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THE ODD-EVEN EFFECT IN MULTIPLICATION: FAMILIARITY WITH EVEN NUMBERS OR A PARITY RULE AFTER ALL?

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This study investigated two possible explanations for the odd-even effect in simple arithmetic. This effect is well documented for multiplication, and implies that an incorrect proposed answer in a verification task is rejected faster if the parity of that answer is different from the parity of the correct answer. The explanation based on the parity rule (Krueger, 1986) contends that subjects (implicitly) use the information that the parity of incorrect and correct answers diverge. The second explanation is based on familiarity with even numbers (Lochy, Seron, Delazer, & Butterworth, 2000). Three fourths of the possible products have indeed an even outcome, so that, according to the familiarity hypothesis, rejecting a wrong even answer is more difficult than rejecting a wrong odd answer. In this study, we conducted a straightforward test of both accounts by presenting as many even as odd incongruent/congruent answers. A main congruency effect which did not interact with problem type was found. The congruency effect on O x O problems, however, was not significant. The study concludes that (1) the evidence argues against the familiarity hypothesis; (2) the parity rule cannot be generalized to all problem types but is restricted to certain types of problems; and (3) the findings on problems with two odd multipliers leave the possibility for a limited familiarity with even numbers.

Cognitive psychology strongly relies on a distinction between rules and similarity. In categorization, for example, some theoretical approaches take similarity between instances as the basis for categorization even in situations in which knowledge of rules appears to be involved (e.g., Nosofsky, 1984, 1992; Nosofsky, Clark, & Shin, 1989), while other theories claim that categorization is a rule-based process, even in situations where similarity seems

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to play an obvious role (e.g., Anderson, Kline, & Beasly, 1979; Ashby, 1992; Ashby & Alfonso-Reese, 1995; Ashby & Gott, 1988). Further theories attempt to bridge the gap between these two broad accounts (e.g., Rosseel, 1996, 2002). A similar situation is observed in the domain of reasoning, which, for a long time, has been considered as a rule-based endeavour. In particular the so-called "rule theory" (e.g., Braine & O'Brien, 1991; Rips, 1989) has been advocated since the beginnings of experimental psychology, but case-based and similarity-based approaches have been present (e.g., Tversky & Kahneman, 1973).

These are just a few examples of situations in which cognitive psychology has called on rules *and* similarity as explanatory mechanisms. According to Pothos (in press), it can be argued that rules and similarities are merely extremes of a continuum, with on the one hand similarity judgements based on a large amount of attributes or features, and on the other hand rules that can be considered as similarity based on only one or two features.

In mental arithmetic, it seems that a similar debate is present with respect to the odd-even effect. This effect is a well documented finding in the literature (Krueger, 1986; Krueger & Hallford, 1984; Lemaire & Fayol, 1995; Lemaire & Reder, 1999; Lochy, Seron, Delazer, & Butterworth, 2000; Masse & Lemaire, 2001). It concerns the faster rejection of an incorrect proposed answer if its parity differs from the parity of the correct answer; for example, 8 x 4 = 33 (incongruent problem) is rejected faster than 8 x 4 = 34 (congruent problem) because 33 is odd whereas 34 is even. Some researchers propose that the odd-even effect in simple multiplication can be explained on the basis of rule usage, whereas others call on familiarity as an explanatory device. Krueger (1986) was the first author to report an odd-even effect for the single x single digit multiplication. His explanation was that subjects used the following parity rule: "The true product must be even if any multiplier is even, otherwise it must be odd". Lochy et al. (2000), however, proposed an alternative explanation based on familiarity with even numbers. For multiplication, 75% of correct possible outcomes are even [Even x Even (E x E), Even x Odd (E x O), and Odd x Even (O x E)], and only 25% are odd [Odd x Odd (O x O)]. Thus, P[correct outcome | even number] is three times larger than P[correct outcome | odd number] in the tables of multiplication. Given the more frequent exposure to even outcomes in multiplication, an even number is more strongly associated with a correct outcome. Because the proportion E x E and Mixed problems (E x O or O x E) was much larger than the proportion O x O problems in the studies about the odd-even effect, more congruent problems were even than odd and vice versa for incongruent problems. According to the familiarity hypothesis (which states that even answers are more strongly associated with a correct outcome), this leads to a slower rejection of congruent answers than incongruent ones.

