Uniwersytet Warmiǹsko-Mazurski w Olsztynie
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A Unifying Program for Defeasible Reasoning Forms: Adaptive Logics

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Outline

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Defeasible Reasoning

- most actual reasoning is *defeasible*
- applications of *methods* require defeasible reasoning
- all knowledge ultimately results from defeasible reasoning

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- all knowledge ultimately results from defeasible reasoning

- logicians: almost exclusively *deductive* logics
- defeasible logics: fragmentary and disparate studies (e.g., non-monotonic logics [Łukaszewicz, 1990])

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Rescher's Weak Consequence relation: monotonic ⋇

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$\Rightarrow$ conclusions revised

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Historical note:

- a unifying framework: Adaptive logics
- First adaptive logic atypical not on known method
- plus new examples (*creative*)
- known methods (*reconstructing*): only later (students)
- *integrating* e.g. Rescher-Manor logics

Outline

Introduction

Computational Stuff

Avoiding Circularity

The Standard Format

Examples and details

Metatheory

References

Computational Stuff

- **CL** and most predicative deductive logics:
semi-recursive but not recursive (positive, no negative test) ⋇

- idea behind defeasible logics: from Δ derive A unless $\Gamma \vdash_L X$
external dyn.: *Mill* on inductive generalization

- defeasible logics: no positive test
$\Rightarrow$ causes the internal dynamics

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- negative test?

- A may be derivable in different ways:

- If $\Delta \subseteq \mathrm{Cn}_{\mathbf{L}}(\Gamma)$, then derive A unless $B \in \mathrm{Cn}_{\mathbf{L}}(\Gamma)$
- If $\Theta \subseteq \mathrm{Cn}_{\mathbf{L}}(\Gamma)$, then derive A unless $B \in \mathrm{Cn}_{\mathbf{L}}(\Gamma)$

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For some defeasible logics, neither $\mathrm{Cn}_{\mathbf{L}}(\Gamma)$ nor $\mathcal{W} \setminus \mathrm{Cn}_{\mathbf{L}}(\Gamma)$ is semi-recursive.
It was proved that, for some **AL**, $\mathrm{Cn}_{\mathbf{AL}}(\Gamma)$ is Π^1_1-complex.

Outline

Avoiding Circularity

- reasoning (and derivations) proceed linearly
- "A is **L**-derivable from Γ unless $B \in \mathrm{Cn}_{\mathbf{L}}(\Gamma)$" is flatly circular
- even "If $\Delta \subseteq \mathrm{Cn}_{\mathbf{L}}(\Gamma)$, then **L** allows one to derive A from Γ unless $B \in \mathrm{Cn}_{\mathbf{L}}(\Gamma)$" easily causes circularity

Solution

- "If $\Gamma \vdash_{\mathbf{LLL}} B$, then **AL** allows one to derive A from Γ unless $\Gamma \vdash_{LLL} C$"
- $\Gamma \vdash_{AL} A$ iff $\Gamma \vdash_{\mathbf{LLL}} B$ and $\Gamma \nvdash_{LLL} C$.

Outline

The Standard Format

large diversity of adaptive logics

every new adaptive logic requires: proof theory, semantics, metatheory (study properties of the logic)

AL: $\wp(\mathcal{W}) \to \wp(\mathcal{W})$ is defined by a triple:

(1) A *lower limit logic* **LLL**: a logic that is defined over some $\mathcal{L}$ and is reflexive, transitive, monotonic, formal and compact. All these logics have a proper semantics $\mathbb{S}$ phrased in a **CL**-metalanguage.

(2) A decidable *set of abnormalities* $\Omega \subseteq \mathcal{W}$: a set of formulas characterized by a (possibly restricted) logical form F; or a union of such sets.

(3) An *adaptive strategy*: Reliability, Minimal Abnormality, ...

