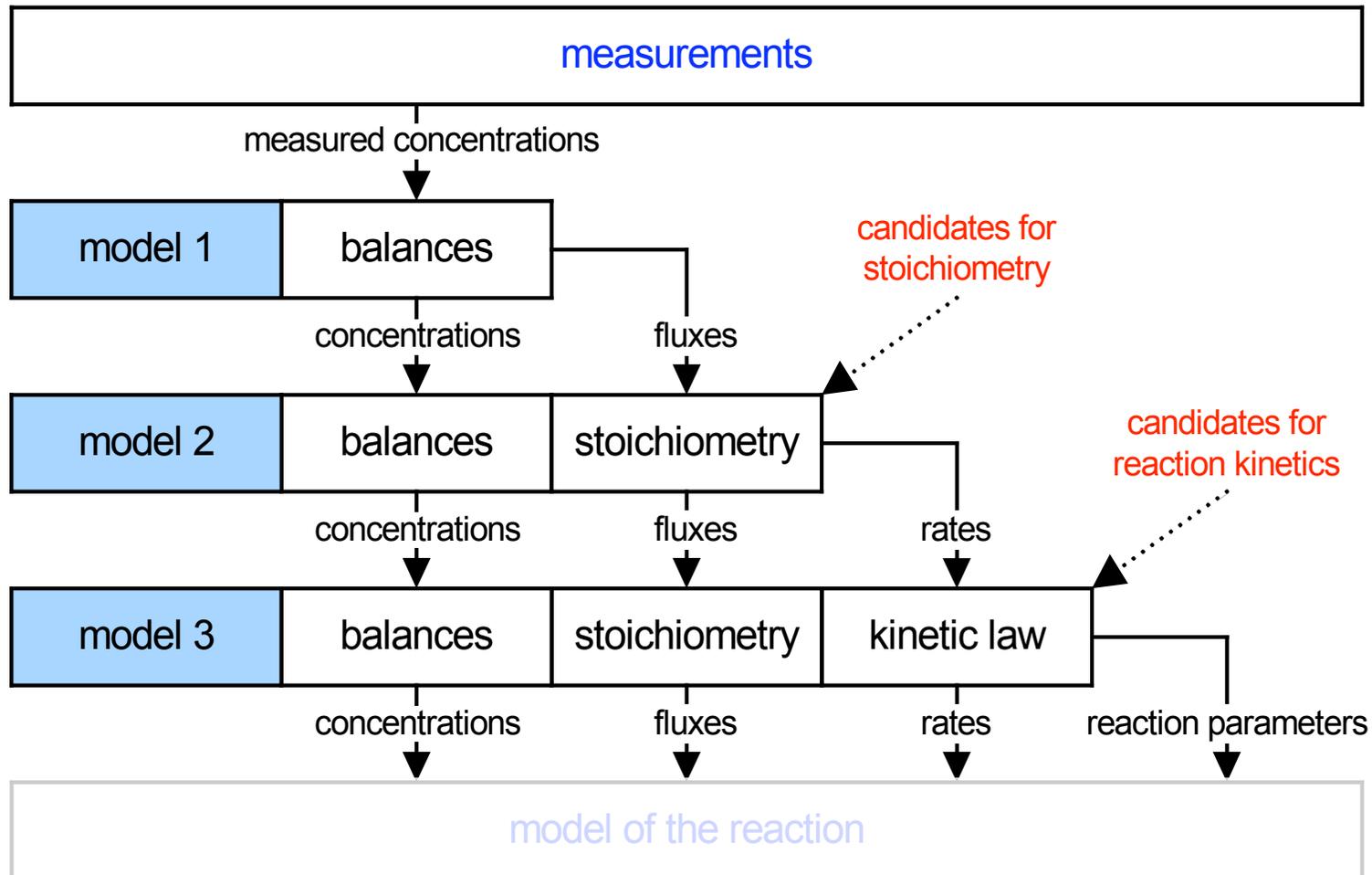


estimated  
reaction rates



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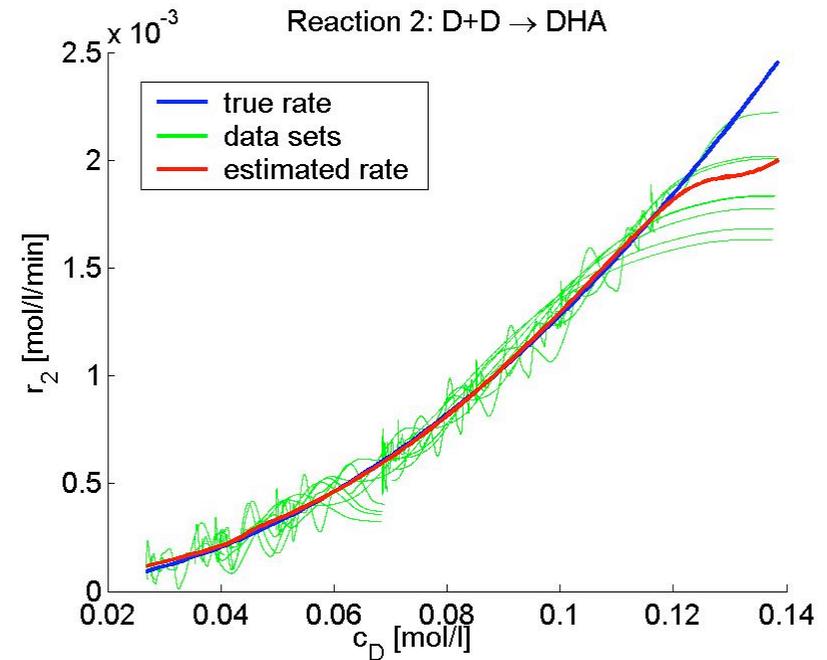
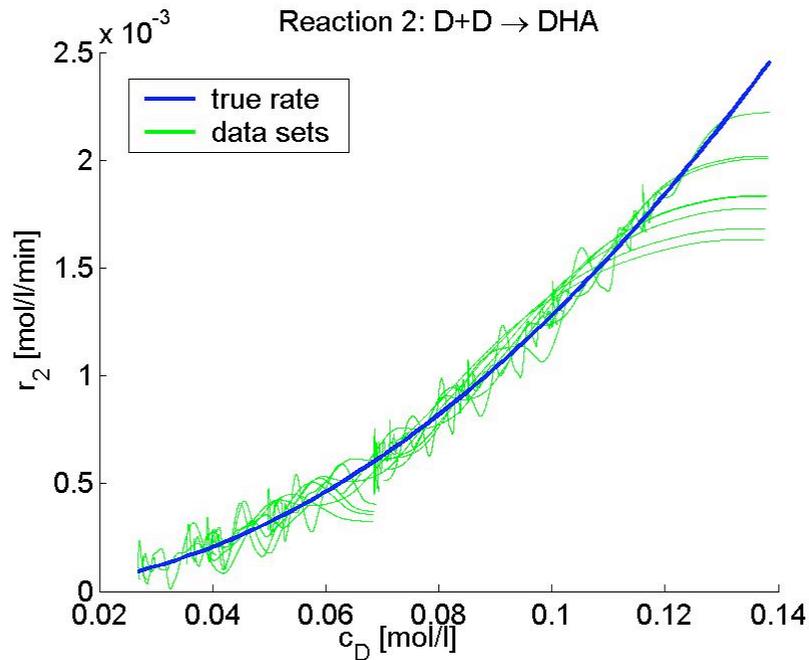
correlation of concentrations and rates

$$\begin{array}{l} \hat{c}(t) \longrightarrow \\ \hat{r}(t) \longrightarrow \end{array} \hat{r}(c)$$

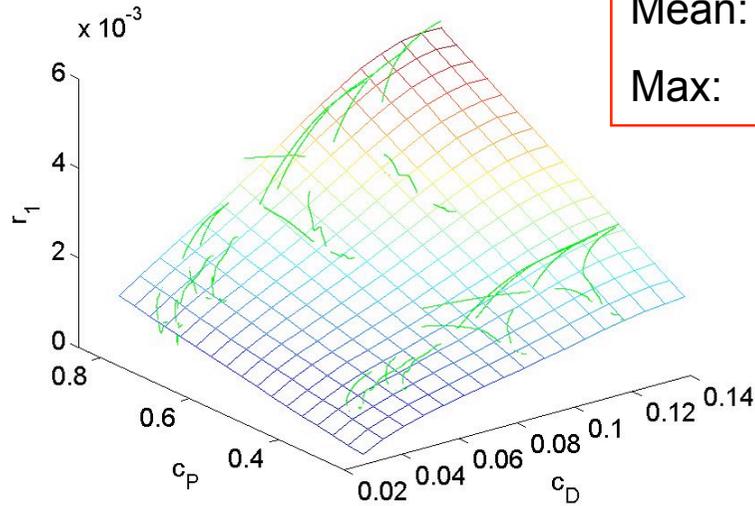
- band of noisy data sets
- physical insight



- correlation with data-driven methods
- NN with Bayesian regularization

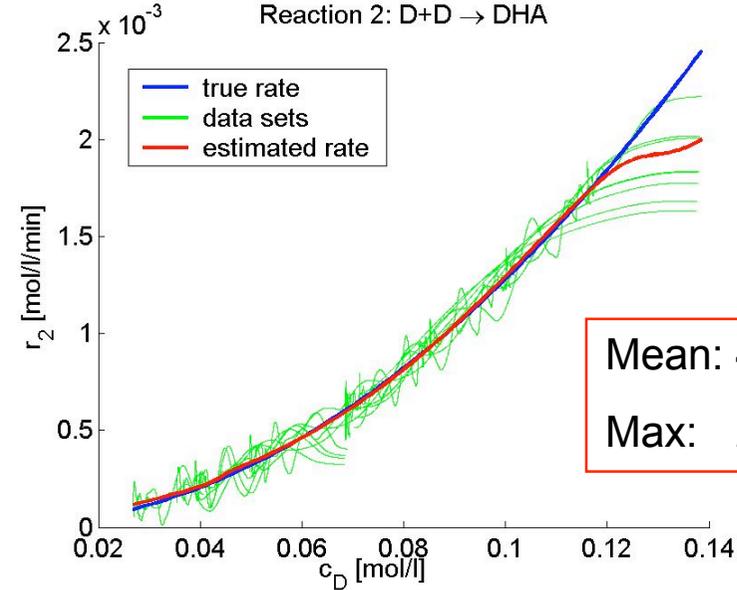


Reaction 1: P+D → PAA



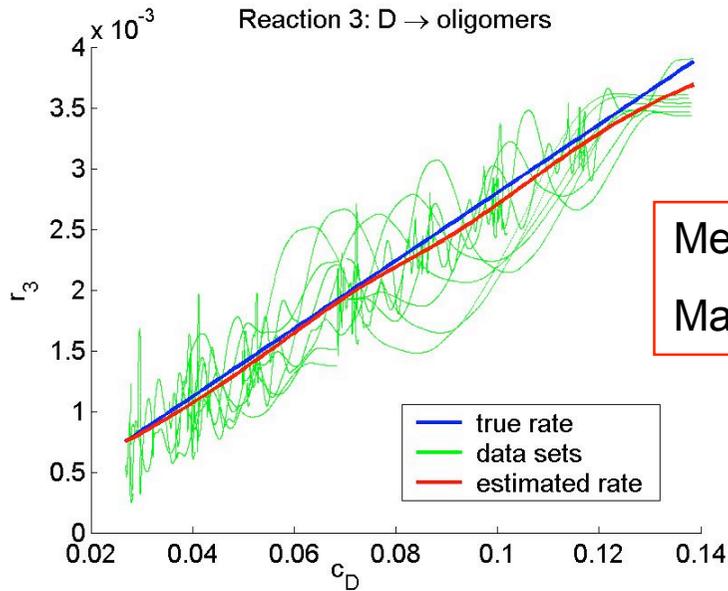
Mean: 2.7%  
Max: 6%

Reaction 2: D+D → DHA



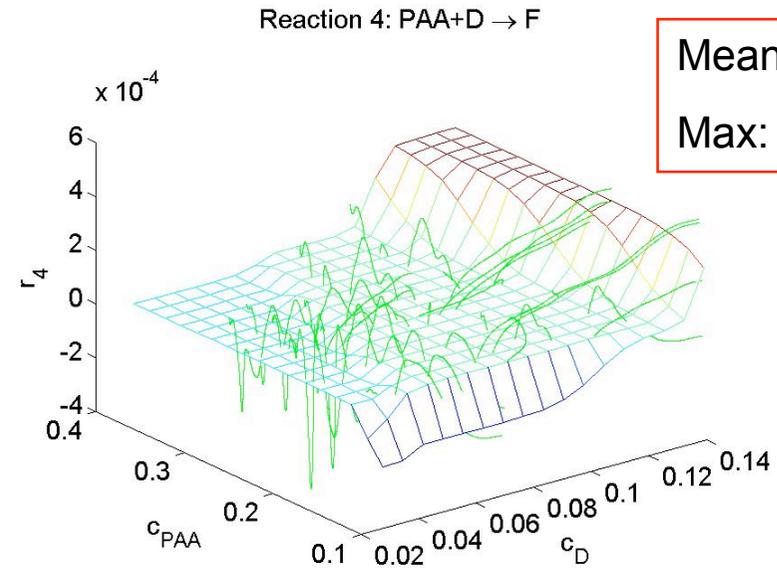
Mean: 4.2%  
Max: 25%

Reaction 3: D → oligomers



Mean: 2.9%  
Max: 5%

Reaction 4: PAA+D → F



Mean: 167%  
Max: 2958%

regression of estimated rates and concentrations for each individual reaction

$$\begin{matrix} r(t) & \longrightarrow & \\ c(t) & \longrightarrow & \end{matrix} r(k, c)$$