Hence, according to Lochy et al., there is no need to assume that subjects use parity information.

A distinction between the parity hypothesis and the familiarity hypothesis is only possible on problems for which the parity of the correct outcome is odd. For these problems, the parity rule predicts, as usual, that congruent answers will be rejected more slowly than incongruent ones, while the familiarity hypothesis predicts the opposite, since congruent answers are odd and incongruent answers are even. The situation is different when the solution to a problem is of even parity. In such cases indeed, congruent answers have an even parity, and hence both the familiarity and the parity hypotheses make the same prediction that congruent answers will be rejected more slowly than incongruent ones. In order to distinguish between the two alternative explanations, Lochy et al. divided their stimulus set into problem types (4 E x E problems, 4 Mixed problems, and 4 O x O problems). They found a main effect of congruency as predicted by the parity rule, but this effect interacted with problem type on reaction times, a result that is not predicted by the parity rule. Incongruent answers were rejected faster than congruent ones for E x E and mixed problems. This effect was reversed for O x O problems: completely contrary to the parity hypothesis, incongruent O x O problems were rejected more slowly than congruent ones. In a second analysis and completely analogous to the just mentioned results, they found a main effect of parity of the proposed answer in favour of odd answers and no interaction with problem type. The authors concluded that it is not the use of the parity rule that causes the odd-even effect, but the use of other arithmetical knowledge, namely the larger proportion of even products in de tables of multiplication.

Parity Against Familiarity

The data of Lochy et al. (2000) suggest that the use of a parity rule as an explanation for the odd-even effect is wrong, but that this could not be discovered because reaction times for congruent and incongruent problems were never analysed separately per problem type in previous studies. The study of Lemaire and Reder (1999) however, is an exception. One of the goals of their Experiment 1 was to investigate the influence of the number of even multiplicators on the magnitude of the odd-even effect. They found a significant interaction effect between number of even multiplicators and parity effect. There was a strong significant odd-even effect on E x E problems (624 ms), and a significant odd-even effect on Mixed problems (188 ms). The odd-even effect on O x O problems however was reversed (-128 ms), but was not significant. Lemaire and Reder gave three possible explanations for the fact that the strongest effects were found on E x E problems. The first possible reason is that the parity rule for problems with two even figures gives the same result for addition and multiplication (E x E = E; E + E = E). The second possible explanation is that the knowledge that the multiplication of two even numbers always leads to an even number is better than any other parity related knowledge. As a third explanation, it is also possible that "is even" as a feature that evokes the parity rule, is twice as possible when the two multiplicators are even. The finding that the odd-even effect was smallest on problems without one even multiplicator is consistent with this reasoning, at least according to the authors, because there was actually a (not significant) reversed odd-even effect on O x O problems.

The findings of Lochy et al. (2000) and, to a lesser extent, those of Lemaire and Reder (1999) on the O x O problems are problematic for the parity rule. The only contra-argument that defenders of the parity rule for multiplication can make is a familiarity bias with even numbers *within* experiments. This bias can exist when the amount of E x E and Mixed problems is larger than the amount of O x O problems in an experiment. In such a situation, P[correct | even] is greater than P[correct | odd], or in other words, there can exist a familiarity with correct even numbers within the experiment. As a consequence, the parity effect could be strongest on E x E problems because of reasons mentioned in the previous paragraph. It could be moderate on Mixed problems, and non-existing or even reversed on O x O problems if these problems evoke no parity knowledge.

