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The sense of Non-Dialetheic Paraconsistency. The compatibility relation as an example, with an application to Connexivity.

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Outline

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Two paraconsistent communities?

Dialetheists: aiming at developping a view (on the world, on knowledge and on logic) that agrees with their philosophical tenets:

· there are true inconsistencies

· there is a single 'vernacular', fit to describe its own full metatheory (opposing a Tarskian hierarchy of languages)

· a specific logic is "**the true** logic of the vernacular", e.g., for Priest **LP**, for Routley this is a weak relevant logic (e.g. missing contraction), for Brady his "universal logic"; Zach Weber has clear requirements on the logic, but did not claim to have located it.

Non-dialetheists

What do they do and why?

general answer:

- often also work on problems in non-paraconsistent logics or on general problems
- each one develops the own view on logic (or no view)
- sometimes use paraconsistent logics in a more pragmatic way (e.g. with classical negation, consistency operator, etc.)
- like intuitionist logic, modal logics, relevant logics, etc., paraconsistent logics enable one to define certain concepts and make certain discriminations that cannot be defined/made in terms of **CL** (and other 'older' logics)

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Yet, they have certain committments:
- inconsistent descriptions/theories are possible, conceivable, sensible and are an informative, usefull, necessary, . . . ingredient of our KS
- inconsistent theories *may* be true (logical, factual, pragmatic, . . . possibility)

and, just as much as dialetheists, have been despised, insulted, scolded at, . . . by those who knitted a simplistic and confused ideology around **CL**

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∗ explicate the reasoning that accompanies replacing inconsistent theories by consistent ones
  · minimally inconsistent interpretations [Batens, 1985]
  · explicate the historical removal of inconsistencies from scientific disciplines
    this removal is a defeasible process and its explication requires (dynamic) proofs
  · clarify possible sources of inconsistency in (mathematical) theories, e.g. **PA**
  · define adaptive *Fregean* set theories
  · ...

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adaptive logics are intended to unify the domain of defeasible reasoning; inconsistency-adaptive logics are merely a special (but typical) case

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list of some properties of adaptive Fregean set theories

∗ presenting two new notions of compatibility
they can only be characterized by means of a paraconsistent logic; I shall compare the notions wrt some applications.

All I shall need and say on adaptive logics:

Let $\mathcal{L}$ be a language schema and $\mathcal{W}$ its set of closed formulas. An adaptive logic in standard format, **AL** : $\wp(\mathcal{W}) \to \wp(\mathcal{W})$ is defined by a triple:

(1) A *lower limit logic* **LLL**: a logic that is defined over $\mathcal{L}$, has *static proofs* in $\mathcal{L}$.
simplification relies on *trusty semantics*' [Batens, 2021].

(2) A decidable *set of abnormalities* $\Omega \subseteq \mathcal{W}^*$: a set of formulas characterized by a (possibly restricted) logical form F; or a decidable union of such sets.

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For all adaptive logics in standard format, the format defines by generic means the semantics, the proof theory, and a very extensive meta-theory (soundness, completeness, stopperedness, etc., etc. — see [Batens, 2007] and several later results).

no further comments on ALs

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Adaptive Fregean set theories (AFS)

Diderik Batens. Adaptive Fregean set theory. *Studia Logica*, 108:903-939, 2020. Published online: 10 November 2019. *cf. Peter Verdée*

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- (local) triviality constant present: $\bot =_{\rm df} \forall x \forall y(y = x \wedge y \neq x \wedge y \in x \wedge y \notin x)$
- *classical negation* definable: $\check{\neg} A =_{\rm df} A \supset \bot$
  $\check{\neg}$ verifies exactly the same rules and semantic clause as the **CL**-negation
- the classical Russell set is definable $R^c =_{\rm df} \{x \mid \check{\neg} x \in x)$
- $\vdash R^c \in R^c$ and $\vdash \neg R^c \in R^c$ but $\nvdash \check{\neg} R^c \in R^c$.

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- for every inconsistent set S, there are *consistent* sets S_0 and S_1 such that $\forall x((x \in S_1 \equiv x \in S) \wedge (x \in S_0 \equiv x \notin S))$. S_0 and S_1 may exist in **NF**, not in **ZF**.

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- If a set theory **ST** is consistent, then there is an adaptive AFS such that (i) all **ST**-sets are consistent AFS-sets and (ii) AFS has further inconsistent and consistent sets. Many definable sets turn out to be consistent but not **ST**-sets. (corrected !)

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let $!A$ abbreviate $A \wedge \neg A$

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- every **CL**-model $M = \langle D, v \rangle$ is described in a language $\mathcal{L}_+$, which extends $\mathcal{L}_s$ with a set of pseudo-constants $\mathcal{O}$ such that $\{v(\alpha) \mid \alpha \in \mathcal{C} \cup \mathcal{O}\} = D \qquad (\#(\mathcal{C} \cup \mathcal{O}) \geq \#D)$

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  (D1) iff $\Gamma \nvdash_{\mathbf{CL}} \neg A$, viz. $\circ(\Gamma \cup \{A\})$, and
  (D2) iff $\not\diamond\, \Gamma$ or $\not\diamond\, A$ or $\circ(\Gamma \cup \{A\})$
  (i) set of $\Gamma \subseteq \mathcal{W}_s$ that have a model is not (even) semi-recursive
  (no positive test for consistency)
  (ii) so (D1) and (D2) are *computationally* weak.
  in view of (i), no Tarski logic characterizes compatibility
  but an adaptive logic in Standard Format does

To save time, I skip the characterization of classical compatibility in terms of an adaptive logic.

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- A **S5**-model $\mathcal{M} = \langle \Sigma_\Delta, M_0 \rangle$, in which $\Delta \subseteq \mathcal{W}_s$, Σ_Δ is the set of **CL**-models of Δ, and $M_0 \in \Sigma_\Delta$

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The valuation $v_\mathcal{M}$ determined by a **S5**-model $\mathcal{M} = \langle \Sigma_\Delta, M_0 \rangle$ is defined by:

C1 where $A \in \mathcal{W}_s$ is an atomic formula, $v_\mathcal{M}(A, M_i) = v_{M_i}(A)$
C2 $v_\mathcal{M}(\neg A, M_i) = 1$ iff $v_\mathcal{M}(A, M_i) = 0$
C3 $v_\mathcal{M}(A \wedge B, M_i) = 1$ iff $v_\mathcal{M}(A, M_i) = v_\mathcal{M}(B, M_i) = 1$
C4 $v_\mathcal{M}((\forall \alpha)A(\alpha), M_i) = 1$ iff $v_\mathcal{M}(A(\beta), M_i) = 1$ for all $\beta \in \mathcal{C} \cup \mathcal{O}$
C5 $v_\mathcal{M}(\Box A, M_i) = 1$ iff $v_\mathcal{M}(A, M_j) = 1$ for all $M_j \in \Sigma_\Delta$

as usual: $\mathcal{M} \Vdash A =_{\text{df}} v_\mathcal{M}(A, M_0) = 1$
$\mathcal{M} = \langle \Sigma_\Delta, M_0 \rangle$ is a model of Γ iff $\Gamma \subseteq \mathrm{Cn}_{\mathbf{CL}}(\Delta)$
$\Gamma \vDash_{\mathbf{S5}} A$ iff A verified by every **S5**-model of Γ.