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ULL obtained by trivializing abnormalities

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Convention

corrective adaptive logics: **LLL** weaker then **CL**; ex. inconsistency-adaptive
ampliative adaptive logics: **LLL** is **CL**; ex. compatibility, inductive generalisation

Proof theory

- rules of inference (determined by **LLL** and Ω)
- a marking definition (determined by Ω and the stategy)

(Generic) Rules of inference

PREM If $A \in \Gamma$:

$$\frac{\begin{array}{ll} \ldots & \ldots \end{array}}{\begin{array}{ll} A & \emptyset \end{array}}$$

RU If $A_1, \ldots, A_n \vdash_{\mathbf{LLL}} B$:

$$\frac{\begin{array}{ll} A_1 & \Delta_1 \\ \ldots & \ldots \\ A_n & \Delta_n \end{array}}{\begin{array}{ll} B & \Delta_1 \cup \ldots \cup \Delta_n \end{array}}$$

RC If $A_1, \ldots, A_n \vdash_{\mathbf{LLL}} B \check{\vee} \mathrm{Dab}(\Theta)$

$$\frac{\begin{array}{ll} A_1 & \Delta_1 \\ \ldots & \ldots \\ A_n & \Delta_n \end{array}}{\begin{array}{ll} B & \Delta_1 \cup \ldots \cup \Delta_n \cup \Theta \end{array}}$$

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Handling Inconsistency

dynamics caused by contradictions derived in the proofs
simplistic example: $\Gamma = \{p, q, \neg q, \neg p \vee r, \neg q \vee s\}$
minimally inconsistent interpretation: r derivable, s not

- LLL: **CLuN** ⋇
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example proof:

1	p	Prem	$\emptyset$
2	q	Prem	$\emptyset$
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6	$(p \wedge \neg p) \vee r$	1, 4; RU	$\emptyset$
7	r	1, 4; RC	$\{p \wedge \neg p\}$
8	s	2, 5; RC	$\{q \wedge \neg q\}$

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6	$(p \wedge \neg p) \vee r$	1, 4; RU	$\emptyset$
7	r	1, 4; RC	$\{p \wedge \neg p\}$
8	s	2, 5; RC	$\{q \wedge \neg q\}$ $\checkmark^{9}$
9	$q \wedge \neg q$	2, 3; RU	$\emptyset$

Inductive Generalisation

Let the data comprise the following literals.

Pa	Pb	Pc		
Qa			Qd	$\neg Qe$
Ra	Rb	$\neg Rc$		

1	Pa	Prem	$\emptyset$	
2	Ra	Prem	$\emptyset$	
3	$\forall x(Px \supset Qx)$	RC	$\{\neg\forall x(Px \supset Qx)\}$	$\checkmark^7$
4.1	Qa	1, 6; RU	$\{\neg\forall x(Px \supset Qx)\}$	$\checkmark^7$
5.2	$\forall x(Px \supset \neg Qx)$	RC	$\{\neg\forall x(Px \supset \neg Qx)\}$	$\checkmark^7$
6.3	$\neg Qa$	1, 6.2; RU	$\{\neg\forall x(Px \supset \neg Qx)\}$	$\checkmark^7$
7	$\neg\forall x(Px \supset Qx) \vee \neg\forall x(Px \supset \neg Qx)$	6.1, 6.3; RD	$\emptyset$	

Content guidance: observations and experiments.
[Batens, 2011]

Strategies

Let $\Gamma = \{p \vee q, \neg p, \neg q, \neg p \vee r, \neg q \vee s\}$

Obviously $\Gamma \vdash_{\mathbf{CLuN}} (p \wedge \neg p) \vee (q \wedge \neg q)$, $\Gamma \nvdash_{\mathbf{CLuN}} (p \wedge \neg p)$ and $\Gamma \nvdash_{\mathbf{CLuN}} (q \wedge \neg q)$

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$\mathrm{Dab}(\Delta)$ is the classical disjunction of the members of a $\Delta \subseteq_{fin} \Omega$.

If $\Gamma \vdash_{\mathbf{LLL}} \mathrm{Dab}(\Delta)$ and there is no $\Delta' \subset \Delta$ such that $\Gamma \vdash_{\mathbf{LLL}} \mathrm{Dab}(\Delta')$, then $\mathrm{Dab}(\Delta)$ is a *minimal* Dab-consequence of Γ.