The familiarity hypothesis, however, has also difficulties to explain the finding of Lemaire and Reder (1999) that the odd-even effect is stronger on E x E problems. Adherents of the parity rule give some possible explanations for this effect (see above), but this is more difficult for defenders of the familiarity hypothesis. Larger familiarity with even numbers is *general*: we see no explanation why this general familiarity would be greater for certain problem types than for other problem types. Lochy et al. (2000) contend that: "What our data show is actually a kind of "degraded rule": The plausibility of even numbers seems to be greater than odd ones in general, and contrary to the mathematical rule in the case of O x O. But it is also greater for E x E than for the other types" (Lochy et al., p. 364). In our opinion, P[correct outcome = E | E x E] = 1, P[correct outcome = E | E x O or O x E] = 1, and P[correct outcome = E | O x O] = 0. The greater subjective/psychological plausibility the authors allude to has no objective basis *in contrast to* a general familiarity with even numbers (P[E |multiplication] = .75). It seems more plausible to us that the presented problem E x E = O is obviously wrong, contrary to the problem E x E = E. This latter remark has nothing to do with familiarity with even numbers, but with use of parity information. We note for completeness and clearness that also the parity rule has no objective criterion for the fact that odd-even effects are greater on E x E problems. However, it concerns a *rule* which, as already mentioned (Lemaire and Reder, 1999), (1) can be activated more strongly by certain problem types, and (2) is in particular also valid with two even operators for addition. Furthermore, the finding of Lemaire and Reder (1999) that the odd-even effect increases (or the reversed odd-even effect on O x O problems decreases) during an experiment is also problematic for the familiarity hypothesis. We see no reason why a general familiarity effect with even numbers should increase for E x E and Mixed problems and decrease for O x O problems. The explanation of Lemaire and Reder that the strategy based on the parity rule is much more applied or becomes faster as the experiment proceeds, sounds much more plausible to us.

In order to shed more light on the controversy between the parity rule on the one hand and familiarity with even numbers on the other hand, we investigated a straightforward test of the two alternative explanations by presenting as many O x O as other multiplication problems. Under these conditions, the familiarity rule and parity rule put forward two clearly different predictions. According to the familiarity hypothesis, there should not be a main effect of congruency because the parity of congruent and incongruent answers is equally divided between odd and even. However, the familiarity hypothesis predicts a strong interaction between congruency and problem type. Congruent problems should be solved slower for E x E and mixed problems than incongruent problems, and vice versa for O x O problems. The parity rule on the other hand, predicts as usual a main effect of congruency and no interaction with problem type.

Method

Participants

Twenty volunteers participated, thirteen women and seven men. The mean age was 22 years and 2 months (range: 18-30 years). All participants had normal or corrected to normal sight.

Material

Stimuli were multiplications, presented in standard form (a x b = c), with a and b as single digits. The basic set existed of 12 problems (see Appendix for the complete list of stimuli). Each problem was presented four times with the correct answer and four times with an incorrect answer. Incorrect answers were the correct answers +/- 1,2,3 or 4[1]. Incongruent answers were thus composed by splits +/- 1 or 3, and congruent ones by split +/- 2 or 4. Of the 12 basic problems, there were 3 E x E problems, 3 mixed problems, and 6 O x O problems. The mean size of the multiplications with an even answer was 41.3, and 37 for those with a odd answer. Following restrictions were put on the operands a and b: (a) no 0 or 1 because problems with such operands are more solved with retrieval of rules than with retrieval of outcomes (for example, Ashcraft, 1982; Baroody, 1985; LeFevre et al., 1996b); (b) no 2 because of the possibility of repeated addition (e.g., 2 x 8 = 8 + 8; LeFevre et al., 1996a); and (c) no 5 because of the possibility of the five-rule and because production problems with a 5 as operator are easier (e.g., Campbell, 1994; Campbell & Oliphant, 1992; Lemaire & Reder, 1995; Masse & Lemaire, 2001). Also ties (e.g. 4 x 4) were not presented. The reason for avoiding ties is that ties seem to have easier access to long term memory in comparison with other problems (for example Graham & Campbell, 1992; Groen & Parkman, 1972; but see Blankenberger, 2001, for another explanation of the tie-effect). Within every split condition, we tried to keep the sum of positive and negative splits as close to 0 as possible. However, this was not completely possible because we also avoided table related products (e.g., 4 x 8 was not presented with 28 or 36 as an incorrect outcome on split +/-4 condition; for the interferential effect of table related products, see for example Galfano, Rusconi, & Umiltà, 2003; LeFevre, Bisanz, & Mrkonjic, 1988; LeFevre, Kulak, & Bisanz, 1991; Lemaire, Barrett, Fayol, & Abdi, 1994; Lemaire, Fayol, & Abdi, 1991). The sum of splits for split +/-1 was -2, and +2 for split +/-4. In half of the problems, operand a was the smallest one. Overall, there were 96 problems [12 x 4 (correct) x 4 (incorrect)] which were, in accordance with Lochy et al. (2000), presented twice in two successive sessions.

