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Leo Apostel on Dialectical Logic

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1. To write the present paper, even to start writing it, has not been an easy task. I have learnt to appreciate Leo Apostel as a very stimulating teacher, as an extremely important and inspiring philosopher, and as a deep and complex human being, whom I have the privilege to consider as a friend. It is difficult for me to write on an aspect of his philosophy. First of all, because I am afraid that I will fail and come up with too superficial a story. But there is a second reason. I shall criticize his views, in part by referring to some of my own work. However, on reading Leo Apostel's older work, I have been constantly amazed by the extent to which I am influenced by him. So, I fear that I shall be criticizing him by relying on ideas that are his own.

Irrespective of any personal feelings, Apostel's philosophy is difficult to write about. He is not a philosopher of a few ideas or of a few theories. He takes philosophy extremely serious. He believes that all major philosophers, even those belonging to schools which he opposes, were dealing with deep and important problems. He is typically a continental European philosopher in that he eschews mere philosophical specialization, considers philosophy as intimately connected with the human condition, and feels the need to take a stand on everything that affects this condition and to communicate his views to fellow human beings, whether philosophers or not. At the same time, however, he is deeply convinced that the solution of any philosophical problem presupposes that one takes into account an extremely extensive body of knowledge provided by the different sciences, including mathematical theories. Finally, he is deeply convinced, and was so even before being a student of Rudolf Carnap, that clarity of expression and exactness are essential to any philosophical discussion.

All this might suggest that Apostel's programme is very demanding from a methodological point of view, but it is even more demanding, and moreover complex and enormous, with respect to content. There are few domains in human knowledge which Apostel has not felt the need to study, and his work covers the whole domain of logic and epistemology - from decision theory to empirical epistemology, from the foundations of mathematics to the foundations of the social sciences - and much more besides, both within philosophy (e.g., ethics) and outside philosophy (e.g., cognitive psychology). He has done work in all these fields, not because he considers himself to have original ideas about all of them, but because he is convinced that none of them can be dealt with separately; and by working in all those fields he indeed come up with original and important ideas, opened fundamentally new perspectives and took or indicated decisive steps towards the solution of central philosophical problems. On the other hand, his work suffers to some extent from the huge methodological and material complexity of his programme and from the extensiveness of his outlook. In most of his papers he did not reach well-structured theories in final formulation. Seeing so many sensible alternatives, so many unsolved problems, and so many relevant aspects, he presumably is discouraged to stick to the articulation of one single theory. It is quite clear that the theories his programme aims at, cannot be arrived at by one single person. In this sense Apostel's extremely stimulating work is at the same time frustrating. With him one might hope that the battle for an interdisciplinary approach in philosophy (by teams of open-minded specialists) will be won, and in this sense hope for a better future. With respect to one's individual work, however, the justification of one's enthousiasm is bound to be somewhat meagre.

The domain of problems most central to Leo Apostel's philosophy concerns concrete human actions. His (very open-minded) nominalist attitude derives from it, and not the other way around. His stress on concrete actions has clearly an anthropological basis. All philosophy is about the world or, more specifically, about the concrete subsystems of the world which humans are. Leo Apostel sees human beings as characterized essentially by actions, whereas the world, in so far as it is accessible to us, is only accessible through actions. I employ the confusing term 'essentially' to indicate that, according to Apostel, the fact that humans are acting beings is not merely a very important property of them. Rather, he seems convinced that any important property of humans concerns the fact that they perform specific actions in specific ways. Action is even the essential clue to our mental processes. The latter are actions themselves in the sense that we transform our own mental states by them; but at the same time they are linked to other (usually "external") actions which they direct, to which they commit, which they cause, etc. Apart from being fundamental for humans and their relations to the world, actions are also objects of concrete and empirical study. Whence philosophy, because it is centered on human action, depends basi-

cally on empirical sciences. This fundamental rejection of *philosophia perennis*, typical for the more advanced offsprings of the anti-dogmatic tradition, is at the same time a powerful intellectual, viz. hypothesis-generating, device and a powerful ideological and motivational factor. The latter combination too is typical for Apostel.

As the reader might easily underrate the impact of the action- theoretic approach, I mention, by way of an example, Leo Apostel's criticism of present-day logic and mathematics. Both disciplines underwent an extremely successful development as far as abstract, theoretical matters are concerned. However, the study of the concrete chains of actions, the result of which we call proofs, is severely neglected. (Even decision procedures are handled in an abstract way, e.g. in terms of such abstract and theoretical entities as Turing machines). Even more neglected is the link of mathematics to such elementary things as counting, and also (apart from a few recent contributions) the study of such concrete activities as (mathematical) hypothesis generation, although it is even essential for the "abstract" mathematician. The impact of the action-theoretic approach to logic (derivability, natural implication) is illustrated by the last pages of [4], which summarize some ideas of a large set of lecture notes Apostel wrote some fifteen years ago.

There is one aspect of Leo Apostel's action-theoretic approach which I did not even mention up to now. Actions not only characterize human beings basically; they are not only the basic means by which human beings approach the world (and, by the results of which they enrich their models of the world), but they are also the basic means available to humans to metaphorically understand the world or, which comes to the same, to generate models about it. The latter function of actions is illustrated in Apostel's theory of causality, which is most extensively dealt with in [1]. There (and I simplify drastically) the act of producing an object from certain materials and with the help of specific instruments, is taken as the basic model of causality, which at the same time determines a criteriology.

In view of the structure of the book for which this paper is meant, I would like to phrase two questions in this first section. They are not specific for the subject of this paper, and would confuse the structure of the paper if I were to postpone them. Yet, they seem too essential to be left out. The first problem concerns the status of epistemology; the second concerns epistemological realism.

According to Apostel, epistemology - and the same obtains for other philosophical disciplines as well - is a scientific discipline, not in the sense that it would be reduced to a merely empirical and descriptive level, but in the sense that it has the task, starting form the synthesis of results of the empirical sciences and within the frame provide by recently developed formal theories, to answer philosophical questions and to generate new problems for the other (empirical and formal) sciences. In defining epistemology thus, Apostel tries (i) to bridge the subject-object gap, which compels most epistemologies to make the rationality of knowledge-acquisition transcendent with respect to the known reality, and (ii) to deal with epistemological questions in their full *philosophical* significance, unlike the old psychologism, certain currents in present-day sociology of science, or other approaches which, reducing epistemological questions to their descriptive aspects, loose any hope for the justification of their answers as well as any possibility to avoid bare relativism. Let me stress that I am in deep agreement with this view and these aims of Apostel. Yet, precisely because of my agreement, I think it important that Apostel should spell out in general the justificatory impact of factual and epistemologically evaluative statements respectively. I would have no objection against some circularity in the justification; the concept of a healthy circle is well-known. It seems to me, however, that, if I may exaggerate somewhat, Apostel implies the priority of the epistemological over the factual in discussing ontological problems, but turns the priority around while dealing with epistemological problems. Given the importance of the matter and the promising character of the approach proposed, my present difficult question, which obviously concerns the Is-Ought relation, seems rather central.

My second question concerns realism. As long as I know Leo Apostel, he has been a realist, both at the ontological level and at the epistemological level. I shall not, of course, involve the whole recent debate on epistemological realism here, but only point to a problem I believe to affect Apostel's views. Given the empirical and hypothetical status which Apostel attaches to methodological theories, and to epistemology in general, it is difficult to see how such theories could *guarantee* the truth of our theories (including themselves), or could guarantee their truth-likeness and their converging towards truth. We may sensibly *intend* some theories as descriptions of reality, and we may *believe* that the problems these theories meet are caused by their partial falsehood, but how could we justify epistemological realism as an empirical hypothesis if we take into account the actual

history of the sciences and the absence of a non-empirical methodology? I trust that Apostel's realism is more sophisticated than the more naive brand, but I fail to see as yet how it overcomes the aforementioned.