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The adaptive logic **COM** is defined by the triple:
(i) lower limit logic: **S5** [specific version simplifies the proofs],
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Conventions: $\vDash_{\mathbf{COM}} \Diamond A =_{\mathrm{df}} \emptyset \vDash_{\mathbf{COM}} \Diamond A$ and $\vDash_{\mathbf{COM}} \Diamond\Gamma =_{\mathrm{df}}$ for all $\Delta \subseteq_{\mathrm{fin}} \Gamma$, $\vDash_{\mathbf{COM}} \Diamond \bigwedge(\Delta)$ viz.∘Γ
Theorem $A \rhd_1^c \Gamma =_{\mathrm{df}} \Gamma^{\Box} \vdash_{\mathbf{COM}} \Diamond A$ $(\mathcal{W}_{s+}/\mathcal{W}_{+}^{\Diamond})$ and
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$\mathrm{Ab}(\Gamma) =_{\mathrm{df}} \mathrm{Cn}_{\mathbf{S5}}(\Gamma) \cap \Omega$
line *i* with condition Δ is marked at stage *s* iff an $A \in \Delta$ is derived on condition $\emptyset$ at *s* (marking for Simple strategy)

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Theorem $\Gamma^{\Box} \vdash_{\mathbf{COM}} \Diamond A$ iff $\Gamma^{\Box} \vDash_{\mathbf{COM}} \Diamond A$ (by the 2001 generic proof [Batens, 2007])

Basic insight in dynamic proofs:

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i	$\Diamond A \vee \neg\Diamond A$	RU	$\emptyset$
$i+1$	$\Diamond A$	i, RC	$\{\neg\Diamond A\}$

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· line $i+1$ marked iff $\neg\Diamond A$ is derived on condition $\emptyset$

· $\Diamond A$ finally **COM**-derivable from $\Gamma^{\Box}$ iff $\Gamma^{\Box} \nvdash_{\mathbf{S5}} \neg\Diamond A$ (defines complexity)

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· $\Diamond A$ finally **COM**-derivable from $\Gamma^{\Box}$ iff $\Gamma^{\Box} \nvdash_{\mathbf{S5}} \neg\Diamond A$ (defines complexity)

properties of $\rhd_1^c$ and $\rhd_2^c$ ·

if $\not\vdash A$, then, for all Γ, $A \not\rhd_1^c \Gamma$ and $A \rhd_2^c \Gamma$ problematic

ex.: $p \wedge \neg p \not\rhd_1^c \{p \wedge \neg p\}$ and $p \wedge \neg p \not\rhd_1^c \{p, \neg p, q, \neg q\}$

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Basic insight in dynamic proofs:

...	...	...	...
i	$\Diamond A \vee \neg\Diamond A$	RU	$\emptyset$
$i+1$	$\Diamond A$	i, RC	$\{\neg\Diamond A\}$

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(D1) also comes to: $\Gamma \cup \{A\}$ has a **CL**-model.

(D2) also comes to: if A and Γ have **CL**-models, then so does $\Gamma \cup \{A\}$.

(D2) leads to 'duals' of problems of (D1): if ϕA, then, for all Γ, $A \rhd_2^c \Gamma$; etc.

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Replace **CL** by a different logic?

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paraconsistent logics have models for inconsistent sets

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Suggestion
A syntactic approach may overcome this: a model of Γ that verifies A may be more inconsistent (verify more inconsistencies) than other models of Γ.

Outline

More Sensible Alternatives

Alternative 1: Minimal inconsistency Comp. Intuitive desiderata

- $A \not\triangleright^m \Gamma$ iff $\Gamma \cup \{A\}$ is *more* inconsistent than Γ.

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- $\triangleright^m$ is possibly asymmetric: $p \triangleright^m \{!q\}$ **but** $!q \not\triangleright^m \{p\}$ and $!q \triangleright^m \{!p \wedge !q\}$ **but** $(!p \wedge !q) \not\triangleright^m \{!q\}$.

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These ideas were behind Meheus' paper on *paraconsistent compatibility* [Meheus, 2003], but there was a mistake in the dynamic proof theory – the SF was not yet articulated in those days.

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These ideas were behind Meheus' paper on *paraconsistent compatibility* [Meheus, 2003], but there was a mistake in the dynamic proof theory – the SF was not yet articulated in those days. This is easily corrected if **L**=**CLuN** extended with a classical negation $\check{\neg}$.

Alternative 2: Relational Comp. Intuitive desiderata

- idea: $\Delta \not\triangleright^r \Gamma$ requires *conflict between* Δ and Γ.
- if no non-logical term occurs in both A and (members of) $\mathrm{Cn}^{\mathcal{L}}_{\mathbf{L}}(\Gamma)$, with $\mathcal{L}$ the language of $\langle \Gamma, \mathbf{L} \rangle$, then $A \triangleright^r \Gamma$ e.g. $p \wedge \neg p \triangleright^r \{q\}$ Ⓢ

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· if $A \in \mathrm{Cn}^{\mathcal{L}}_{\mathbf{L}}(\Gamma)$, then $A \triangleright^r \Gamma$
· $A \not\triangleright^r \Gamma$ iff $\neg A \in \mathrm{Cn}^{\mathcal{L}}_{\mathbf{L}}(\Gamma)$ and $A \notin \mathrm{Cn}^{\mathcal{L}}_{\mathbf{L}}(\Gamma)$

example: avoid that $\neg(p \vee \neg p) \not\triangleright^r \Gamma$ iff $\vdash_{\mathbf{L}} A \vee \neg A$ and $\neg(p \vee \neg p) \notin \mathrm{Cn}_{\mathbf{L}}(\Gamma)$
- define $A \triangleright^r \Gamma =_{\mathrm{df}} A \in \mathrm{Cn}^{\mathcal{L}}_{\mathbf{L}}(\Gamma)$ *or* $\neg A \notin \mathrm{Cn}^{\mathcal{L}}_{\mathbf{L}}(\Gamma)$ and define $\Delta \triangleright^r \Gamma$ iff, for all $A \in \mathrm{Cn}^{\mathcal{L}'}_{\mathbf{L}'}(\Delta)$, $A \triangleright^r \Gamma$.

$\triangleright^r$ is non-symmetric (like $\triangleright^m$). Example: $p \triangleright^r p \wedge \neg p$ but $p \wedge \neg p \not\triangleright^r p$

Some insights on proofs / propositional (= decidable) examples
WEG

Are $\neg q$, q, s, $\neg s$, $\neg p$, $p \wedge s$, $r \wedge \neg r$ and s compatible with $\Gamma = \{\neg p, q \wedge \neg q\}$?

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(A) $\mathbf{CLuN}^m$-consequences of $\Gamma^{\check{\neg}\neg}$:

1	$\check{\neg}(r \wedge \neg r) \vee (r \wedge \neg r)$	RU	$\emptyset$
2	$\check{\neg}(r \wedge \neg r)$	RC	$\{r \wedge \neg r\}$
3	$\check{\neg}\neg\neg p$	Prem	$\emptyset$
4	$\check{\neg}\neg(q \wedge \neg q)$	Prem	$\emptyset$

As r does not occur in Γ, line (2) will not be marked in any extension of the proof: so $\check{\neg}(r \wedge \neg r)$ is a final $\mathbf{CLuNs}^m$-consequence of $\Gamma^{\check{\neg}\neg}$.
lines (3) and (4) will not be marked either; on them (transformed) premises are stated unconditionally.