Procedure

Every trial started with a centrally presented fixation point (***) during 500 ms. Then, the multiplication problems appeared centrally on the screen until the participant answered with a left or right manual response on a response box or until 10000 ms had elapsed. The ITI was 2000 ms. The sequence of presentation was randomized per participant. The mapping of correct and incorrect to the left or right responses was counterbalanced over

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[1] In some cases, we had to take another split in order to avoid table related products. See Appendix for all the details.

participants. Within every session (after 48 trials) and between the two sessions, there was a short break which participants could end with an arbitrary press on the keyboard. In order to get confident with the manner of presentation and the equipment, the experiment started with 10 training trials which were removed from analysis. The experiment lasted about 15 minutes.

Analyses/Design

We conducted three kinds of analyses. The first analysis compared correct and incorrect answers in a 2 (session) x 2 (proposed answer: correct or incorrect) x 2 (problem-type: ExE/Mixed and OxO) design. The two following analyses concerned only trials with an incorrect proposed answer. The most important analysis compared congruent with incongruent problems in a 2 (session) x 2 (proposed answer: congruent or incongruent) x 2 (problem type) design. The third analysis is only for completeness and the ease of the reader[2], and compares even proposed answers with odd ones in a 2 (even/odd) x 2 (problem type) design. All factors are within-subject. Dependent variables are reaction times and accuracy.

Results

The results were analysed with the repeated measures module of the General Linear Model (GLM) in SPSS. The reaction times were only analysed for the correctly solved trials. Outliers (+/-2 standard deviation) were excluded.

Correct Versus Incorrect Problems

The data are summarized in Table 1. The accuracy data revealed no significant effect. The analyses of the reaction times revealed that correct problems were solved faster than incorrect problems, $F(1,19) = 10.49$, $p < .01$. There was also a main effect of problem type, $F(1,19) = 9.60$, $p < .01$: O x O

[2] Given the specific properties of a 2 x 2 design and the fact that incongruent versus congruent is the same as even/odd for one level of problem-type and vice versa for the other level of problem-type, it follows that (1) the main effect of congruent/incongruent is equal to the interaction-effect of problem-type with the factor odd/even, and (2) the interaction-effect of congruent/incongruent with problem-type is equal to the main-effect of odd/even. The effect of problem-type is in both designs of course the same.

problems were solved faster than E x E/mixed problems, but this difference was larger in session 1 than in session 2. The interaction between problem type and session was significant, $F(1,19) = 5.12$, $p < .05$. Reaction times were also shorter in session 2 than in session 1, $F(1,19) = 32.90$, $p < .001$, probably due to practice. Finally, there was also a 3-way interaction between correctness of the presented answer, problem type and session, $F(1,19) = 4.55$, $p < .05$. The difference between correct E x E/Mixed problems and incorrect E x E/Mixed problems was smaller in session 1 than the difference between correct and incorrect O x O problems. The reverse was true for session 2.