candidate kinetic laws

$$r_1^{(1)} = k_1^{(1)}$$

$$r_1^{(2)} = k_1^{(2)} c_D$$

$$r_1^{(3)} = k_1^{(3)} c_P$$

$$r_1^{(4)} = k_1^{(4)} c_K$$

$$r_1^{(5)} = k_1^{(5)} c_D c_P$$

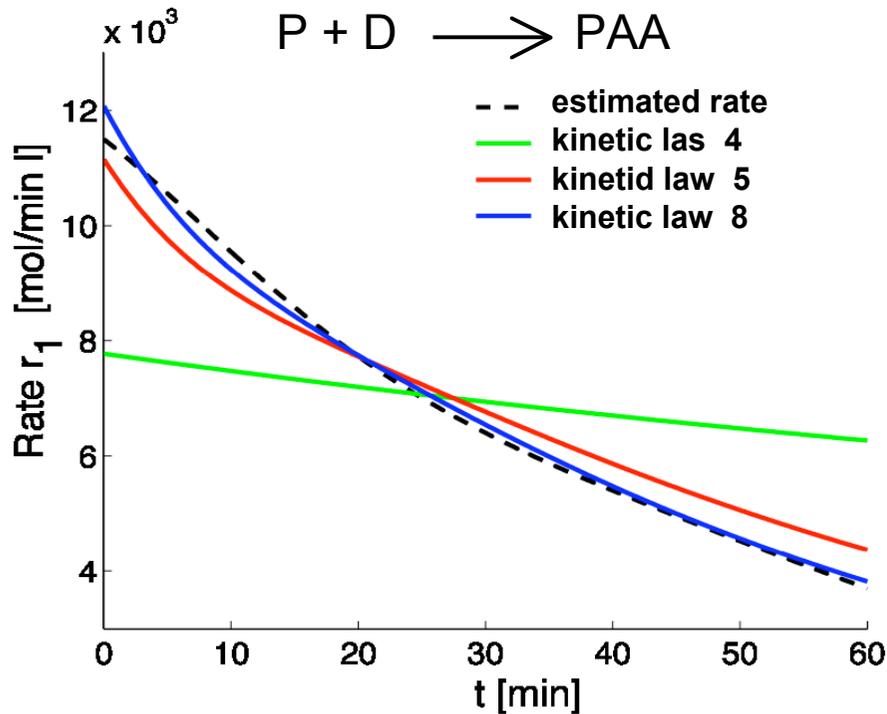
$$r_1^{(6)} = k_1^{(6)} c_P c_K$$

$$r_1^{(7)} = k_1^{(7)} c_D c_K$$

$$r_1^{(8)} = k_1^{(8)} c_D c_P c_K$$

$$r_1^{(9)} = k_1^{(9)} c_D c_P^2$$

$$r_1^{(10)} = k_1^{(10)} c_D^2 c_P$$

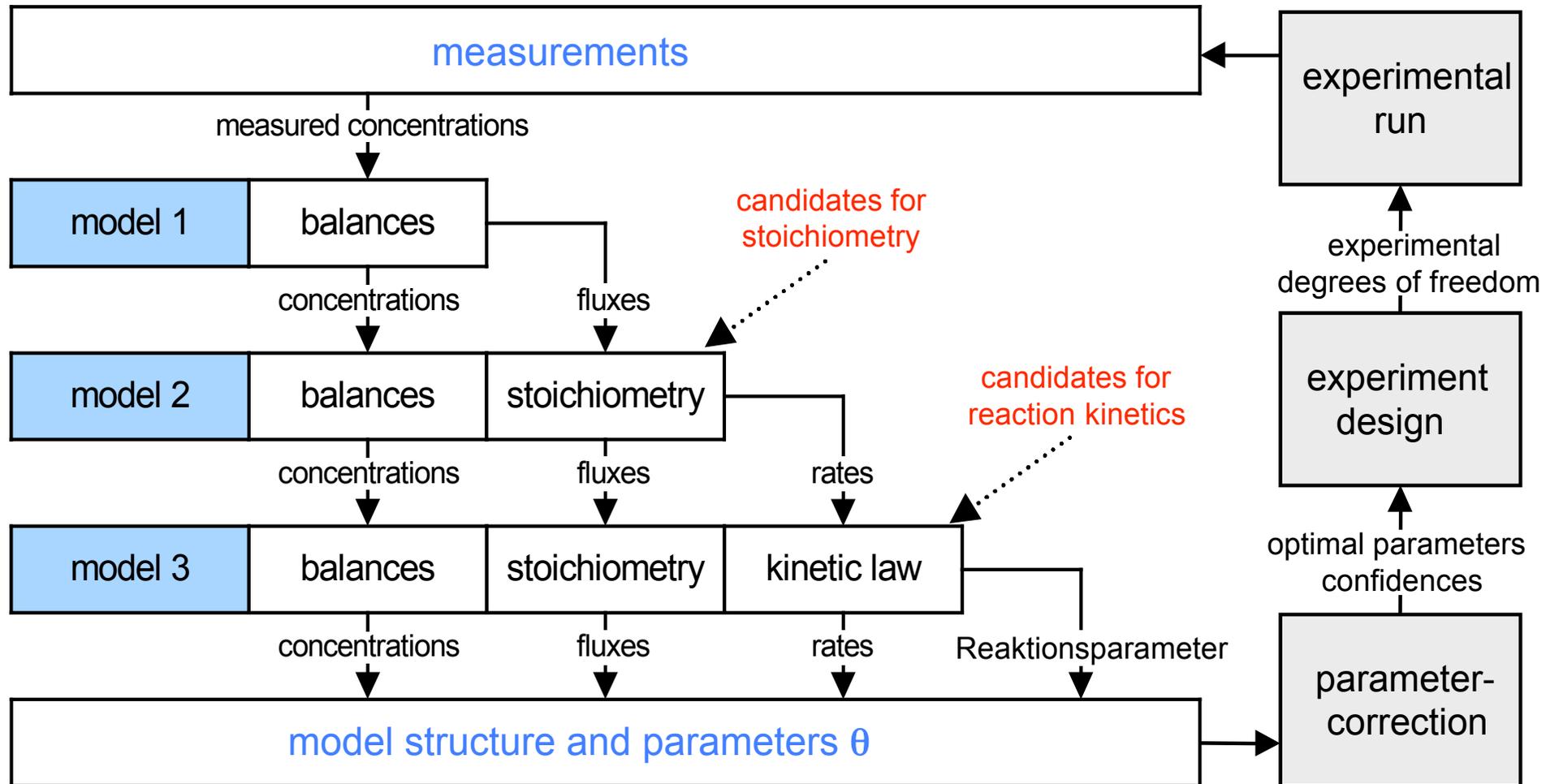


?

simultaneous identification for promising model candidates



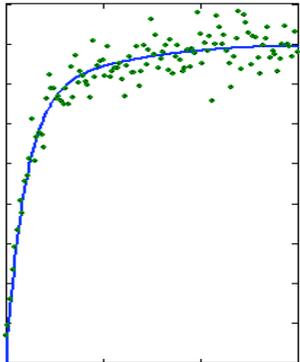
best model structure with statistically optimal parameters



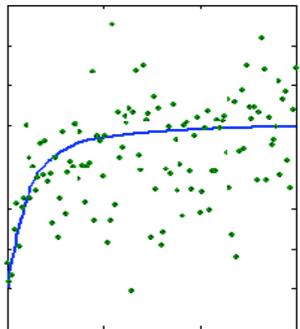
scenario: 3600 data points in each experiment ( $T_s=1$  s), 5% noise

	stoichiometry	kinetic law	validation	parameters
1	1: P + D $\longrightarrow$ PAA 2: D + D $\longrightarrow$ DHA 3: D $\longrightarrow$ OL	$r_1 = k_1 c_D c_P c_K$ $r_2 = k_2 c_D$ $r_3 = k_3$	—	N/A
experimental design for model discrimination				
2	1: P + D $\longrightarrow$ PAA 2: D + D $\longrightarrow$ DHA 3: D $\longrightarrow$ OL	$r_1 = k_1 c_D c_P c_K$ $r_2 = k_2 c_D^2 c_K$ $r_3 = k_3 c_D$	✓	Parametersatz $k_1, k_2, k_3$
experimental design for best model parameters				
3	1: P + D $\longrightarrow$ PAA 2: D + D $\longrightarrow$ DHA 3: D $\longrightarrow$ OL	$r_1 = k_1 c_D c_P c_K$ $r_2 = k_2 c_D^2 c_K$ $r_3 = k_3 c_D$	✓	Verbesserter Parametersatz $k_1, k_2, k_3$
requested accuracy achieved				

STDV = 2%



STDV = 10%



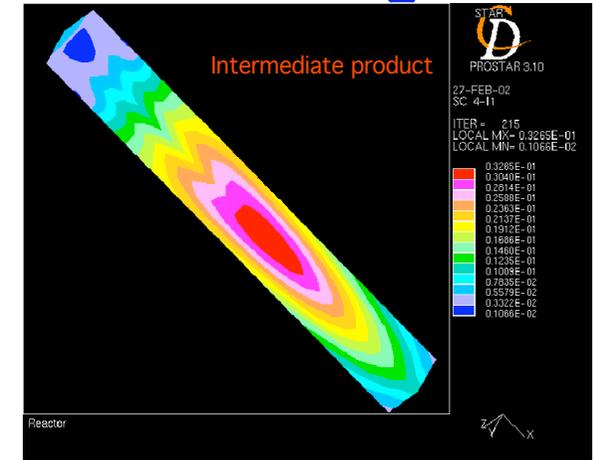
60 min

Incremental Method				Simultaneous Method			
Resolution	# Models	CPU Time	Ident. Corr.	Resolution	# Models	CPU Time	Ident. Corr.
3 sec	1	30 min	100 %	3 sec	3600	1.7 d	100 %
30 sec	≈ 1	15 sec	100 %	30 sec	3600	3.8 h	100 %
5 min	≈ 7	6 sec	100 %	5 min	3600	1.7 h	100 %
3 sec	≈ 100	40 min	100 %	3 sec	3600	1.6 d	100 %
30 sec	≈ 160	3 min	50 %	30 sec	3600	3.9 h	50 %
5 min	≈ 200	2 min	10 %	5 min	3600	1.1 h	10 %

