Psychologica Belgica
2003, 43-4, 259-268.

NUMBER COMPARISON UNDER EXECUTIVE DUAL-TASK

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Several studies have shown that random interval generation (RIG) interferes with simple mental arithmetic, suggesting that executive processes are involved in simple arithmetic. However, an alternative interpretation would be that RIG calls on quantitative instead of executive processes. The present study explored this possibility. We investigated a typical quantitative effect, the distance effect, under single and dual task. We reasoned that if it is true that both number comparison and RIG are based on quantitative processes, then the distance effect should disappear or at least decrease under dual task. Results showed that the distance effect did not decrease when performed simultaneously with RIG, ruling out this alternative interpretation. Moreover, our data provide additional evidence for the robustness of the distance effect in numerical cognition.

Introduction

In the last decade, several research groups studied the interaction of mental arithmetic and working memory. With a variety of tasks loading the executive subsystem of working memory as defined by Baddeley and colleagues (e.g., Baddeley & Hitch, 1974; Baddeley, 1986; Baddeley, 1996), these studies have concluded that the central executive is involved in simple addition (e.g., 7 + 5) and simple multiplication (e.g., 6 x 9) in both verification and production tasks (De Rammelaere, Stuyven & Vandierendonck, 1999, 2001; De Rammelaere & Vandierendonck, 2001; Lemaire, Abdi & Fayol, 1996; Seitz & Schumann-Hengsteler, 2000), while the majority of these studies have concluded that the phonological loop is not involved in any detectable way in these simple arithmetic tasks.

One important problem in such dual-task designs is that the task used to load the central executive may also interfere with one of the slave systems. For instance, random letter generation not only hinders the functioning of the central executive but also interferes with the phonological loop, which makes clean-cut conclusions not easy to obtain. In an attempt to solve this problem,

This research was carried out while Stijn De Rammelaere was a research assistant of the Fund for Scientific Research – Flanders. We are grateful to Wim Fias for his comments and to Filip Van Opstal for running the experiment.

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Vandierendonck, De Vooght and Van der Goten (1998a) introduced the "Random time Interval Generation" task (abbreviated: RIG; also see Vandierendonck, 2000a, 2000b, 2000c). In the RIG-task, participants are requested to tap a randomly spaced sequence of time intervals on a key so as to produce an unpredictable, random "rhythm". Vandierendonck et al. (1998a) have demonstrated that in the RIG-task, the requirement to be random and to avoid automaticity loads the central executive, while there are neither logical nor empirical grounds to assume that there is interference with one of the slave systems of working memory, such as the phonological loop or the visuo-spatial sketch pad (also see Vandierendonck, De Vooght, & Van der Goten, 1998b and Szmalec, Vandierendonck, & Kemps, 2003, for an other executive task with similar characteristics).

In a series of experiments, De Rammelaere and colleagues have shown that RIG interferes with simple mental arithmetic. First, it was shown that the verification of simple arithmetic sums (e.g., 8 + 4 = 13. True or false?) was hindered by the RIG-task (De Rammelaere et al., 1999). However, as only a subset of all possible simple sums was studied, no general conclusions could be drawn. Therefore, De Rammelaere et al. (2001) conducted a follow-up study in which they investigated whether the effect is still obtained when all possible simple arithmetical sums are investigated (Experiment 1) and when another arithmetic operation (multiplication) has to be performed (Experiment 2). De Rammelaere et al. (2001) obtained the same results as De Rammelaere et al. (1999). This corroborated the conclusion that simple mental arithmetic calls on executive processes.

However, three criticisms were formulated against this conclusion. First, the effect was repeatedly shown for verification tasks (e.g., 8 + 4 = 13. True or False?), but never for production tasks (e.g., 8 + 4 = ???) (also see Ashcraft, Donley, Halas, & Vakali, 1992; Lemaire, Abdi, & Fayol, 1996). If executive processes contribute to simple arithmetic, this should also be demonstrated for production and not only for verification tasks. However, we have shown that RIG also hinders the production of simple mental arithmetic sums and products (De Rammelaere & Vandierendonck, 2001).

A second criticism concerns the finding that the interaction of executive load by problem size was never significant (see e.g., Noël, Désert, Aubrun & Seron, 2001; Oberauer, Demmrich, Mayr & Kliegl, 2001). The possibility thus remains that RIG added a constant amount of time to the latencies, regardless the difficulty of the problem. This criticism has been countered by the demonstration that the effect of RIG increases when the step of solving a simple arithmetic problem has to be executed more than once as in two-digit (24 + 33) and three-digit (281 + 314) problems (De Rammelaere, Vandierendonck, & Deschuyteneer, 2002).