Table 1. Mean Reaction Times (RT, in ms) and Accuracy (Acc, in %) for Correct and Incorrect Problems per Problem Type[a]

	Correct problem				Incorrect problem			
	Session 1		Session 2		Session 1		Session 2	
Outcome	RT	Acc	RT	Acc	RT	Acc	RT	Acc
Even[b]	1209 (505)	93.6 (5.3)	952 (329)	96.3 (3.8)	1258 (489)	94.4 (6.4)	1044 (398)	94.4 (8.4)
Odd	1110 (391)	95.7 (4.5)	943 (306)	95.1 (5.1)	1212 (463)	95.9 (4.9)	990 (347)	96.8 (4.0)

[a] Standard deviations between brackets.
[b] These are E x E *and* Mixed problems.

Congruent Versus Incongruent Problems

The data are summarized in Table 2. Incongruent trials were solved faster than congruent trials, $F(1,19) = 8.94$, $p < .01$. The interaction between congruency and problem type was not significant, $F(1,19) = 2.65$, $p > .10$, although the congruency effect was much larger for E x E/Mixed problems than for O x O problems (121 versus 19 ms for session 1; and 64 ms versus 20 ms for session 2). Separate analyses revealed that the congruency-effect was significant for E x E/Mixed problems, $F(1,19) = 7.14$, $p < .05$, but not for O x O problems, $F(1,19) < 1$. Finally, just as in the correct/incorrect analysis, O x O problems were solved faster than E x E/Mixed problems, $F(1,19) = 13.23$, $p < .01$, and there was an effect of session, $F(1,19) = 30.25$, $p < .001$. Accuracy data were high and analyses revealed no significant effects.

Odd/Even Analyses

The main effect of odd/even on reaction times was not significant, $F(1,19) = 2.65$, $p > .10$, but interacted with problem-type, $F(1,19) = 8.94$, $p < .01$. There was also a main effect of problem type, $F(1,19) = 13.32$, $p < .01$ and

Table 2. Mean Reaction Times (RT, in ms) and Accuracy (Acc, in %) for Incongruent and Congruent Problems per Problem Type (based on the parity of the correct solution)[a].

	Incongruent problem				Congruent problem			
	Session 1		Session 2		Session 1		Session 2	
Outcome	RT	Acc	RT	Acc	RT	Acc	RT	Acc
Even[b]	1197 (463)	94.2 (8.6)	1013 (374)	93.8 (10.8)	1318 (551)	94.7 (6.8)	1077 (429)	95.1 (8.2)
Odd	1201 (477)	97.5 (5.5)	979 (335)	96.3 (5.0)	1220 (461)	94.2 (7.2)	999 (360)	97.2 (4.9)

[a] Standard deviations between brackets.
[b] These are E x E *and* Mixed problems.

of session, $F(1,19) = 30.25$, $p < .001$. There were no significant effects nor trends with respect to accuracy.

Discussion

The present study conducted a straightforward test of the use of a parity rule on the one hand and the familiarity with even numbers hypothesis on the other hand as explanations for the odd-even effect in multiplication. We used a stimulus set of which half of the problems were O x O problems. Under these conditions, both explanations make distinct predictions. The use of a parity rule predicts a main effect of congruency and no interaction with problem type. The familiarity with even numbers hypothesis on the other hand predicts no main effect of congruency but a strong interaction of congruency with problem type. Our main results pointed to a main effect of congruency and no interaction with problem type. The congruency-effect on O x O problems, however, was not significant.

These findings can be summarized into three conclusions: (1) the present findings contradict the familiarity hypothesis as a complete alternative for the parity rule; (2) the parity rule is not universal but is restricted to certain problem types; and (3) the findings on the O x O problems leave the possibility or suggest a limited familiarity with even numbers. We develop each of these conclusions.