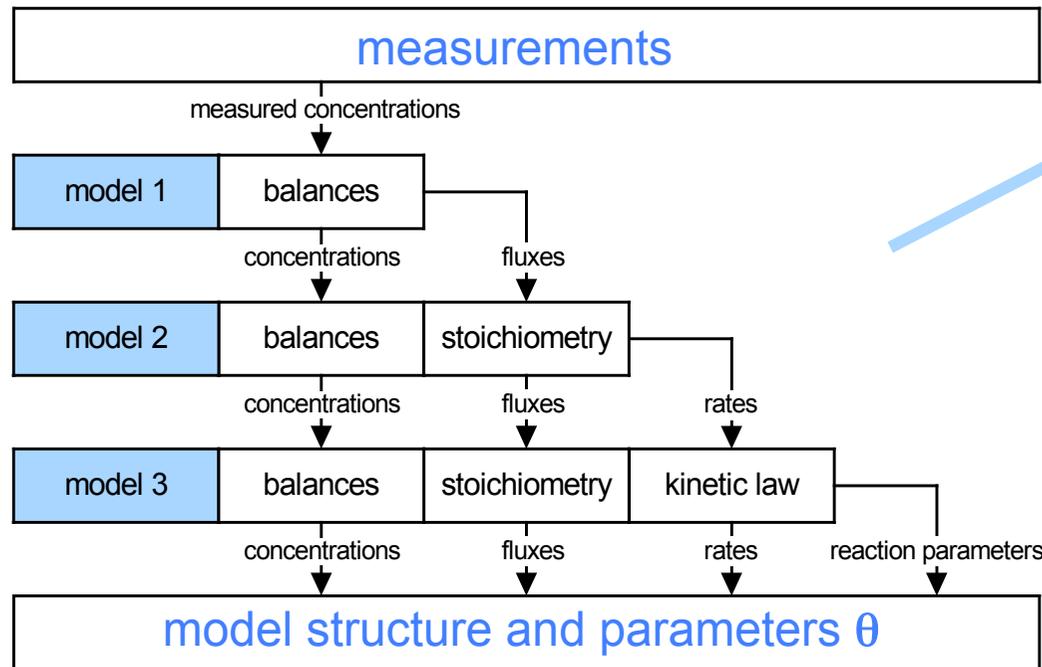
much faster identification

### Why studying diffusion ?

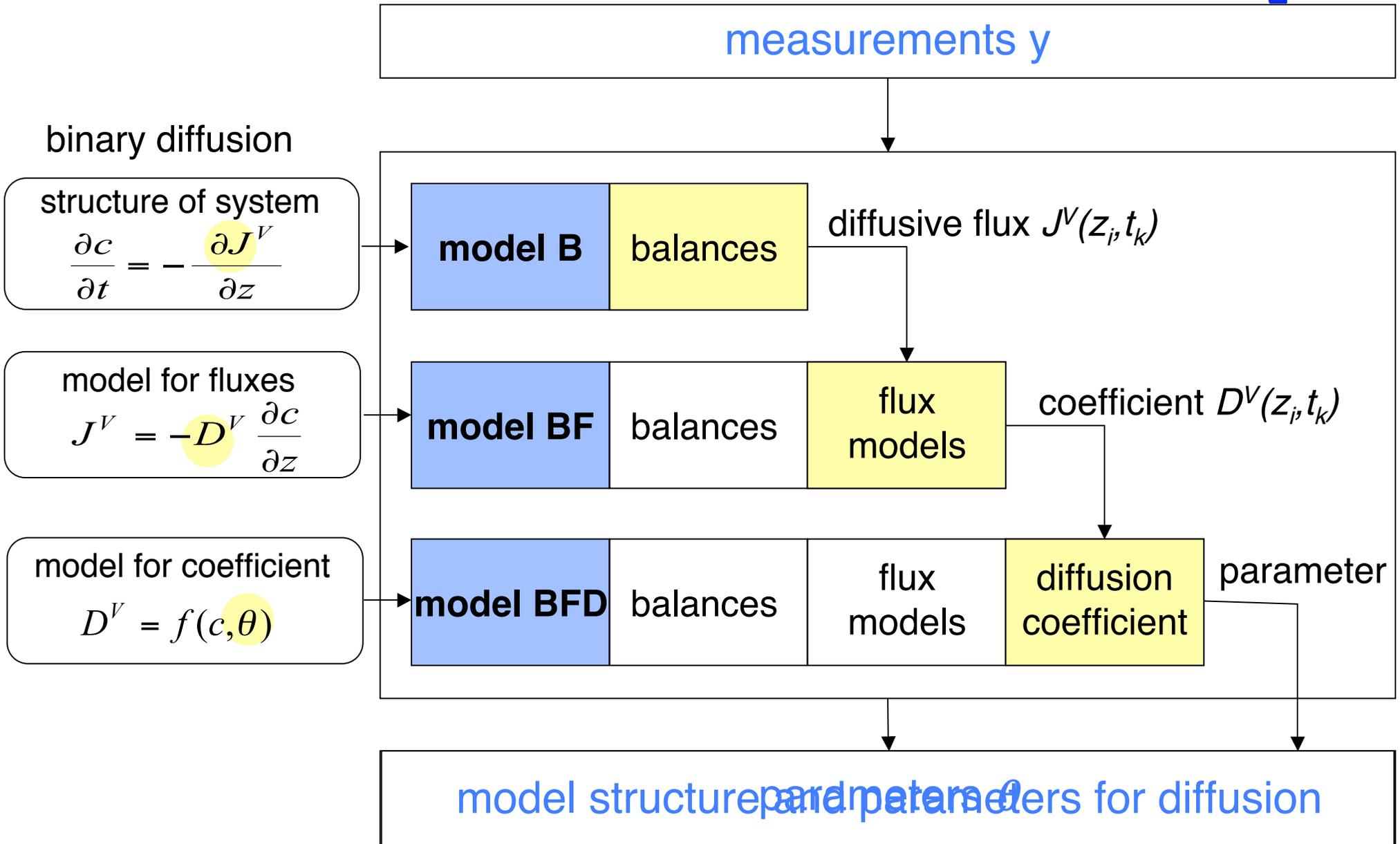
- detrimental for product and process design
- very high experimental effort
- very few multi-component diffusion data available
- validity of diffusion models still a matter of debate

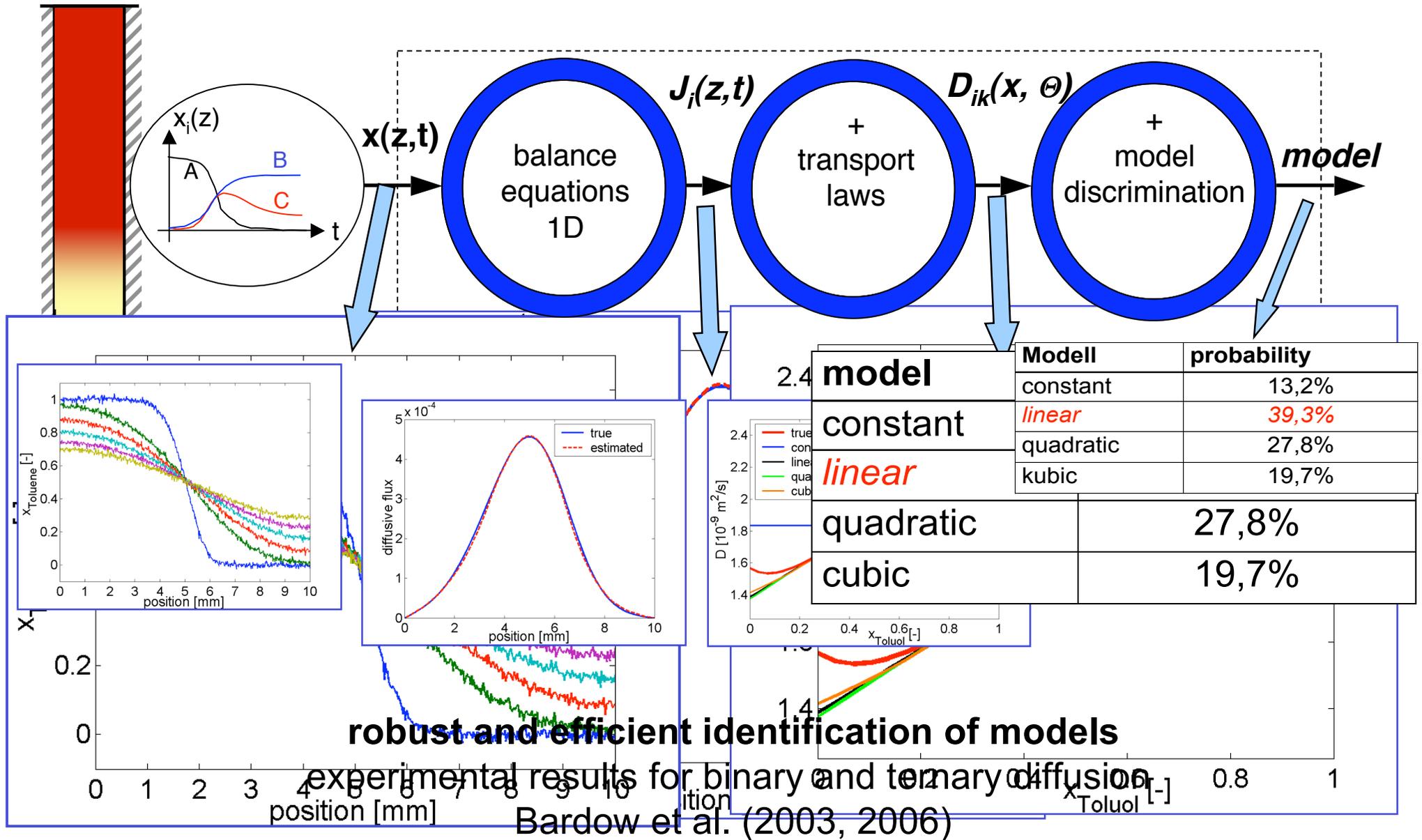


selectivity of heterogeneously catalyzed reactions (Pantelides & Urban, 2004)



- a good model problem for the development of MEXA methodology





<b>model B</b>	balances
----------------	----------

$$\frac{\partial c_i}{\partial t} = -\frac{\partial J_i^V}{\partial z}; i = 1,2$$

→ problem decouples  
 → solve binary problem twice

<b>model BF</b>	balances	flux models
-----------------	----------	-------------

$$J_1^V = -D_{11}^V \frac{\partial c_1}{\partial z} - D_{12}^V \frac{\partial c_2}{\partial z}$$

→ coefficients not identifiable

→ error-in-variables regression

<b>model BFD</b>	balances	flux models	diffusion coefficient
------------------	----------	-------------	-----------------------

Example: simulated experiment as in Arnold & Toor (1967), noise level  $\sigma=0.01$

Coefficient	$D_{11}$	$D_{12}$	$D_{21}$	$D_{22}$
Error [%]	-9.0	-33.4	-10.8	-10.4

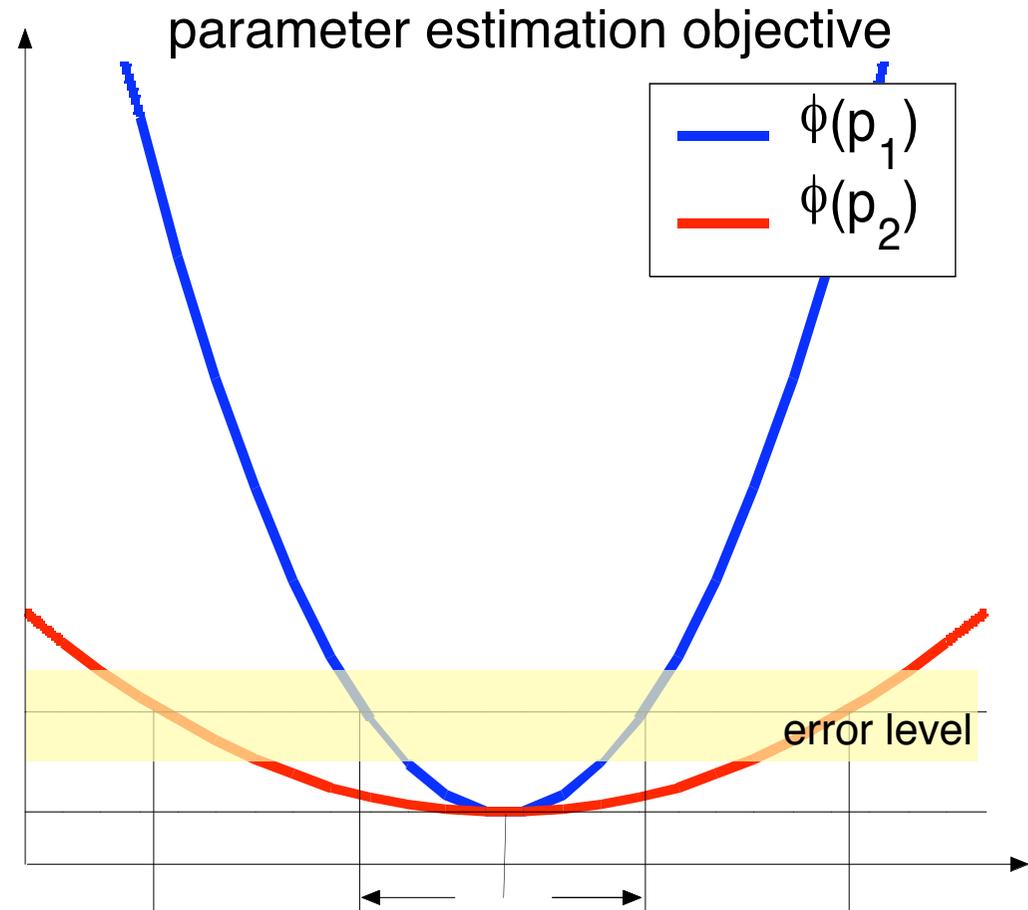
estimation of constant diffusion coefficients from a single experiment with good precision