Where $\mathrm{Dab}(\Delta_1), \mathrm{Dab}(\Delta_2), \ldots$ – the Δ_i may overlap – are the *minimal* Dab-consequences of Γ:
$U(\Gamma) =_{\mathrm{df}} \Delta_1 \cup \Delta_2 \cup \ldots$.
$\Phi(\Gamma)$ is the set of *minimal choice sets* of $\{\Delta_1, \Delta_2, \ldots\}$.※

Marking for Reliability: where Θ is the condition of line l of a proof from Γ, line l is *marked* iff $\Theta \cap U(\Gamma) \neq \emptyset$.

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Semantically: $\mathrm{Ab}(M) = \{B \in \Omega \mid M \Vdash B\}$; a **LLL**-model M of Γ is reliable iff $\mathrm{Ab}(M) \subseteq U(\Gamma)$. $\Gamma \vDash_{\textbf{AL}} A$ iff, for every reliable model of Γ, $M \Vdash A$.

Strategies

Marking for Minimal Abnormality: Where A is derived in line l, on the condition Θ, line l is *unmarked* iff, (i) $\Theta \cap \Delta = \emptyset$ for some $\Delta \in \Phi(\Gamma)$ and (ii) for each $\Delta \in \Phi(\Gamma)$, A is derived in a line with condition Θ' such that $\Theta' \cap \Delta = \emptyset$.

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Semantically: a **LLL**-model M of Γ is Minimally Abnormal iff there is no **LLL**-model M' of Γ such that $\mathrm{Ab}(M') \subset \mathrm{Ab}(M)$.
$\Gamma \vDash_{\mathbf{AL}} A$ iff, for every minimally abnormal model of Γ, $M \Vdash A$.

Hint: it was proved that M is a minimally abnormal model of Γ iff $\mathrm{Ab}(M) \in \Phi(\Gamma)$.

Illustrating Marking

Let $\Gamma = \{p \vee q, \neg p, \neg q, \neg p \vee r, \neg q \vee s\}$

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3	$\neg q$	Prem	$\emptyset$
4	$p \vee r$	Prem	$\emptyset$
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$U(\Gamma) = \{p \wedge \neg p, q \wedge \neg q\}$ and $\Phi(\Gamma) = \{\{p \wedge \neg p\}, \{q \wedge \neg q\}\}$

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10	$r \vee s$	7; ADD	$\{q \wedge \neg q\}$	$\checkmark^{10}$

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marking for Reliability

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marking for Minimal Abnormality from stage 10 on

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marking for Minimal Abnormality from stage 10 on

$\Gamma \vdash_{\mathbf{CLuN}^m} r \vee s$ but $\Gamma \nvdash_{\mathbf{CLuN}^r} r \vee s$

Final Derivability

if proponent can answer any move by the opponent, then she can answer any (finite or infinite) set of consecutive moves by the opponent

Outline

Standard format and metatheory

Theorem
$\Gamma \vDash_{\mathbf{AL}^r} A$ iff $\Gamma \vDash_{\mathbf{LLL}} A \check{\vee} \mathrm{Dab}(\Delta)$ and $\Delta \cap U(\Gamma) = \emptyset$ for a finite $\Delta \subset \Omega$.

...

Corollary
$\Gamma \vdash_{\mathbf{AL}^r} A$ iff $\Gamma \vDash_{\mathbf{AL}^r} A$. (Soundness and Completeness)

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$\Gamma \vdash_{\mathbf{AL}^r} A$ iff $\Gamma \vDash_{\mathbf{AL}^r} A$. (Soundness and Completeness)

Lemma
$M \in \mathcal{M}^m_\Gamma$ iff $M \in \mathcal{M}^{\mathbf{LLL}}_\Gamma$ and $\mathrm{Ab}(M) \in \Phi_\Gamma$.

...