(1) The familiarity hypothesis already had to deal with serious problems on basis of the existing literature. A general familiarity strategy can not directly explain why effects on certain problem types are larger (Lemaire & Reder, 1999; Lochy et al., 2000), or why the odd-even effect increases during an experiment (Lemaire & Reder). The straightforward test of the familiarity hypothesis in this study raises serious additional problems for a general familiarity with even numbers as an explanation for the odd-even effect. The congruency-effect on reaction times and the absence of an interaction effect between congruency and problem type are completely in line with the

parity rule and falsify the familiarity hypothesis. Completely at variance with the latter hypothesis is that, if anything, incongruent O x O problems are solved somewhat faster than congruent ones.

(2) All the results of the present research and some findings available in the literature support a revision of the original parity rule for multiplication. It seems that subjects do not use the second part of the rule: "A multiplication must be even if any multiplier is even, otherwise it must be odd". The congruency effect on O x O problems was not significant in our study, was reversed but not significant in the study of Lemaire and Reder (1999), and was reversed and significant in the study of Lochy et al. (2000). The behavioural data suggest that subjects rather use the rule: "A multiplication must be even when any multiplier is even". The finding that odd-even effects are usually greater on E x E problems than on mixed problems can be explained by such a parity rule and the parallel-race model. The detection of one even multiplier is enough to reject an incongruent answer on basis of parity information. This strategy can be faster or the parity rule can be activated more quickly when there are two even multipliers in the problem presented in comparison with problems with one even multiplier. As a consequence, the strategy based on parity information has within a parallel-race model more chance to be completed *before* a strategy based on retrieval and comparison when there are two even multipliers in the problem. This remark is in agreement with the possibility that Lemaire and Reder posit that "even" as a feature that evokes the parity rule is twice as possible if the two multipliers are even.

(3) As a third conclusion, we need to say something about the findings concerning the O x O problems. When we argued in this article against the familiarity hypothesis, it was against the strong form of the hypothesis that the complete odd-even effect could be explained by a general familiarity with even numbers. Neither the rewritten nor the original parity rule, however, provide an answer to the question why no congruency effect is found on O x O problems. This leaves the possibility of an influence of familiarity with even numbers. However, this influence seems to be limited by the use of parity information. Contrary to the study of Lochy et al. (2000), we did not find a reversed congruency effect on O x O problems. An important difference with the study of Lochy et al., however, was the larger amount and proportion of O x O problems. As a consequence, it is possible that participants got more confident with those problems during the experiment and that they more strongly realized that this operation has an odd outcome. This is in line with the finding of Lemaire and Reder (1999) that the parity effect increases with practice in the experimental session. Therefore, one could explain the contrasting results of our study and Lochy et al. by the larger proportion of O x O problems in our experiment.

The avoidance of table related products in the present study and in many other studies on multiplication leaves the possibility of still another strategy to reject an incorrect proposed answer. Maybe subjects or at least some of them control the table of products so well that they can reject a false problem purely on an outcome that does not exist in the tables without considering the part to the left of the equal sign (a strategy spontaneously reported after the experimental session by one participant). Judging whether a presented outcome is present in the tables of multiplication can be more difficult for even than for odd outcomes because 75% of the outcomes in the tables are even. In other words, the avoidance of table related products can favour a familiarity effect. To investigate this possibility, it could be interesting to run an experiment where some table related products as incorrect outcomes are included, and to look at the effect of this manipulation on the reaction times for congruent and incongruent O x O problems.