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CLuNs nodig!

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5	$\check{\neg}\neg\neg q$	2, RU	$\emptyset$ CLuNs nodig!

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8	$\neg\check{\neg}s$	RC	$\{\check{\neg}s\}$
9	$\neg\check{\neg}\neg s$	similarly	$\{\check{\neg}\neg s\}$

· lines (1)–(9) not marked in any extension of the proof: so $s \triangleright^m \Gamma$ and $\neg s \triangleright^m \Gamma$; ...

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13	$\neg\check{\neg}(p \wedge s)$	RC	$\{\check{\neg}(p \wedge s)\}$	

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9	$\neg\check{\neg}\neg s$	similarly	$\{\check{\neg}\neg s\}$	
10	$\neg\check{\neg}q$	similarly	$\{\check{\neg}q\}$	$\checkmark^{11}$
11	$\check{\neg}q$	5, RU	$\emptyset$	
12	$\neg\check{\neg}q$	4; RU	$\emptyset$	
13	$\neg\check{\neg}(p \wedge s)$	RC	$\{\check{\neg}(p \wedge s)\}$	
14	$\neg p$	1; RU	$\emptyset$	

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WEG

Are $\neg q$, q, s, $\neg s$, $\neg p$, $p \wedge s$, $r \wedge \neg r$ and s compatible with $\Gamma = \{\neg p, q \wedge \neg q\}$?

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1	$\check{\neg}\neg\neg p$	Prem	$\emptyset$	
2	$\check{\neg}\neg(q \wedge \neg q)$	Prem	$\emptyset$	
3	$\neg\check{\neg}\neg p$	1, RU	$\emptyset$	
4	$\check{\neg}\neg q$	2, RU	$\emptyset$ CLuNs nodig!	
5	$\check{\neg}\neg\neg q$	2, RU	$\emptyset$ CLuNs nodig!	
6	$\neg\check{\neg}\neg q$	5, RU	$\emptyset$	
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16	$\neg\check{\neg}(r \wedge \neg r)$	RC	$\{\check{\neg}(r \wedge \neg r)\}$	

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16	$\neg\check{\neg}(r \wedge \neg r)$	RC	$\{\check{\neg}(r \wedge \neg r)\}$	$\checkmark^{3}$
17	$\check{\neg}(r \wedge \neg r)$	Prem	$\emptyset$	

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te herzien

WEG

Some *'laws'* about relational compatibility: (Γ, Δ and Θ sets of formulas) verder

(i) $((A \rhd^m \Gamma) \vee (\neg A \rhd^m \Gamma))$ and $((\Gamma \rhd^m B) \vee (\Gamma \rhd^m \neg B))$

(ii) if $\circ\Gamma$ and $\circ\Delta$, then $(\Gamma \rhd^m \Delta$ iff $\Delta \rhd^m \Gamma)$ and $(\Gamma \not\rhd^m \Delta$ iff $\Delta \not\rhd^m \Gamma)$.

(iii) if $\circledcirc\Gamma$ and $\circledcirc\Delta$, then $(\Gamma \rhd^m \Delta$ iff $\neg\Delta \not\rhd^m \neg\Gamma)$ and $(\Gamma \not\rhd^m \Delta$ iff $\neg\Delta \not\rhd^m \neg\Gamma)$.

(iv) if $\circledcirc\Gamma$, $\circledcirc\Delta$ and $\circledcirc\Theta$, then $(\neg\Delta \not\rhd^m \Gamma), (\neg\Theta \not\rhd^m \Delta) \vdash_{\mathbf{AL}} (\neg\Theta \not\rhd^m \Gamma)$

(v) $A \rhd^m \Gamma$ iff $(\Gamma \nvdash_{\mathbf{CLuN}_*} \neg A$ or $\Gamma \vdash_{\mathbf{CLuN}_*} A)$.

$$A \rhd^m \Gamma =_{\rm df} \neg\check{\neg} A \in \mathrm{Cn}_{\mathbf{RC}}(\Gamma^{\check{\neg}\neg}) =_{\rm df} \neg\check{\neg} A \in \mathrm{Cn}_{\mathbf{CO}}(\mathrm{Cn}_{\mathbf{CLuNs}^m}(\Gamma^{\check{\neg}\neg}))$$

probleem met (v)? $B \nvdash_{\mathbf{CLuN}_*} \neg A$ iff $\neg(A \to \neg B)$

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WEG

The first two ‘applications of relational compatibility’: generalizing *falsification* and *content* to the inconsistent case.

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plus, elsewhere, I argued that removing inconsistencies *ceteris paribus* causes a richer or a more precise description (also [Meheus, 2002]; comments by Priest [Priest, 2014]) advantages of $\rhd^r$ wrt extending:

· if the best theory in a domain is inconsistent, it can still be extended with compatible statements/theories

· even if a statement A / theory T in a domain is inconsistent, another theory T' / our KS can be extended with T provided it is compatible (derivable, not conflicting, phrased in different language)

in both respects $\rhd^r$ does better than $\rhd^c$ and $\rhd^m$

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Generalizing the notion of *Falsification*

Popper's view: where Γ is the set of non-logical axioms of $T = \langle \Gamma, \mathbf{CL} \rangle$, [the description A of] an *observation* falsifies a theory T, iff A contradicts $\mathrm{Cn}_{\mathbf{CL}}(\Gamma)$, viz. $\neg A \in \mathrm{Cn}_{\mathbf{CL}}(\Gamma)$.

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Proposal: where Γ is the set of non-logical axioms of $T = \langle \Gamma, \mathbf{L} \rangle$ and $\mathbf{L}$ is its underlying logic, A [the description of] an observation falsifies a theory T, iff A is incompatible with Γ, viz. $A \not\triangleright^{r} \Gamma$.

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· if $\mathrm{Cn}_{\mathbf{L}}(\Gamma)$ is consistent, then $A \not\rhd^r \mathrm{Cn}_{\mathbf{L}}(\Gamma)$ iff A contradicts $\mathrm{Cn}_{\mathbf{CL}}(\Gamma)$
· if $\mathrm{Cn}_{\mathbf{L}}(\Gamma)$ is inconsistent but non-trivial, then $A \not\rhd^r \mathrm{Cn}_{\mathbf{L}}(\Gamma)$ offers a sensible notion of falsification. *A conflicts* with the theory: A belongs to the language of the theory, is not a theorem of it, but contradicts the theory's prediction. If both the theory T and A were true, T would still be incomplete (fail to predict a correct observation in its domain).

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Popper: $\mathrm{C}^c(T)$ the Classical content of a (statement or) theory $T = \langle \Gamma, \mathbf{CL} \rangle$ is what T forbids, viz. $\{A \mid \neg A \in \mathrm{Cn}_{\mathbf{CL}}(\Gamma)\}$.
Related: Aristotle: Metaphysics Γ4, in Priest's paraphrase: "For an assertion to have determinate content, it must rule something out. The content is, as it were, what is left open when the possibilities ruled out by the assertion are deleted."

