Markovian Imprecise Jump Processes: Foundations, Algorithms and Applications

Alexander Erreygers

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Ghent University, ELIS, Foundations Lab for imprecise probabilities

A jump process is a mathematical model for a dynamical system whose state X_t

- takes values in a finite state space $\mathscr{X}$
- evolves over continuous time in an uncertain manner.

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dynamical system

state

state space

$$\{0, 1, 2, 3, 4, 5\}$$

Interested in things like

- the event that there is precisely 1 🙋 at 16:00 tomorrow;
- the event that at some point in the next 24 hours there are no 🙋;
- the number of 🙋 at 16:00 tomorrow;
- the average number of 🙋 over the next 24 hours.

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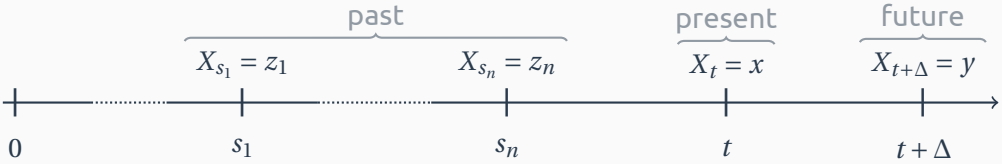

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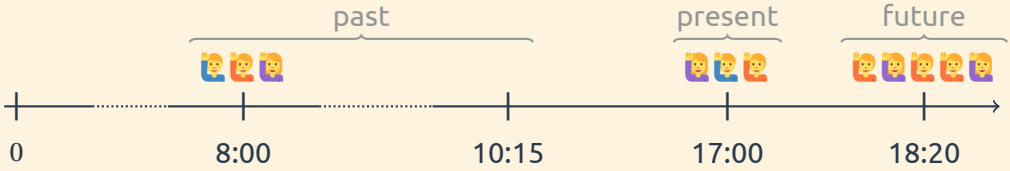

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🗝️ continuity 🗝️ 🧮 algorithms 🧮

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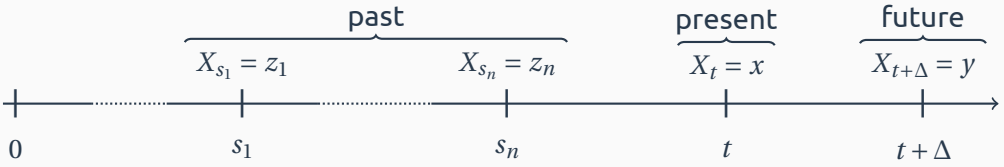
Markovian jump process

Specifying a jump process P is simpler under the Markov assumption that

$$P(X_{t+\Delta} = y \mid X_t = x, X_{s_1} = z_1, \ldots, X_{s_n} = z_n) = P(X_{t+\Delta} = y \mid X_t = x).$$



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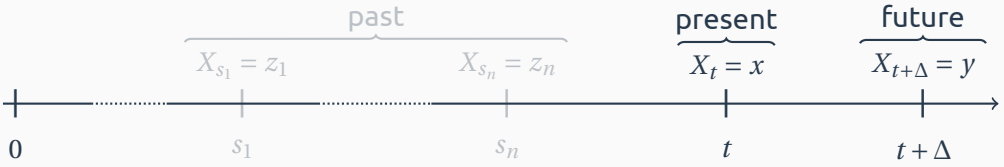

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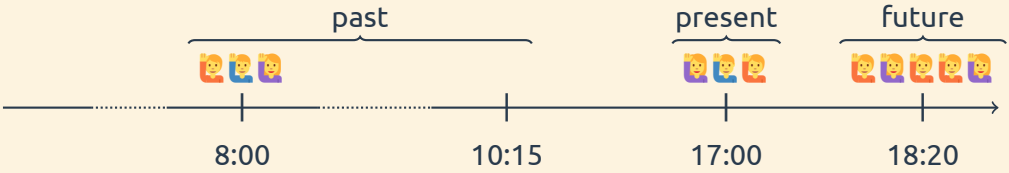

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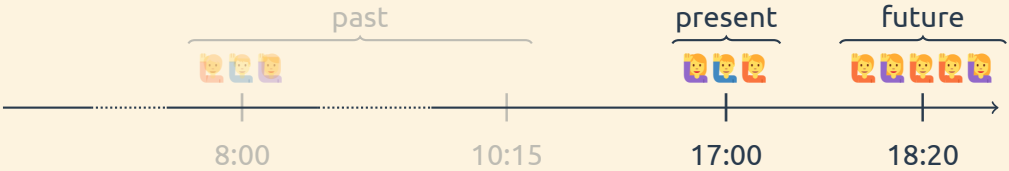

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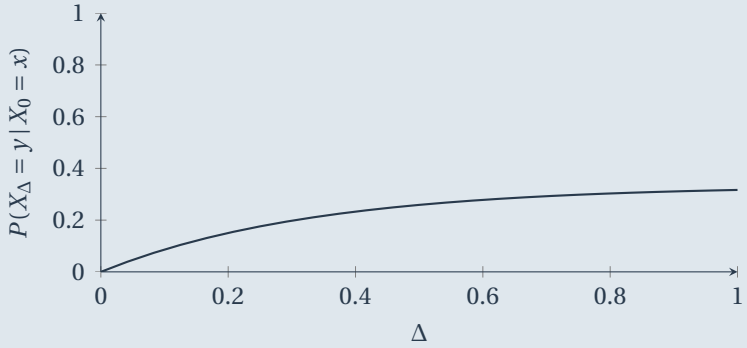

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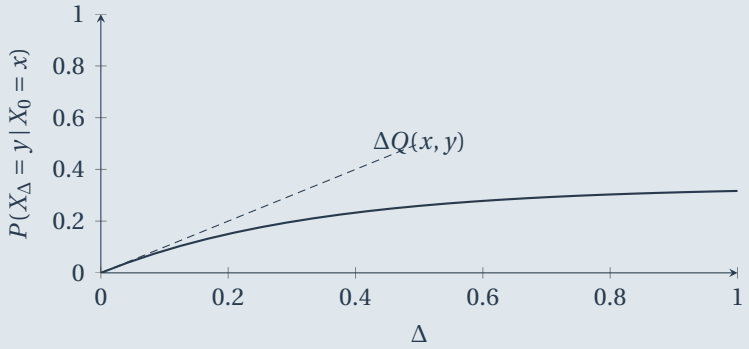