2. In this section I shall briefly consider Apostel's work on dialectics outside the domain of logic. Two distinct domains are involved: on the one hand, the domain that is directly related to human consciousness: belief-systems, preferences, actions, etc.; and on the other hand, the other hand, the other aspects of reality (cf. the so- called dialectics of nature). I shall start with the latter.

Only a short passage of [2], Leo Apostel's most recent paper on dialectics, concerns the dialectics of nature. The passage in mainly a summary of some theses from Milton Fisk's *Dialectics and Ontology*. Apostel considers Fisk's treatment of the problem extremely important, but time and space prevent him from offering a personal rejoinder.

[3], an older paper, is concerned explicitly the dialectics of nature and the resulting dialectics in the sciences. One of the aims of this paper is to demonstrate that dialectical processes may be described by means developed within the tradition of Western science, viz. cybernetics, information theory, game theory, and even thermodynamics. In doing so, Apostel shows that dialectical processes should for this very reason be considered as serious and respectable by scientists and philosophers from the analytic tradition; viewed under this angle, dialectical processes are not merely intelligible, but actually occur.

With respect to the dialectics of belief-systems, preference- systems, etc. Apostel follows basically the same strategy: he shows that the basic characteristics of dialectics, as described by Hegel, may be formulated within the analytic tradition. The status of actions is somewhat special in this respect, on the one hand for reasons I explained in the first section, on the other hand because actions do not merely pertain to human beings but to the specific "concrete relation" between human beings and the world in general. In a sense, the dialectics in actions is threefold: human beliefs, goals and deliberations behave dialectically; actions in the sense of purposeful changes to the world behave dialectically; and the world itself behaves dialectically. The specific case of human action is studied extensively in, whereas the general problem of dialectics within the human sphere is

studied in.

It seems to me that the analysis in is pushed further than in the papers to which I referred up to now. Let us consider the specific case of "intellectual contradictions"; i.e. contradictions in knowledge-systems and belief-systems. Summarizing heavily, I may render Leo Apostel's claims as follows. (i) All belief systems are inconsistent at any time, in that inconsistencies may be derived from the set of statements believed by some (rational) person at some time. (ii) Some of these inconsistencies are necessary. The argument is that, on the one hand, as Hintikka has shown in *Knowledge and Belief*, no one can reasonably defend to have conclusive reasons to defend p, unless he also defends to have conclusive reasons to defend this, but, on the other hand, in view of, e.g., the numerous mistakes that appear from the history of science, any person should realize that he or she has no conclusive reasons for his or her knowledge; hence, at least for some p, both KaKap and ~KaKap. (iii) The "direction" in which the system - in our case, the belief-system - is changed depends on the specificity of the inconsistency. With respect to "intellectual contradictions" this condition is guaranteed by the fact that the person who faces the inconsistency, will try to save as much as possible from the previous set of beliefs. The reader will have noticed that (ii) and (iii) are absolutely necessary to convince any adherent of Hegelian dialectics that the process described is indeed dialectical.

Before the discussion of the aforementioned statements is begun, it should be emphasized that Leo Apostel rejects explicitly the current reliance on dialectics that is found both with obscurantists of all sorts and with well-meaning but confused progressive social workers. as well as contain severe rejections of and attacks on defensive moves in terms of unanalyzed dialectics. It seems to me that the people attacked here might be attacked on two accounts; they do not take dialectics seriously enough, or they fail to meet the criteria imposed by the present state of the art; the latter refer both to contemporary requirements of clarity of expression as well as to requirements related to content, e.g., to take account of contemporary results of relevant domains. (Herbert Marcuse's well-known "dialectical" critique of "contemporary science" fails especially in the latter respect). I shall soon come to some questions related to taking dialectics seriously.

Let us return to the aforementioned analysis of "intellectual contradictions". I cannot see any reason to question (i); the actual inconsistency of belief-

systems seems inescapable in view of the multiplicity of domains covered by such systems, the fact that changes to such systems occur locally, and the fact that such changes are very frequent. Incidentally, the occurrence of inconsistent scientific theories and of inconsistent sets of such theories has recently been documented by historical studies.

I am less confident of the impact of the argument in (ii). The argument itself is quite all right, and the necessity of inconsistencies follows within a globalistic framework, but I doubt whether such framework is correct. In [5] I defended a rather radical approach to meaning and belief (and, implicitly, to knowledge, but I was not interested there in the specificity of knowledge with respect to belief). Within this approach it seems clear that *KaKap* and *~KaKap*, as construed before, belong to distinct contexts. By a context I roughly mean a "communication situation", centering around some problem and in which a set of statements is taken as beyond discussion. (As i cannot elaborate on the matter here, I beg the reader to believe that the following vague statements are capable of being formulated in an exact way). Let us accept that Hintikka is right in taking $Kap \supset KaKap$ to be a theorem for the reasons stated before. The conclusive reason to hold *Kap*, depend on the context in which one holds *Kap*; in other words, they depend on the purpose *Kap* has to serve in this context, on the consequences which holding *Kap* with respect to a specific problem to be solved. E.g., where *p* is some law of nature, an engineer may defend that he has conclusive reasons to hold *Kap* with respect to a context in which he applies *p* in, say, planning a bridge, and a physicist may do the same with respect to a context in which he needs to reply on *p* in order to set up an experiment. This, however, does by no means entail that either of them should defend to have conclusive reasons to hold *Kap* in the foundational sense in which Descartes thought he could hold his *cogito*. The matter is even more transparent with respect to the "necessary inconsistency" *Kap* and *~Kap*. In so far as the argument for *~KaKap* is conclusive - and I am convinced that is is - it follows (in view of the theorem $Kap \supset KaKap$) that *~Kap*. Again, it seems preferable to me to say that we do not know *p* in the sense that we cannot defend to have an absolute foundation for *p*, but that we may nevertheless have conclusive reasons to hold *p* with respect to some contexts. The reason for this preference is not that I wish to talk inconsistencies away, but rather that I am convinced that there is a real difference in this respect concerning *Kap* and *~Kap*. Lest the reader believe that the distinction is *ad hoc*, I refer him to for completely independent arguments to introduce distinct contexts. Incidentally, I also

offer an argument there which, under the present presuppositions, may be reconstructed to the effect that, for some *p*, *Kap* and *Ka~p*, again with respect to different contexts.

I now turn to (iii): the way in which the specificity of the inconsistency determines the direction of the change. This is *par excellence* the aspect of dialectics I always found either obscure or false, and, I must confess, Leo Apostel's analysis does not help me out. His point is that the person who faces an inconsistency will try to save as much as possible from the previous set of beliefs. As I have a bit more information about scientific change than about change to belief-systems, I shall mainly refer to the former (I realize, though, that a negative conclusion pertaining to the former need not entail a negative conclusion as to the latter).

Let me first state up to which point I understand the claim and agree with it. Problems are localized; they are attributed to specific domains; whenever we try to solve a problem, we question only part of our belief-system, and keep considering the rest as provisionally beyond doubt. There is, of course, the riddle of Quine's (and Kuhn's) holism, but in [6] Larry Laudan presents an analysis of scientific problems that enables us to get beyond holism in some cases and to some extent (see e.g., his distinction between unsolved problems and anomalies), and chapter 4 of [7] bears the picturesque title "Dissecting the Holist Picture of Scientific Change". So, let us maximally concede to Apostel and suppose that only one single theory *T* is under strain, whereas all (previously accepted) theories about other domains are beyond discussion. Moreover, depending on the state of the research-tradition (and on some other factors which are not too well articulated in the present-day literature) several parts of *T* itself (and perhaps of some rejected competitors of *T*) are more or less beyond discussion. All those theories and parts of theories form *constraints* on the solution of the problem under consideration, viz. to find an alternative to *T* in which the inconsistency is lost. The heuristic guidance offered by such constraints, and the way in which such constraints may rationally be violated has been studied in recent literature. I refer especially to Thomas Nickles's main paper in his [8], to the references given therein, and to other papers in this book. Up to this point, I understand what is meant by (iii).

