

# lavaan: an R package for structural equation modeling and more

## Version 0.5-12 (BETA)

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### Abstract

In this document, we illustrate the use of **lavaan** by providing several examples. If you are new to **lavaan**, this is the first document to read.

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# 1 Before you start

Before you start, please read these points carefully:

- First of all, you must have a recent version (2.14.0 or higher) of R installed. You can download the latest version of R from this page: <http://cran.r-project.org/>.
- The **lavaan** package is not finished yet. But it is already very useful for most users, or so we hope. However, some important features that are currently *NOT* available in **lavaan** are:
  - support for hierarchical/multilevel datasets (multilevel cfa, multilevel sem)
  - support for discrete latent variables (mixture models, latent classes)
  - Bayesian estimation

We hope to add these features in the next (two?) year(s) or so.

- We consider the current version as ‘beta’ software. This does NOT mean that you can not trust the results. We believe the results are accurate. It does mean that things may change when new versions come out. For example, we may change the name of the arguments in function calls. And we change the internals of the source code constantly. However, the model syntax is fairly mature and has been stable for a while.
- We do not expect you to be an expert in R. In fact, the **lavaan** package is designed to be used by users that would normally never use R. Nevertheless, it may help to familiarize yourself a bit with R, just to be comfortable with it. Perhaps the most important skill that you may need to learn is how to import your own datasets (perhaps in an SPSS format) into R. There are many tutorials on the web to teach you just that. Once you have your data in R, you can start specifying your model. We have tried very hard to make it as easy as possible for users to fit their models. Of course, if you have suggestions on how we can improve things, please let us know.
- This document is written for first-time users (and beta-testers) of the **lavaan** package. It is not a reference manual, nor does it contain technical material on how things are done in the **lavaan** package. These documents are currently under preparation.
- The **lavaan** package is free open-source software. This means (among other things) that there is no warranty whatsoever.
- The numerical results of the **lavaan** package are typically very close, if not identical, to the results of the commercial package Mplus. If you wish to compare the results with those obtained by other SEM packages, you can use the optional argument `mimic="EQS"` when calling the `cfa`, `sem` or `growth` functions (see section 9.2).

- (New since 12 September 2012). If you need help, you can ask questions in the lavaan discussion group. Go to <https://groups.google.com/d/forum/lavaan/> and join the group. Once you have joined the group, you can email your questions to [lavaan@googlegroups.com](mailto:lavaan@googlegroups.com). If you think you have found a bug, or if you have a suggestion for improvement, you can open an issue on github (see <https://github.com/yrosseel/lavaan/issues>). If you open an issue to report a bug, it is always very useful to provide a reproducible example (a short R script and some data).
- This document is not up to date (as of 12 September 2012). We are working on a web-based user manual that will gradually replace this document. The 'lavaan paper' (<http://www.jstatsoft.org/v48/i02/>) is more up to date (at least up to version 0.4-14).

## 2 Installation of the lavaan package

Since May 2010, the **lavaan** package is available on CRAN. Therefore, to install **lavaan**, simply start up R, and type:

```
> install.packages("lavaan", dependencies=TRUE)
```

You can check if the installation was successful by typing

```
> library(lavaan)

This is lavaan 0.5-12
lavaan is BETA software! Please report any bugs.
```

When the package is loaded, a startup message will be displayed showing the version number, and a reminder that this is beta software.

## 3 The model syntax

At the heart of the **lavaan** package is the 'model syntax'. The model syntax is a description of the model to be estimated. In this section, we briefly explain the elements of the **lavaan** model syntax. More details are given in the examples that follow.

In the R environment, a regression formula has the following form:

$$y \sim x1 + x2 + x3 + x4$$

In this formula, the tilde sign (" $\sim$ ") is the regression operator. On the left-hand side of the operator, we have the dependent variable ( $y$ ), and on the right-hand side, we have the independent variables, separated by the "+" operator. In **lavaan**, a typical model is simply a set (or system) of regression formulas, where some variables (starting with an 'f' below) may be latent. For example:

$$\begin{aligned} y &\sim f1 + f2 + x1 + x2 \\ f1 &\sim f2 + f3 \\ f2 &\sim f3 + x1 + x2 \end{aligned}$$

If we have latent variables in any of the regression formulas, we must 'define' them by listing their manifest indicators. We do this by using the special operator " $=\sim$ ", which can be read as *is manifested by*. For example, to define the three latent variables  $f1$ ,  $f2$  and  $f3$ , we can use something like:

$$\begin{aligned} f1 &=\sim y1 + y2 + y3 \\ f2 &=\sim y4 + y5 + y6 \\ f3 &=\sim y7 + y8 + y9 + y10 \end{aligned}$$

Furthermore, variances and covariances are specified using a 'double tilde' operator, for example:

$$\begin{aligned} y1 &\sim\sim y1 \\ y1 &\sim\sim y2 \\ f1 &\sim\sim f2 \end{aligned}$$

And finally, intercepts for observed and latent variables are simple regression formulas with only an intercept (explicitly denoted by the number '1') as the only predictor:

$$\begin{aligned} y1 &\sim 1 \\ f1 &\sim 1 \end{aligned}$$

Using these four *formula types*, a large variety of latent variable models can be described. But new formula types may be added in the future. The current set of formula types is summarized in the table below.