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Priest claims "We can think of the content of a sentence as *the information it carries.* It is then quite possible for sentences *a* and *b* to have different and determinate contents [. . .] if *a* carries *information* that *b* does not, or vice versa."(my italics)

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(1) empirical content: "*a* rules out *A*" neen $\Leftrightarrow$ *observing* that *A* commits one to "*a* is falsified" (commits one to *rejecting a* in Priest's sense).non-falsifiable theories
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Some statements miss all empirical content, whatever they entail.

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if $\mathrm{Cn}_{\mathbf{L}}(\Gamma)$ is inconsistent but non-trivial, then $A \not\between^r \mathrm{Cn}_{\mathbf{L}}(\Gamma)$ offers a sensible notion of falsification: $\mathrm{Cn}_{\mathbf{L}}(\Gamma) \cup \{A\}$ entails a disjunction of contradictions that is not a theorem of T. As no minimally abnormal model M of $\mathrm{Cn}_{\mathbf{L}}(\Gamma)$ verifies A, extending M into a model M' of $\mathrm{Cn}_{\mathbf{L}}(\Gamma) \cup \{A\}$ causes M' to verify abnormalities not verified by M.

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if $\mathrm{Cn}_{\mathbf{L}}(\Gamma)$ is consistent, then $C^c(\Gamma) = C^r(\Gamma)$.

if $\mathrm{Cn}_{\mathbf{L}}(\Gamma)$ is inconsistent, then $C^r(\Gamma)$ delivers a sensible approach: the content of Γ is what falsifies Γ.

$\neg p \not\triangleright^r \{p\}$

but $p \triangleright^r \{p, \neg p\}$, $\neg p \triangleright^r \{p, \neg p\}$ and $p \wedge \neg p \triangleright^r \{p, \neg p\}$.

Outline

Connexive implication(s)

Taken from Heinrich Wansing ("Connexive logic" in *Stanford Encyclopedia of Philosophy*):

And those who introduce the notion of connection say that a conditional is sound *when* the contradictory of its consequent is incompatible with its antecedent. (Sextus Empiricus, translated in Kneale and Kneale 1962, p. 129.) my italics

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(i) sufficient or necessary condition? Or both?

(ii) connexive implication depends on compatibility, which depends on a logic (here **CLuN**).

(iii) is $B \not\triangleright^m A$ a condition for $\vdash \neg(A \rightarrow B)$ or for $\nvdash (A \rightarrow B)$?

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Some sources:

- **S**: Wansing, Heinrich, "Connexive Logic", The *Stanford Encyclopedia of Philosophy* (Summer 2022 Edition), Edward N. Zalta (ed.). - Omori, H. and Wansing, H., "An Extension of Connexive Logic C", in Olivietti, N., Verbrugge, R., Negri, S. & Sandu, G. (eds), Advances in Modal Logic, Vol 13, pp. 503–522, College Publications, 2020.

classical compatibility is not suitable to define $\rightarrow$

$$A \rightarrow A =_{\mathrm{df}} \neg A \ntriangleright^{m} A$$

$$\neg(A \rightarrow \neg A) =_{\mathrm{df}} \neg\neg A \triangleright^{m} A$$

$$\neg(\neg A \rightarrow A) =_{\mathrm{df}} \neg A \triangleright^{m} \neg A$$

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$A \rightarrow A =_{\mathrm{df}} \neg A \not\triangleright^{m} A$
but if $\oslash$ A, then $\neg A \triangleright^{c}_{2} A$:
ex.: $\neg(p \wedge \neg p) \triangleright^{c}_{2} (p \wedge \neg p)$ and $\neg(p \vee \neg p) \triangleright^{c}_{2} (p \vee \neg p)$

$\neg(A \rightarrow \neg A) =_{\mathrm{df}} \neg\neg A \triangleright^{m} A$
but if $\vdash_{\mathbf{CL}} \neg A$, then $\neg\neg A \not\triangleright^{c}_{1} A$:
ex.: $\neg\neg(p \wedge \neg p) \not\triangleright^{c}_{1} (p \wedge \neg p)$
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ex.: $\neg(p \supset p) \not\triangleright^{c}_{1} \neg(p \supset p)$

LP and CLuNs are not suitable replacements of CLuN for defining $\triangleright^m$

if $\triangleright^m$ defined in terms of **LP** or **CLuNs** then

(i) if $\vdash \neg A$, then $\neg A \triangleright^m A$, whence $\vdash \neg(A \to A)$.

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ex.: $(A \wedge \neg A) \wedge \neg(A \wedge \neg A) \dashv\vdash_{\mathbf{LP}} A \wedge \neg A$ because $\vdash_{\mathbf{LP}} \neg(A \wedge \neg A)$ and the irrelevance of Priest's $\vdash_{\mathbf{LP}}$
Compare $\not\models_{\mathbf{CLuN}} \neg(A \wedge \neg A)$; even $\not\models_{\mathbf{CLuN}} \neg A$.

LP and CLuNs are not suitable replacements of CLuN for defining $\triangleright^m$

if $\triangleright^m$ defined in terms of **LP** or **CLuNs** then

(i) if $\vdash \neg A$, then $\neg A \triangleright^m A$, whence $\vdash \neg(A \to A)$.

Indeed, as $\models \neg A$, *all* minab models of A verify $\neg A$.

ex.: $(A \wedge \neg A) \wedge \neg(A \wedge \neg A) \dashv\vdash_{\mathbf{LP}} A \wedge \neg A$ because $\vdash_{\mathbf{LP}} \neg(A \wedge \neg A)$ and the irrelevance of Priest's $\vdash_{\mathbf{LP}}$

Compare $\nvDash_{\mathbf{CLuN}} \neg(A \wedge \neg A)$; even $\nvDash_{\mathbf{CLuN}} \neg A$.

So, w.r.t. **CLuN** $\{(A \wedge \neg A) \wedge \neg(A \wedge \neg A)\}$ is more inconsistent than $\{A \wedge \neg A\}$, whence $\{(A \wedge \neg A) \wedge \neg(A \wedge \neg A)\} \not\triangleright^m A \wedge \neg A$.

LP and CLuNs are not suitable replacements of CLuN for defining $\rhd^m$

if $\rhd^m$ defined in terms of **LP** or **CLuNs** then

(i) if $\vdash \neg A$, then $\neg A \rhd^m A$, whence $\vdash \neg(A \rightarrow A)$.
Indeed, as $\models \neg A$, *all* minab models of A verify $\neg A$.
ex.: $(A \wedge \neg A) \wedge \neg(A \wedge \neg A) \dashv\vdash_{\mathbf{LP}} A \wedge \neg A$ because $\vdash_{\mathbf{LP}} \neg(A \wedge \neg A)$ and the irrelevance of Priest's $\vdash_{\mathbf{LP}}$
Compare $\nvDash_{\mathbf{CLuN}} \neg(A \wedge \neg A)$; even $\nvDash_{\mathbf{CLuN}} \neg A$.
So, w.r.t. **CLuN** $\{(A \wedge \neg A) \wedge \neg(A \wedge \neg A)\}$ is more inconsistent than $\{A \wedge \neg A\}$, whence $\{(A \wedge \neg A) \wedge \neg(A \wedge \neg A)\} \not\rhd^m A \wedge \neg A$.