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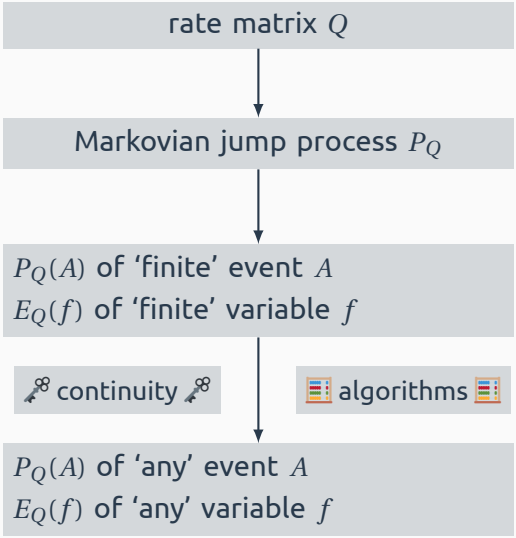
because a Markovian jump process P is uniquely determined by its rate matrix Q:

$$P(X_\Delta = y \mid X_0 = x) \approx P(X_0 = y \mid X_0 = x) + \Delta Q(x, y) \quad \text{for sufficiently small } \Delta.$$


rate matrix Q
Markovian jump process P_Q
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continuity
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What if we cannot specify the rate matrix Q precisely?

cannot spe rate matri v?

Markovian imprecise jump process

Given is a set of rate matrices $\mathcal{Q}$.

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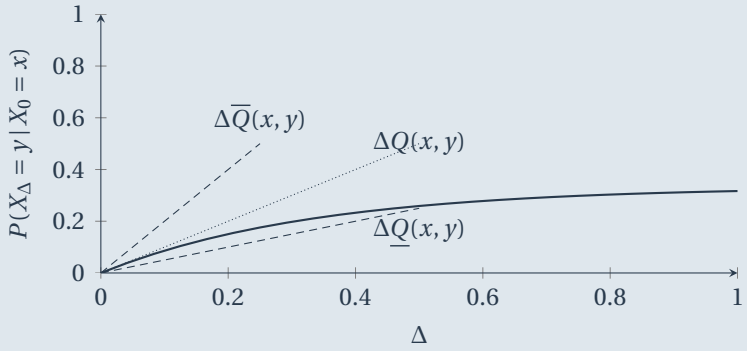
Let $\mathscr{P}^{\mathrm{M}}_{\mathcal{Q}}$ be the set of Markovian jump processes that are 'consistent' with $\mathcal{Q}$.



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We are interested in the lower/upper probability of the event A

– so in $\underline{P}^{\mathrm{M}}_{\mathcal{Q}}(A) := \inf_{P \in \mathscr{P}^{\mathrm{M}}_{\mathcal{Q}}} P(A)$ and $\overline{P}^{\mathrm{M}}_{\mathcal{Q}}(A) := \sup_{P \in \mathscr{P}^{\mathrm{M}}_{\mathcal{Q}}} P(A)$ –

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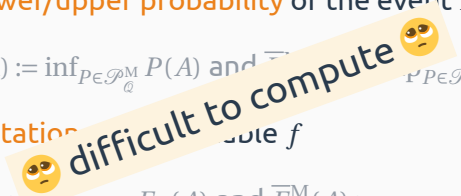

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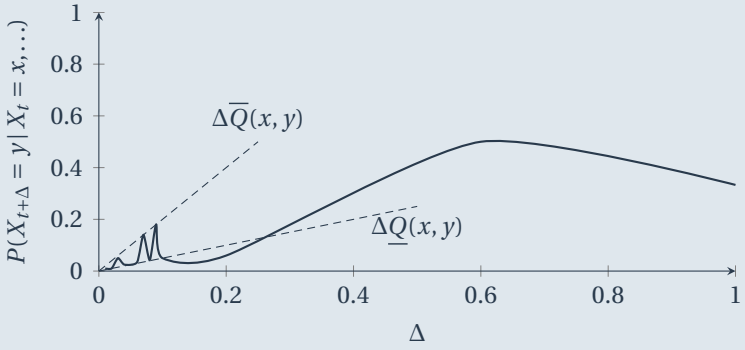

Markovian imprecise jump process

Given is a set of rate matrices $\mathcal{Q}$.

Let $\mathscr{P}_{\mathcal{Q}}$ be the set of jump processes that are 'consistent' with $\mathcal{Q}$.

Then we can compute the lower/upper probability of any finite event A

$$-\text{ so } \underline{P}_{\mathcal{Q}}(A) := \inf_{P \in \mathscr{P}_{\mathcal{Q}}} P(A) \text{ and } \overline{P}_{\mathcal{Q}}(A) := \sup_{P \in \mathscr{P}_{\mathcal{Q}}} P(A) -$$

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Markovian imprecise jump pro

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Then we can comput vent A

– so A) –

and the lower/upper

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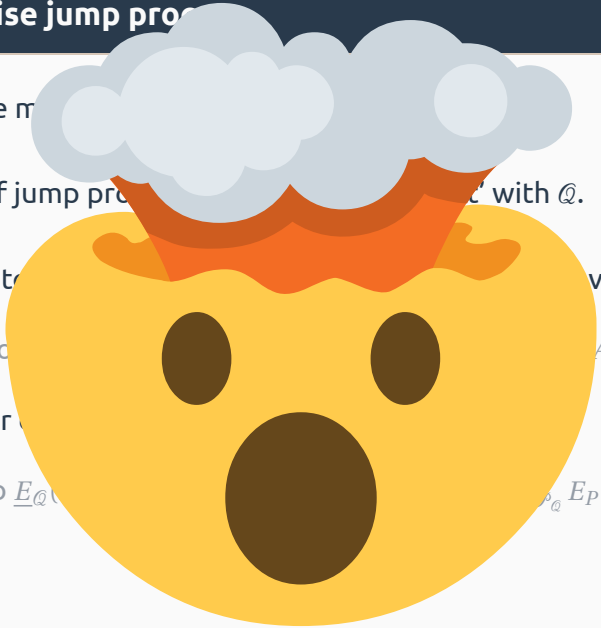

Foundations

rate matrix Q

↓

Markovian jump process P_Q

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$P_Q(A)$ of 'finite' event A
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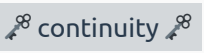

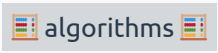

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set of rate matrices $\mathbb{Q}$

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Markovian imprecise
jump process $\mathscr{P}_{\mathbb{Q}}$

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🗝️ continuity 🗝️ | 🧮 algorithms 🧮