formula type	operator	mnemonic
latent variable definition	=~	is measured by
regression	~	is regressed on
(residual) (co)variance	~~	is correlated with
intercept	~ 1	intercept

### 3.1 Entering the model syntax as a string literal

If the model syntax is fairly short, you can specify it interactively at the R prompt by enclosing the formulas with single quotes. For example:

```
> myModel <- ' # regressions
  y1 + y2 ~ f1 + f2 + x1 + x2
    f1 ~ f2 + f3
    f2 ~ f3 + x1 + x2

# latent variable definitions
f1 =~ y1 + y2 + y3
f2 =~ y4 + y5 + y6
f3 =~ y7 + y8 +
    y9 + y10

# variances and covariances
y1 ~~ y1
y1 ~~ y2
f1 ~~ f2

# intercepts
y1 ~ 1
f1 ~ 1
'
```

Of course, instead of typing this interactively at the R prompt, you may prefer to type the whole model first in an external text editor, and when you are done, you can copy/paste it to the R console. This piece of code will produce a model syntax object, called `myModel` that can be used later when calling a function that actually estimates this model given a dataset. Note that formulas can be split over multiple lines, and you can use comments (starting with the `#` character) and blank lines within the single quotes to improve readability of the model syntax.

### 3.2 Reading the model syntax from an external file

If your model syntax is rather long, or you need to reuse the model syntax over and over again, you may prefer to type it in a separate text file called, say, `myModel.lav`. This text file should be in a human readable format (not a Word document). Within R, you can then read the model syntax from the file as follows:

```
> myModel <- readLines("/mydirectory/myModel.lav")
```

The argument of `readLines` is the full path to the file containing the model syntax. Again, the model syntax object `myModel` can be used later to fit this model given a dataset.

## 4 Fitting latent variable models: two examples

### 4.1 A first example: confirmatory factor analysis (CFA)

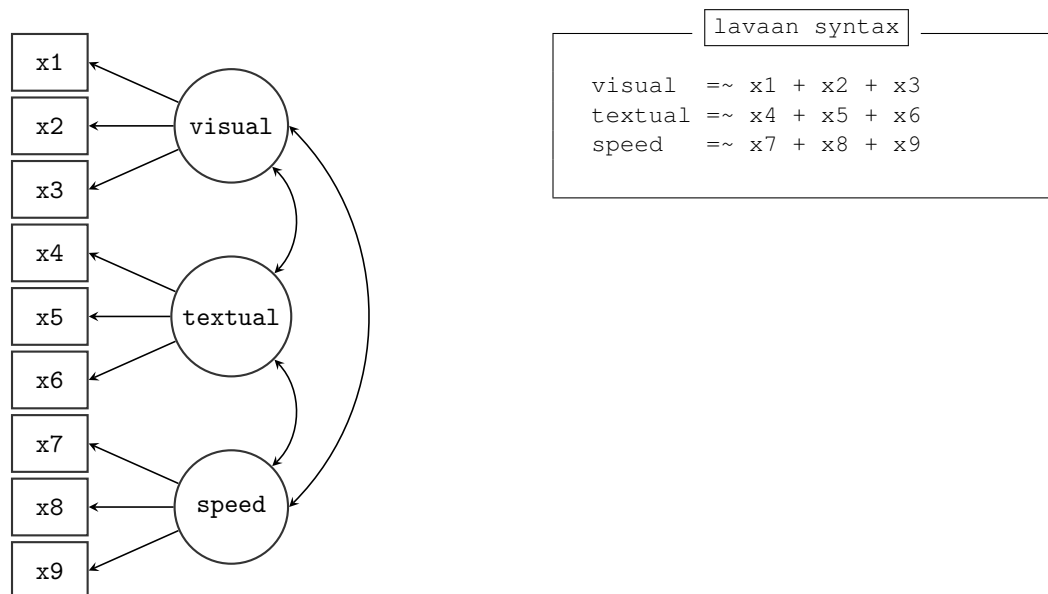
We start with a simple example of confirmatory factor analysis, using the `cfa` function, which is a user-friendly function for fitting CFA models. The `lavaan` package contains a built-in dataset called `HolzingerSwineford1939`. See the help page for this dataset by typing

```
> ?HolzingerSwineford1939
```

at the R prompt. This is a ‘classic’ dataset that is used in many papers and books on Structural Equation Modeling (SEM), including some manuals of commercial SEM software packages. The data consists of mental ability test scores of seventh- and eighth-grade children from two different schools (Pasteur and Grant-White). In our version of the dataset, only 9 out of the original 26 tests are included. A CFA model that is often proposed for these 9 variables consists of three latent variables (or factors), each with three indicators:

- a *visual* factor measured by 3 variables:  $x_1$ ,  $x_2$  and  $x_3$
- a *textual* factor measured by 3 variables:  $x_4$ ,  $x_5$  and  $x_6$
- a *speed* factor measured by 3 variables:  $x_7$ ,  $x_8$  and  $x_9$

The left panel of the figure below contains a simplified graphical representation of the three-factor model. The right panel contains the corresponding **lavaan** syntax for specifying this model.