(ii) if $A \wedge B \vdash_{\mathbf{LP}} \neg A$, then $\neg A \rhd^m (A \wedge B)$; so $\neg((A \wedge B) \rightarrow A)$.
ex.: $\neg((p \wedge \neg p) \rightarrow p)$ because $p \rhd^m (p \wedge \neg p)$.
Compare: $\nvdash_{\mathbf{CLuN}} \neg A$ for all A

LP and CLuNs are not suitable replacements of CLuN for defining $\triangleright^m$

if $\triangleright^m$ defined in terms of **LP** or **CLuNs** then

(i) if $\vdash \neg A$, then $\neg A \triangleright^m A$, whence $\vdash \neg(A \rightarrow A)$.
Indeed, as $\vDash \neg A$, *all* minab models of A verify $\neg A$.
ex.: $(A \wedge \neg A) \wedge \neg(A \wedge \neg A) \dashv\vdash_{\mathbf{LP}} A \wedge \neg A$ because $\vdash_{\mathbf{LP}} \neg(A \wedge \neg A)$ and the irrelevance of Priest's $\vdash_{\mathbf{LP}}$
Compare $\nvDash_{\mathbf{CLuN}} \neg(A \wedge \neg A)$; even $\nvDash_{\mathbf{CLuN}} \neg A$.
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(ii) if $A \wedge B \vdash_{\mathbf{LP}} \neg A$, then $\neg A \triangleright^m (A \wedge B)$; so $\neg((A \wedge B) \rightarrow A)$.
ex.: $\neg((p \wedge \neg p) \rightarrow p)$ because $p \triangleright^m (p \wedge \neg p)$.
Compare: $\nvdash_{\mathbf{CLuN}} \neg A$ for all A

(iii) if $\oslash A$, then $\neg A \triangleright^m A$; so $(A \rightarrow \neg A)$. NEEN
ex.: $(p \wedge \neg p) \rightarrow \neg(p \wedge \neg p))$ because $\neg\neg(p \wedge \neg p) \not\triangleright^m (p \wedge \neg p)$. NEEN
Compare: $\nvdash_{\mathbf{CLuN}} \neg A$ for all A

LP and CLuNs are not suitable replacements of CLuN for defining $\rhd^m$

if $\rhd^m$ defined in terms of **LP** or **CLuNs** then

(i) if $\vdash \neg A$, then $\neg A \rhd^m A$, whence $\vdash \neg(A \rightarrow A)$.
Indeed, as $\vDash \neg A$, *all* minab models of A verify $\neg A$.
ex.: $(A \wedge \neg A) \wedge \neg(A \wedge \neg A) \dashv\vdash_{\mathbf{LP}} A \wedge \neg A$ because $\vdash_{\mathbf{LP}} \neg(A \wedge \neg A)$ and the irrelevance of Priest's $\vdash_{\mathbf{LP}}$
Compare $\nvDash_{\mathbf{CLuN}} \neg(A \wedge \neg A)$; even $\nvDash_{\mathbf{CLuN}} \neg A$.
So, w.r.t. **CLuN** $\{(A \wedge \neg A) \wedge \neg(A \wedge \neg A)\}$ is more inconsistent than $\{A \wedge \neg A\}$, whence $\{(A \wedge \neg A) \wedge \neg(A \wedge \neg A)\} \not\rhd^m A \wedge \neg A$.

(ii) if $A \wedge B \vdash_{\mathbf{LP}} \neg A$, then $\neg A \rhd^m (A \wedge B)$; so $\neg((A \wedge B) \rightarrow A)$.
ex.: $\neg((p \wedge \neg p) \rightarrow p)$ because $p \rhd^m (p \wedge \neg p)$.
Compare: $\nvdash_{\mathbf{CLuN}} \neg A$ for all A

(iii) if $\not{\textcircled{p}}\, A$, then $\neg A \rhd^m A$; so $(A \rightarrow \neg A)$. NEEN
ex.: $(p \wedge \neg p) \rightarrow \neg(p \wedge \neg p))$ because $\neg\neg(p \wedge \neg p) \not\rhd^m (p \wedge \neg p)$. NEEN
Compare: $\nvdash_{\mathbf{CLuN}} \neg A$ for all A

(iv) if $\not{\textcircled{p}}\, A$, then $\neg A \rhd^m \neg A$; so $(\neg A \rightarrow A)$. NEEN
ex.: $\neg(p \vee \neg p) \rightarrow (p \vee \neg p))$ because $\neg(p \vee \neg p) \rhd^m \neg(p \vee \neg p)$ and $\neg(p \wedge \neg p) \rightarrow (p \wedge \neg p)$ because $\neg(p \wedge \neg p) \rhd^m \neg(p \wedge \neg p)$.
Compare: $\nvdash_{\mathbf{CLuN}} \neg A$ for all A

even with $\triangleright^m$ defined in terms of CLuN, many $\rightarrow$-schemas mentioned in the literature do *not* hold generally

Often papers on connexive logic [Estrada-GonzÃ¡lez and RamÃrez-CÃ¡mara, 2019] mention a number of valid schemas, some of the famous anti-classical ones and some classical, and rely on these to develop their arguments.

Yet, several schemas not justifiable on the basis of the incompatibility criterion. Some examples:

even with $\triangleright^m$ defined in terms of CLuN, many $\rightarrow$-schemas mentioned in the literature do *not* hold generally

Often papers on connexive logic [Estrada-GonzÃ¡lez and RamÃrez-CÃ¡mara, 2019] mention a number of valid schemas, some of the famous anti-classical ones and some classical, and rely on these to develop their arguments.

Yet, several schemas not justifiable on the basis of the incompatibility criterion. Some examples:

if $B \wedge \neg B \vdash_{\mathbf{CLuN}} \neg A$, then $\neg A \triangleright^m (B \wedge \neg B)$; so $\neg((B \wedge \neg B) \rightarrow A)$.
ex.: $p \triangleright^m (p \wedge \neg p)$ and $\neg p \triangleright^m (p \wedge \neg p)$. So $\neg((p \wedge \neg p) \rightarrow p)$ and $\neg((p \wedge \neg p) \rightarrow \neg p)$
different *compatibility* notion or mistakes?