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🔑 continuity 🔑

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Theorem 5.19. *If a jump process P has uniformly bounded rate, then it is countably additive.*

belongs to $\sigma(\bar{\mathscr{F}}_u)$. Furthermore, it follows from repeated application of Eq. (5.41)$_{\curvearrowright}$ and (PM8)$_{461}$ that for all k in $\mathbb{N}$,

$$\bar{P}_\sigma\Big(\bigcap_{n=1}^{k}\bar{A}_n\Big)=\bar{P}_\sigma\Big(\bigcap_{n=1}^{k}\{\theta^D_{[s,r_n]}\ll+\infty\}\Big)=1; \tag{5.42}$$

from this equality and Lemma C.3$_{463}$ (CA3), it follows immediately that

$$\bar{P}_\sigma(\bar{A}_D)=\lim_{k\to+\infty}\bar{P}_\sigma\Big(\bigcap_{n=1}^{k}\bar{A}_n\Big)=1.$$

The key point is that by construction, every path ω in $\bar{A}_D$ satisfies the conditions of Lemma 5.51$_{260}$. Consequently, for every t in $]s,+\infty[$ we can define the $\mathscr{X}$-valued variable $\bar{Y}_t$ by

$$\bar{Y}_t(\omega):=\begin{cases}\lim_{D\ni r\searrow t}\omega(r) & \text{if } \omega\in\bar{A}_D\\ \omega(s) & \text{otherwise}\end{cases}\quad\text{for all } \omega\in\bar{\Omega}.$$

Observe that (i)$_{\curvearrowright}$ follows immediately from the definition above and Lemma 5.51$_{260}$. Thus, what is left, is to prove the three remaining properties of $(\bar{Y}_t)_{t\in]s,+\infty[}$.

To prove (ii)$_{\curvearrowright}$, we fix a future time point t in $]s,+\infty[$ and a state x. Let $(r_n)_{n\in\mathbb{N}}$ be any decreasing sequence in $]t,+\infty[\cap D$ that converges to t. Then it follows immediately from the definition of the variable $\bar{Y}_t$ that for all ω in $\bar{\Omega}$,

$$\bar{Y}_t(\omega)=\begin{cases}\lim_{n\to+\infty}\omega(r_n) & \text{if } \omega\in\bar{A}_D,\\ \omega(s) & \text{otherwise.}\end{cases} \tag{5.43}$$

Consequently,

$$[\bar{Y}_t=x]=\Big(\bar{A}_D\cap\Big(\bigcup_{n\in\mathbb{N}}\bigcap_{k=n}^{+\infty}[\bar{X}_{r_k}=x]\Big)\Big)\cup\big(\bar{A}^c_D\cap[\bar{X}_s=x]\big).$$

Because $\bar{A}_D$ and all $[\bar{X}_{r_n}=x]$ and $[\bar{X}_s=x]$ belong to $\sigma(\bar{\mathscr{F}}_u)$, it follows from this and the properties of σ-fields that $[\bar{Y}_t=x]$ belongs to $\sigma(\bar{\mathscr{F}}_u)$.

Next, we prove (iii)$_{\curvearrowright}$. Again, we fix any future time point t in $]s,+\infty[$, and let $(r_n)_{n\in\mathbb{N}}$ be any decreasing sequence in $]t,+\infty[\cap D$ that converges to t. Recall that for all x in $\mathscr{X}$, $[\bar{X}_t=x]$ belongs to $\sigma(\bar{\mathscr{F}}_u)$ by construction of $\bar{\mathscr{F}}_u$ and $[\bar{Y}_t=x]$ belongs to $\sigma(\bar{\mathscr{F}}_u)$ by (ii)$_{\curvearrowright}$. Consequently,

$$[\bar{X}_t=\bar{Y}_t]=\bigcup_{x\in\mathscr{X}}[\bar{X}_t=x]\cap[\bar{Y}_t=x]$$

belongs to $\sigma(\bar{\mathscr{F}}_u)$ as well because this σ-field is closed under countable – and hence finite – unions. Consequently, $[\bar{X}_t\neq\bar{Y}_t]:=[\bar{X}_t=\bar{Y}_t]^c$ belongs to $\sigma(\bar{\mathscr{F}}_u)$ due to (F2)$_{32}$, and it follows from (PM6)$_{461}$ that

$$\bar{P}_{\sigma'}(\bar{X}_t=\bar{Y}_t)=1-\bar{P}_{\sigma'}(\bar{X}_t\neq\bar{Y}_t). \tag{5.44}$$

Observe that

$$[\bar{X}_t\neq\bar{Y}_t]=\big(\bar{A}_D\cap[\bar{X}_t\neq\bar{Y}_t]\big)\cup\big(\bar{A}^c_D\cap[\bar{X}_t\neq\bar{Y}_t]\big).$$

From this and Eq. (5.43)$_{\curvearrowright}$, it follows that

$$\mathbb{I}_{[\bar{X}_t\neq\bar{Y}_t]}=\mathbb{I}_{\big(A_D\cap[X_t\neq\bar{Y}_t]\big)\cup\big(A^c_D\cap[X_t\neq\bar{Y}_t]\big)}=\mathbb{P}\text{-w}\lim_{n\to+\infty}\mathbb{I}_{\big(A_D\cap[X_t\neq X_{r_n}]\big)\cup\big(A^c_D\cap[X_t\neq X_s]\big)}.$$

Because $\bar{A}_D$, $[\bar{X}_t\neq\bar{X}_{r_n}]$ and $[\bar{X}_t\neq\bar{X}_s]$ belong to $\sigma(\bar{\mathscr{F}}_u)$, it follows from this, (DE1)$_{225}$ and Theorem 5.11$_{226}$, that

$$\begin{aligned}\bar{P}_\sigma(\bar{X}_t\neq\bar{Y}_t)&=E^{\mathbb{D}}_{\bar{P}_\sigma}\big(\mathbb{I}_{[\bar{X}_t\neq\bar{Y}_t]}\big)=\lim_{n\to+\infty}E^{\mathbb{D}}_{\bar{P}_\sigma}\Big(\mathbb{I}_{\big(A_D\cap[X_t\neq X_{r_n}]\big)\cup\big(A^c_D\cap[X_t\neq X_s]\big)}\Big)\\&=\lim_{n\to+\infty}P_\sigma\big(\big(A_D\cap[X_t\neq X_{r_n}]\big)\cup\big(A^c_D\cap[X_t\neq X_s]\big)\big).\end{aligned}$$

Observe that because $\bar{P}_\sigma(\bar{A}_D)=1$, it follows from (PM8)$_{461}$ that for all n in $\mathbb{N}$,

$$\bar{P}_\sigma\big(\big(\bar{A}_D\cap[\bar{X}_t\neq\bar{X}_{r_n}]\big)\cup\big(\bar{A}^c_D\cap[\bar{X}_t\neq\bar{X}_s]\big)\big)=\bar{P}_\sigma\big(\bar{A}_D\cap[\bar{X}_t\neq\bar{X}_{r_n}]\big)=\bar{P}_\sigma\big([\bar{X}_t\neq\bar{X}_{r_n}]\big).$$

We substitute this equality in the preceding equality, to yield

$$\bar{P}_\sigma(\bar{X}_t\neq\bar{Y}_t)=\lim_{n\to+\infty}\bar{P}_\sigma\big(\bar{X}_t\neq\bar{X}_{r_n}\big).$$

Finally, because $(r_n)_{n\in\mathbb{N}}$ decreases to t and $\bar{P}_{\sigma'}$ has uniformly bounded rate, we can conclude that

$$\bar{P}_{\sigma'}(\bar{X}_t\neq\bar{Y}_t)=\lim_{n\to+\infty}\bar{P}_{\sigma'}(\bar{X}_t\neq\bar{X}_{r_n})=0.$$

The statement follows immediately from this and Eq. (5.44)$_{\curvearrowright}$.