In this example, the model syntax only contains three ‘latent variable definitions’. Each formula has the following format:

```
latent variable =~ indicator1 + indicator2 + indicator3
```

We call these expressions *latent variable definitions* because they define how the latent variables are ‘manifested by’ a set of observed (or manifest) variables, often called ‘indicators’. Note that the special “=~” operator in the middle consists of a sign (“=”) character and a tilde (“~”) character next to each other. The reason why this model syntax is so short, is that behind the scenes, the `cfa` function will take care of several things. First, by default, the factor loading of the first indicator of a latent variable is fixed to 1, thereby fixing the scale of the latent variable. Second, residual variances are added automatically. And third, all exogenous latent variables are correlated by default. This way, the model syntax can be kept concise. On the other hand, the user remains in control, since all this ‘default’ behavior can be overridden and/or switched off.

We can enter the model syntax using the single quotes:

```
> HS.model <- '
+   visual =~ x1 + x2 + x3
+   textual =~ x4 + x5 + x6
+   speed  =~ x7 + x8 + x9
+ '
```

We can now fit the model as follows:

```
> fit <- cfa(HS.model, data=HolzingerSwineford1939)
```

The **lavaan** function `cfa` is a dedicated function for fitting confirmatory factor analysis models. The first argument is the user-specified model. The second argument is the dataset that contains the observed variables. Once the model has been fitted, the `summary` method provides a nice summary of the fitted model:

```

> summary(fit, fit.measures=TRUE)
lavaan (0.5-12) converged normally after 41 iterations

Number of observations                    301

Estimator                                ML
Minimum Function Test Statistic          85.306
Degrees of freedom                        24
P-value (Chi-square)                     0.000

Model test baseline model:

Minimum Function Test Statistic          918.852
Degrees of freedom                        36
P-value                                   0.000

Full model versus baseline model:

Comparative Fit Index (CFI)              0.931
Tucker-Lewis Index (TLI)                0.896

Loglikelihood and Information Criteria:

Loglikelihood user model (H0)             -3737.745
Loglikelihood unrestricted model (H1)     -3695.092

Number of free parameters                 21
Akaike (AIC)                             7517.490
Bayesian (BIC)                           7595.339
Sample-size adjusted Bayesian (BIC)      7528.739

Root Mean Square Error of Approximation:

RMSEA                                    0.092
90 Percent Confidence Interval            0.071 0.114
P-value RMSEA <= 0.05                    0.001

Standardized Root Mean Square Residual:

SRMR                                      0.065

Parameter estimates:

Information                                Expected
Standard Errors                            Standard

Estimate  Std.err  Z-value  P(>|z|)

Latent variables:
visual =~
  x1          1.000
  x2          0.553    0.100    5.554    0.000
  x3          0.729    0.109    6.685    0.000
textual =~
  x4          1.000
  x5          1.113    0.065   17.014    0.000
  x6          0.926    0.055   16.703    0.000
speed =~
  x7          1.000
  x8          1.180    0.165    7.152    0.000
  x9          1.082    0.151    7.155    0.000

Covariances:
visual ~~
  textual    0.408    0.074    5.552    0.000
  speed      0.262    0.056    4.660    0.000
textual ~~
  speed      0.173    0.049    3.518    0.000

Variances:
  x1          0.549    0.114
  x2          1.134    0.102
  x3          0.844    0.091
  x4          0.371    0.048
  x5          0.446    0.058
  x6          0.356    0.043

```

x7	0.799	0.081
x8	0.488	0.074
x9	0.566	0.071
visual	0.809	0.145
textual	0.979	0.112
speed	0.384	0.086

The output should look familiar to users of other SEM software. If you find it confusing or esthetically unpleasing, again, please let us know, and we will try to improve it. To wrap up this first example, we summarize the code that was needed to fit this three-factor model:

R code

```
# load the lavaan package (only needed once per session)
library(lavaan)

# specify the model
HS.model <- ' visual  =~ x1 + x2 + x3
             textual  =~ x4 + x5 + x6
             speed    =~ x7 + x8 + x9 '

# fit the model
fit <- cfa(HS.model, data=HolzingerSwineford1939)

# display summary output
summary(fit, fit.measures=TRUE)
```