The finding that certain effects are problem type dependent or are stronger on certain problem types must invite researchers to investigate whether global effects in simple arithmetic are a real effect for every problem type, and vice versa, if no effect is found, whether this is the case for every problem type. An example concerns a study of Masse and Lemaire (2001). They compared problems which violate the parity rule, problems which violate the five-rule, and problems which violate both rules. One of their conclusions was that problems which violated the five-rule were rejected faster than problems which violated the parity rule. A possible restriction to this conclusion is that problems which are in agreement with the five rule but in violation with the parity rule (for example 26 x 5 = 125) can never be E x E problems because of the digit 5. Also a comparison between a five-problem that violates no rule (for example 67 x 5 = 345) and no five-problem that is also with no rule in violation (for example 16 x 8 = 138) is problematic if one wants to explain a difference in reaction times in terms of the presence or absence of the multiplier 5 in the problems. The first example is after all a congruent O x O problem and the second example a congruent E x E problem. We have to mention that the examples given are some of which Masse and Lemaire reported in their study, and it is not clear if they had controlled as well as possible for the different problem types. As such, this paragraph mentions only a few potential problems of interpreting the results and is certainly not a straightforward criticism of the study concerned. The point we want to make is that problem type can be an important moderator of simple arithmetic results or effects.

Several studies point to the flexibility and variability of strategies in the development of arithmetic skills in children (for example Geary & Burlingham-Dubree, 1989; Goldman, Mertz, & Pellegrino, 1989; Lemaire & Siegler, 1995; Siegler, 1988). Lemaire and Fayol (1995) investigated in their Experiment 2 whether the odd-even effect was also found with children of grade two, three, and four. The answer was positive, but the effect varied with age, difficulty of the problems, and the time between the presentation of the two multipliers and the presentation of the outcome. It would be interesting to replicate the study of Lemaire and Fayol with an analysis per problem type to see whether there were different patterns on different types of problems, with again a special attention to O x O problems.

Finally, an interesting result of a study of Lemaire and Siegler (1995) should be mentioned. They investigated reaction times, accuracy and use of strategies in solving multiplications during three different times in the year in which French children of the second grade learned multiplications. The odd-even status of mistakes on problems solved with retrieval showed an interesting evolution. As the year passed, the agreement between the odd-even status of the mistakes with the odd-even pattern of multiplication increased, and the agreement with the odd-even pattern for addition decreased[3]. According to Lemaire and Siegler, the change in the pattern of mistakes showed that children learned in an increasing way that a multiplication of two odd figures results in an odd outcome and adding two odd figures in an even outcome. They argued it was not plausible that teachers learned this systematically to their pupils. Therefore, one of their conclusions was that a lot of features of acquiring arithmetic skills were reflecting more the nature of the learning system than the instructions as such. In light of our findings concerning the odd-even effect on O x O problems, we have to moderate this conclusion. It may be useful for a teacher of simple arithmetic to point to the fact that the outcome of O x O problems is odd.

To conclude, the use of parity information plays an important role within cognitive processes concerning simple arithmetic. This role can be different for different problem types. Models that attempt to formalize these processes have to take this role into account. Concerning the odd-even effect, we conclude that the familiarity hypothesis is inadequate to explain all the results of this study and those available in the literature. Subjects seem to make use of a parity rule, of which the effect differs across problem types.

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[3] For a good understanding: The production of a wrong even answer on a O x O problem is an answer of which the odd-even status is in agreement with the odd-even pattern of addition, because O + O is even.

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Appendix

Stimuli Experiment[a]

	split +/-1	split +/-2	split +/-3	split +/-4
E x E				
4 x 8	33	34	29	38[b]
8 x 4	33	34	29	38[b]
6 x 8	47	46	51	44
Mixed				
6 x 9	53	52	51	58
9 x 6	53	52	51	58
7 x 4	29	26	31	22
O x O				
9 x 7	62	61	66	67
7 x 9	62	61	66	67
3 x 9	26	29	22[b]	23
9 x 3	26	29	22[b]	23
3 x 7	22	23	26[b]	17
7 x 3	22	23	26[b]	17

[a] Every problem was also presented four times with the correct answer.

[b] Because of avoiding table related products, we used split 5 and split 6 for respectively split 3 and split 4. These preserve the congruence/incongruence of the presented answer.

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