Finally, we prove (iv)$_{261}$. The statement holds trivially in case v is the empty sequence of time points, so we may assume without loss of generality that v is non-empty. Because in the definition

$$[\bar{Y}_v\in B]=\bigcup_{x_e\in B}\bigcap_{t\in v}[\bar{Y}_t=x_t],$$

all events on the right-hand side belong to $\sigma(\bar{\mathscr{F}})$ due to (iii)$_{261}$, so does $[\bar{Y}_v\in B]$. This verifies the first part of (iv)$_{261}$. To verify the second part, we observe that

$$[\bar{X}_v=\bar{Y}_v]:=\bigcap_{t\in v}[\bar{X}_t=\bar{Y}_t]$$

belongs to $\sigma(\bar{\mathscr{F}}_u)$ because all events on the right-hand side belong to $\sigma(\bar{\mathscr{F}}_u)$, this time by construction of $\bar{\mathscr{F}}_u$ and due to (iii)$_{261}$. Because furthermore $\bar{P}_{\sigma'}(\bar{X}_t=\bar{Y}_t)=1$ for all t in v due to (iii)$_{261}$, it follows from (PM8)$_{461}$ that $\bar{P}_{\sigma'}(\bar{X}_v=\bar{Y}_v)=1$. For this reason, it follows from (PM8)$_{461}$ that

$$\bar{P}_{\sigma'}(\bar{Y}_v\in B)=\bar{P}_{\sigma'}\big([\bar{Y}_v\in B]\cap[\bar{X}_v=\bar{Y}_v]\big)=\bar{P}_{\sigma'}\big([\bar{X}_v=\bar{Y}_v]\cap[\bar{X}_v\in B]\big)=\bar{P}_{\sigma'}(\bar{X}_v\in B),$$

which verifies the second part of (iv)$_{261}$. □

Algorithms

rate matrix Q

Markovian jump process P_Q

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continuity

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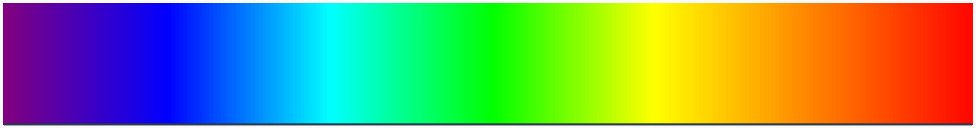
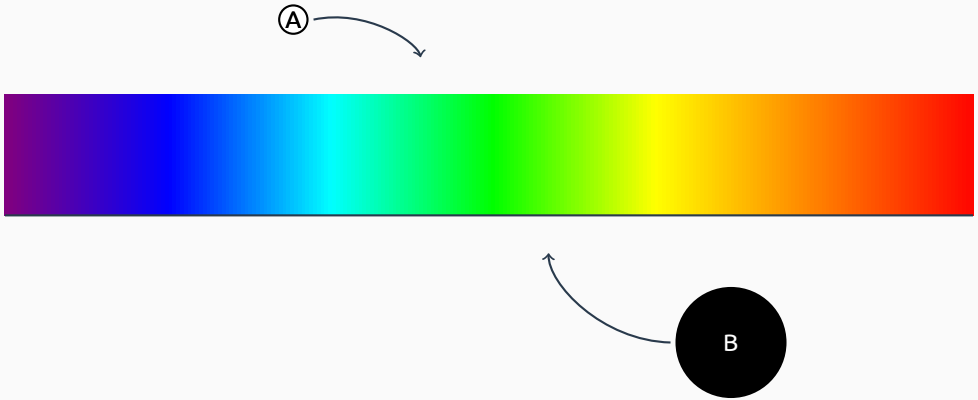
Applications