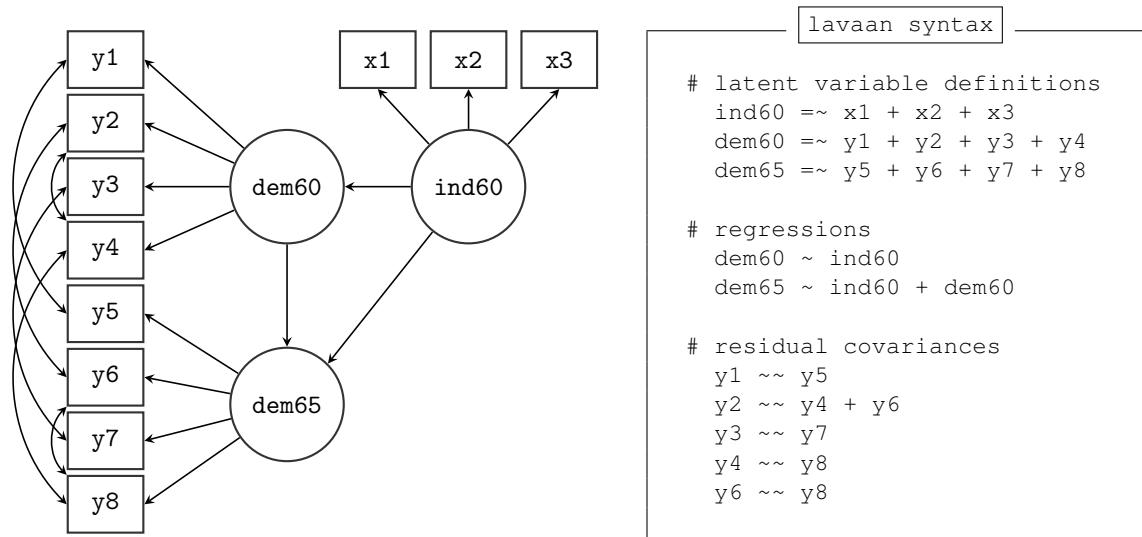
Simply copying this code and pasting it in R should work. The syntax illustrates the typical workflow in the **lavaan** package:

1. Specify your model using the **lavaan** model syntax. In this example, only *latent variable definitions* have been used. In the following examples, other formula types will be used.
2. Fit the model. This requires a dataset containing the observed variables (or alternatively the sample covariance matrix and the number of observations; see section 9.1). In this example, we have used the **cfa** function. Other functions in the **lavaan** package are **sem** and **growth** for fitting full structural equation models and growth curve models respectively. All three functions are so-called user-friendly functions, in the sense that they take care of many details automatically, so we can keep the model syntax simple and concise. If you wish to fit non-standard models or if you don't like the idea that things are done for you automatically, you can use the lower-level function **lavaan**, where you have full control.
3. Extract information from the fitted model. This can be a long verbose summary, or it can be a single number only (say, the RMSEA value). In the spirit of R, you only get what you asked for. We do not print out unnecessary information that you would ignore anyway.

## 4.2 A second example: a structural equation model (SEM)

In our second example, we will use the built-in **PoliticalDemocracy** dataset. This is a dataset that has been used by Bollen in his 1989 book on structural equation modeling (and elsewhere). To learn more about the dataset, see the help page and the references therein.

The left panel of the figure below contains a graphical representation of the model that we want to fit. The right panel contains the corresponding model syntax.



In this example, we use three different formula types: latent variable definitions, regression formulas, and (co)variance formulas. The regression formulas are similar to ordinary formulas in R. The (co)variance formulas typically have the following form:

variable ~~ variable

The variables can be either observed or latent variables. If the two variable names are the same, the expression refers to the variance (or residual variance) of that variable. If the two variable names are different, the expression refers to the (residual) covariance among these two variables. The **lavaan** package automatically makes the distinction between variances and residual variances.

In our example, the expression `y1 ~~ y5` allows the residual variances of the two observed variables to be correlated. This is sometimes done if it is believed that the two variables have something in common that is not captured by the latent variables. In this case, the two variables refer to identical scores, but measured in two different years (1960 and 1965, respectively). Note that the two expressions `y2 ~~ y4` and `y2 ~~ y6`, can be combined into the expression `y2 ~~ y4 + y6`. This is just a shorthand notation.

We enter the model syntax as follows:

```

> model <- '
+ # measurement model
+   ind60 =~ x1 + x2 + x3
+   dem60 =~ y1 + y2 + y3 + y4
+   dem65 =~ y5 + y6 + y7 + y8
+
+ # regressions
+   dem60 ~ ind60
+   dem65 ~ ind60 + dem60
+
+ # residual correlations
+   y1 ~~ y5
+   y2 ~~ y4 + y6
+   y3 ~~ y7
+   y4 ~~ y8
+   y6 ~~ y8
+ '

```

To fit the model and see the results we can type:

```

> fit <- sem(model, data=PoliticalDemocracy)
> summary(fit, standardized=TRUE)

lavaan (0.5-12) converged normally after 70 iterations

Number of observations              75

Estimator                          ML

```



```

Minimum Function Test Statistic      38.125
Degrees of freedom                    35
P-value (Chi-square)                 0.329

```

Parameter estimates:

```

Information                               Expected
Standard Errors                           Standard

Estimate  Std.err  Z-value  P(>|z|)  Std.lv  Std.all

Latent variables:
ind60 =~
  x1      1.000
  x2      2.180   0.139  15.742   0.000   1.460   0.973
  x3      1.819   0.152  11.967   0.000   1.218   0.872
dem60 =~
  y1      1.000
  y2      1.257   0.182   6.889   0.000   2.794   0.717
  y3      1.058   0.151   6.987   0.000   2.351   0.722
  y4      1.265   0.145   8.722   0.000   2.812   0.846
dem65 =~
  y5      1.000
  y6      1.186   0.169   7.024   0.000   2.493   0.746
  y7      1.280   0.160   8.002   0.000   2.691   0.824
  y8      1.266   0.158   8.007   0.000   2.662   0.828

Regressions:
dem60 ~
  ind60      1.483   0.399   3.715   0.000   0.447   0.447
dem65 ~
  ind60      0.572   0.221   2.586   0.010   0.182   0.182
  dem60      0.837   0.098   8.514   0.000   0.885   0.885

Covariances:
y1 ~~
  y5      0.624   0.358   1.741   0.082   0.624   0.296
y2 ~~
  y4      1.313   0.702   1.871   0.061   1.313   0.273
  y6      2.153   0.734   2.934   0.003   2.153   0.356
y3 ~~
  y7      0.795   0.608   1.308   0.191   0.795   0.191
y4 ~~
  y8      0.348   0.442   0.787   0.431   0.348   0.109
y6 ~~
  y8      1.356   0.568   2.386   0.017   1.356   0.338

Variances:
x1      0.082   0.019
x2      0.120   0.070
x3      0.467   0.090
y1      1.891   0.444
y2      7.373   1.374
y3      5.067   0.952
y4      3.148   0.739
y5      2.351   0.480
y6      4.954   0.914
y7      3.431   0.713
y8      3.254   0.695
ind60   0.448   0.087
dem60   3.956   0.921
dem65   0.172   0.215