set free variables  $p$  such that information on model parameters is maximized

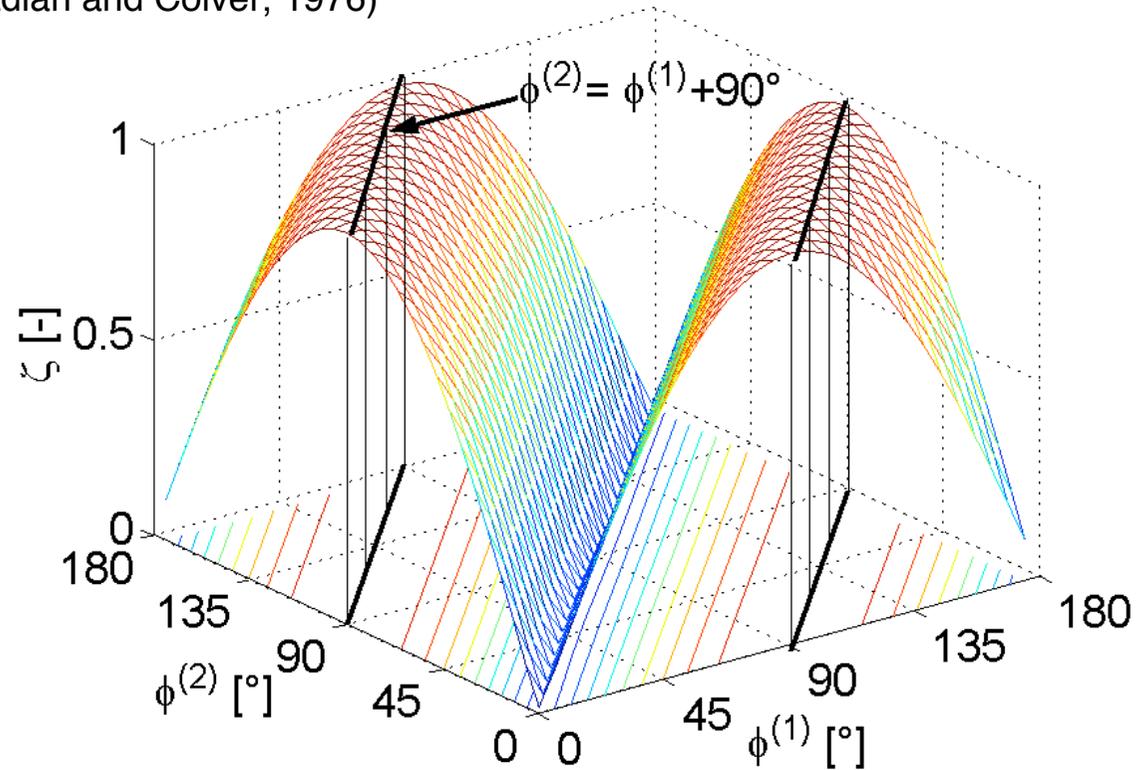
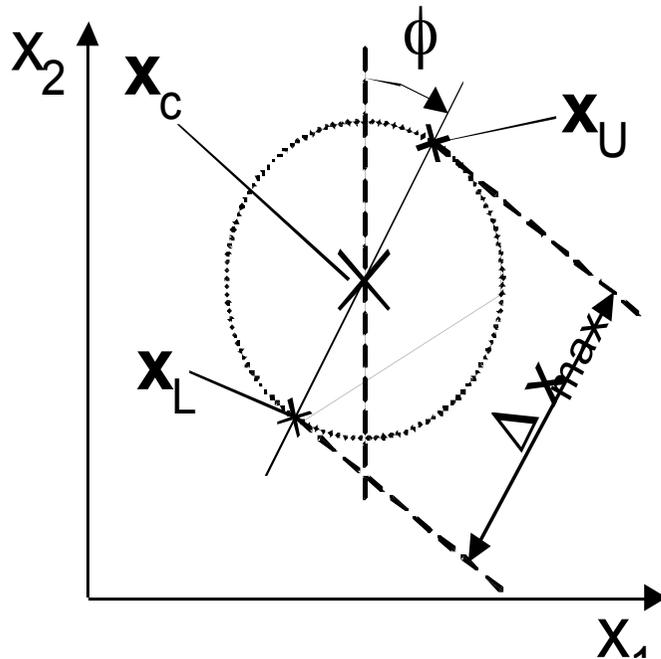
=

maximize curvature of parameter estimation objective

$$\max_p \frac{\partial^2 \phi}{\partial \theta^2}$$

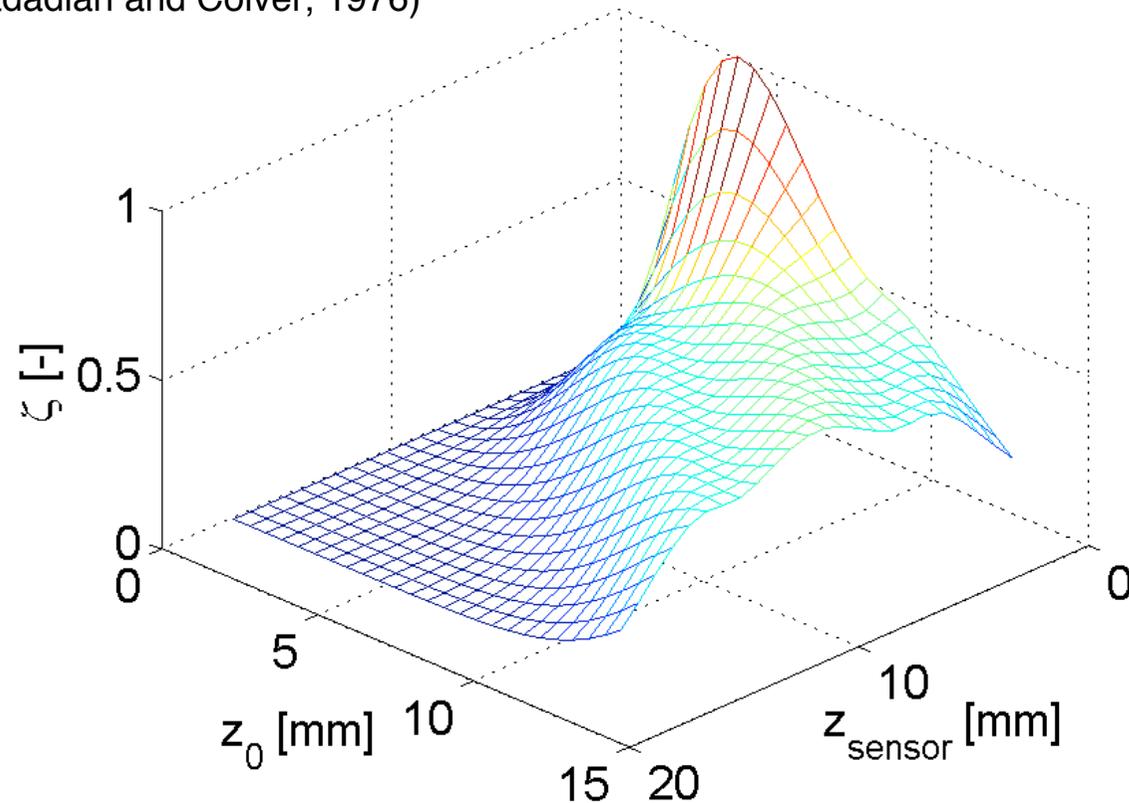
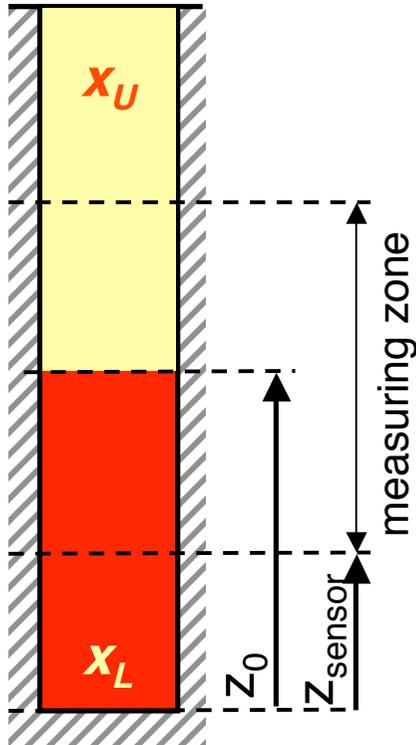


Example: acetone-benzene-methanol at  $\mathbf{x}_c = [0.35; 0.302; 0.348]^T$ ,  
(Alimadadian and Colver, 1976)



- scaled objective  $\zeta$ -efficiency measures information per parameter
- one Raman experiment suffices to determine ternary Fick matrix
- two different experiments result in substantial improvement
- experiments should be as distinct as possible ( $\phi^{(2)} = \phi^{(1)} + 90^\circ$ )

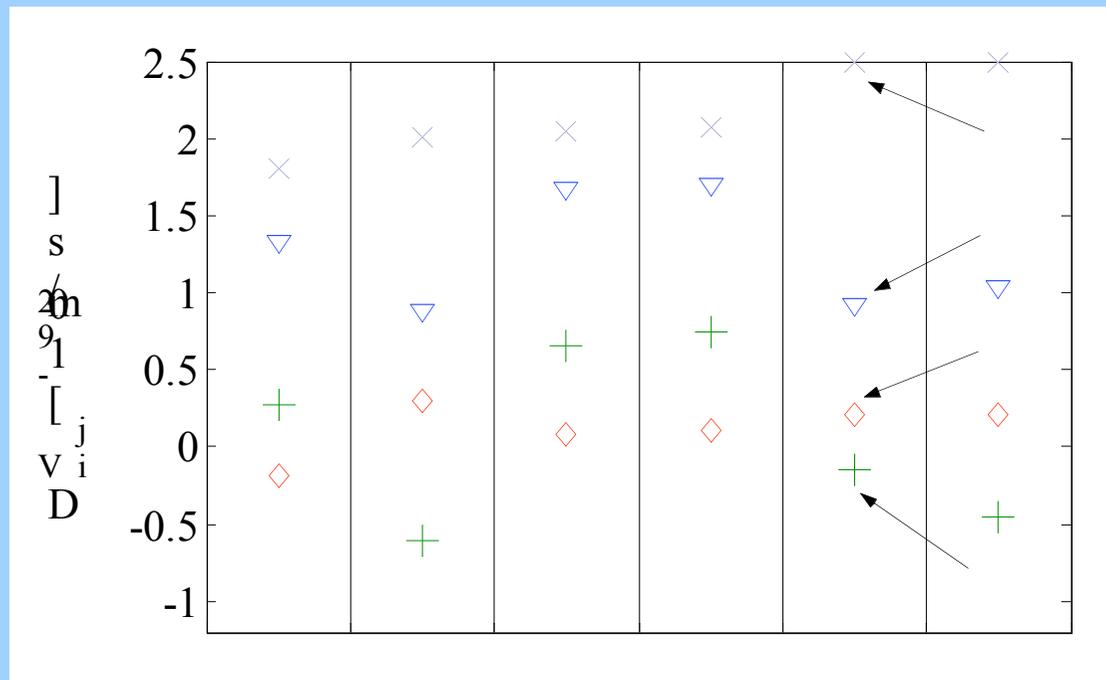
Example: acetone-benzene-methanol at  $\mathbf{x}_c = [0.35; 0.302; 0.348]^T$ ,  
(Alimadadian and Colver, 1976)