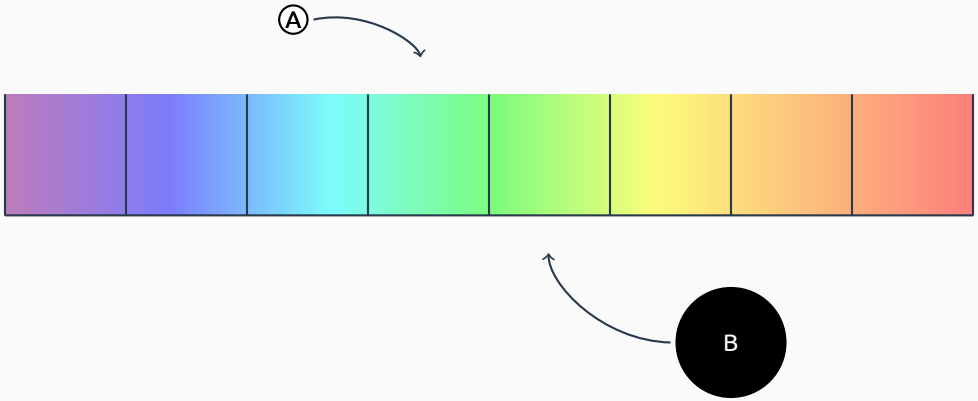
Spectrum allocation in an optical link



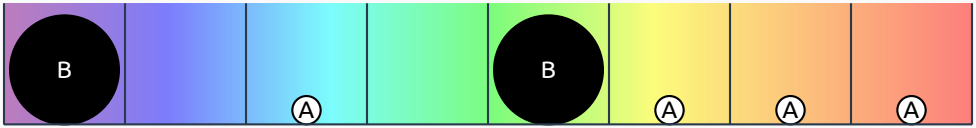
Spectrum allocation in an optical link



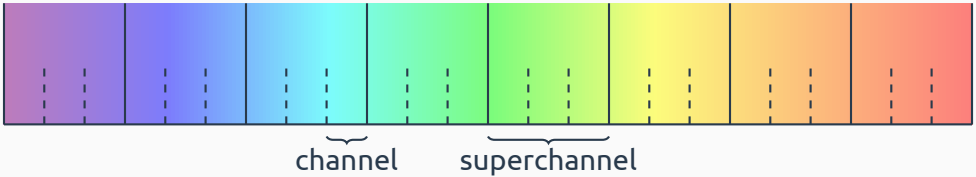
Spectrum allocation in an optical link



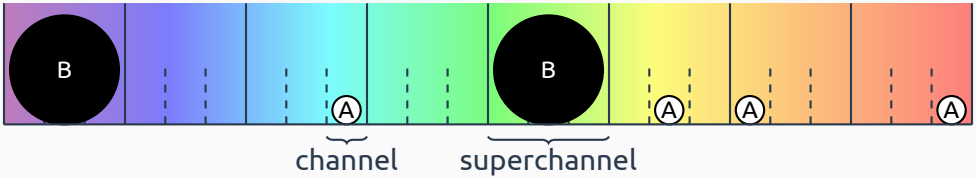
Spectrum allocation in an optical link



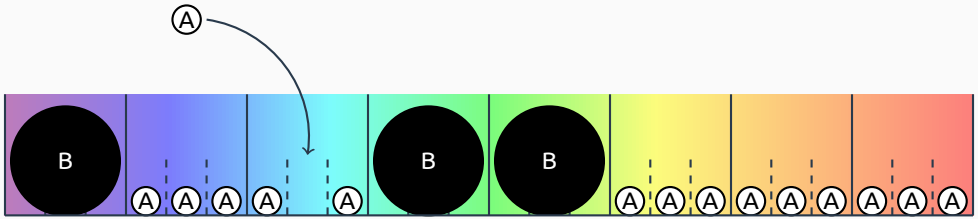
Spectrum allocation in an optical link



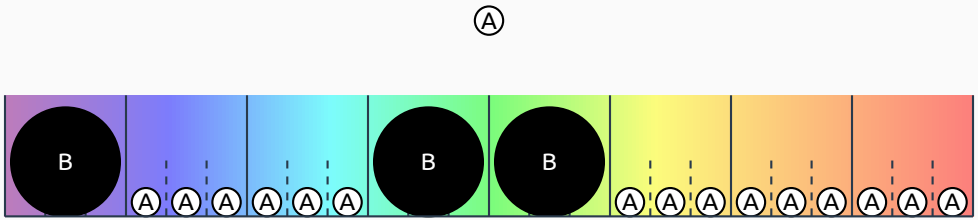
Spectrum allocation in an optical link



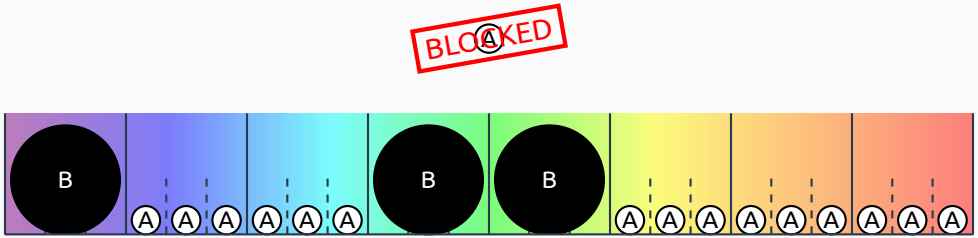
Blocking



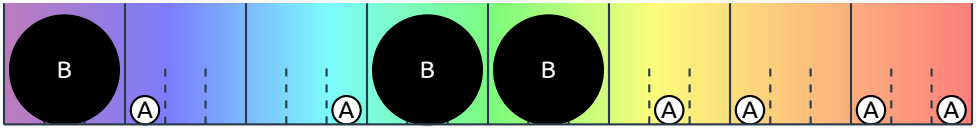
Blocking



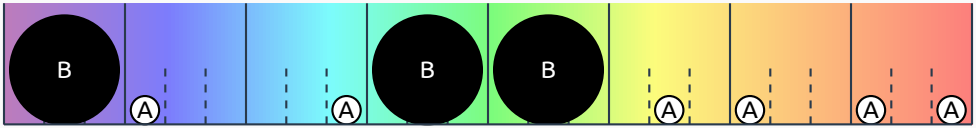
Blocking



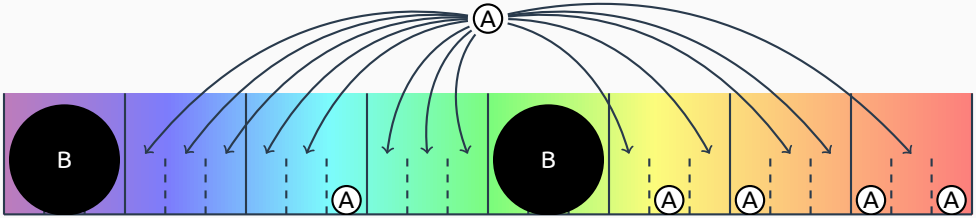
Blocking



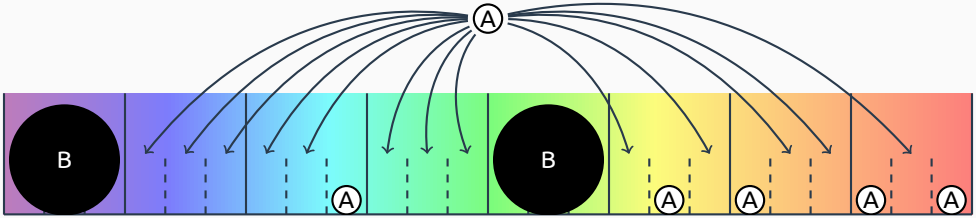
Blocking



Allocation policies

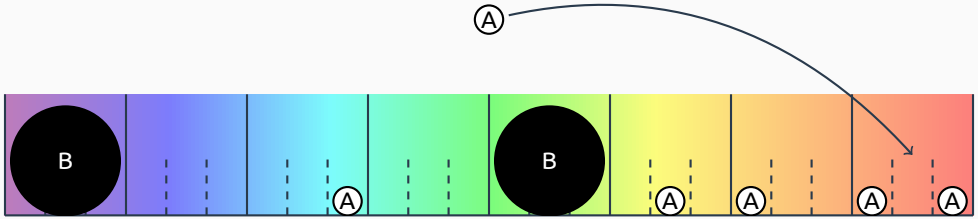


Allocation policies



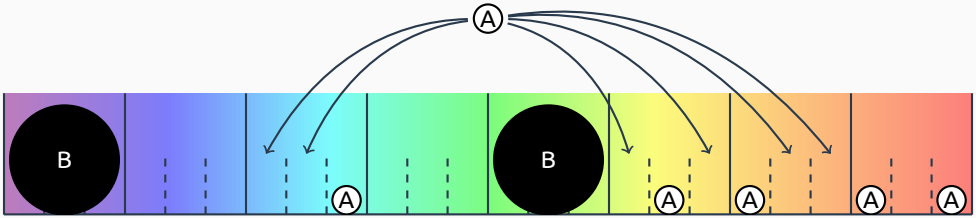
random allocation policy

Allocation policies



