## **Extending the Domain of Imprecise Jump Processes** from Simple Variables to Measurable Ones

 $\begin{array}{l} \Omega \\ \text{set of càdlàg paths} \\ \omega \colon \mathbb{R}_{\geq 0} \to \mathscr{X} \end{array}$ 

 $C = \{X_{t_{1:n}} = x_{1:n}\}$ <br/>conditioning event

 $A = \{X_{t_{1:m}} \in B\}$ algebra  $\mathscr{A}_C$  of finitary events

> $f = \sum_{k=1}^{n} a_k \mathbb{I}_{A_k}$  $\mathscr{A}_C$ -simple variable

Jumps to  $A \subseteq \mathscr{X}$ 

Then number of *jumps to* A over (s, r] is

$$\eta^{A}_{(s,r]} \coloneqq \left| \left\{ t \in (s,r] \colon X_t \in A, \lim_{\Delta \searrow 0} X_{t-\Delta} \notin A \right\} \right|$$

For all  $n \in \mathbb{N}$ , let

$$\Delta_n \coloneqq \frac{r-s}{n}$$
 and  $h_{n,0} \coloneqq 0$ ,

and for all  $k \in \{1, \ldots, n\}$ , let

$$h_{n,k} \coloneqq h_{n,k-1} + \Delta_n d_{n,k},$$

with

$$d_{n,k}: \mathscr{X} \to \mathbb{R} \\ : x \mapsto \left[\underline{Q}(\mathbb{I}_{A^{c}}(x)\mathbb{I}_{A} + h_{n,k-1})\right](x).$$

Then

$$\underline{E}^{\sigma}_{\mathscr{P}}(\eta^{A}_{(s,r]} | X_{s} = x) = \lim_{n \to +\infty} h_{n,n}(x)$$

 $\{f > \alpha\} \in \sigma(\mathscr{A}_C)$  $\sigma(\mathscr{A}_C)$ -measurable variable

## Integral of $g(X_t)$

For all  $n \in \mathbb{N}$ , let

 $\Delta_{i}$ 

$$n \coloneqq \frac{r-s}{n}$$
 and

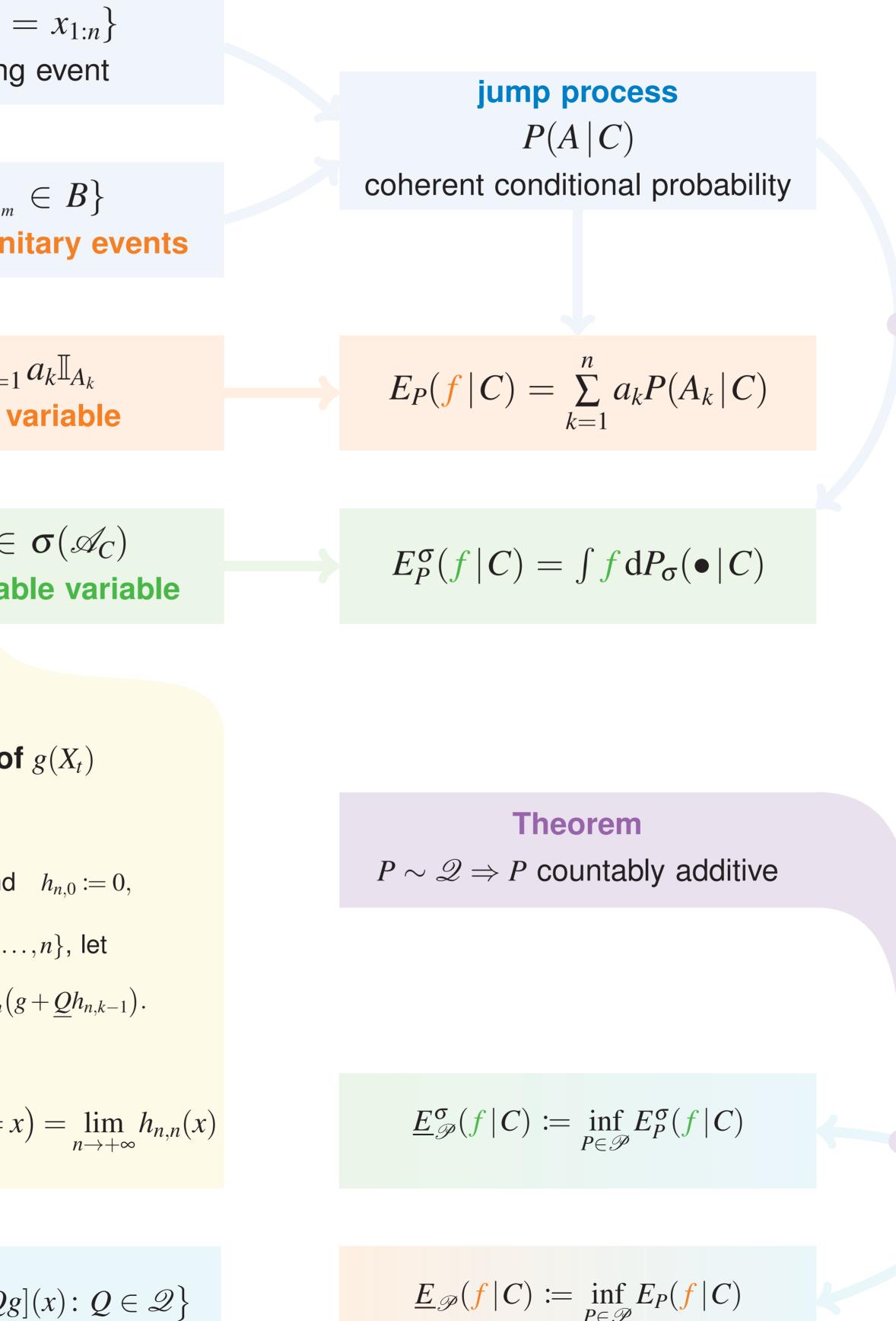
and for all and  $k \in \{1, \ldots, n\}$ , let

$$h_{n,k} := h_{n,k-1} + \Delta_n$$

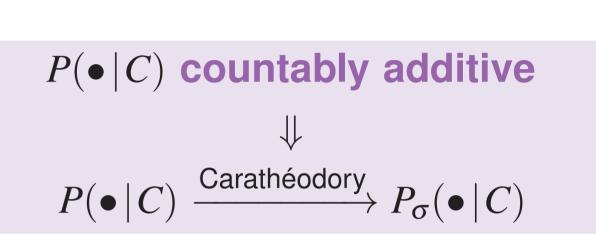
Then

$$\underline{E}^{\sigma}_{\mathscr{P}}\left(\int_{s}^{r}g(X_{t})\mathrm{d}t\,\Big|\,X_{s}=$$

 $\underline{Q} \colon \mathbb{R}^{\mathscr{X}} \to \mathbb{R}^{\mathscr{X}} \colon g \mapsto \underline{Q}g, \text{ where for all } x \in \mathscr{X}, \ [\underline{Q}g](x) \coloneqq \inf\{[Qg](x) \colon Q \in \mathscr{Q}\}$ 







## Homogeneous Markovian jump process

initial p.m.f.

rate operator

## Imprecise jump process