- measurements at the wall, i.e. restricted diffusion experiments
- unequal volume of both phases, almost independent of concrete mixture
- short experiments are beneficial

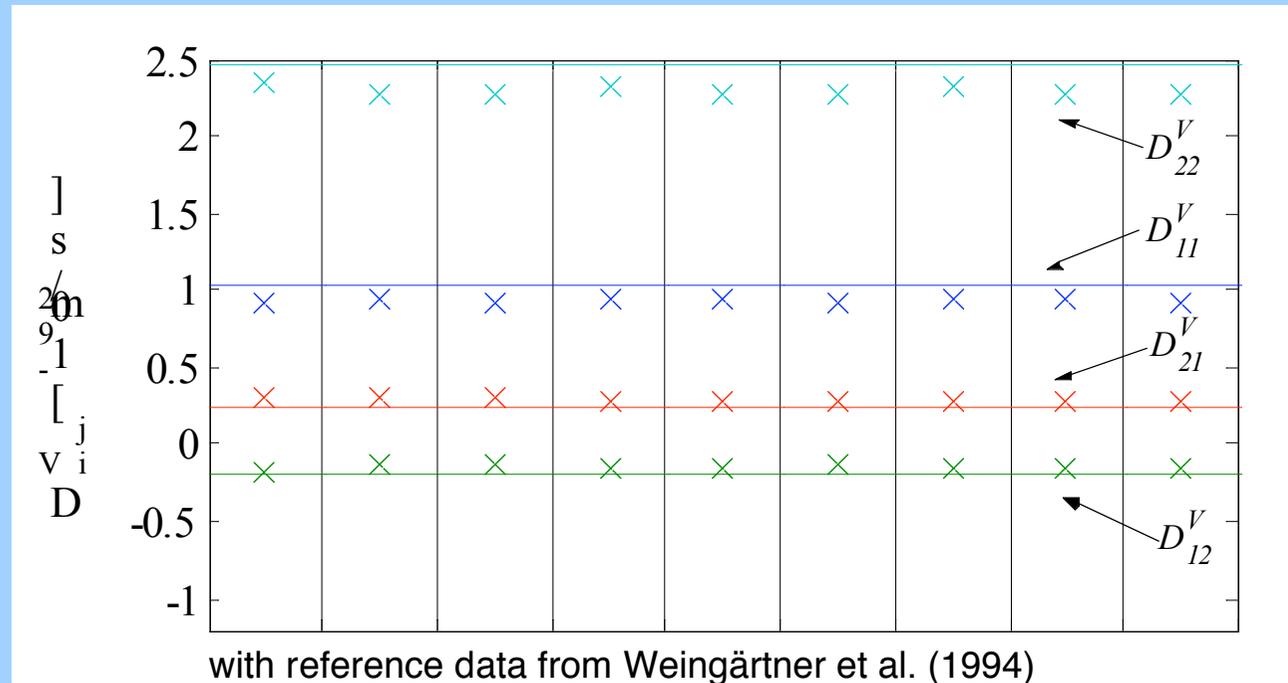
**System:** 1-Propanol - 1-Chlorobutane - n-Heptane

Diffusivities from a single Raman experiment



- one Raman experiment gives full diffusion matrix
- currently scatter in data is still significant

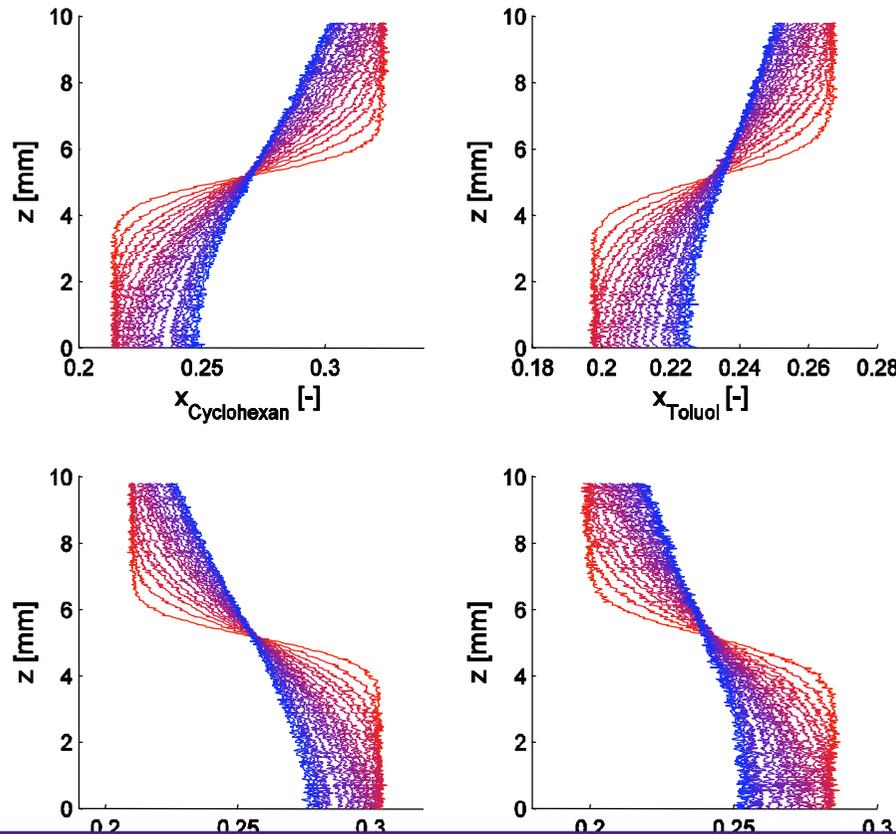
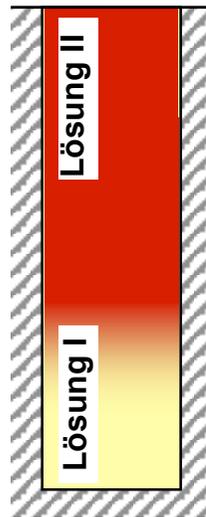
### Diffusion coefficients from optimized Raman experiments



- one Raman experiment gives full diffusion matrix
- good precision from 2 optimized runs
- robust & efficient measurement
- quantitative validation of design predictions

concentration measurements with Raman spectroscopy and model-based design and evaluation of diffusion experiments

(cyclohexane – toluene – dioxane – chlorobutane)



matrix of Fick's diffusion coefficients

$$D_{ij} \left[ 10^{-9} \frac{m^2}{s} \right]$$

1,708	0,086	0,244
0,019	2,066	0,028
0,296	0,111	1,74

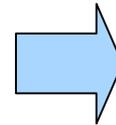
thermodynamical factor – accuracy improvements – electrolytes

high resolution  
measurements

model-based  
methods

### reduces experimental effort

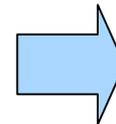
- ✓ fewer experiments
- ✓ maximum precision
- ✓ simple design and preparation



- experimentally validated for binary, ternary, quaternary and quaternary mixtures
- concentration dependence of diffusion coefficients

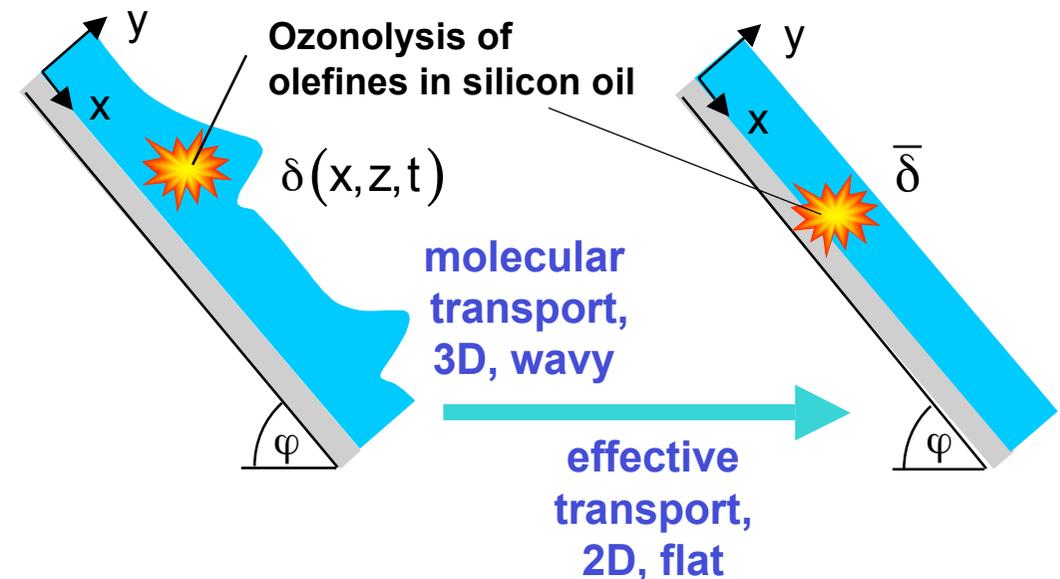
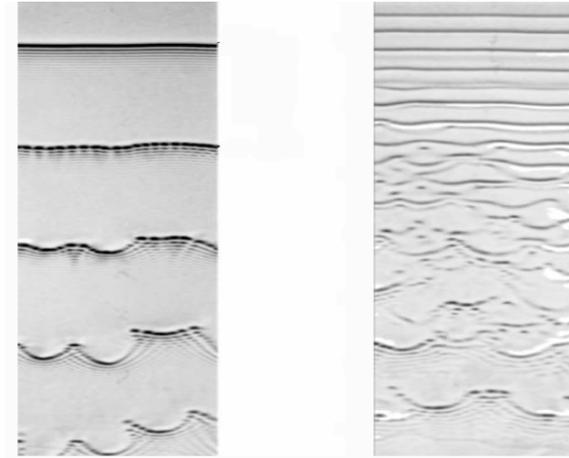
### diffusion experiments for

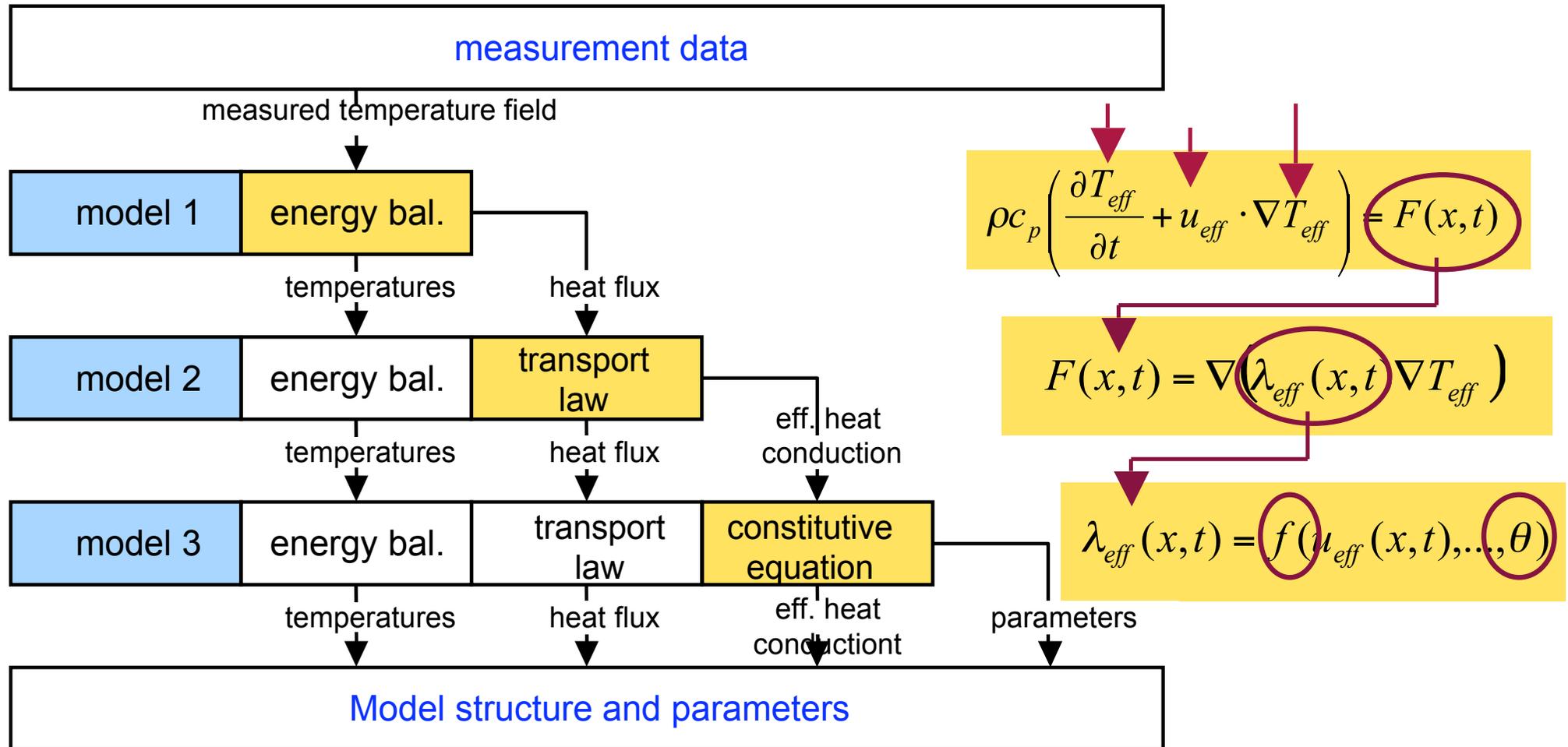
- ✓ - multi-component mixtures
- reactive systems
- electrolytes