A third comment states that the effect of RIG on simple arithmetic is possibly due to the fact that participants solve the RIG-task with the aid of quantitative processes[1]. For instance, participants could estimate the length of a given time interval on each key-press, so as to produce intervals of various lengths. Still other participants may accomplish RIG by internally counting (e.g., "one, two, three", PRESS, "one, two", PRESS, "one, two, three, four, five", PRESS, and so on). If the effect of RIG on simple arithmetic is solely due to the fact that RIG calls on quantitative processes, then it may not necessarily be true that executive functions contribute to simple mental arithmetic. This last criticism has not yet been investigated directly and the present article deals with this issue.

Note that this alternative explanation in terms of quantitative rather than executive functions does not specify *which* quantitative processes could eventually be responsible for the repeatedly found effect of RIG on simple arithmetic. We define a "quantitative process" as any cognitive process in which quantities are involved such as (not exhaustively) subitizing, estimating, counting, calculating, retrieving arithmetical facts, comparing numbers, et cetera. Simple mental arithmetic, which is the focus of the present study, is thus also a quantitative process.

To study whether the RIG-task is accomplished by means of quantitative processes, we investigated a "typical" quantitative effect in the numerical cognition literature. The distance effect reflects the fact that, when participants have to decide whether a number is smaller or larger than another number, latencies are inversely related to the numerical distance (e.g., Dehaene, 1989; Moyer & Landauer, 1967). For instance, participants will be faster in deciding that 9 is larger than 5 than in deciding that 6 is larger than 5 because the numerical distance is 4 (9 – 5) in the former and 1 (6 – 5) in the latter case. We reasoned that, if it is true that the reported effects of RIG on simple mental arithmetic are solely due to quantitative processes, then a typical quantitative effect (the distance effect) should disappear or at least diminish when numbers are compared in a RIG-condition. On the contrary, if the distance effect remains unchanged in a RIG-condition, then this explanation seems rather unlikely: Indeed, it would be strange that a quantitative effect remains unchanged when investigated under a quantitative dual task. In our experiment, participants were presented one-digit numbers that were smaller or larger than 5 (i.e., 1, 2, 3, 4 and 6, 7, 8, 9). Their task was to decide whether the number was smaller or larger than the standard "5". Note that the present experiment is not only important to address the third criticism, but the research question itself is also new: to our knowledge, no study has explored the effect of a secondary task on number comparison.

[1] We are grateful to Wim Fias for drawing our attention to this possibility.

Method

Participants. Twenty first-year psychology students (14 females) from Ghent University participated for course requirements.

Procedure, stimuli and design. The stimuli were the numbers 1, 2, 3, 4, 6, 7, 8 and 9, presented in Arabic format in the center of a computer screen, in bright white on a black background. The task of the participants was to decide as accurately and as fast as possible whether the number was smaller or larger than the standard 5. Accuracy and speed of responding were equally stressed. Participants were instructed to use their index and middle finger of the right hand to press respectively the left or right button of a response box (connected to the gameport of the computer). If the number was smaller than 5, they had to press the left button and if the number was larger than 5, they had to press the right button.

Each subject participated in two counterbalanced conditions. In the control condition, participants performed the number comparison task only. In the RIG-condition, participants had to press the zero-key of the numeric keypad while they were performing the number comparison task. They were instructed to use their left index finger to do so. The experimenter instructed that the key-presses had to be as random and as unpredictable as possible and emphasized that it was important to give equal attention to both tasks. The procedure was thus exactly the same as in De Rammelaere et al. (1999, 2001), except that the primary task here was number comparison instead of verifying simple arithmetic problems. To assess whether there appeared to be any trade-off between the primary and the secondary task, participants also had to perform the secondary task alone for two minutes.

Each digit was presented 64 times in each condition, so that there were 512 trials in each condition (64 trials x 8 numbers). There was a break of about 3 minutes after 256 trials and between the conditions. When the participant pressed a response key, a black screen was presented for 1000ms. Next, the following stimulus appeared. Thus, each participant solved 1024 trials (2 conditions x 64 trials x 8 numbers). Participants were tested individually in a quiet room, with the same experimenter present during the whole experimental session. Each session lasted about 45 minutes.

Results

Latencies

Outliers were defined as latencies smaller then 150ms or larger than 1500ms (as in other number comparison studies such as Ratinckx, Brysbaert, & Reynvoet, 2001; and Ratinckx & Brysbaert, 2001). Only 198 of the 20480 (20 subjects x 1024 trials) experimental trials were outliers, which is less than 1%. Means were 506, 482, 464, and 461ms for distances 1, 2, 3, and 4 respectively in control and 629, 605, 580, and 573ms in RIG (see Figure 1). Only correctly solved trials were included in the latency analysis. In order to avoid problems that typically may arise in the analysis of repeated measures designs, the data were analyzed by means of the multivariate general linear hypothesis with contrasts in the dependent variables. *F*-approximations of Wilks λ are reported. As all *F*-approximations were exact, only the *F*-values are reported here.