```

The function `sem` is very similar to the `cfa` function. In fact, the two functions are currently almost identical, but this may change in the future. In the `summary` method, we omitted the `fit.measures=TRUE` argument. Therefore, you only get the basic chi-square statistic. The argument `standardized=TRUE` augments the output with standardized parameter values. Two extra columns of standardized parameter values are printed. In the first column (labeled `Std.lv`), only the latent variables are standardized. In the second column (labeled `Std.all`), both latent and observed variables are standardized. The latter is often called the ‘completely standardized solution’.

The complete code to specify and fit this model is printed again below:

```

library(lavaan) # only needed once per session
model <- '
  # measurement model
  ind60 =~ x1 + x2 + x3
  dem60 =~ y1 + y2 + y3 + y4
  dem65 =~ y5 + y6 + y7 + y8

  # regressions
  dem60 ~ ind60
  dem65 ~ ind60 + dem60

  # residual correlations
  y1 ~~ y5
  y2 ~~ y4 + y6
  y3 ~~ y7
  y4 ~~ y8
  y6 ~~ y8
'

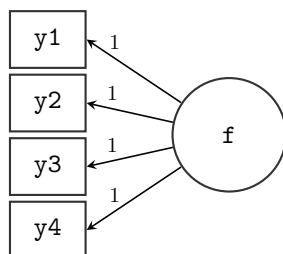
fit <- sem(model, data=PoliticalDemocracy)
summary(fit, standardized=TRUE)

```

## 5 Fixing parameters, starting values and equality constraints

### 5.1 Fixing parameters

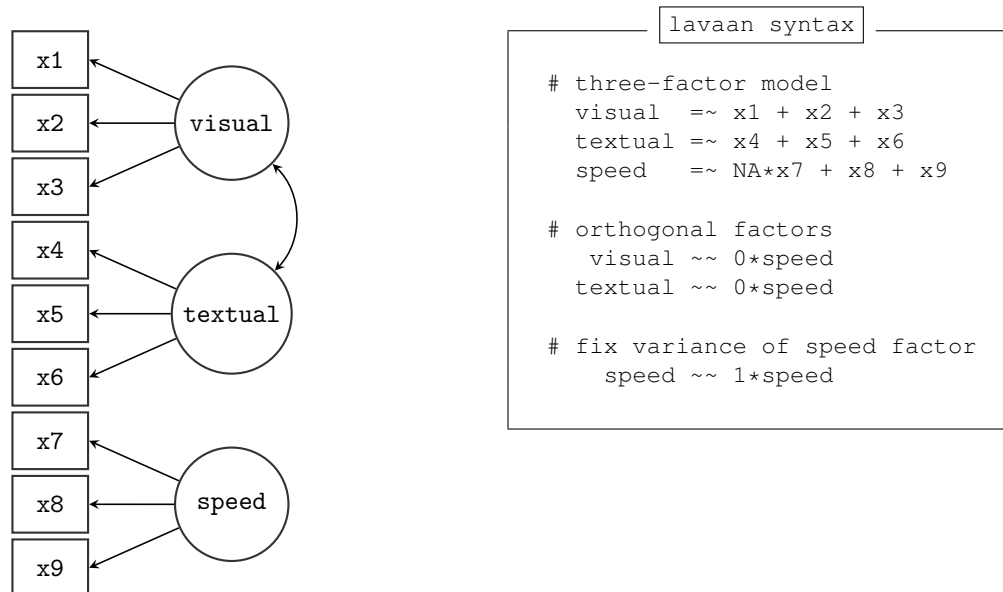
Consider a simple one-factor model with 4 indicators. By default, **lavaan** will always fix the factor loading of the first indicator to 1. The other three factor loadings are free, and their values are estimated by the model. But suppose that you have good reasons the fix all the factor loadings to 1. The syntax below illustrates how this can be done:



lavaan syntax

```
f =~ y1 + 1*y2 + 1*y3 + 1*y4
```

In general, to fix a parameter in a **lavaan** formula, you need to pre-multiply the corresponding variable in the formula by a numerical value. This is called the pre-multiplication mechanism and will be used for many purposes. As another example, consider again the three-factor Holzinger and Swineford CFA model. Recall that, by default, all exogenous latent variables in a CFA model are correlated. But if you wish to fix the correlation (or covariance) between a pair of latent variables to zero, you need to explicitly add a covariance-formula for this pair, and fix the parameter to zero. In the figure below, we allow the covariance between the latent variables `visual` and `textual` to be free, but the two other covariances are fixed to zero. In addition, we fix the variance of the `speed` factor to unity. Therefore, there is no need anymore to set the factor loading of its first indicator (`x7`) equal to one. To force this factor loading to be free, we pre-multiply it with `NA`, as a hint to **lavaan** that the value of this parameter is still unknown.