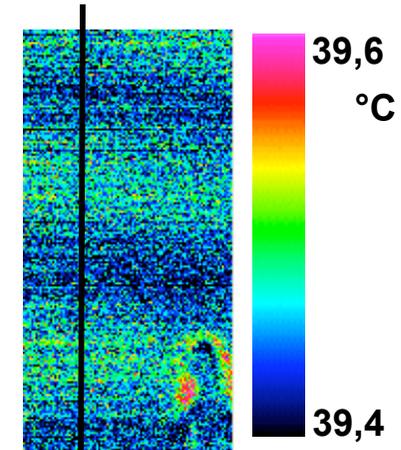
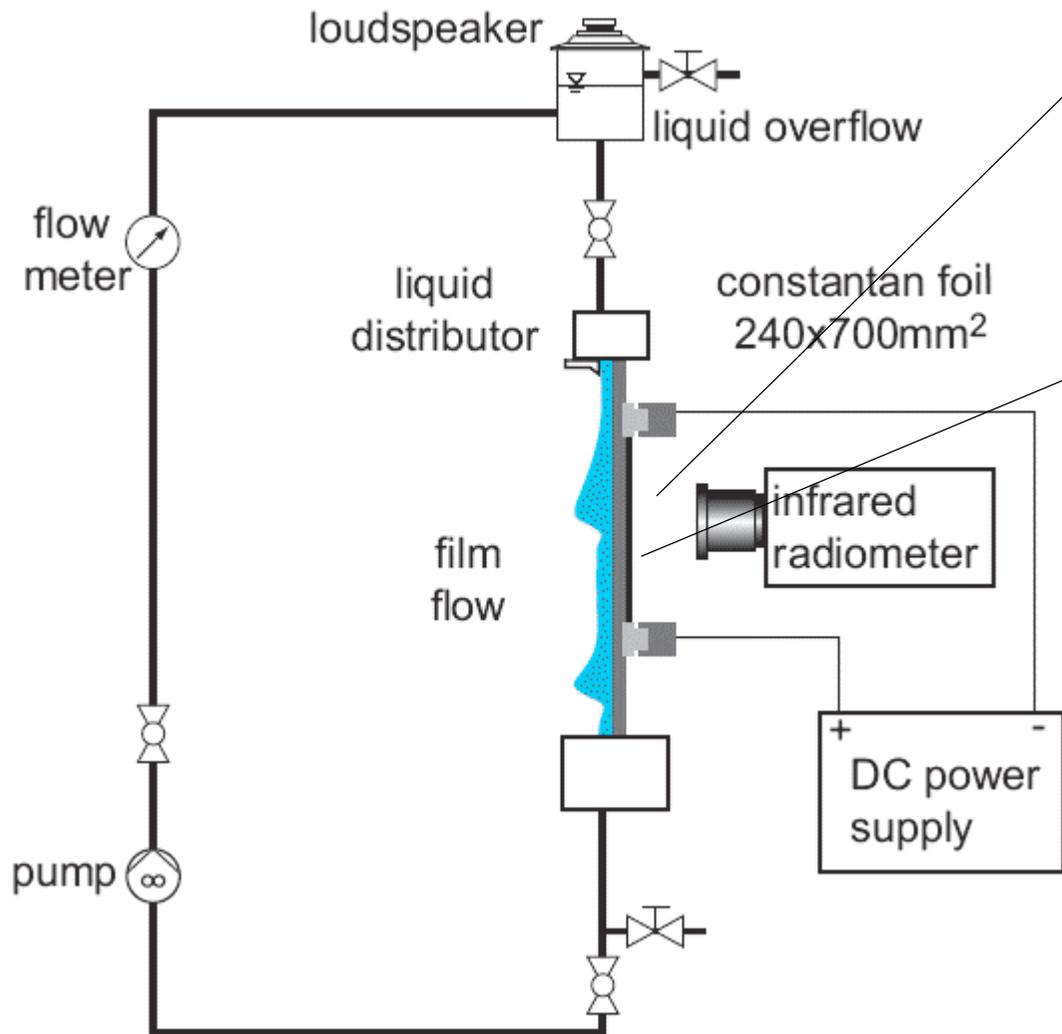
- towards reactive and electrolytes mixtures
- further improvement of method
- diffusion modeling

- falling films are all around:
  - falling film cooler
  - falling film evaporator
  - falling film absorber
  - falling film reactors
- transport phenomena are hardly understood, interaction between
  - fluid dynamics with free surface
  - heat and mass transfer
  - chemical reaction
- first milestone: modelling of heat transfer with effective transport coefficients

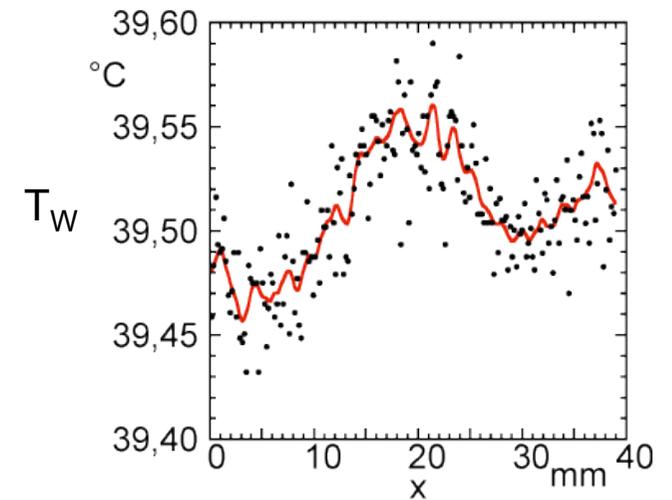




experiments:  
 F. Al-Sibai, A. Leefken, R. Kneer, U. Renz, SFB 540



wall temperature measurements

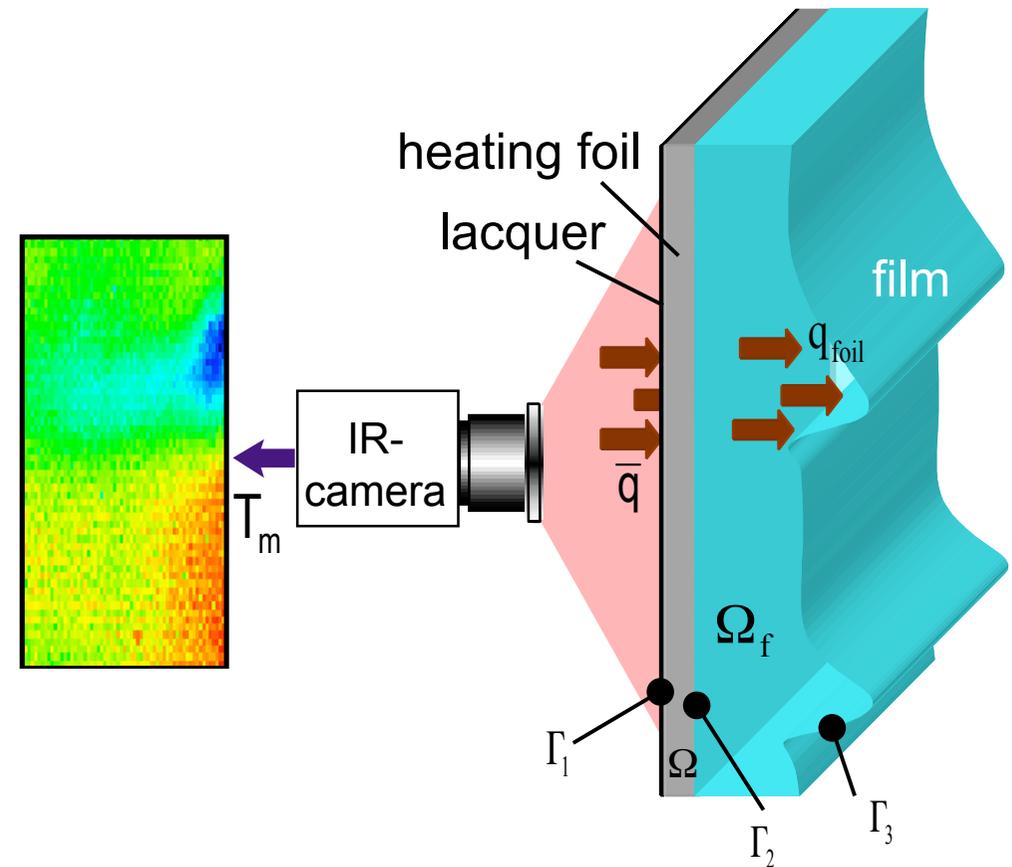


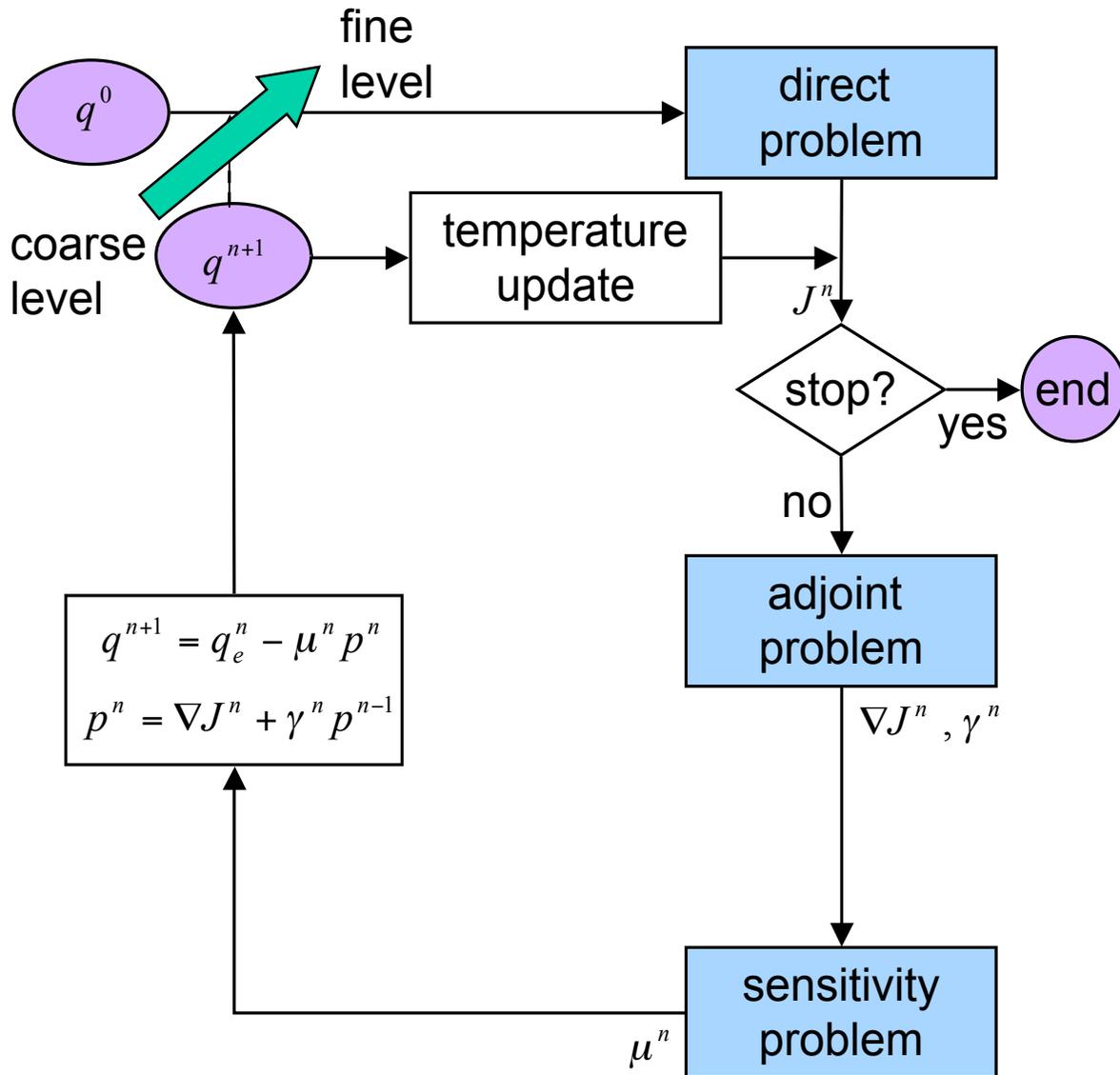
Inverse heat conduction problem as a minimization problem:

$$J(q_{foil}) := \int_0^{t_f} \int_{\Gamma_1} [T(q_{foil}, x, t) - T_m(x, t)]^2 dx dt + R(q_{foil}) \rightarrow \min$$