Theorem
$\Gamma \vdash_{\mathbf{AL}^m} A$ iff $\Gamma \vDash_{\mathbf{AL}^m} A$. (Soundness and Completeness)

Strong Reassurance (Stopperedness, Smoothness)

if a **LLL**- model *M* of Γ is not selected, this is justified by the fact that a selected model of Γ is less abnormal than *M* REF

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Theorem
If $M \in \mathcal{M}_{\Gamma}^{\mathbf{LLL}} - \mathcal{M}_{\Gamma}^{m}$, then there is a $M' \in \mathcal{M}_{\Gamma}^{m}$ such that $\mathrm{Ab}(M') \subset \mathrm{Ab}(M)$. (Strong Reassurance for Minimal Abnormality.)

Theorem
If $M \in \mathcal{M}_{\Gamma}^{\mathbf{LLL}} - \mathcal{M}_{\Gamma}^{r}$, then there is a $M' \in \mathcal{M}_{\Gamma}^{r}$ such that $\mathrm{Ab}(M') \subset \mathrm{Ab}(M)$. (Strong Reassurance for Reliability.)

Theorem each of the following obtains:

1. $\mathcal{M}^m_\Gamma \subseteq \mathcal{M}^r_\Gamma$. Hence $\mathrm{Cn}_{\mathbf{AL}^r}(\Gamma) \subseteq \mathrm{Cn}_{\mathbf{AL}^m}(\Gamma)$. •
2. If $A \in \Omega - U(\Gamma)$, then $\check{\neg} A \in \mathrm{Cn}_{\mathbf{AL}^r}(\Gamma)$.
3. If $\mathrm{Dab}(\Delta)$ is a minimal Dab-consequence of Γ and $A \in \Delta$, then some $M \in \mathcal{M}^m_\Gamma$ verifies A and falsifies all members (if any) of $\Delta - \{A\}$.
4. $\mathcal{M}^m_\Gamma = \mathcal{M}^m_{\mathrm{Cn}_{\mathbf{AL}^m}(\Gamma)}$ whence $\mathrm{Cn}_{\mathbf{AL}^m}(\Gamma) = \mathrm{Cn}_{\mathbf{AL}^m}(\mathrm{Cn}_{\mathbf{AL}^m}(\Gamma))$. •(Fixed Point.)
5. $\mathcal{M}^r_\Gamma = \mathcal{M}^r_{\mathrm{Cn}_{\mathbf{AL}^r}(\Gamma)}$ whence $\mathrm{Cn}_{\mathbf{AL}^r}(\Gamma) = \mathrm{Cn}_{\mathbf{AL}^r}(\mathrm{Cn}_{\mathbf{AL}^r}(\Gamma))$. •(Fixed Point.)
6. For all $\Delta \subseteq \Omega$, $\mathrm{Dab}(\Delta) \in \mathrm{Cn}_{\mathbf{AL}}(\Gamma)$ iff $\mathrm{Dab}(\Delta) \in \mathrm{Cn}_{\mathbf{LLL}}(\Gamma)$. (Immunity.)
7. If $\Gamma' \subseteq \mathrm{Cn}_{\mathbf{AL}}(\Gamma)$, then $\mathrm{Cn}_{\mathbf{AL}}(\Gamma') \subseteq \mathrm{Cn}_{\mathbf{AL}}(\Gamma)$. •(Cautious Cut.)
8. If $\Gamma' \subseteq \mathrm{Cn}_{\mathbf{AL}}(\Gamma)$, then $\mathrm{Cn}_{\mathbf{AL}}(\Gamma \cup \Gamma') \subseteq \mathrm{Cn}_{\mathbf{AL}}(\Gamma)$. •(Cautious Monotonicity.)