If you need to constrain all covariances of the latent variables in a CFA model to be orthogonal, there is a shortcut. You can omit the covariance formulas in the model syntax and simply add an `orthogonal=TRUE` argument to the `cfa` function call:

```
> HS.model <- ' visual =~ x1 + x2 + x3
+           textual =~ x4 + x5 + x6
+           speed  =~ x7 + x8 + x9 '
> fit.HS.ortho <- cfa(HS.model, data=HolzingerSwineford1939, orthogonal=TRUE)
```

Similarly, if you want to fix the variances of *all* the latent variables in a CFA model to unity, there is again a shortcut. Simply add a `std.lv=TRUE` argument to the `cfa` function call:

```
> HS.model <- ' visual =~ x1 + x2 + x3
+           textual =~ x4 + x5 + x6
+           speed  =~ x7 + x8 + x9 '
> fit <- cfa(HS.model, data=HolzingerSwineford1939, std.lv=TRUE)
```

If the `std.lv=TRUE` argument is used, the factor loadings of the first indicator of each latent variable will no longer be fixed to 1.

## 5.2 Starting values

The `lavaan` package automatically generates starting values for all free parameters. Normally, this works fine. But if you must provide your own starting values, you are free to do so. The way it works is based on the pre-multiplication mechanism that we discussed before. But the numeric constant is now the argument of a special function `start()`. An example will make this clear:

```
lavaan syntax
visual =~ x1 + start(0.8)*x2 + start(1.2)*x3
textual =~ x4 + start(0.5)*x5 + start(1.0)*x6
speed  =~ x7 + start(0.7)*x8 + start(1.8)*x9
```

The factor loadings of the first indicators (`x1`, `x4` and `x7`) are fixed, so no starting values are needed. But for all other factor loadings, starting values are provided in this example.

## 5.3 Parameter labels

A nice property of the `lavaan` package is that all free parameters are automatically named according to a simple set of rules. This is convenient, for example, if equality constraints are needed (see the next subsection). To see how the naming mechanism works, we will use the model that we used for the Political Democracy data.

```

> model <- '
+ # latent variable definitions
+   ind60 =~ x1 + x2 + x3
+   dem60 =~ y1 + y2 + y3 + y4
+   dem65 =~ y5 + y6 + y7 + y8
+ # regressions
+   dem60 ~ ind60
+   dem65 ~ ind60 + dem60
+ # residual (co)variances
+   y1 ~~ y5
+   y2 ~~ y4 + y6
+   y3 ~~ y7
+   y4 ~~ y8
+   y6 ~~ y8
+ '
> fit <- sem(model, data=PoliticalDemocracy)
> coef(fit)

      ind60=~x2      ind60=~x3      dem60=~y2      dem60=~y3      dem60=~y4      dem65=~y6
      2.180         1.819         1.257         1.058         1.265         1.186
dem65=~y7      dem65=~y8      dem60~ind60      dem65~ind60      dem65~dem60      y1~~y5
      1.280         1.266         1.483         0.572         0.837         0.624
      y2~~y4         y2~~y6         y3~~y7         y4~~y8         y6~~y8         x1~~x1
      1.313         2.153         0.795         0.348         1.356         0.082
      x2~~x2         x3~~x3         y1~~y1         y2~~y2         y3~~y3         y4~~y4
      0.120         0.467         1.891         7.373         5.067         3.148
      y5~~y5         y6~~y6         y7~~y7         y8~~y8      ind60~~ind60      dem60~~dem60
      2.351         4.954         3.431         3.254         0.448         3.956
dem65~~dem65
      0.172

```

The `coef` function extracts the estimated values of the free parameters in the model, together with their names. Each name consists of three parts and reflects the part of the formula where the parameter was involved. The first part is the variable name that appears on the left-hand side of the formula. The middle part is the operator type of the formula, and the third part is the variable in the right-hand side of the formula that corresponds with the parameter.

If you want, you can provide custom parameter names or *labels* simply by pre-multiplying a variable name with that label. An example will make this clear:

```

> model <- '
+ # latent variable definitions
+   ind60 =~ x1 + x2 + myLabel*x3
+   dem60 =~ y1 + y2 + y3 + y4
+   dem65 =~ y5 + y6 + y7 + y8
+ # regressions
+   dem60 ~ ind60
+   dem65 ~ ind60 + dem60
+ # residual (co)variances
+   y1 ~~ y5
+   y2 ~~ y4 + y6
+   y3 ~~ y7
+   y4 ~~ y8
+   y6 ~~ y8
+ '

```

It is important that labels start with a letter (a-zA-Z), and certainly not with a digit. For example '13bis' is not a valid label, and will confuse the lavaan syntax parser. Note: before version 0.4-8, it was necessary to use the `label()` modifier to specify a custom label. Although it is still supported, it is not recommended anymore. The only reason why it should be used in new syntax is if the label contains an operator like " " or "=".