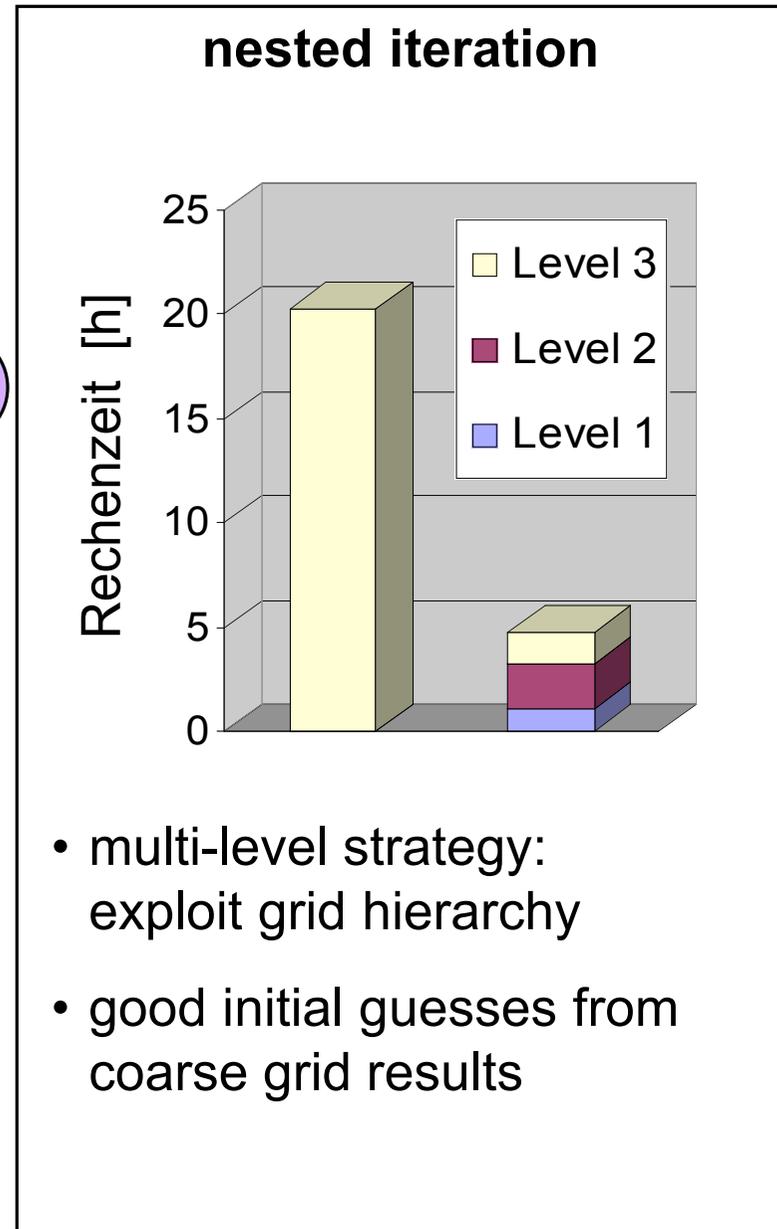
s. t.  $T$  satisfies the **direct heat problem**:

$$\begin{aligned} \frac{\partial T}{\partial t}(x, t) &= a \Delta T(x, t) \\ T(x, 0) &= T_0(x) \\ -\lambda \frac{\partial T}{\partial n}(x, t) &= \bar{q}(x, t) \quad \text{on } \Gamma_1 \\ -\lambda \frac{\partial T}{\partial n}(x, t) &= q_{foil}(x, t) \quad \text{on } \Gamma_2 \end{aligned}$$



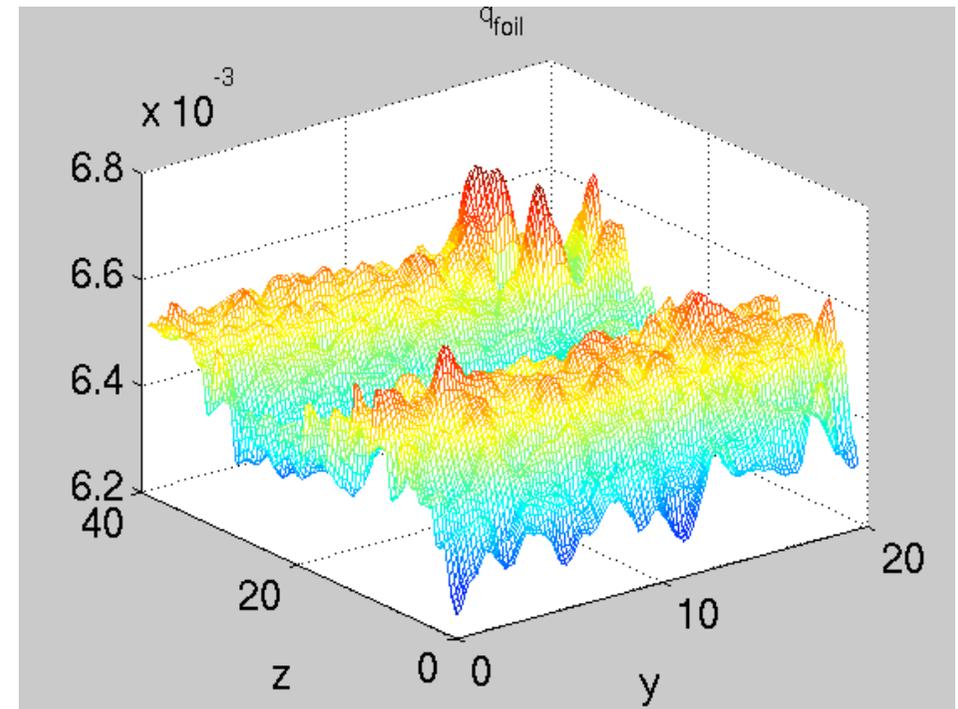
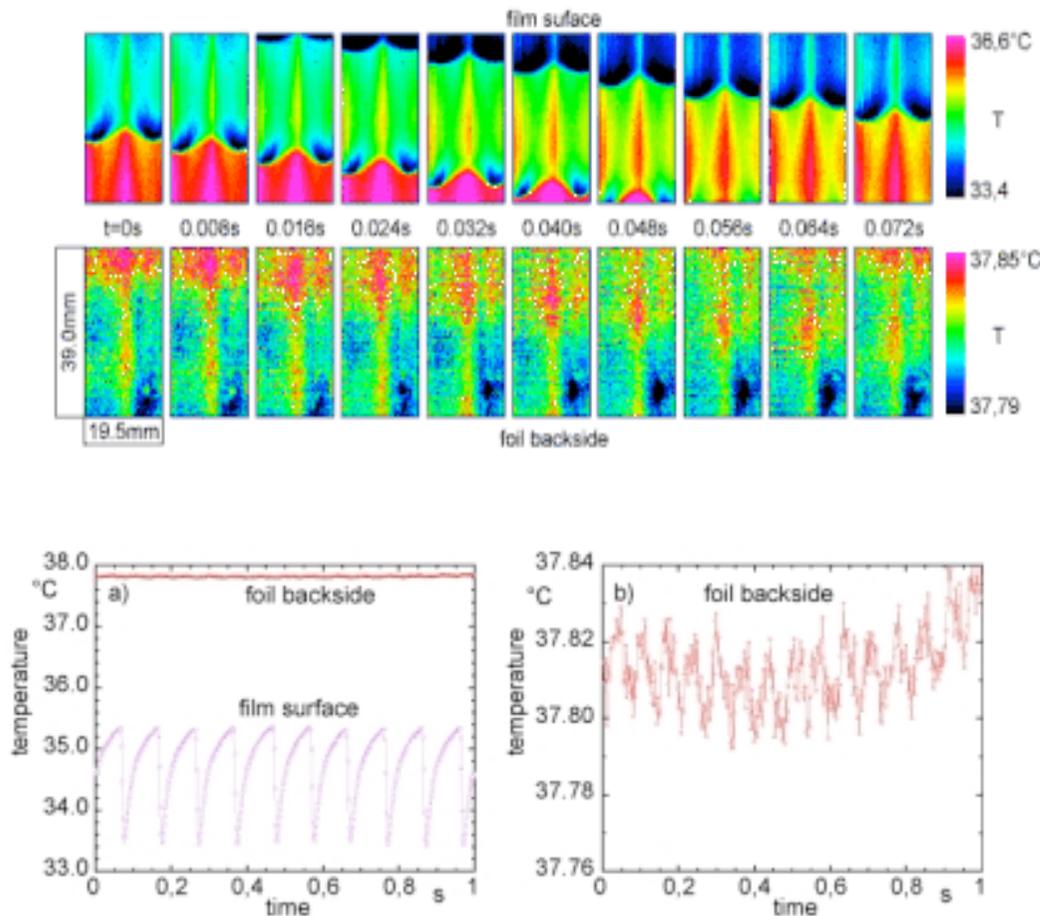


DROPS: adaptive FEM, 3D, anisotropic grids:  
Soemers, Groß, Reichelt, Reusken, CRC 540