most-filled allocation policy

Allocation policies



least-filled allocation policy

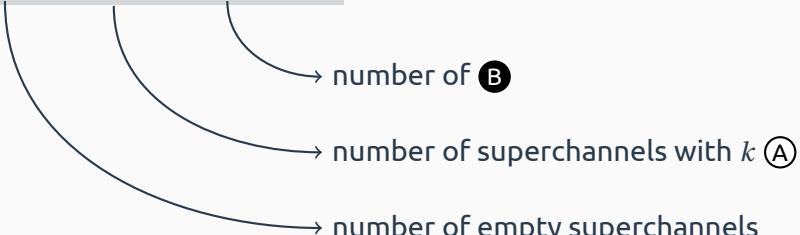
Allocation policies

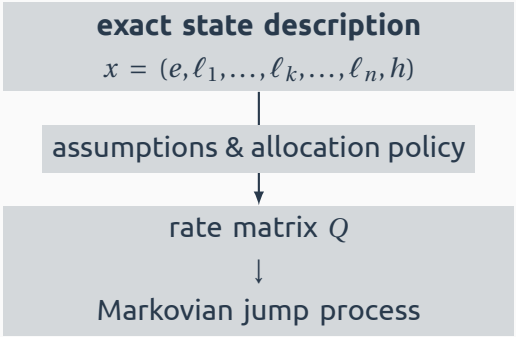
performance measures for the allocation policies

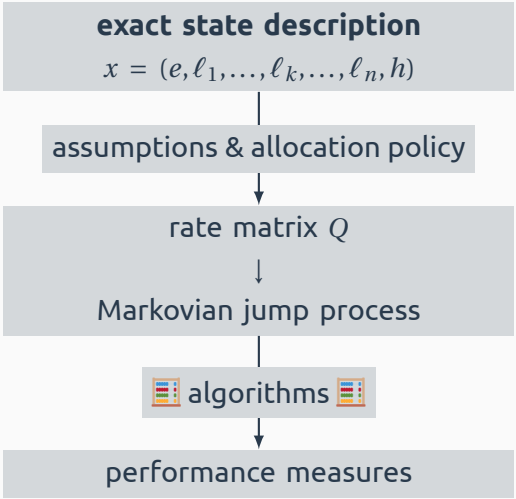
- expected number of BLOCKED Ⓐ/🅑 over the following hour
- expected long-term proportion BLOCKED Ⓐ/🅑
- ...

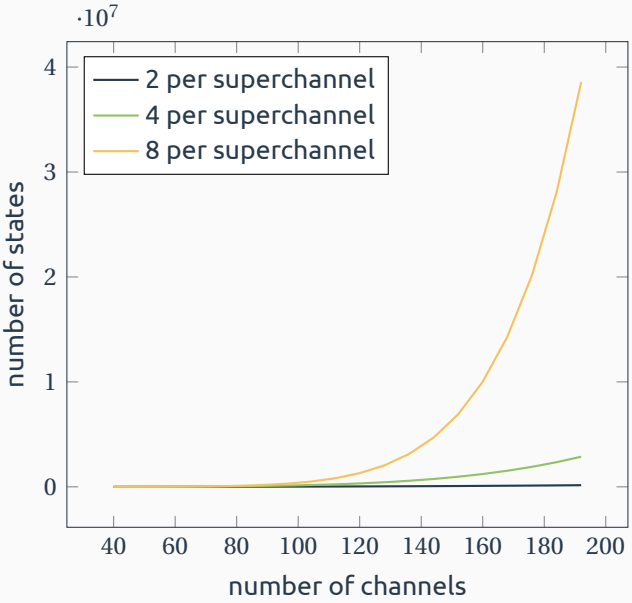
exact state description

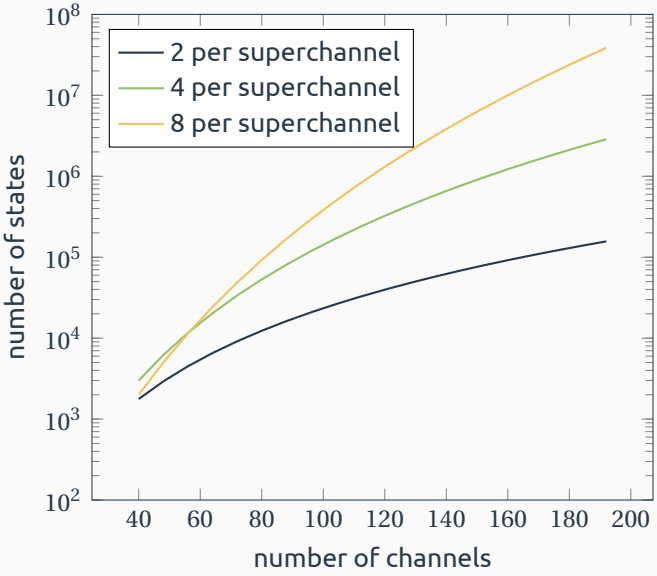
$x = (e, \ell_1, \ldots, \ell_k, \ldots, \ell_n, h)$

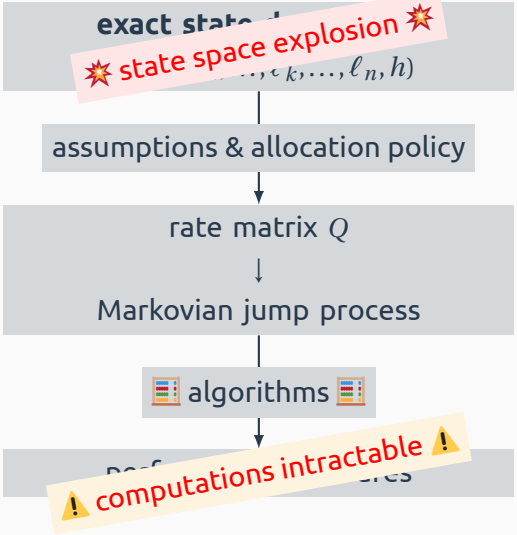



exact state description
$x = (e, \ell_1, \ldots, \ell_k, \ldots, \ell_n, h)$
assumptions & allocation policy
rate matrix Q
Markovian jump process


exact state description
$x = (e, \ell_1, \ldots, \ell_k, \ldots, \ell_n, h)$
assumptions & allocation policy
rate matrix Q
Markovian jump process
algorithms
performance measures