Theorem each of the following obtains:

1. If Γ is normal, then $\mathcal{M}^{\mathbf{ULL}}_\Gamma = \mathcal{M}^m_\Gamma = \mathcal{M}^r_\Gamma$ whence $\mathrm{Cn}_{\mathbf{AL}^r}(\Gamma) = \mathrm{Cn}_{\mathbf{AL}^m}(\Gamma) = \mathrm{Cn}_{\mathbf{ULL}}(\Gamma)$. •
2. If Γ is abnormal and $\mathcal{M}^{\mathbf{LLL}}_\Gamma \neq \emptyset$, then $\mathcal{M}^{\mathbf{ULL}}_\Gamma \subset \mathcal{M}^m_\Gamma$ and hence $\mathrm{Cn}_{\mathbf{AL}^r}(\Gamma) \subseteq \mathrm{Cn}_{\mathbf{AL}^m}(\Gamma) \subset \mathrm{Cn}_{\mathbf{ULL}}(\Gamma)$.
3. $\mathcal{M}^{\mathbf{ULL}}_\Gamma \subseteq \mathcal{M}^m_\Gamma \subseteq \mathcal{M}^r_\Gamma \subseteq \mathcal{M}^{\mathbf{LLL}}_\Gamma$ whence $\mathrm{Cn}_{\mathbf{LLL}}(\Gamma) \subseteq \mathrm{Cn}_{\mathbf{AL}^r}(\Gamma) \subseteq \mathrm{Cn}_{\mathbf{AL}^m}(\Gamma) \subseteq \mathrm{Cn}_{\mathbf{ULL}}(\Gamma)$. •
4. $\mathcal{M}^r_\Gamma \subset \mathcal{M}^{\mathbf{LLL}}_\Gamma$ iff $\Gamma \cup \{A\}$ is **LLL**-satisfiable for some $A \in \Omega - U(\Gamma)$.
5. $\mathrm{Cn}_{\mathbf{LLL}}(\Gamma) \subset \mathrm{Cn}_{\mathbf{AL}^r}(\Gamma)$ iff $\mathcal{M}^r_\Gamma \subset \mathcal{M}^{\mathbf{LLL}}_\Gamma$.
6. $\mathcal{M}^m_\Gamma \subset \mathcal{M}^{\mathbf{LLL}}_\Gamma$ iff there is a (possibly infinite) $\Delta \subseteq \Omega$ such that $\Gamma \cup \Delta$ is **LLL**-satisfiable and there is no $\varphi \in \Phi_\Gamma$ for which $\Delta \subseteq \varphi$.
7. If there are $A_1, \ldots, A_n \in \Omega$ ($n \geq 1$) such that $\Gamma \cup \{A_1, \ldots, A_n\}$ is **LLL**-satisfiable and, for every $\varphi \in \Phi_\Gamma$, $\{A_1, \ldots, A_n\} \nsubseteq \varphi$, then $\mathrm{Cn}_{\mathbf{LLL}}(\Gamma) \subset \mathrm{Cn}_{\mathbf{AL}^m}(\Gamma)$.
8. $\mathrm{Cn}_{\mathbf{AL}^m}(\Gamma)$ and $\mathrm{Cn}_{\mathbf{AL}^r}(\Gamma)$ are non-trivial iff $\mathrm{Cn}_{\mathbf{AL}^m}(\Gamma)$ is non-trivial. • (Reassurance)

Theorem
If $\Gamma' \subseteq \mathrm{Cn}_{\mathbf{AL}}(\Gamma)$, then $\mathrm{Cn}_{\mathbf{AL}}(\Gamma \cup \Gamma') = \mathrm{Cn}_{\mathbf{AL}}(\Gamma)$. (Cumulative Indifference.)

Theorem
If $\Gamma \vdash_{\mathbf{AL}} A$, then every **AL**-proof from Γ can be extended in such a way that A is finally derived in it. (Proof Invariance)

Theorem
If $\Gamma' \in \mathrm{Cn}_{\mathbf{AL}}(\Gamma)$ and $\Gamma \in \mathrm{Cn}_{\mathbf{AL}}(\Gamma')$, then $\mathrm{Cn}_{\mathbf{AL}}(\Gamma) = \mathrm{Cn}_{\mathbf{AL}}(\Gamma')$. (Equivalent Premise Sets) REF

...

Nog niet behandeld

-relatie met Graham
- meer voorbeelden (alleen tripels) cf. J18ALs
- referenties tussenvoegen
- geprioriteerde - theorieën

Questions?

Outline

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