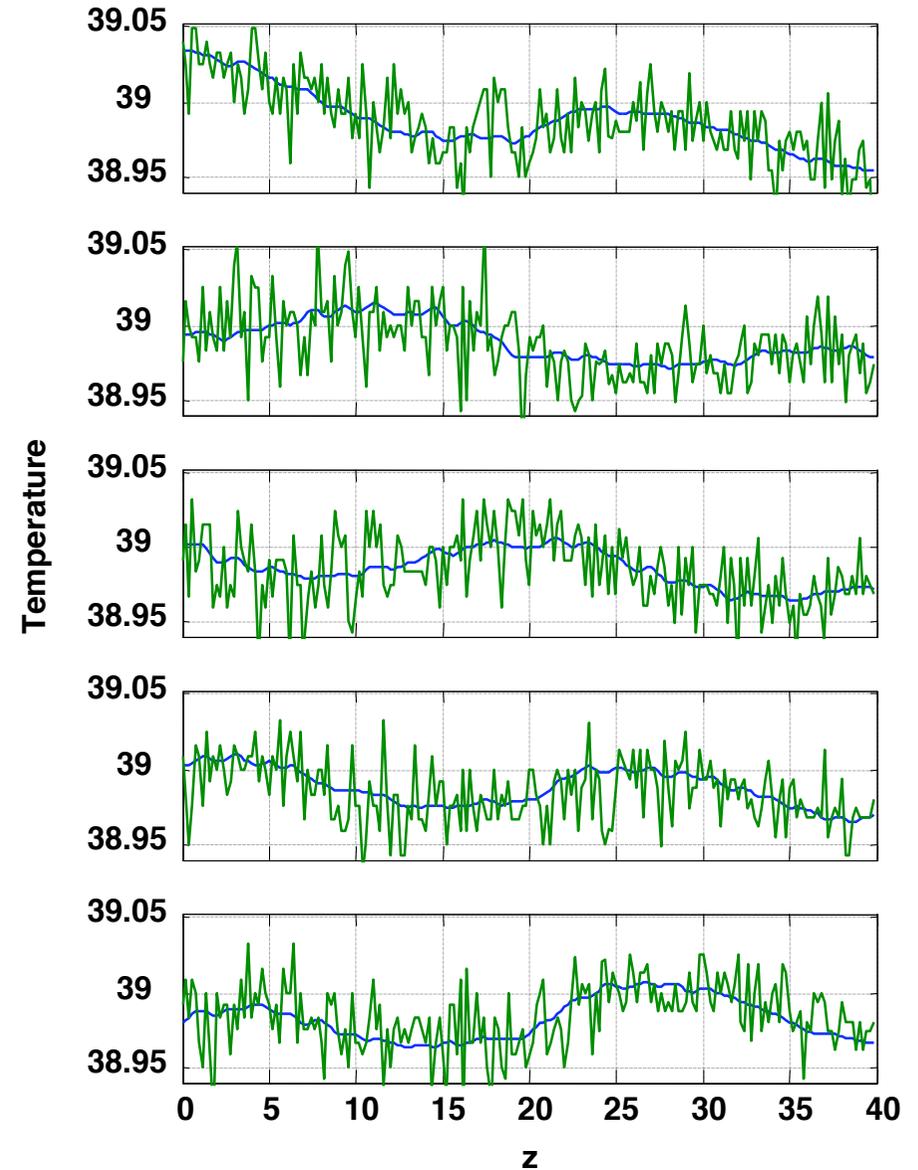
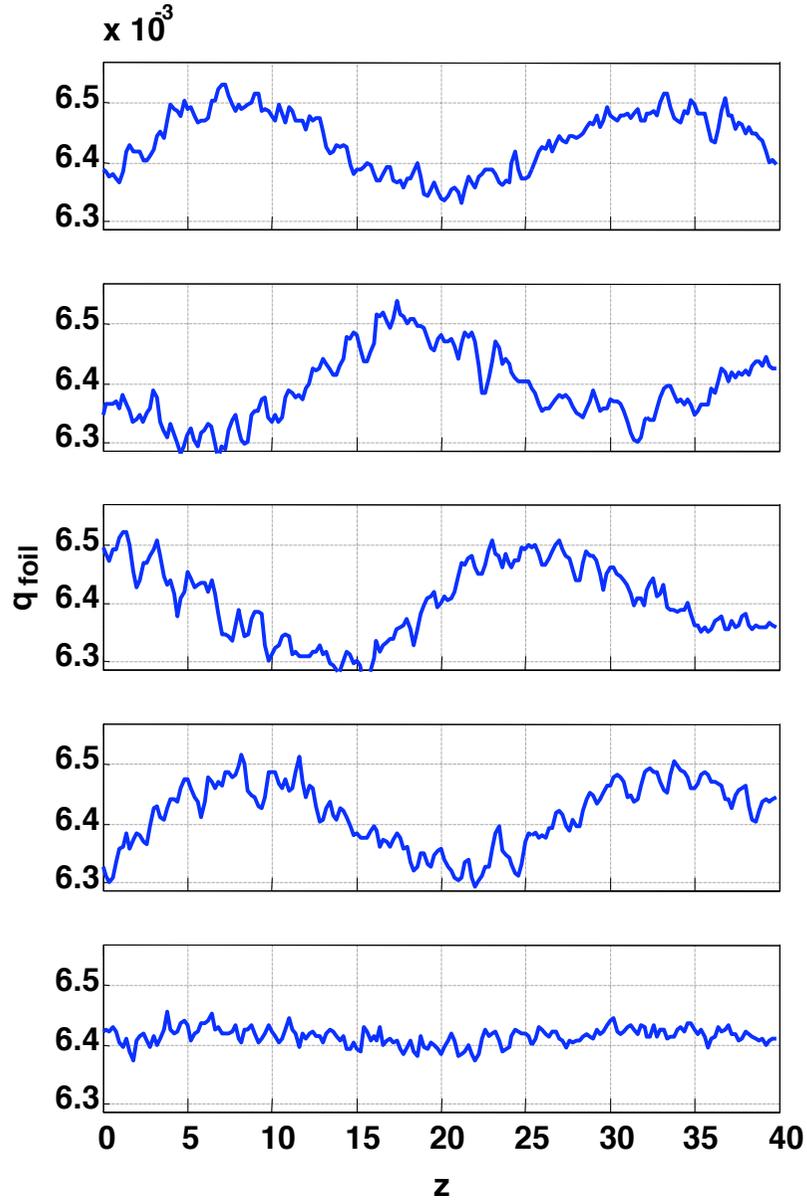


measured temperature distributions

estimated heat flux distribution



(Groß, Soemers, Mhamdi, Al-Sibai, Reusken, Marquardt, Renz, 2005)



**CG method** for the solution of inverse problem embeds DROPS (Reusken u.a., SFB 540) ein.

**DROPS** employs

*adaptive multi grid methods,*  
*finite element discretization,*  
*levelset method*

and facilitates the numerical simulation of multi-phase flow problems in 3D

*at high resolution*

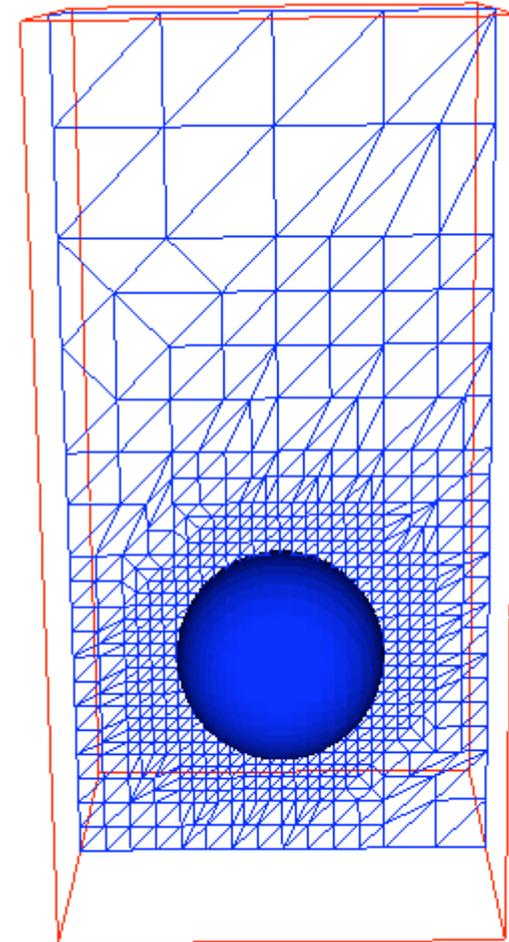
of the phenomena at the phase interface,

*efficient and error-controlled*

with

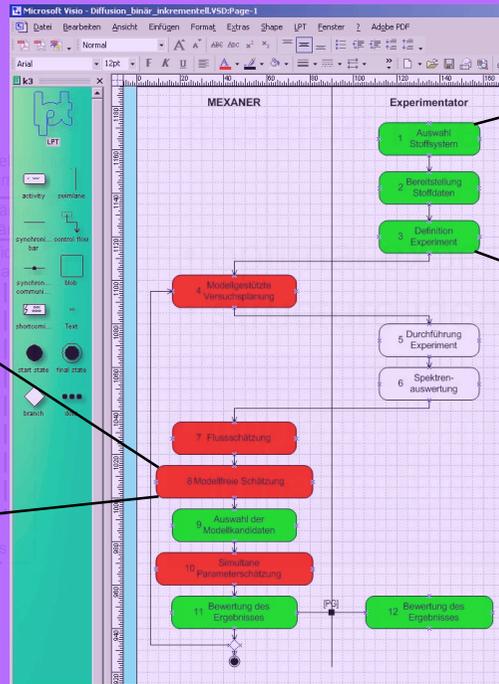
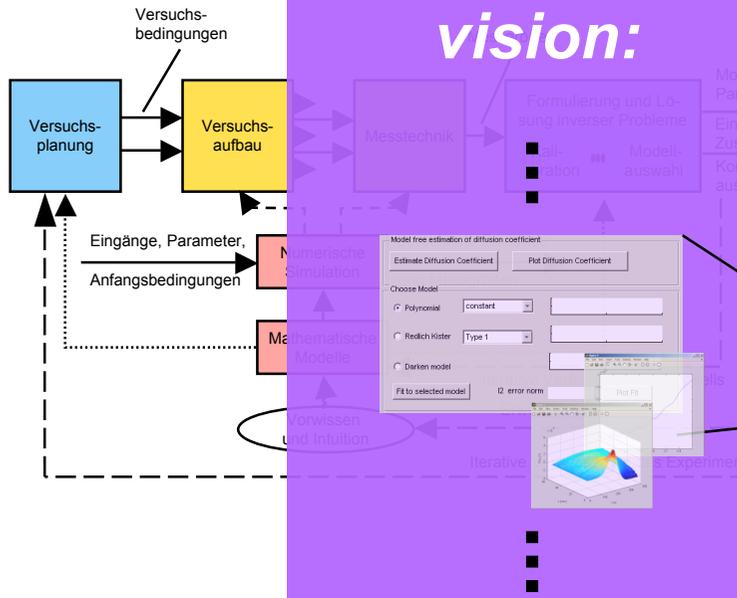
*appropriate flexibility*

for model extensions.



a droplet rising in a stagnant liquid  
Pfennig, Reusken u.a., CRC 540

our vision:



... has potential!

... reduces effort  
... identification leads to mechanisms

... of method integration  
... for distributed problems

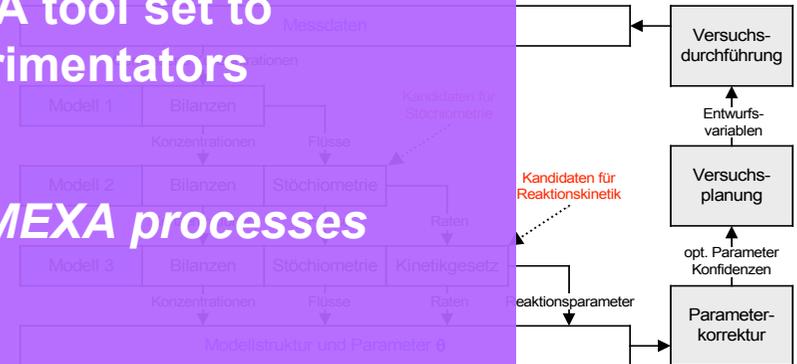
incremental refinement has high potential!

- homogenous reaction, multi-component diffusion, diffusion & bioreaction in general
- drastic reduction of experimental and engg. effort
- significantly improved transparency
- further development

development of an integrated MEXA tool set to transfer systems methods to experimentators

- ⇒ *planning and coordination...*
- ⇒ *documentation...*
- ⇒ *guidance ...*
- ⇒ *process reengineering ...*

...of MEXA processes



- **accept interactions** between kinetic phenomena **in experiments**,  
but  
**isolate** them **during identification** by a suitable decomposition strategy
- **high precision calibration** of high-resolution measurements  
(PIV, LIC, LCSM, NMR imaging, Raman / IR spectroscopy etc.)  
often is a difficult modeling problem in itself
- **statistics of measurement errors** need to be included in the analysis
- **flux estimation** is the key to reliable identification
- **tremendous improvements** are possible by systematic  
cross-disciplinary linking of **process systems** and **experimental skills**

- **refinement of MEXA** work process
  - flux estimation (estimation quality, numerical efficiency)
  - exploit error statistics
  - integrated calibration and kinetics identification
  - tailor optimal experimental design methods to incremental identification
  - assessment of identifiability
  - adjust model parameterization to information content of experiments
- **roll-out MEXA** strategy from meso- to micro- and macro-scale
  - hybrid modeling on macro-scale
  - model structure generation from molecular simulation results
- **application and benchmarking of MEXA** work process
  - complicated reaction and transport problems
  - population systems (crystallization, ...)
  - biological systems (metabolic pathways, ...)

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## Cooperating partners

F. Al-Sibai

J. Koß

F. Alsmeyer

K. Lucas

A. Bardow

T. Lüttich

D. Bonvin, EPFL

A. Mhamdi

J. Blum

A. Pfennig

M. Brendel

U. Renz

V. Göke

A. Reusken

S. Groß

A. Schuppert, BTS

O. Kahrs

M. Soemers

R. Kneer

S. Stapf

and others