Does it follow from all this that a person who faces an inconsistent theory would try to retain as much of this theory as he can? By no means so, People

usually don't and, moreover, shouldn't do so. This is argued at length in [9], now reworked as chapter 5 of . Consequently, I think we should reject even the final statement of (iii), which, as Leo Apostel explicitly states in , is only a very weak echo of Hegel's *Aufhebung*. Of course, I did go a long way with Apostel in the previous paragraph, but what I stated there concerning constraints was still much weaker than Apostel's very weak echo: the determination of the *Aufhebung* by the specificity of the contradiction is lost, were it only because a large multiplicity of ideas may be taken as the point of departure of the *aufheben-de Theorie*, each of which might lead to giving up distinct constraints, and because there is no reason to believe that the combination fo some idea with some set of constraints uniquely determines the final formulation of the theory. The fact that there is no historical documentation for such a multiplicity of points of departure is *obviously* the consequence of the limited character of human imagination, *not* of some inherent dialectical determination.

Before ending this section I would like to formulate some more general questions about dialectics. By answering them, it seems to me, Leo Apostel would clarify his views. The questions concern the status of Hegel's theory about dialectics. Is this merely a source of inspiration, useful with respect to the articulation of theories which might then be evaluated independently? Or is it more? And, if so, is Hegel's dialectics - and I only mean the formal properties, not the fact that it is intertwingled with idealism - a correct theory, which has perhaps to be reformulated with respect to present-day standards? Let me phrase the question in another way. Leo Apostel shows, e.g., that some dialectical processes may be formulated in system-theoretic terms. Does this count as a confirmation for (part of) Hegel's theory? Moreover, does the fact, that some processes, that are describable in system-theoretic terms, are dialectical, entail that Hegel's theory provides us with information which is not already provided by other theories? The same questions may be asked concerning Marx's (or Marxian) dialectics, in as far as it is different from Hegel's.

3. A large part of [10] is devoted to a formal study of Marx's *Das Kapital*. As one might expect, Leo Apostel needs a very complex logical basis to perform this task. This logical basis is not fully presented, but is clearly indicated by referring to existing logical systems and to the changes that have to be induced to these. In doing so, Apostel makes it quite clear how the logical basis has to be reconstructed, although it is by no means obvious that

such a unique reconstruction is possible, or that it will not give rise to unforeseen problems.

The basic idea is to combine Chisholm's M-operator (intentional action) with von Wright's operators *T* ("and next") and *I* ("instead") in such a way that each of these may occur within the scope of any number of the other operators or of itself. The resulting system is embedded in a second-order predicate logic (including propositional quantifiers). In order to formalize Marx's *Capital*, (i) each of these elements has to be transformed in such a way that it becomes paraconsistent, (ii) the result has to be adapted to the effect that different actors may be referred to, and (iii) the result has to be combined in a rather "deep" way with a temporal logic. Again, a number of technical and philosophical problems will evolve when this programme is pursued, but it is quite obvious that it should not be too difficult to work it out, because the elements, the way in which they need to be combined, and the required transformations are clearly stipulated.

Presupposing the aforementioned system, and employing its language, Apostel actually formally expresses several central concepts and conceptual relations from *Capital*. These include such essentials as exchange value and use value, labour, its force, dimensions, and relations to exchange value, etc. The main "contradiction" studied is the one that obtains between use value and exchange value. Typically, the formal machinery enables Apostel to distinguish several "contradictions" and to characterize some of them as logical contradictions and others as oppositions between properties that originate under distinct respects.

I am rather incompetent about *Capital*, viz. fully incompetent, to dare formulate any appreciation about the accuracy of the formalization or of its clarifying, problem-generating and problem- solving character. Nevertheless, I would like to stress that this enterprise of Apostel's is important for two independent reasons. The first is that, given the complexity of Marx's concepts and theory, the formalization of this theory at the *logical* level (and not only on the mathematical level) is absolutely necessary to demonstrate that this theory may be rendered within a perspicuous and exact conceptual system. The second reason is more general. The logical basis that is required to formalize economic action will have at least the complexity of the system indicated by Apostel, and will moreover consist of the same or equivalent (probably more complex) elements. Straightforward "consistent logi-

cians" might dispute the paraconsistent character of the logical system, but even if they were able to uphold this claim in connection with a realistic theory of human action, this would not change much in the degree of complexity of the logical system.

In [4] Leo Apostel offers, among other things, a very detailed analysis of different aspects of the logic of action, which are directly relevant to dialectics. This contribution forms an important rejoinder to the first part of . I choose not to comment on Apostel's action-theoretic approach to dialectics. One of my reasons for this decision is that I am more familiar with the problems of propositional dialectical logic and that I think to be able to offer a more interesting contribution to this volume by concentrating on this topic, which moreover is a necessary preliminary to all sorts of modal dialectical logics. Another reason is that Apostel's contributions to the dialectical logic of action cannot be separated from his other work on the logic of action. Even a glance at Apostel's bibliography, however, will convince one that this topic has been Apostel's main concern during his whole career, and that he has written an enormous number of papers on it. Although the results of are very convincing and illuminating, I consider them neither very difficult to obtain nor the most important parts of Apostel's work in the logic of action, and for this reason too I shall now concentrate on propositional dialectical logic.

The second part of is devoted to this type of logic (with propositional quantifiers), except for some excursions in temporal logic and in logics concerning less familiar modalities. I shall not pay any attention to these excursions either. I hope that the reader by now realizes that my subsequent discussion of Apostel's dialectical logic concerns a very truncated version. Nevertheless, I believe that even this version contains Apostel's most profound contribution in the field, and that it allows for a discussion of the merits of his dialectical logic and of its programme.

4. As a starting point for the discussion of propositional dialectical logic, I give a brief survey (based on [11]) of paraconsistent logics based on material implication. I realize that Leo Apostel in several places expresses the need to replace material implication by some relevant implication. However, he never specified in a precise way the relevant implication he prefers in this connection. Moreover, I shall demonstrate in this paper that several ideas propounded by Apostel, even if they may sound somewhat bizarre at first sight, lead to interesting formal theories; I have been unable up to now to reach such results with respect to relevant logics. Let us start with the paraconsistent logic PI, which I consider to be minimal.

Axiomatic system

A⊃1	$p \supset (q \supset p)$
A⊃2	$(p \supset (q \supset r)) \supset ((p \supset q) \supset (p \supset r))$
A⊃3	$((p \supset q) \supset p) \supset p$
A&1	$(p\&q) \supset p$
A&2	$(p\&q) \supset q$
A&3	$p \supset (q \supset (p\&q)$
Av1	$p \supset (pvq)$
Av2	$q \supset (pvq)$
Av3	$(p \supset r) \supset ((q \supset r) \supset ((pvq) \supset r)$
A~	$(p \supset \sim p) \supset \sim p$

Rules: Modus Ponens and uniform Substitution.