$\cdot 10^7$
4
3
2
1
0
number of states
2 per superchannel
4 per superchannel
8 per superchannel
40
60
80
100
120
140
160
180
200
number of channels


2 per superchannel
4 per superchannel
8 per superchannel
number of states
10^8
10^7
10^6
10^5
10^4
10^3
10^2
40
60
80
100
120
140
160
180
200
number of channels


exact state
$\ell_k, \ldots, \ell_n, h)$
state space explosion
assumptions & allocation policy
rate matrix Q
Markovian jump process
algorithms
computations intractable

exact state description

$x = (e, \ell_1, \ldots, \ell_k, \ldots, \ell_n, h)$

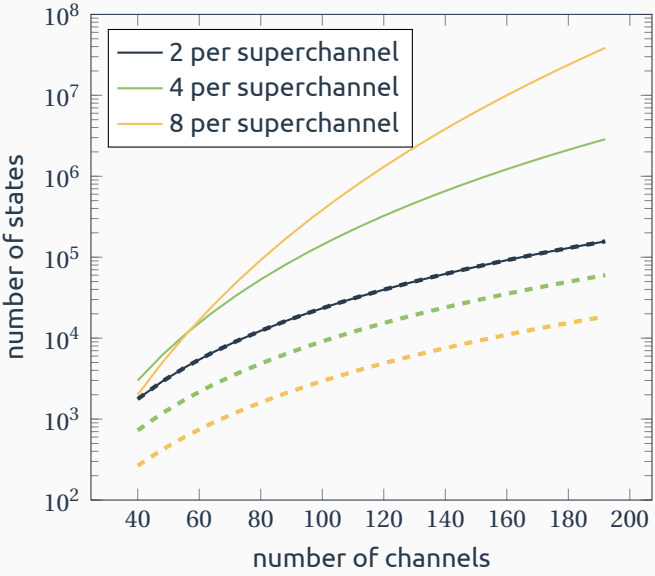
lumped state description

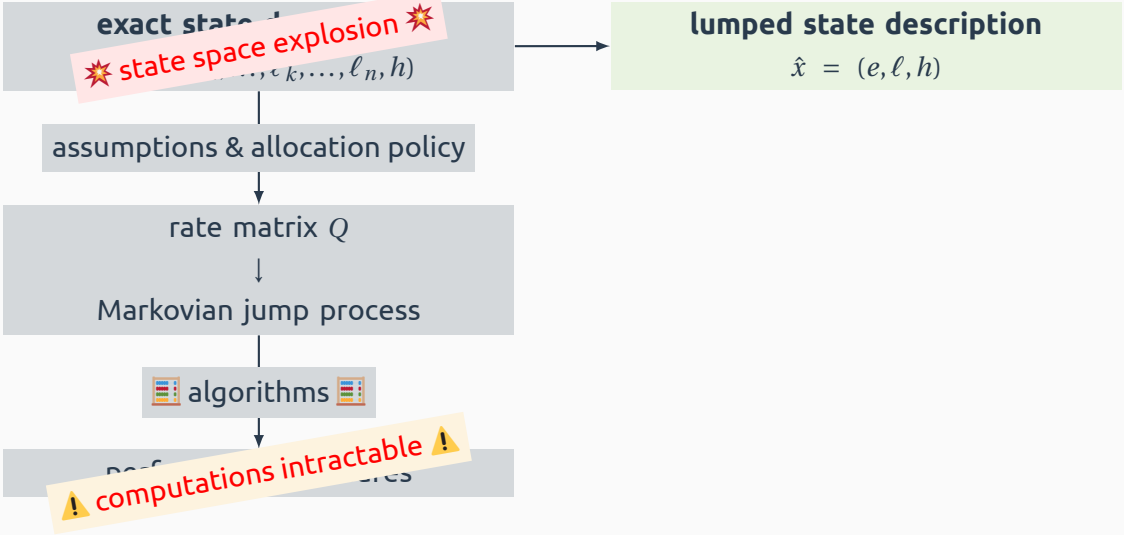
$\hat{x} = (e, \ell, h)$

number of empty superchannels

number of Ⓐ

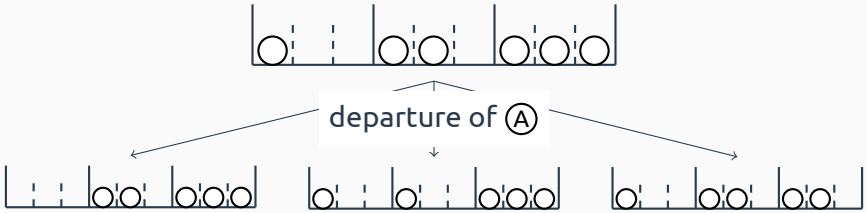
number of ⓑ


2 per superchannel
4 per superchannel
8 per superchannel
number of states
10^8
10^7
10^6
10^5
10^4
10^3
10^2
40
60
80
100
120
140
160
180
200
number of channels


exact state d
$\ell_k, \ldots, \ell_n, h)$
state space explosion
lumped state description
$\hat{x} = (e, \ell, h)$
assumptions & allocation policy
rate matrix Q
Markovian jump process
algorithms
computations intractable