The content criterion of Pierre Abélard Pierre Esbaillart

$A \to B$ iff $C^r(B) \subseteq C^r(A)$

one implication

	$(A \to A)$	$\neg A \not\triangleright^m A$	(1)
	$\neg(\neg A \to A)$	$\neg A \triangleright^m \neg A$	
	$\neg(A \to \neg A)$	$\neg\neg A \triangleright^m A$	
	$(A \to (A \vee B))$	$\neg(A \vee B) \not\triangleright^m A$	
	$(A \to (B \supset A))$	$\neg(B \supset A) \not\triangleright^m A$	
if $B \vdash_{\textbf{CLuN}} \neg(A \vee \neg A)$	$\neg(B \to (A \vee \neg A))$	$\neg(A \vee \neg A) \triangleright^m B$	
if $B \nvdash_{\textbf{CLuN}} \neg(A \vee \neg A)$	$B \to (A \vee \neg A)$	$\neg(A \vee \neg A) \not\triangleright^m B$	
if $B \wedge \neg B \vdash_{\textbf{CLuN}} \neg A$	$\neg((B \wedge \neg B) \to A)$	$\neg A \triangleright^m (B \wedge \neg B)$	
if $B \wedge \neg B \nvdash_{\textbf{CLuN}} \neg A$	$\neg((B \wedge \neg B) \to A)$	$\neg A \not\triangleright^m (B \wedge \neg B)$	

(1) if $\not\textcircled{p}\ A$, then $\neg A \triangleright_2^c A$.
if $\triangleright^m$ defined in terms of **LP** or **CLuNs** and $\vdash \neg A$, then $\neg A \triangleright^m A$ and $\neg(A \to A)$.
Indeed, $\vDash \neg A$ whence all minab models of A verify $\neg A$.

one implication

	$(A \to A)$	$\neg A \not\triangleright^m A$	(1)
	$\neg(\neg A \to A)$	$\neg A \triangleright^m \neg A$	(2)
	$\neg(A \to \neg A)$	$\neg\neg A \triangleright^m A$	
	$(A \to (A \vee B))$	$\neg(A \vee B) \not\triangleright^m A$	
	$(A \to (B \supset A))$	$\neg(B \supset A) \not\triangleright^m A$	
if $B \vdash_{\mathbf{CLuN}} \neg(A \vee \neg A)$	$\neg(B \to (A \vee \neg A))$	$\neg(A \vee \neg A) \triangleright^m B$	
if $B \nvdash_{\mathbf{CLuN}} \neg(A \vee \neg A)$	$B \to (A \vee \neg A)$	$\neg(A \vee \neg A) \not\triangleright^m B$	
if $B \wedge \neg B \vdash_{\mathbf{CLuN}} \neg A$	$\neg((B \wedge \neg B) \to A)$	$\neg A \triangleright^m (B \wedge \neg B)$	
if $B \wedge \neg B \nvdash_{\mathbf{CLuN}} \neg A$	$\neg((B \wedge \neg B) \to A)$	$\neg A \not\triangleright^m (B \wedge \neg B)$	

(2) if $\vdash_{\mathbf{CL}} A$, then $\neg A \not\triangleright^c_1 \neg A$

one implication

	$(A \to A)$	$\neg A \ntriangleright^m A$	(1)
	$\neg(\neg A \to A)$	$\neg A \triangleright^m \neg A$	(2)
	$\neg(A \to \neg A)$	$\neg\neg A \triangleright^m A$	(3)
	$(A \to (A \vee B))$	$\neg(A \vee B) \ntriangleright^m A$	
	$(A \to (B \supset A))$	$\neg(B \supset A) \ntriangleright^m A$	
if $B \vdash_{\mathbf{CLuN}} \neg(A \vee \neg A)$	$\neg(B \to (A \vee \neg A))$	$\neg(A \vee \neg A) \triangleright^m B$	
if $B \nvdash_{\mathbf{CLuN}} \neg(A \vee \neg A)$	$B \to (A \vee \neg A)$	$\neg(A \vee \neg A) \ntriangleright^m B$	
if $B \wedge \neg B \vdash_{\mathbf{CLuN}} \neg A$	$\neg((B \wedge \neg B) \to A)$	$\neg A \triangleright^m (B \wedge \neg B)$	
if $B \wedge \neg B \nvdash_{\mathbf{CLuN}} \neg A$	$\neg((B \wedge \neg B) \to A)$	$\neg A \ntriangleright^m (B \wedge \neg B)$	

(3) if $\vdash_{\mathbf{CL}} \neg A$, then $\neg\neg A \ntriangleright^c_1 A$

one implication

	$(A \rightarrow A)$	$\neg A \not\triangleright^m A$	(1)
	$\neg(\neg A \rightarrow A)$	$\neg A \triangleright^m \neg A$	(2)
	$\neg(A \rightarrow \neg A)$	$\neg\neg A \triangleright^m A$	(3)
	$(A \rightarrow (A \vee B))$	$\neg(A \vee B) \not\triangleright^m A$	(4)
	$(A \rightarrow (B \supset A))$	$\neg(B \supset A) \not\triangleright^m A$	
if $B \vdash_{\mathbf{CLuN}} \neg(A \vee \neg A)$	$\neg(B \rightarrow (A \vee \neg A))$	$\neg(A \vee \neg A) \triangleright^m B$	
if $B \nvdash_{\mathbf{CLuN}} \neg(A \vee \neg A)$	$B \rightarrow (A \vee \neg A)$	$\neg(A \vee \neg A) \not\triangleright^m B$	
if $B \wedge \neg B \vdash_{\mathbf{CLuN}} \neg A$	$\neg((B \wedge \neg B) \rightarrow A)$	$\neg A \triangleright^m (B \wedge \neg B)$	
if $B \wedge \neg B \nvdash_{\mathbf{CLuN}} \neg A$	$\neg((B \wedge \neg B) \rightarrow A)$	$\neg A \not\triangleright^m (B \wedge \neg B)$	

(4) $\neg(A \vee B) \not\triangleright^c_1 A$ and $\neg(A \vee B) \not\triangleright^c_2 A$

one implication

	$(A \to A)$	$\neg A \not\rhd^m A$	(1)
	$\neg(\neg A \to A)$	$\neg A \rhd^m \neg A$	(2)
	$\neg(A \to \neg A)$	$\neg\neg A \rhd^m A$	(3)
	$(A \to (A \vee B))$	$\neg(A \vee B) \not\rhd^m A$	(4)
	$(A \to (B \supset A))$	$\neg(B \supset A) \not\rhd^m A$	(5)
if $B \vdash_{\mathbf{CLuN}} \neg(A \vee \neg A)$	$\neg(B \to (A \vee \neg A))$	$\neg(A \vee \neg A) \rhd^m B$	
if $B \nvdash_{\mathbf{CLuN}} \neg(A \vee \neg A)$	$B \to (A \vee \neg A)$	$\neg(A \vee \neg A) \not\rhd^m B$	
if $B \wedge \neg B \vdash_{\mathbf{CLuN}} \neg A$	$\neg((B \wedge \neg B) \to A)$	$\neg A \rhd^m (B \wedge \neg B)$	
if $B \wedge \neg B \nvdash_{\mathbf{CLuN}} \neg A$	$\neg((B \wedge \neg B) \to A)$	$\neg A \not\rhd^m (B \wedge \neg B)$	