Semantics

CO	$v : F \rightarrow \{1,O\}$ (F is the set of formulas)
C⊃	$v(A \supset B)=1$ iff $v(A)=0$ or $v(B)=1$
C&	$v(AvB)= 1$ iff $v(A)= 1$ or $v(B) = 1$
Cv	$v(AvB)=1$ iff $v(A)=1$ or $v(B)=1$
C~	If $v(A)=0$, then $v(\sim A)= 1$

There are two obvious ways to strengthen PI in such a way that the result is still paraconsistent; with respect to the present system this means that not all formulas of the form $(A\&\sim A)\supset B$ are theses. The first way to do so is by relating negations of complex formulas to truth- functions of less complex formulas. All the following belong to this family of theses (the corresponding semantic clauses are perfectly

$A\sim\supset 1^s$	$\sim(p \supset q) \supset (p\&\sim q)$
$A\sim\supset 2^s$	$(p\&\sim q) \supset \sim(p\supset q)$
$A\sim\&1^1$	$\sim(p\&q) \supset (\sim pv\sim q)$
$A\sim\&2^s$	$(\sim v\sim q) \supset \sim(p\&q)$
$A\sim v1^s$	$\sim(pvq) \supset (\sim p\&\sim q)$
$A\sim v2^s$	$(\sim p\&\sim q) \supset \sim(pvq)$

$A\text{-}\text{-}1^{s}$ $\quad\text{-}\text{-}p \supset p$
$A\text{-}\text{-}2^{s}$ $\quad p \supset \text{-}\text{-}p$

All these axioms may be added together to PI. The resulting logic, which in I called PI^{s} because it was first formulated by Schütte, is *strictly* paraconsistent (for no A, $\vdash (A\&\text{-}A) \supset B$) and is *maximally* paraconsistent (the addition of any non-derivable rule or axiom, except for those containing non-logical propositional constants, leads to a non-paraconsistent logic, viz. either the classical propositional calculus, PC, or the trivial calculus).

The second obvious way to strengthen PI is to add axioms which lead to specific forms of $(A\&\text{-}A)\supset B$, i.e. axioms which stipulate that negation behaves classically in front of formulas that have some or other complex form. Here they are (the corresponding semantic clauses are again obvious):

$A\text{-}\supset^{v}$ $\quad\text{-}(p \supset q) \supset ((p \supset q)\supset r)$
$A\text{-}\&^{v}$ $\quad\text{-}(p\&q) \supset ((p\&q) \supset r)$
$A\text{-}v^{v}$ $\quad\text{-}(pvq) \supset ((pvq) \supset r)$
$A\text{-}\text{-}^{v}$ $\quad\text{-}\text{-}p \supset (\text{-}p \supset q)$

If all these axioms are together added to PI, we obtain a system, called in PI^{v} because it was forged by Ayda I. Arruda after ideas of N.A. Vasil'ev, which is maximally paraconsistent, but not strictly paraconsistent.

As, according to some marxists, all laws of classical logic stay valid in dialectical logic -- I take this claim from [12] -- it is worth mentioning that PI^{s} and several of its fragments contain all *theses* of PC (because they have the same disjunction-negation fragment), but not all of its rules (e.g., Disjunctive Syllogism is not valid). PI^{v} and any of its fragments which have at least one of the four axioms as theses, contain both all theses and all rules of PC (because the classical negation may be defined, e.g. , by $\neg A :_{df} \text{-}(A\&A)$ in the presence of $A\text{-}\&^{v}$).

Intuitionistic counterparts (as far as implication is concerned) of all these logics may be obtained by dropping $A\supset 3$; these counterparts are more elegant from the point of view of natural deduction. The *predicative* extensions are most easily obtained by adding the usual axiom-scheme and rule for the universal quantifier and the corresponding axiom-scheme and rule for the existential quantifier. The extensions are served well by:

$A\text{-}\forall 1^{s}$ $\quad\text{-}(\forall\alpha)A \supset (\exists\alpha)\text{-}A$
$A\text{-}\forall 2^{s}$ $\quad(\exists\alpha)\text{-}A \supset \text{-}(\forall\alpha)A$
$A\text{-}\exists 1^{s}$ $\quad\text{-}(\exists\alpha) \supset (\forall\alpha)\text{-}A$
$A\text{-}\exists 2^{s}$ $\quad\forall\alpha)\text{-}A \supset \text{-}(\exists\alpha)A$
$A\text{-}\forall^{v}$ $\quad\text{-}(\forall\alpha)A \supset ((\forall\alpha)A \supset B)$
$A\text{-}\exists^{v}$ $\quad\text{-}(\exists\alpha)A \supset ((\exists\alpha)A \supset B)$

PI may also be extended by adding some axioms from the Schütteseries and some from the Vasil'ev-series. Some of these logics are again maximally paraconsistent. Several of the aforementioned logics (and of their fragments) have been studied in the literature.

The logic which Apostel takes as a starting point in consists of $A\supset 1$, $A\supset 2$, A&1-3, Av1-3, A~, and $A\text{-}\text{-}1^{s}$, which is Newton C.A. da Costa's system $C\omega$ (see, e.g., [13]). Instead of the C_n-axioms ($1\geq n>\omega$), such as the non-contradiction spreading

$$(\text{-}(p\&\text{-}p)\&\text{-}(q\&\text{-}q))\supset\text{-}((p\supset q)\&\text{-}(p\supset q))$$

Apostel supplements the previous system with axioms which I shall consider soon.

The choice of the aforementioned system is somewhat arbitrary, and even somewhat determined by the absence of a better, viz. relevant, alternative. In order to express his interest in the multiplicity of means which enable us to deal with paraconsistent theories (i.e., to combine inconsistency with non-triviality), Apostel studies a large number of theses which are problematic from the paraconsistent point of view (pp. 53-58--all references by page numbers are to , unless stated otherwise). In this section (which is somewhat messed up by the printer and contains at least one mistake by the author) Apostel considers two distinct kinds of problems.

The first kind of problems concerns combinations of theses and rules from which the *exfalso quodlibet* is derivable; it is demonstrated that some theses have to be rejected in the presence of some other theses and rules. The latter are not subscribed to, and are not all valid in the aforementioned system (from which Apostel starts) or even in PC. The second kind of problems are claimed to be related to the *falsehood* of certain formulas. So, e.g., on p. 54, Apostel argues that, although $(\forall p)\text{-}(p\&\text{-}p)$ need not result in $(p\&\text{-}p)\supset q$, it

is *false* because we accepted that $p_1\&\sim p_1$ might be true in the first place.

It seems to me that such uses of 'falsehood' are mistaken. Let me present my argument in the form of a dilemma. According to the view which I defended in , and which is implicit in the work of da Costa and his school and even in Apostel's (p. 48), inconsistencies occur, if they occur, on the object level and not on the semantic meta-level. In other words, an inconsistency is rendered by saying that *both A* and *~A* are *true*, and *not* by saying that A is both *true and false*. One of the arguments I adduced in concerns the fact that, if we give up the consistency of our meta- language, the object-level theory fails to be a theory about some domain (for we are unable to tell what it states about the domain), even if the meta-level theory may be a theory about the object-level theory (provided we describe the meta-theory in a consistent way in the meta-meta-theory). Simplifying: "the world", including mathematical domains and suchlike constructions, may force us, possibly in view of the inadequacy of our conceptual systems, to consider inconsistent theories as true, but in order to describe the world by such theories we need to keep our thinking about (not in) such theories consistent. From this point of view, there is nothing wrong with saying that $p_1\&\sim p_1$ is true for factual reasons whereas $(\forall p)\sim(p\& \sim p)$, and hence also $\sim(p_1\&\sim p_1)$, is true for logical reasons. (In general, the negation of any logical truth may turn out to be factually true). The alternative view on truth and falsehood is held by Graham Priest, e.g., in [14], with whom I had some instructive but undecided discussions on the matter. Priest renders $p\&\sim p$ by saying that *p* (and also *~p*) is both true and false. He keeps '$v(\sim p)= 1$ iff $v(p)=0$', but has to allow for multiple truth-values: some statements have only one truth-value, others have two. Syntactically his system from [14] is identical to PI^s. The semantic interpretation, however, is different from mine; His interpretation has the advantage of keeping negation a truth-function, but so does mine in a specific sense. (His interpretation uses the values {1},{1,0} and {0}, whereas I may use the "couplevalues" [1,0], [1,1], [0,1], where $V(A)=[v(A), v(\sim A)]$; see [15]). On the all-level- paraconsistency view of Priest - Nuel D. Belnap, Jr., seems to subscribe to it in [16] - Apostel is right to claim that $(\forall p)\sim(p\&\sim p)$ is *false*, for $\sim(p_1\&\sim p_1$ is false in view of the truth of $p_1\&\sim p_1$. But, alas, the fact that $(\forall p)\sim(p\&\sim p)$ is false does not in any way exclude that it be a theorem, for falsehood in *this* sense does not exclude truth. Hence, neither of the two conceptions of truth and falsehood justifies the rejection of the theoremhood of $\sim(p\&\sim p)$.