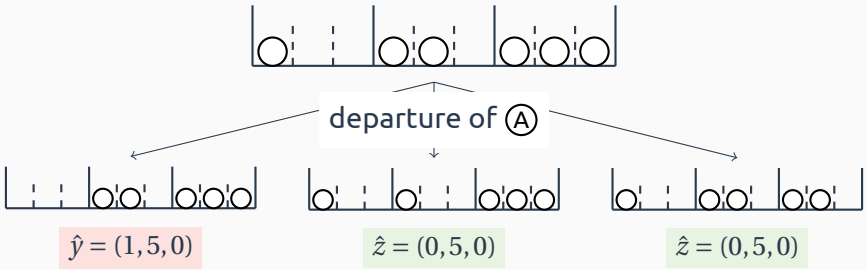
$$\hat{x} = (e, \ell, h) = (0, 6, 0)$$

$\hat{x} = (e, \ell, h) = (0, 6, 0)$

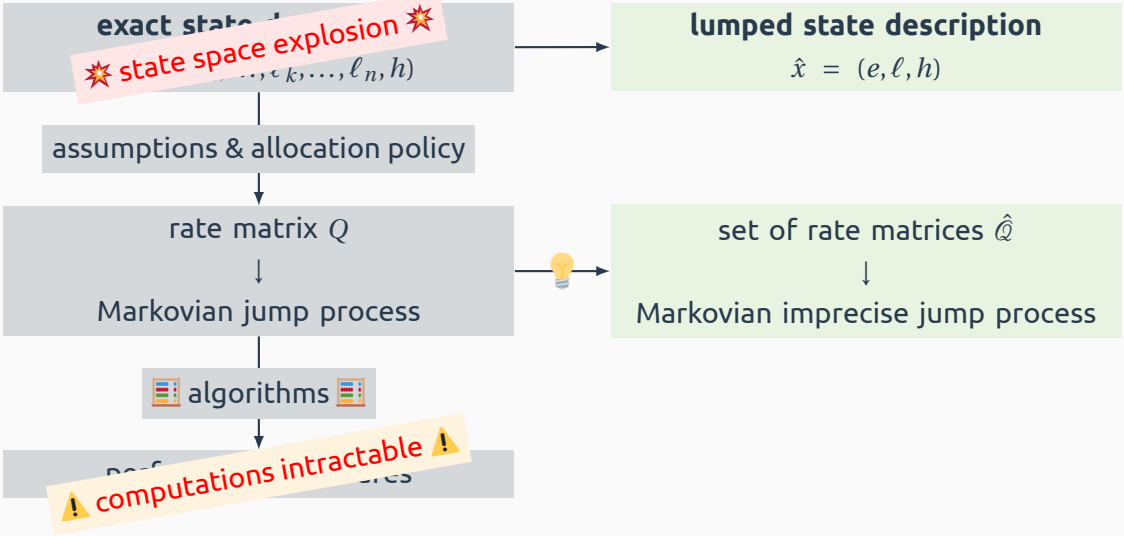


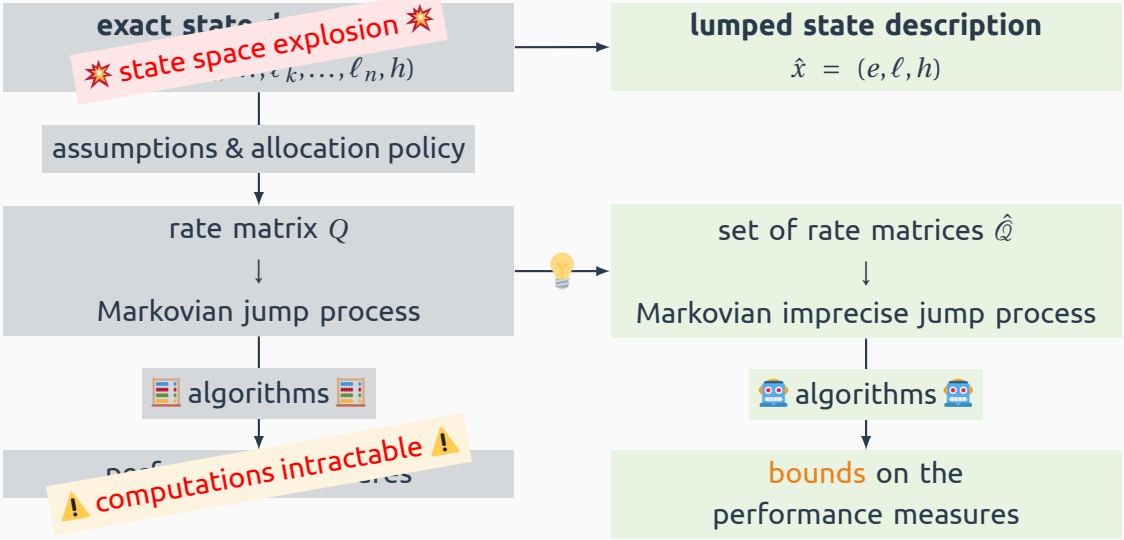
$\hat{y} = (1, 5, 0)$ $\hat{z} = (0, 5, 0)$ $\hat{z} = (0, 5, 0)$

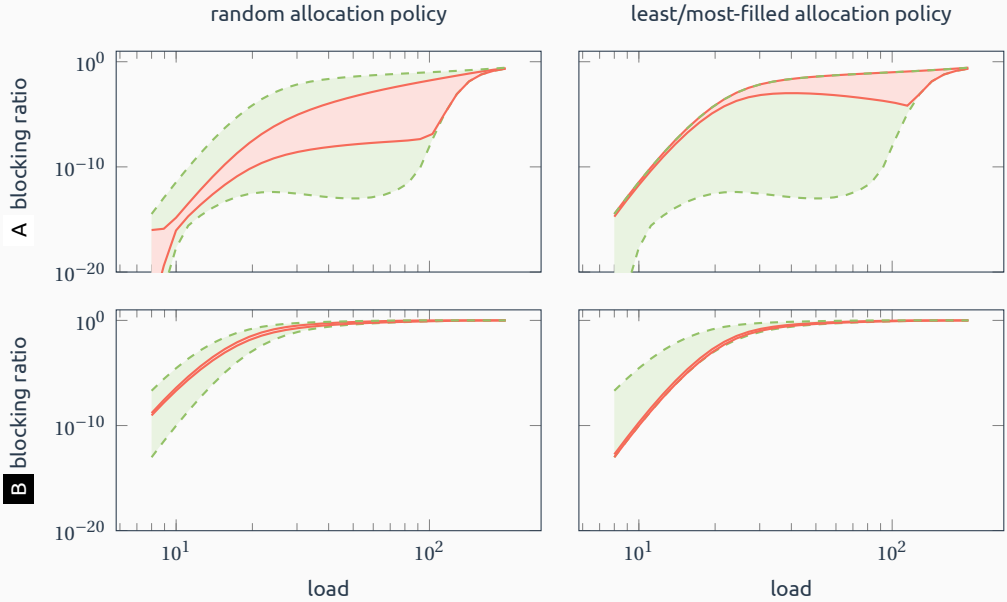
$$\hat{x} = (e, \ell, h) = (0, 6, 0)$$



Due to the loss of information,
we cannot specify the transition rates exactly!


exact state d
$\ell_k, \ldots, \ell_n, h)$
state space explosion
lumped state description
$\hat{x} = (e, \ell, h)$
assumptions & allocation policy
rate matrix Q
Markovian jump process
set of rate matrices $\hat{\mathcal{Q}}$
Markovian imprecise jump process
algorithms
computations intractable


exact state
$\ldots, \ell_k, \ldots, \ell_n, h)$
state space explosion
lumped state description
$\hat{x} = (e, \ell, h)$
assumptions & allocation policy
rate matrix Q
Markovian jump process
algorithms
computations intractable
set of rate matrices $\hat{\mathcal{Q}}$
Markovian imprecise jump process
algorithms
bounds on the
performance measures


random allocation policy
least/most-filled allocation policy
A
B
blocking ratio
blocking ratio
10^{0}
10^{-10}
10^{-20}
10^{1}
10^{2}
load
load