There were significant main effects of Distance, $F(3,17) = 33.18, p < .001$, and of Load, $F(1,19) = 90.70, p < .001$. The interaction between Distance and Load was not significant, $F<1$. The linear and quadratic trends over Distance were statistically significant, $F(1,19) = 93.62$ and $F(1,19) = 19.15, p < .001$ respectively, but these trends did not interact with Load, $F(1,19) = 2.03, p > .15$ and $F < 1$ respectively.

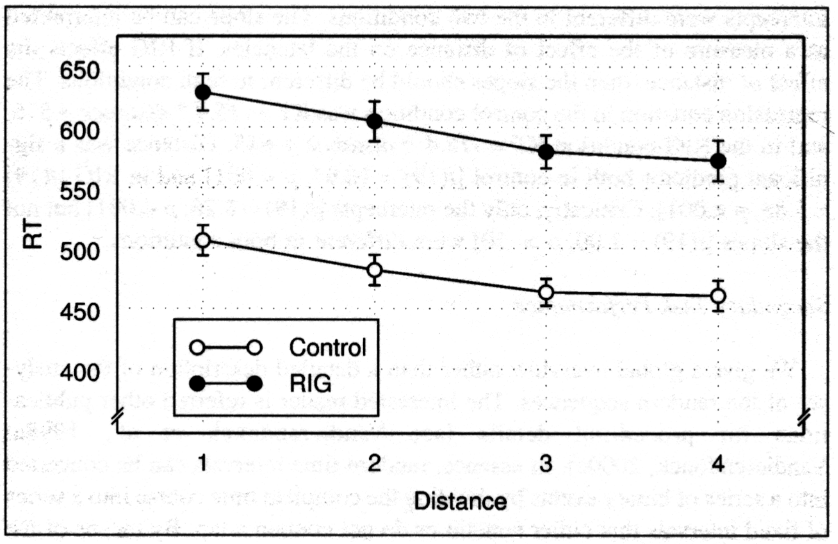

Figure 1. Mean latencies in ms as a function of numerical distance and condition (RIG=Random Interval Generation).

Accuracy

Mean proportions of correctly solved trials were .94, .97, .99 and .99 respectively for the numerical distances 1, 2, 3 and 4 in the control condition and .92, .95, .97 and .98 in the RIG-condition. To check whether a speed-accuracy trade-off occurred, mean accuracy data and mean latencies of each subject in the two conditions were correlated, yielding a significant negative correlation of $r = -0.33$ ($p < .05$). This shows that fast latencies were accompanied by high accuracy and suggests the absence of a speed-accuracy trade-off. The analyses showed main effects of Load, $F(1,19) = 4.68$, $p < .05$, and of Distance, $F(3,57) = 30.57$, $p < .001$. The interaction Load by Distance was not significant, $F(3,57) < 1$. The linear and quadratic trends over Distance were significant, respectively $F(1,19) = 38.02$ and $F(1,19) = 22.69$, $p < .001$, but these trends did not interact with Load, both $F < 1$.

Individual Regression Analyses

As a further test for the stability of the distance effect in the RIG-condition, individual regression analyses were run for each subject in each condition, with distance as the predictor and latency of correctly solved trials as the dependent variable. Following the suggestions of Lorch and Myers (1990), we obtained a slope and an intercept for each subject in the two conditions and next, we used dependent *t*-tests to test whether the slopes and the intercepts were different in the two conditions. The slope can be interpreted as a measure of the effect of distance on the latencies. If RIG affects the effect of distance, then the slopes should be different in both conditions. The regression equation in the control condition was RT = *-15.4 * distance* + 516, and in the RIG-condition RT = *-18.4 * distance* + 645. Distance was a significant predictor both in control [$t(19) = 10.97$, $p <.001$] and in RIG [$t(19) = 5.83$, $p <.001$]. Critically, only the intercepts [$t(19) = 8.26$, $p <.001$] but not the slopes [$t(19) = 1.00$, $p > .30$] were different in both conditions.

Secondary Task Performance

We give a global overview rather than a detailed description of the analyses of the random sequences. The interested reader is referred other publications for procedural details (see Vandierendonck et al., 1998a; Vandierendonck, 2000c). In essence, random time intervals can be converted into a series of binary events by dividing the complete time course into a series of fixed intervals that either contain or do not contain a tap. By means of the appropriate statistics on the probability of the taps, the tendency to deviate from randomness (i.e., to alternate or to perseverate) can be estimated. These statistics and the analyses revealed that the RIG-task was performed equally well in the experimental condition as in the single secondary task control condition. This indicates that participants complied with task instructions and that no trade-offs between the primary and the secondary task occurred.