(5) $\neg(B \supset A) \not\rhd^c_1 A$

one implication

	$(A \to A)$	$\neg A \not\triangleright^m A$	(1)
	$\neg(\neg A \to A)$	$\neg A \triangleright^m \neg A$	(2)
	$\neg(A \to \neg A)$	$\neg\neg A \triangleright^m A$	(3)
	$(A \to (A \vee B))$	$\neg(A \vee B) \not\triangleright^m A$	(4)
	$(A \to (B \supset A))$	$\neg(B \supset A) \not\triangleright^m A$	(5)
if $B \vdash_{\mathbf{CLuN}} \neg(A \vee \neg A)$	$\neg(B \to (A \vee \neg A))$	$\neg(A \vee \neg A) \triangleright^m B$	(6)
if $B \nvdash_{\mathbf{CLuN}} \neg(A \vee \neg A)$	$B \to (A \vee \neg A)$	$\neg(A \vee \neg A) \not\triangleright^m B$	(6)
if $B \wedge \neg B \vdash_{\mathbf{CLuN}} \neg A$	$\neg((B \wedge \neg B) \to A)$	$\neg A \triangleright^m (B \wedge \neg B)$	
if $B \wedge \neg B \nvdash_{\mathbf{CLuN}} \neg A$	$\neg((B \wedge \neg B) \to A)$	$\neg A \not\triangleright^m (B \wedge \neg B)$	

(6) example $\neg(p \vee \neg p) \triangleright^m \neg(p \vee \neg p)$, whence $\neg(\neg(p \vee \neg p) \to (p \vee \neg p))$
for all A and B, $\neg(A \vee \neg A) \not\triangleright_1^c B$ and $\neg(A \vee \neg A) \triangleright_2^c B$

one implication

	$(A \to A)$	$\neg A \not\triangleright^m A$	(1)
	$\neg(\neg A \to A)$	$\neg A \triangleright^m \neg A$	(2)
	$\neg(A \to \neg A)$	$\neg\neg A \triangleright^m A$	(3)
	$(A \to (A \vee B))$	$\neg(A \vee B) \not\triangleright^m A$	(4)
	$(A \to (B \supset A))$	$\neg(B \supset A) \not\triangleright^m A$	(5)
if $B \vdash_{\mathbf{CLuN}} \neg(A \vee \neg A)$	$\neg(B \to (A \vee \neg A))$	$\neg(A \vee \neg A) \triangleright^m B$	(6)
if $B \nvdash_{\mathbf{CLuN}} \neg(A \vee \neg A)$	$B \to (A \vee \neg A)$	$\neg(A \vee \neg A) \not\triangleright^m B$	(6)
if $B \wedge \neg B \vdash_{\mathbf{CLuN}} \neg A$	$\neg((B \wedge \neg B) \to A)$	$\neg A \triangleright^m (B \wedge \neg B)$	(7)
if $B \wedge \neg B \nvdash_{\mathbf{CLuN}} \neg A$	$\neg((B \wedge \neg B) \to A)$	$\neg A \not\triangleright^m (B \wedge \neg B)$	(7)

(7) example $\neg(p \wedge \neg p) \triangleright^m \neg(p \wedge \neg p)$ whence
$\neg(\neg(p \wedge \neg p) \to (p \wedge \neg p))$
for all A and B, $(B \wedge B) \not\triangleright^c_1 A$ and $(B \wedge B) \triangleright^c_2 A$

one implication

	$(A \to A)$	$\neg A \not\triangleright^m A$	(1)
	$\neg(\neg A \to A)$	$\neg A \triangleright^m \neg A$	(2)
	$\neg(A \to \neg A)$	$\neg\neg A \triangleright^m A$	(3)
	$(A \to (A \vee B))$	$\neg(A \vee B) \not\triangleright^m A$	(4)
	$(A \to (B \supset A))$	$\neg(B \supset A) \not\triangleright^m A$	(5)
if $B \vdash_{\mathbf{CLuN}} \neg(A \vee \neg A)$	$\neg(B \to (A \vee \neg A))$	$\neg(A \vee \neg A) \triangleright^m B$	(6)
if $B \nvdash_{\mathbf{CLuN}} \neg(A \vee \neg A)$	$B \to (A \vee \neg A)$	$\neg(A \vee \neg A) \not\triangleright^m B$	(6)
if $B \wedge \neg B \vdash_{\mathbf{CLuN}} \neg A$	$\neg((B \wedge \neg B) \to A)$	$\neg A \triangleright^m (B \wedge \neg B)$	(7)
if $B \wedge \neg B \nvdash_{\mathbf{CLuN}} \neg A$	$\neg((B \wedge \neg B) \to A)$	$\neg A \not\triangleright^m (B \wedge \neg B)$	(7)

$*$ **LP** and **CLuNs** not suitable for defining $\triangleright^m$.

$*$ Only $\triangleright^m$ ensures the schemas $A \to A$, $\neg(\neg A \to A)$ and $\neg(A \to \neg A)$.

one implication; with condition

—	$((p \wedge q) \rightarrow p)$	$\neg p \ntriangleright (p \wedge q)$
—	$((p \wedge q) \rightarrow q)$	$\neg q \ntriangleright (p \wedge q)$
—	$\neg((p \wedge \neg p) \rightarrow p)$	$\neg p \triangleright !p$
—	$\neg(!p \rightarrow \neg p)$	$\neg\neg p \triangleright !p$
$\circ(A \wedge B)$	$((A \wedge B) \rightarrow A)$	$\neg A \ntriangleright (A \wedge B)$
$\circ(A \wedge B)$	$((A \wedge B) \rightarrow B)$	$\neg B \ntriangleright (A \wedge B)$
—	$(!p \vee q) \wedge \neg q) \rightarrow !p)$	$\neg !p \triangleright (!p \vee q) \wedge \neg q)$
—	$((p \vee q) \wedge \neg q) \rightarrow p)$	$\neg p \ntriangleright ((p \vee q) \wedge \neg q)$
$\circ A$	$((A \vee B) \wedge \neg B) \rightarrow A)$	$\neg A \ntriangleright ((A \vee B) \wedge \neg B)$

implication between implications

S	$(A \rightarrow B) \rightarrow \neg(A \rightarrow \neg B)$	$\neg(\neg\neg B \not\triangleright A) \not\triangleright \neg(\neg\neg B \not\triangleright A)$
S	$(A \rightarrow \neg B) \rightarrow \neg(A \rightarrow B)$	$\neg(\neg\neg B \not\triangleright A) \not\triangleright \neg(\neg\neg B \not\triangleright A)$
S^*	$\nvdash ((A \rightarrow B) \rightarrow (B \rightarrow A))$	$\nvdash ((\neg B \not\triangleright A) \not\triangleright \neg(\neg\neg B \not\triangleright A)$
S	$(A \rightarrow B) \rightarrow (\neg B \rightarrow \neg A)$	$\neg(\neg\neg A \not\triangleright \neg B) \not\triangleright (\neg B \not\triangleright A)$
S	$(p \rightarrow q) \rightarrow (\neg q \rightarrow \neg p)$	$\neg(\neg\neg p \not\triangleright \neg q) \not\triangleright (\neg q \not\triangleright p)$
S	$(!p \rightarrow q) \rightarrow (\neg q \rightarrow \neg !p)$	$\neg(\neg\neg !p \not\triangleright \neg q) \not\triangleright (\neg q \not\triangleright !p)$
2	$\nvdash (A \rightarrow B) \rightarrow (B \rightarrow A)$	$\nvdash \neg(\neg B \not\triangleright A) \not\triangleright (\neg A \not\triangleright B)$