I now return to the axioms which Apostel adds to da Costa's propositional logic c_ω. These axioms are formulated with the help of propositional quantifiers. Some examples are given, not an enumerative list. There are two reasons for introducing them. The first is that a dialectical logic should contain a specific theory about the relation between contradictory and non-contradictory statements, and about the specific deductive behaviour of both kinds of statements. The second is that a dialectical logic should specify the relation between different types of contradictions. The first reason brings Apostel to such theses as

(4.1) $(\forall p)(\exists q)[(q\&\sim q)\supset p]$

and the second to such theses as

(4.2) $(\forall p)[(p\&\sim p)\supset(\exists q)(\exists r)((q\&r)\&\sim(q\&r))]$

The formulation of these theses is not adequate. Indeed, Apostel requires that restrictions be imposed on (4.1) and (4.2) in order to prevent that, e.g. *p* be identical to, or equivalent with *q* in (4.1). The axioms may probably be reformulated in a way which is adequate with respect to some suitable interpretation of the propositional variables. However, I fail to see the reason to consider either (4.1) or (4.2) as general theses. (4.1) is interpreted by saying that any proposition is implied by some contradiction. Apostel clearly does not mean that this be so because a contradiction implies anything or because any proposition is in some obvious (paraconsistent) deductive way related to some contradiction. But why then should any proposition be implied by some contradiction? Apostel claims (p. 61) that theses like (4.1) and (4.2) are within propositional logic closest approximations we can get of Marxian and Hegelian dialectics. One way to understand (4.1) would be to say that any proposition is the "dialectical synthesis" of some contradiction, but I doubt whether this makes sense. For this reason I am unable to judge the merits of the typology (according to logical form), which Apostel formulates. Clearly, some of his suggestions make sense under the above interpretation, e.g., that any proposition implies some contradiction, whence

(4.3) $((p\supset q)\&(p\supset\sim q))\supset\sim p$

should not be a theorem. Some caution is at hand, however, for if 'any proposition produces its negation' is a dialectical principle, to render this as $(\forall p)(p\supset\sim p)$ would force us to give up either $p\supset p$, or Dilemma, or excluded middle. If Apostel takes the matter, viz. the typology of contradictions and the interpretation of implication, to be important enough, he might clarify it in his reply.

Before coming to *dynamic* dialectical logic, and in preparation of it, I now consider two proposals by Apostel which he takes to be preliminary to dynamics. The *first proposal* concerns series of dialectical logics. The idea is that each dialectical logic resolves some contradictions not resolved by its predecessor, and at the same time produces some new contradictions. Some meta-logic orders the logics into series and decides about the time of the transition. Apostel requires this meta-logic itself to be dialectical. I am not sure this should be the case, but I note that this does not contradict my aforementioned argument in favour of the consistent use of truth and falsehood in the meta-language. Indeed, the aforementioned meta-logic is intended to serve another purpose than a semantic meta-language. The *second proposal* concerns paraconsistent variants of Rescher and Urquhart's temporal logic and suchlike logics. They are introduced because dynamics presupposes time.

The logic presented at the outset of the next section is directly inspired on such ideas as the one contained in the first proposal. Yet, it seems to me that the first proposal, if taken literally, has two serious drawbacks. First of all this proposal avoids the central difficulty from which interests it, viz. the resolution of contradictions. Nowhere, in the book under discussion, does Apostel show in a concrete way how a formal logic might lead to the dialectical resolution of a contradiction. I return on this point in the subsequent section, but even in a previous section I argued that it is extremely difficult to explicate the relation between a contradiction and its resolution (*Aufhebung*) in a concrete way. The other problem with the first proposal is that the transition from one dialectical logic to the other should clearly depend on the specific contradictions that are reached at some point in time. Formal logics, however, may only be different with respect to the *forms* of contradictions they resolve, and (i) it will be impossible in general to decide whether all contradictions of a given form have been resolved (especially in view of the rather wild axioms of Apostel concerning implications between contradictions of different forms), (ii) it is not clear at all that there is anything sensible about the resolution (and production) of some *forms* of contradictions preceding the resolution (and production) of other forms of contradictions, and (iii) it is not clear how the series should end, or even to which point it should converge (and note that to make the series circular, would be abortive to the whole enterprise).

Paraconsistent variants of, e.g., Rescher and Urquhart's temporal logic are of course sensible and interesting formal systems. Moreover, they are needed to describe the world if the dynamics of the world is dialectical itself, i.e. if actual contradictions are produced and resolved in the world. Although such logics are in this sense required to describe dialectical evolutions, I doubt whether they bring us any closer to dialectics itself. Stated more explicitly, I doubt whether there is any use in making logics dialectical themselves, e.g., by ordering them in a series, or by strengthening such logics to the effect that their application causes the production and resolution of contradictions.

For the sake of argument, suppose first that the evolution of the world is dialectical, and that we dispose of criteria to produce instant truth. In this case, it would clearly be wrong to adopt a logic which itself produces or resolves contradictions (unless, of course, in case we were Hegelian idealists). Next, suppose that the world is consistent, and that we believe so, but that our relation to the world is such that the dynamics of our theories need to be dialectical (Our theories in a sense produce their own falsifications on the empirical level, and produce their own competitors on the conceptual level. Moreover, these oppositions and tensions in a sense produce their own resolution). In this case, it does make sense to have our logic produce and resolve contradictions. I do not claim that it is possible to articulate such logics, I just say it would be sensible that they do if they can. However, although the contradictions are produced and resolved in our theories at a given time, they are not true and false with respect to the world at those times. As a consequence, the dialectical dynamics concerns our beliefs about the facts (at some point in time), but not the facts that obtain (at some point in time) according to our beliefs. Finally, let there be dialectics in both the world and in our beliefs (by which, in the absence of instant truth, we try to grasp the dialectical evolution of the world). The contradictions produces and resolved by our theories are clearly distinct from the contradictions produced and resolved in the world. We need a paraconsistent logic of time to describe the world, and our logic produces and resolves contradictions, but this production and resolution happens to our theories about the dynamical world, not within the world as we describe it. As a consequence, the dynamics of our logic lies on the propositional level (*outside* the time- operators), not at the level of the logic of time itself (inside the time-operators).

I did not make something disappear up my sleeve while discussing the last case. If we describe the dynamics of our beliefs which is typical for the last

case, we are either back again in the second cade (if we have instant knowledge about the aforesaid evolution) or in the third cade. In general, if the aim of some reflection is a dialectical process, this reflection does not itself produce *this* dialectics, although the reflection itself may be dialectical. Naive minds may see this as a further reason to give up the object-subject distinction (or object-subject "splitting" as they call it), but they fail to see that, in the few good moments in which we are able to do so, we have no need for productions or resolutions of inconsistencies. They just happen.