General Discussion

The goal of the present study was to investigate whether the effect of the Random time Interval Generation task (RIG; Vandierendonck et al., 1998a) on simple mental arithmetic as reported in several studies (De Rammelaere et al., 1999, 2001; De Rammelaere & Vandierendonck, 2001), is due to the fact that the RIG-task is solved by means of quantitative processes. To that end, we investigated a typical quantitative effect, the distance effect, in a control condition and in a condition with the RIG-task. If RIG calls on quantitative processes, then we should find that the distance effect decreases or even disappears under dual task. On the contrary, if the distance effect remains unchanged or even increases under dual-task, then it would be rather unlikely that RIG is solved by means of quantitative processes.

The data are very clear in showing that the distance effect remained unchanged under executive dual task, ruling out the alternative interpretation of the effect of RIG on simple arithmetic in terms of quantitative processes. Figure 1 shows that the distance effects in the control and the RIG condition were completely similar; the lines in the graph are almost parallel. This finding is in agreement with other indirect evidence against the claim that RIG is solved by means of quantitative processes. Indeed, two other quantitative effects, the "problem size" effect (i.e., large problems such as 9 + 8 are harder to solve than small problems such as 2 + 3) and the "split" effect (i.e., large-split problems such as 5 + 7 = 19 are solved faster than small-split problems such as 5 + 7 = 13) remain unchanged under RIG (De Rammelaere et al., 1999, 2001; De Rammelaere & Vandierendonck, 2001). In short, all data seem to indicate that the effect of RIG on simple mental arithmetic is not due to quantitative interference.

One could argue that we are defending a null result (i.e., the same distance effect in control and RIG) that could be due to a lack of power. However, the power of the study was sufficient to detect the distance effect in both conditions separately. It is clear for the data reported and from Figure 1 that there is no reliable interaction of condition by distance. In fact, as Figure 1 shows the difference between the control and the RIG condition tends to increase with distance rather than to decrease. In short, our data contained no trend at all for a *decreased* distance effect under dual-task conditions.

By showing that the distance effect is still observed under executive dual task, we have evidence for the fact that even under these conditions, it is possible to access the numerical semantic system. This is compatible with the converging evidence for a distance effect in a variety of tasks and situations (for an overview, see Butterworth, 1999; Dehaene, 1997; Gallistel & Gelman, 2000). Indeed, the distance effect has also been demonstrated for dot arrays compared for numerosity (Buckley & Gillman, 1974), for objects compared for size (Moyer, 1973), for two-digit numerals (Dehaene, Dupoux, & Mehler, 1990), for word numerals and bar graphs (Ratinckx, Brysbaert, & Reynvoet, 2001), in priming studies (e.g., Brysbaert, 1995) and even in non-quantitative tasks, such a linear reasoning (e.g., Potts, 1974, 1976). The effect has also been found in young children (e.g., Sekuler & Mierkiewicz, 1977), and even in Rhesus Monkeys (e.g., Brannon & Terrace, 2000).

The present findings also add to the accumulating body of evidence supporting the view that that the RIG task involves executive control (Vandierendonck et al., 1998a, Vandierendonck, 2000b). Vandierendonck (2000a) has suggested that several fundamental processes may be involved in random interval generation. Under the assumption that interval generation is driven by clock pulses as it is proposed in temporal information processing theories (e.g., Church, 1984, Treisman, 1963, Wearden, 1991), generation of time intervals requires monitoring of these clock pulses (input monitoring), response selection (decide to respond now or later), maintaining a memory window for previously produced intervals, updating of this memory after each new interval emitted, and possibly also monitoring of the output to check for deviations from randomness. Several of these processes lay at the heart of executive functions as commonly described in neuropsychology (e.g., Burgess, 1997) and in models of working memory (e.g., Baddeley, 1996). Whether all of the processes listed above are in the core of executive control is a matter for further research. There are indications, however, that at least the component of response selection plays a central role in many tasks used in dual-task studies (see e.g., Hegarty, Shah & Miyake, 2000) and in tasks that involve retrieval (e.g., Naveh-Benjamin, Craik, Gavrilescu, & Anderson, 2000; Rohrer, Pashler & Etchegaray, 1998; Szmalec, Vandierendonck & Kemps, 2003). How this and other the components mentioned above interact with mental arithmetic is an issue that is further developed in research presently performed in our laboratory.

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Received February, 2003
Revised and accepted January, 2004