implication between implications

S	$(A \to B) \to \neg(A \to \neg B)$	$\neg(\neg\neg B \not\rhd A) \not\rhd \neg(\neg\neg B \not\rhd A)$
S	$(A \to \neg B) \to \neg(A \to B)$	$\neg(\neg\neg B \not\rhd A) \not\rhd \neg(\neg\neg B \not\rhd A)$
S^*	$\nvdash ((A \to B) \to (B \to A))$	$\nvdash ((\neg B \not\rhd A) \not\rhd \neg(\neg\neg B \not\rhd A)$
S	$(A \to B) \to (\neg B \to \neg A)$	$\neg(\neg\neg A \not\rhd \neg B) \not\rhd (\neg B \not\rhd A)$
S	$(p \to q) \to (\neg q \to \neg p)$	$\neg(\neg\neg p \not\rhd \neg q) \not\rhd (\neg q \not\rhd p)$
S	$(!p \to q) \to (\neg q \to \neg !p)$	$\neg(\neg\neg !p \not\rhd \neg q) \not\rhd (\neg q \not\rhd !p)$
2	$\nvdash (A \to B) \to (B \to A)$	$\nvdash \neg(\neg B \not\rhd A) \not\rhd (\neg A \not\rhd B)$

contraposition

–	$((p \to q) \to (\neg q \to \neg p)$	$\neg(\neg\neg p \not\rhd \neg q) \not\rhd (\neg q \not\rhd p)$
–	$((p \to (q \vee \neg q)) \to (\neg(q \vee \neg q) \to \neg p)$	$\neg(\neg\neg p \not\rhd \neg(q \vee \neg q)) \not\rhd (\neg(q \vee \neg q) \not\rhd p)$
$\circledcirc A, \circledcirc B$	$((A \to B) \to (\neg B \to \neg A)$	$\neg(\neg\neg A \not\rhd \neg B) \not\rhd (\neg B \not\rhd A)$

Outline

Variations

probleem hier en al eerder: ook wffs van vorm $!A$ in Ω?
Opgelost !

Outline

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Outline

$\star$ Zach Weber's Fregean set theories (with relevant implication): Extensionality is phrased as $\forall x \forall y \forall z((z \in x \leftrightarrow z \in y) \to x = y)$ in which $\to$ is a detachable relevant implication and $\leftrightarrow$ a detachable and *contraposable* relevant equivalence.

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Abstraction $\forall y\, y \in \{x \mid A(x)\} \leftrightarrow A(y)$ turns identity into an *intensional* operator: for proving $\{x \mid A(x)\} = \{x \mid B(x)\}$ it is insufficient that $\{x \mid A(x)\}$ and $\{x \mid B(x)\}$ have the same members as well as the same non-members; one needs that $A(x) \leftrightarrow B(x)$ holds (that $A(x)$ and $B(x)$ are *relevantly equivalent*.

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Example: $\{x \mid \exists z(z \notin z \wedge z \in z)\}$ and $\{x \mid \forall z(z \in z \vee z \notin z)\}$

Let $\vdash_{\mathbf{WF}} A$. So clearly both $\exists z(z \notin z \wedge z \in z) \wedge A$ and its negation $\forall z(z \in z \vee z \notin z) \vee \neg A$ are theorems, whence both $\{x \mid \exists z(z \notin z \wedge z \in z) \wedge A\}$ and $\{x \mid \forall z(z \in z \vee z \notin z) \vee \neg A\}$ have all sets as members as well as non-members. So, if **WF** is truly extensional, $(\exists z(z \notin z \wedge z \in z) \wedge A) \leftrightarrow (\forall z(z \in z \vee z \notin z) \vee \neg A)$ and $(\exists z(z \notin z \wedge z \in z) \wedge A) \leftrightarrow \exists z(z \notin z \wedge z \in z)$ etc.

$\star$ Graham Priest's 'material' Fregean set theory, say **PFS**, phrased in **LP**-language:
here Ext reads: $\forall x \forall y(\forall z((z \in x \supset z \in y) \wedge (z \in y \supset z \in x)) \supset x = y)$,
equivalently: $\forall x \forall y(\exists z((z \in x \wedge \neg z \in y) \vee (z \in y \wedge \neg z \in x)) \vee x = y$

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Let $G = \{x \mid \exists y(y \in y \wedge \neg y \in y)\}$ and $G^* = \{x \mid \forall y(\neg y \in y \wedge y \in y)\}$
G and G^* have the same members (viz. all sets) as well as the same non-members (viz. all sets)

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The relevant Ext-instance:
$(\exists z((z \in G \wedge \neg z \in G^*) \vee (z \in G^* \wedge \neg z \in G)) \vee G = G^*)$

as the subformula in red is a theorem of **PFS**, the Ext-instance does not contribute anything to showing that $G = G^*$; so Ext is *too weak* to serve its purpose

the same holds for many extensionally identical sets

every **LP**-model of {Abs, Ext} verifies the disjunct in red. So it verifies Ext even if it falsifies $G = G^*$. klopt niet vanaf volgende lijn

every **LP**-model of {Abs, Ext} falsifies $\forall z(z \in G \supset z \in G^*) \wedge \forall z(z \in G^* \supset z \in G)$.
plus: every **LP**-model of $\{$Abs, Ext, $\neg G = G^*\}$ verifies Ext (viz. verifies the formula in red) but also verifies $\neg G = G^*$. I challenge material dialetheists to demonstrate that **PFS** is truly extensional, for example that all **LP**-models of **PFS** verify $G = G^*$.

all of the following ‘implications’ are **CL**-equivalent as well as **LP**-equivalent: $A \supset B$, $\neg A \vee B$, $\neg B \supset \neg A$
but no two are **CLuNs**-equivalent

similarly for $B \supset A$, $\neg B \vee A$, $\neg A \supset \neg B$

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so at least 9 ‘equivalences’ are **CL**-equivalent as well as **LP**-equivalent, but not **CLuNs**-equivalent:
$(A \supset B) \wedge (B \supset A)$
$(A \supset B) \wedge (\neg B \vee A)$
$(A \supset B) \wedge (\neg A \supset \neg B)$
$\ldots$

and their meanings in **CL** and in **LP** are different: detachable / non-detachable

For Ext and Abs, Priest keeps literally the same string of symbols, but with their **LP**-meanings where Frege intends the **CL**-meanings. (VERDER)

Outline

App: Modalities in paraconsistent context

SKIP

where $\mathcal{M} = \langle \Sigma_\Delta, M_0 \rangle$:

· $\mathcal{M} \Vdash \Diamond A$ corresponds to: $M \Vdash A$ for a $M \in \Sigma_\Delta$.

· $\mathcal{M} \Vdash \Diamond\neg A$ and $\mathcal{M} \Vdash \neg\Box A$ correspond to: $M \Vdash \neg A$ for a $M \in \Sigma_\Delta$.

· $\mathcal{M} \Vdash \Diamond A$ and $\mathcal{M} \Vdash \Diamond\neg A$ are exhaustive: every $M \in \Sigma_\Delta$ verifies A or verifies $\neg A$.

· $\mathcal{M} \Vdash \Diamond A$ and $\mathcal{M} \Vdash \Diamond\neg A$ are not exclusive: every $M \in \Sigma_\Delta$ verifies A or verifies $\neg A$.