5. I now come to the discussion of dynamic dialectical logics. As argued in the previous section, such logics may be applied in case our theories about some domains, whether empirical or mathematical, are inconsistent, and on the condition that this inconsistency is not present within the domain itself, but is a consequence of the inadequacy of our knowledge about the domain, and hence need to be eliminated. The dynamic character of the logic has two different aspects. The first concerns the changes which are introduced when inconsistencies are discovered, the second concerns the elimination of inconsistencies. It is rather clear what is meant by the second aspect, although it might be problematic to articulate a logic which accounts for it. The first aspect, however, requires some more attention. I shall discuss this aspect first and present the solution I proposed in [17]. Proofs of all claims made here about the intended logic may be found there.

It seems to me that, at the level of deductive logic, the production of an inconsistency reduces to the fact that an inconsistency is derived from some theory (or set of premisses). If this is correct, there is no need for a special contradiction-producing feature of the logic. The question is how one might react to the discovery of an inconsistency. If one was employing classical logic up to that point, one might replace it by some paraconsistent logic, and in this way avoid triviality. However, by doing so, one weakens the theory extremely: all applications of inferential rules which are classically correct but incorrect according to the paraconsistent logic, are rejected. From the viewpoint of dialectics, one would expect that it makes a difference which specific contradiction has been discovered, and I think this requirement is sensible and justified in most contexts. Indeed, consider a theory T which was intended to be consistent, but proved to be inconsistent. If we try to find a consistent improvement T^* for T, we need to be able to define "what was intended by T". This obviously was not T itself, for, being based on classical logic, it is trivial; but neither was it the result of replacing classical logic by some paraconsistent logic, for this result is obviously much weaker than what was intended by T". To take a simplistic example, if the axioms of the theory are {~p, s, ~r, qvp, s⊃r}, then no (static) paraconsistent logic will enable us to derive q from it, but nevertheless the derivable contradiction r&~r is not in any way related to the derivation of q from ~p and qvp. Hence, it seems sensible to say that the theory was intended to contain q.

What we are looking for is the theory *in its full strength, except for the pernicious consequences of its inconsistency.* In order to define this, we need to be able to *localize* inconsistencies, and retain all classical rules in deriving consequences from theorems of the theory whenever the inconsistencies are not relevant. One of the troubles with this program is that a circularity is involved. The fact that some statements, viz. contradictions, are derivable, will prevent other statements from being derivable. I discovered the logic presented below by looking at this problem in a way which is typical for Leo Apostel's philosophy, viz. by phrasing the problem with respect to concrete steps made within concrete proofs. After the problem was solved this way, I was able to characterize the notion of derivability in a systematic way, and to show that, with respect to some dynamic dialectical logics, the set of derivable statements is independent of the arbitrary way in which the proof may proceed (see below for a more exact statement).

Consider the rule of Disjunctive Syllogism:
~A, AvB / B
According to all paraconsistent logics of the preceding section, Disjunctive Syllogism is invalid (if both ~A and A are true, ~A and ~ AvB are true even if B is false). Hence, according to the aforementioned idea, we would like to be able to apply Disjunctive Syllogism just in case ~A and A are not both derived. Now notice that, in PI as well as in all of its extensions,

$$\vdash (A\&\sim A)\vee((\sim A\&(A\vee B))\supset B)$$

which may be read as: either both A and ~A are true (according to some theory) or else B is derivable from (the theory if) ~A and AvB (are derivable from it). In general, whenever

$$A_1, \ldots A_n \vdash_{PC} B$$

we have either (5.1.) or (5.2).

(5.1) $\vdash_{PI} A_1 \supset (...\supset(A_n \supset B)...)$

(5.2) For some subformulas $C_1, ..., C_m$ of $\{A_1,...A_n,B\}$,
$\vdash_{PI} (C_1 \& \sim C_1) v ... v (C_m \& \sim C_m) v (A_1 \supset (...\supset (A_n \supset B)...))$.

In the case of (5.1) B is (unconditionally) derivable from A_1, ..., and A_n; in the case of (5.2) it is derivable from them if each of the C_i behave consistently. The conditional rule to be applied in the proof will be: derive B from the A_i, unless one of the contradictory disjuncts of (5.2) has already been derived.

After we applied some conditional rule to derive some conclusion B from some set of statements in the proof, we might derive one of the contradictory disjuncts of (5.2). At this later time, the conclusion B will not be derivable any more. To keep track of this, we shall delete the line where B occurs and all lines containing statements derived with the help of B occur. In this way we ensure that all statements which occur in some (non-deleted) line of a proof, are derivable "at the time" of the last line. Notice that inconsistencies might be deleted. Just as statements derivable at some time, which may not be derivable at some later time, it is equally possible that statements not derivable at some time may become derivable at a later time. In this way we arrive at a logic which reacts to specific inconsistencies and the proofs of which have a dynamic character that is directly related to the occurrence of inconsistencies.

One slight complication has to be considered. In view of the previous story, some proofs from {~p,~q, pvq} will contain p and not q, whereas others will contain q and not p. The accidental way in which the proof proceeds, determines the statements derivable in the proof. In order to avoid this, we require that one should not rely on the consistent behaviour of some statement C, whenever some permutation of

(5.3) $(D_1 \& \sim D_1) v ... v (D_k \& \sim D_k) \& (C \& \sim C)$

has been derived whereas

(5.4) $(D_1 \& \sim D_1) v ... v (D_k \& \sim D_k)$

(or some permutation of it) has not been derived. In such cases we say that $D_1, ..., D_n$ and C are *connected* with respect to their consistent behaviour. According to this modification, neither p nor q is finally derivable (cf. below) from the aforementioned set. The propositional logic I just described is called DPI in [17]. I refer the reader to that paper for an exact statement of the rules and for examples of proofs. By an *intelligent extension* of some proof I mean an extension in which no statement of the form (5.3) is derived if the corresponding statement of the form of (5.4) is derivable (and this is decidable!). A statement is *finally derived* in some DPI-proof if there is no intelligent extension of this proof in which it is deleted. A statement is *finally* DPI- *derivable* from some set of premisses if there "is" a DPI-proof from this set in which the statement is finally derived. Let $Cn_{DPI}(\alpha)$ be the set of statements that are finally DPI-derivable from α. This set is decidable (for finite α) and may be *characterized in terms of the static logic* PI. If α is consistent, $Cn_{DNP}(\alpha) = Cn_{PC}(\alpha)$; if it is inconsistent $Cn_{PI}(\alpha)$ is a subset (in all but some uninteresting vorder cases a real subset) of $Cn_{DPI}(\alpha)$. All proofs are in [17], and so is a semantics, its metatheory, etc. A further interesting point demonstrated there is that it is preferable in general to start from PI, rather than from some propositional extension of PI. Incidentally, the minimal predicative extension of PI and its extensions do not present any specific problems, except for the usual lack of decidability (which has consequences for the notion of final derivability).

All this shows that it is possible to articulate a nice, full- grown formal deductive logic that accounts for the first dynamic aspect of dialectical logic, viz. the changes to be introduced after the discovery of inconsistencies. Moreover, the solution I presented is deterministic in the sense that the premisses (or nonlogical axioms) alone, and not the accidental way in which the proof proceeds, determine the set of final consequences. At the end of the preceding section I mentioned Leo Apostel's idea of a series of dynamic logics, ordered by some metalogic. DPI may be seen as an explication of this idea. At any stage (or "time") of the proof, there is a set of correct inferences (specified with respect to specific statements), and in this sense a "logic". Whenever a contradiction is derived (or some "new" disjunction of contradictions) this "logic" is modified and possibly leads to new contradictions (and other statements) as well as to the deletion of old contradictions (and other statements). This corresponds to Apostel's statement that each logic produces new contradictions and resolves old contradictions. DPI functions as the aforementioned "metalogic": it generates the next "logic" in function of the state of the proof; hence, the production and resolution of contradictions is strictly determined by the contradictions that occur in the proof and caused the transition to the new "logic".

I now turn to the second dynamic aspect of deductive dialectical logic, viz. the resolution of inconsistencies. I already used this term in the preceding paragraph, but the resolution of contradictions discussed there is a *weak* resolution. The *strong* resolution, which we should consider now, concerns the fact that the derivation of a contradiction itself starts a dynamics which aims at and eventually in the resolution of the contradiction. Leo Apostel tries to specify the strong resolution (p. 83-84) and it seems worthwhile to paraphrase and comment his proposal. He introduces the following eight conditions.

(1) The strongest contradiction(s) of the theory T_i are eliminated.
(2) The inconsistent T_i is replaced by an inconsistent T_{i+1}.
(3) The relation between T_i and T_{i+1} must be analogous to the relation between T_{i-1} and T_i (pragmatic notion dependent on time).
(4) T_{i+1} is not a subtheory of T_i but of some extension ofT_i.
(5) T_{i+1} contains statements and proofs that are on the average less distant from the statements and proofs of T_i than are the statements and proofs of all alternative T'_{I+1} which also eliminate the strongest contradiction(s) of T_i.
(This distance should be measured in a way independent of any specific axiomatizations of the theories).
(6) More specifically, T_{i+1} should contain (in comparison to T'_{i+1}) as much as possible of the proofs that lead in T_i to "parts" of the strongest contradiction(s) or in which such "parts" are employed as premisses.
(7) T_{i+1} should be weakly maximal with respect to the extension of T_i mentioned in (4). This means that, if any nontheorem of T_{i+1} which is a theorem of this extension, is added to T_{i+1}, then the result contains a contradiction stronger than any contradiction in T_{i+1}.
(8) T_{i+1} should neither be the union nor the intersection of the consequence-sets of all weakly maximal subtheories of the extension (see 4) of T_i . This is an implicit rejection of a Rescher-like solution of the problem (see "weak consequence" and "strong consequence" in (*18*).

In justification of these conditions, Leo Apostel refers to Hegel's and Marx's ideas of transcendence with conservation (as much as possible is retained from previous stages, and the evolution tends asymptotically towards, but never reaches, some final coherent theory).

Before discussing these conditions for the *strong* resolution of contradictions, let me return for a moment to DPI and consider the extent to which the *weak* resolution arrived at by means of this logic satisfies the conditions. Let us define a theory T_i for each stage (time) of some DPI-proof in the most natural way available, viz. as the PI-consequence set of the union of the set of premisses and of the set of (nondeleted) statements in the proof. (The subsequent reasoning would also work, although in a different way, if we considered DPI-consequence sets, but it seems more natural to define the theories from the sequence in terms of some static logic). If we apply a rule of inference which is PI-valid and the result of this application does not lead to the deletion of any previously derived statement, we stay within the previous theory (strictly, T_{i+1}=T_i). If we apply a rule which is only conditionally valid, i.e. invalid in PI but valid in DPI in view of the absence of one or more (disjunctions of) inconsistencies, we face an *enrichment* of the previous theory. Moreover, new inconsistencies may become derivable as a consequence of this enrichment. Finally, if we apply a rule which is PI-valid, and the result of which leads to the deletion of previously derived statements, the resulting theory is a *subtheory* of its predecessor, and possibly eliminates some of its inconsistencies. If the rule is only conditionally valid, but leads to the deletion of a previously derived statement, we face an enrichment as before, but, as we immediately delete, we at once move to a subtheory of this extension. I leave it to the technically somewhat skilled reader to verify the following: under this interpretation of 'theory', DPI fails to satisfy (1), but in a specific and precise sense, and with respect to some proofs each of (2) and (4) to (8) is exemplified (although not true for any couple of consecutive theories). I fail to understand (3), and hence cannot decide whether it may be exemplified.

There is one *caveat*. On the level of propositional logic, a clever person might first make some moves within the original theory, and next run through the complete series of consecutive extensions without deleting anything. Nevertheless, I have shown that all but two of the conditions listed by Leo Apostel may be exemplified within strictly defined formal-logical deductive proofs, and this notwithstanding the fact that these conditions must sound absolutely weird to narrow-minded classical logicians. One may even do a lot better (viz. fulfil (1) also, and fulfil some other conditions in a "deeper" way) by introducing external preferences, apart from the formal-logical rules, but I take it that I already made my present point, and that it was important to make it with respect to mere formal logic.

I now return to the discussion of the conditions (1) to (8) in connection with the strong resolution of inconsistencies. Having been laudatory before, I shall now try to be maximally critical (but brief, as I already tried to spell out analogous problems in the present section and in section 2). In order to set things clear, I interpret the theories as consecutive successful scientific theories, or sets of such theories, about some domain. I provisionally disregard complications connected with the fact that, in the most interesting cases, there are contemporary competitors.

Ad (1). This and several other conditions should be rephrased with respect to the state of our knowledge. No one tries or even may try to eliminate and undiscovered inconsistency. Moreover, I would advice people to try to eliminate not the strongest, but rather the most embarrassing contradiction. Worse, if this contradiction seems beyond one's reach, there is nothing wrong in attacking a less embarrassing contradiction. E.g., to eliminate Russell's paradox by some paraconsistent set-theory seems sensible, even if the by far more embarrassing (and stronger) Curry-paradox remains unsolved.

Ad (2). However, if you see the chance to get a (as far as you know) consistent theory, take it.

Ad (3). I already said that I do not understand this condition. Whatever it means, it seems to me that I should add: or do something better if you have enough imagination.

Ad (4). This is, of course, the BIG PROBLEM discussed in section 2. How do we generate such an extension? The extension produces by applications of conditional rules in DPI (see the penultimate paragraph) are clearly too weak in the present context. I still return to this point in the sequel.

Ad (5), (6) *et* (7). I would reduce these conditions to *ceteris paribus* conditions. If T_{i+1} is not merely a somewhat less inconsistent version of the preceding theory, but a really progressive alternative, these conditions are minor worries.

Ad (8). I agree.

I realize that my criticism is unjust to Apostel in the sense that one of his main aims was to clarify Hegel's and Marx's dialectics. I have the excuse

of being not a historian of philosophy. Still, I want to Point to the following problem. Since Apostel is convinced of the present-day importance of those authors' work on dialectics, he tries to support his interpretation of dialectics by adducing arguments which refer to our contemporary views on science and knowledge. However, our insight in and view on science, and human knowledge in general, has chanted and improved a lot since the days of Marx and, especially Hegel. In view of this and in view of the fact that the standards of clarity and explicitness also have changed, Apostel has to disentangle Hegel's and Marx's theories on dialectics, in order to separate their "essence" from the historical formulation. Such an enterprise is always somewhat risky. Moreover, especially as Apostel himself stresses the importance of furthering and improving dialectical logic and dialectics in general, I am not sure whether dialectical logic should at least offer a precise formulation of the things the great dialecticians Hegel and Marx were actually doing (p. 86). If the dialectical tradition is to have a future, especially after the long years of mainly mere lip- service, it will gain this by the development of new theories within the tradition, not merely by the study and refinement of old theories. In this sense, it seems to me, my criticism is not completely unfair.

There are other situations to which one might apply Apostel's conditions (1)-(8). One of these concerns cases in which an inconsistent theory of some research-tradition is replaced by another, consistent or less inconsistent, theory within the same tradition. In some such cases, it seems sensible to suppose that all the conditions, except probably for (3), do obtain. The reason why I believe that the condition hold in some such cases, is that I believe that the dialectical type of evolution may be adequate if there is no need to reject some part of a theory, but merely a need to reformulate it in view of the elimination of some unwanted consequences, viz. contradictions which, although they affect central parts of the theory, are seen as more deriving from its formulation than from its contents. Perhaps the reformulation of Newtonian mechanics with the help of a decent analysis is a good example, but I must confess that I cannot offer the describe historical documentation right now.

Irrespective of the correctness of my conjecture in the preceding paragraph, it is clear that the most important, fruitful and dynamic episodes of the history of some science are those in which theories about the same domain but belonging to different research traditions coexist, or where a new research

tradition comes into being as a consequence of the formulation of a new, promising theory about the domain (see, of course, especially,). Ever since Thomas Kuhn's *The Structure of Scientific Revolutions*, people have been fascinated by the drastic conceptual differences between "the old theory" and "the new theory". Whether such changes occur all of a sudden on all levels, as Kuhn wants it, or are the result of smaller, justifiable shifts, as I believe with Laudan , is not so important. The main point is that, at least after some period of time, old beliefs are fully rejected and not, as dialectics requires, retained and incorporated. Leo Apostel is aware of this and for this reason localizes retention in the relation between two successive theories only, and moreover replaces the absolute requirement of retention by the relativized requirement ("less distant from the old theory than the alternatives"). I have tried to argue, however (and referred to arguments in the literature in section 2) that even this weakened retention should not be taken too serious.

My criticism of Apostel's view on the dialectical resolution of contradictions within a dialectical logic mainly concerns the notion of an enrichment and the weaker, incorporated notion of retention. I have shown that a weak resolution is found in proofs governed by DPI, and is unobjectionable. In [19] I have even shown that some variants of DPI, which may be adequate in some contexts, exemplify forms of weak resolution which are closer to the dialectical view. On the other hand, my conclusions were rather negative in connection with what I called the strong resolution of contradictions (see the paragraph preceding the list of the eight conditions). Yet, and although I realize that the reader might think my fascination for the resolution of contradictions has turned into a neurosis, I again wend to come back to the problem in the next section.

6. Leo Apostel has shown that dialectical mechanisms are exemplified in modern science and may be described in accordance with contemporary standards. He offered an essential contribution to the development of the logical theories needed for the formalization of the dialectics of nature. In attacking the formalization of *Capital* by means of action logic, he launched a promising research programme which is essential to the human sciences. Finally, he laid the basis for a dialectical description of both inconsistency-caused changes of belief at specific points and the overall dynamics of scientific theories and of belief systems in general.

After having stated all this, it seems to me, I should repeat the question from the second section, viz. about the status of Hegel's and Marx's dialectics. One might specify the question into three subquestions: Is there something fundamentally, even if only approximatively, correct in dialectics, in that it more or less accurately describes actual processes? Is dialectics interesting as a source of inspiration? Is dialectics as a research tradition promising enough to deserve being kept alive by the creation of new dialectical theories? In all the questions I mean dialectics as it is seen and was presented by Apostel, and I direct these questions to him. In participation of his answers, I formulate the conclusions to which I came after studying his work.

I have no doubts that the first question has to be answered in the positive. At the outset of this section I stated already that Apostel succeeds in showing that dialectics is in a more or less precise way exemplified in different domains.

My answer to the second question is positive in a specific respect only. Dialectics is not an important source of inspiration in the sense that it offers clear guidance to those who confront a new problem of some type. Let me refer to two examples. Some of the people working in paraconsistent logic were motivated by dialectics, but the so-called static dialectical logics they came up with do not seem to do much justice to the great laws of dialectics. Also, the problem of the discovery of new theories seems an excellent case for dialectics to prove its inspiring force. Yet, dialectical inspiration has led to little or nothing in this respect, and even Leo Apostel, who could not escape from touching upon the problem at least in an implicit way, comes up with only a poor set of formal guidelines some of which I argued to be mistaken.

Nevertheless, it seems to me that dialectics may be used as a source of inspiration in a weaker way. Once one arrived at a sketchy solution which appears to involve dialectical properties, e.g. to contain contradictory stages which are in some more or less determined way connected to subsequent consistent or less inconsistent stages, then dialectics might play a useful hypothesis-generating role. In this sense dialectics played a role in my articulation of PDI. In the same sense dialectics might be useful to the study of the logic of discovery to someone who starts from Thomas Nickles's main paper in . Drastically simplified, the situation may be described by saying that a fullfledged theory is reached by starting from some idea or minitheory,

combined with a set of *constraints* which derive from previous, rejected theories (from their theoretical structure, or from their factual generalizations, or from their supporting evidence, etc.). The constraints are accepted as "setting the problem" by the author of the new theory; they may by all means be inconsistent (either separately or in combination). The new theory is worked out step by step. The constraints do not play a merely negative heuristic role, but also a positive one in that new parts of the (new) theory may derived from the previous stage of the theory together with one or more constraints, even if these by themselves contradict either the result or later stages of the theory. The dialectical aspects of the situation are quite clear, and even the notions of retainment and enrichment may be given a clear sense. For this reason, it seems obvious that anyone who has the time to devise dialectical logics specific for such situations, should immediately engage in doing so.

I now come to the last subquestion, viz. the one concerning the promising character of the dialectical research tradition. Again my main worry concerns enrichment and retainment. It seems to me that I have conceded as much as I could up to now. Even with respect to the logic of discovery programme from the last paragraph (i) the source of the enrichment, viz. the mini-theory, is not determined by the constraints, (ii) rather, the acceptability of something as a constraint, and its importance as a constraint, is determined to a large extent by the mini-theory (especially if the "something" is a theoretical statement, and (iii) the final result (the full-fledged theory) will be largely dependent on the presumably largely accidental order in which the constraints are integrated in the theory. In other words, I concede that our imagination is too limited to come up with completely new theories, and that we and the world are such that we are unable to gain any new knowledge if we question the totality of our previous knowledge. Yet, both the enrichment and its retained parts are severely underdetermined by the previous contradiction.

It seems to me that one may easily substantiate the same thesis in connection with dialectics outside of our thinking, say in economics or biology. Even if the world is completely deterministic, I see no reasons to believe that (real) contradictions alone, i.e. independent of the rest of the state of affairs, strictly determine the evolution of the world. Moreover, it seems to me that the facts cry out that not all resolutions of contradictions are enrichments, and that even the better aspects of the previous stage are not always

retained.

These are the reasons why I refuse to believe that dialectics is a viable research tradition, if a deterministic view on enrichment and retainment is essential to it. Taken in a vague sense (and considered of restricted application only), it is a truism. If it is weakened to relative requirements, hardly anything of a general dialectical theory will remain. If it is taken to hold strictly and in general, it leads to an extremely simplistic view on evolution, a view which is moreover both false and empty. Indeed, to say that anything produces its negation, and that any contradiction produces its resolution, which contains both sides of the contradiction, is about the most simplistic view on evolution one might imagine, on condition that one is willing to accept true contradictions. Furthermore, this view is clearly false. Finally, as "its negation" and "its resolution" are completely undefined, it is empty.

I realize that Leo Apostel will disagree, especially with my answer to the last question, and that he has arguments for defending his position. I only hope that the strongest criticism I could produce will enable him to offer a bold reply, from which the reader will profit.

7. Here ends a paper which will at best be *eine Unvollendete Kakaphonie*. Unfinished, because it is only a far cry, even on the descriptive level, of a full discussion of Leo Apostel's ideas on dialectics. A *Kakaphonie* because I am rather incompetent on dialectics, unfamiliar with writing this type of paper, and unable to find an equilibrium between my admiration and agreement on the one hand, and my profound disagreement on the other. Having jumped from one point to the other, I am not sure whether I familiarized the reader with this aspect of Apostel's philosophy and with the problems it is concerned with and I am not sure whether this paper will stimulate Leo Apostel to write the replies which will be most helpful for the clarification of this aspect of his philosophy. Nevertheless, I hope at least to have induced in the reader to some degree the stimulation I derived by writing this paper, and to some extent the admiration and love I feel.

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