## 5.4 Simple equality constraints

In some applications, it is useful to impose equality constraints on one or more otherwise free parameters. Consider again the three-factor H&S CFA model. Suppose a user has a priori reasons to believe that the factor loadings of the `x2` and `x3` indicators are equal to each other. Instead of estimating two free parameters, **lavaan**

should only estimate a single free parameter, and use that value for both factor loadings. The main mechanism to specify this type of (simple) equality constraints is by using labels: if two parameters have the same label, they will be considered to be the same, and only one value will be computed for them. This is illustrated in the following syntax:

lavaan syntax

```
visual  =~ x1 + v2*x2 + v2*x3
textual =~ x4 + x5 + x6
speed   =~ x7 + x8 + x9
```

All parameters having the same label will be constrained to be equal.

An alternative approach is to use the `equal()` modifier. This is useful if no custom label has been specified, and one needs to refer to the automatic label. For example:

lavaan syntax

```
visual  =~ x1 + x2 + equal("visual=~x2")*x3
textual =~ x4 + x5 + x6
speed   =~ x7 + x8 + x9
```

The parameter corresponding to the factor loading of the `x2` variable is (automatically) called `"visual=~x2"`. By using the `equal()` modifier for `x3`, the corresponding parameter value will be set equal to the factor loading of `x2`.

## 5.5 Nonlinear equality and inequality constraints

In version 0.4-8, initial support has been added for general nonlinear equality and inequality constraints. Consider for example the following regression:

lavaan syntax

```
y ~ b1*x1 + b2*x2 + b3*x3
```

where we have explicitly labeled the regression coefficients as `b1`, `b2` and `b3`. We create a toy dataset containing these four variables and fit the regression model:

```
> set.seed(1234)
> Data <- data.frame(y = rnorm(100), x1 = rnorm(100), x2 = rnorm(100),
+                   x3 = rnorm(100))
> model <- ' y ~ b1*x1 + b2*x2 + b3*x3 '
> fit <- sem(model, data=Data)
> coef(fit)

      b1      b2      b3  y~~y
-0.052  0.084  0.139  0.970
```

Suppose that we need to impose the following two (nonlinear) constraints on  $b_1$ :  $b_1 = (b_2 + b_3)^2$  and  $b_1 \geq \exp(b_2 + b_3)$ . The first constraint is an equality constraint. The second is an inequality constraint. To specify these constraints, you can use the following syntax:

lavaan syntax

```
model.constr <- ' # model with labeled parameters
                  y ~ b1*x1 + b2*x2 + b3*x3
                  # constraints
                  b1 == (b2 + b3)^2
                  b1 > exp(b2 + b3) '
```

To see the effect of the constraints, we refit the model:

```
> model.constr <- ' # model with labeled parameters
+                 y ~ b1*x1 + b2*x2 + b3*x3
+                 # constraints
+                 b1 == (b2 + b3)^2
+                 b1 > exp(b2 + b3) '
> fit <- sem(model.constr, data=Data)
> coef(fit)

      b1      b2      b3  y~~y
0.495 -0.405 -0.299  1.610
```

The reader can verify that the constraints are indeed respected. The equality constraint holds exactly. The inequality constraint has resulted in an equality between the left-hand side ( $b_1$ ) and the right-hand side ( $\exp(b_2 + b_3)$ ).

## 6 Mean structures and multiple groups

### 6.1 Bringing in the means

By and large, structural equation models are used to model the covariance matrix of the observed variables in a dataset. But in some applications, it is useful to bring in the means of the observed variables too. One way to do this is to explicitly refer to intercepts in the **lavaan** syntax. This can be done by including ‘intercept formulas’ in the model syntax. An intercept formula has the following form:

$$\text{variable} \sim 1$$

The left part of the expression contains the name of the observed or latent variable. The right part contains the number 1, representing the intercept. For example, in the three-factor H&S CFA model, we can add the intercepts of the observed variables as follows:

lavaan syntax

```
# three-factor model
visual  =~ x1 + x2 + x3
textual =~ x4 + x5 + x6
speed   =~ x7 + x8 + x9

# intercepts
x1 ~ 1
x2 ~ 1
x3 ~ 1
x4 ~ 1
x5 ~ 1
x6 ~ 1
x7 ~ 1
x8 ~ 1
x9 ~ 1
```

However, it is more convenient to omit the intercept formulas in the model syntax (unless you want to fix their values), and to add the `meanstructure = TRUE` argument in the `cfa` and `sem` function calls. For example, we can refit the three-factor H&S CFA model as follows:

```
> fit <- cfa(HS.model, data=HolzingerSwineford1939, meanstructure=TRUE)
> summary(fit)

lavaan (0.5-12) converged normally after 41 iterations

Number of observations              301

Estimator                          ML
Minimum Function Test Statistic     85.306
Degrees of freedom                   24
P-value (Chi-square)                 0.000
```

Parameter estimates:

Information				Expected
Standard Errors				Standard
	Estimate	Std.err	Z-value	P (> z )
Latent variables:				
visual =~				
x1	1.000			
x2	0.553	0.100	5.554	0.000
x3	0.729	0.109	6.685	0.000
textual =~				
x4	1.000			
x5	1.113	0.065	17.014	0.000
x6	0.926	0.055	16.703	0.000
speed =~				
x7	1.000			
x8	1.180	0.165	7.152	0.000
x9	1.082	0.151	7.155	0.000
Covariances:				
visual ~~				
textual	0.408	0.074	5.552	0.000
speed	0.262	0.056	4.660	0.000
textual ~~				
speed	0.173	0.049	3.518	0.000
Intercepts:				
x1	4.936	0.067	73.473	0.000
x2	6.088	0.068	89.855	0.000
x3	2.250	0.065	34.579	0.000
x4	3.061	0.067	45.694	0.000
x5	4.341	0.074	58.452	0.000
x6	2.186	0.063	34.667	0.000
x7	4.186	0.063	66.766	0.000
x8	5.527	0.058	94.854	0.000
x9	5.374	0.058	92.546	0.000
visual	0.000			
textual	0.000			
speed	0.000			
Variances:				
x1	0.549	0.114		
x2	1.134	0.102		
x3	0.844	0.091		
x4	0.371	0.048		
x5	0.446	0.058		
x6	0.356	0.043		
x7	0.799	0.081		
x8	0.488	0.074		
x9	0.566	0.071		
visual	0.809	0.145		
textual	0.979	0.112		
speed	0.384	0.086		

As you can see in the output, the model includes intercept parameters for both the observed and latent variables. By default, the `cfa` and `sem` functions fix the latent variable intercepts (which in this case correspond to the latent *means*) to zero. Otherwise, the model would not be estimable. Note that the chi-square statistic and the number of degrees of freedom is the same as in the original model (without a mean structure). The reason is that we brought in some new data (a mean value for each of the 9 observed variables), but we also added 9 additional parameters to the model (an intercept for each of the 9 observed variables). The end result is an identical fit. In practice, the only reason why a user would add intercept-formulas in the model syntax, is because some constraints must be specified on them. For example, suppose that we wish to fix the intercepts of the variables `x1`, `x2`, `x3` and `x4` to, say, 0.5. We would write the model syntax as follows:

lavaan syntax

```
# three-factor model
visual =~ x1 + x2 + x3
textual =~ x4 + x5 + x6
speed =~ x7 + x8 + x9
```

```
# intercepts with fixed values
x1 + x2 + x3 + x4 ~ 0.5*1
```

where we have used the left-hand side of the formula to ‘repeat’ the right-hand side for each element of the left-hand side.

## 6.2 Multiple groups

The `lavaan` package has full support for multiple groups. To request a multiple group analysis, you need to add the name of the group variable in your dataset to the `group` argument in the `cfa` and `sem` function calls. By default, the same model is fitted in all groups. In the following example, we fit the H&S CFA model for the two schools (Pasteur and Grant-White).

```
> HS.model <- ' visual =~ x1 + x2 + x3
+             textual =~ x4 + x5 + x6
+             speed  =~ x7 + x8 + x9 '
> fit <- cfa(HS.model, data=HolzingerSwineford1939, group="school")
> summary(fit)
```

lavaan (0.5-12) converged normally after 63 iterations

Number of observations per group	
Pasteur	156
Grant-White	145
Estimator	ML
Minimum Function Test Statistic	115.851
Degrees of freedom	48
P-value (Chi-square)	0.000

Chi-square for each group:

Pasteur	64.309
Grant-White	51.542

Parameter estimates:

Information	Expected
Standard Errors	Standard

Group 1 [Pasteur]:

	Estimate	Std.err	Z-value	P(> z )
Latent variables:				
visual =~				
x1	1.000			
x2	0.394	0.122	3.220	0.001
x3	0.570	0.140	4.076	0.000
textual =~				
x4	1.000			
x5	1.183	0.102	11.613	0.000
x6	0.875	0.077	11.421	0.000
speed =~				
x7	1.000			
x8	1.125	0.277	4.057	0.000
x9	0.922	0.225	4.104	0.000

Covariances:

visual ~~				
textual	0.479	0.106	4.531	0.000
speed	0.185	0.077	2.397	0.017
textual ~~				
speed	0.182	0.069	2.628	0.009

Intercepts:

x1	4.941	0.095	52.249	0.000
x2	5.984	0.098	60.949	0.000
x3	2.487	0.093	26.778	0.000
x4	2.823	0.092	30.689	0.000



x5	3.995	0.105	38.183	0.000
x6	1.922	0.079	24.321	0.000
x7	4.432	0.087	51.181	0.000
x8	5.563	0.078	71.214	0.000
x9	5.418	0.079	68.440	0.000
visual	0.000			
textual	0.000			
speed	0.000			

Variances:

x1	0.298	0.232
x2	1.334	0.158
x3	0.989	0.136
x4	0.425	0.069
x5	0.456	0.086
x6	0.290	0.050
x7	0.820	0.125
x8	0.510	0.116
x9	0.680	0.104
visual	1.097	0.276
textual	0.894	0.150
speed	0.350	0.126

Group 2 [Grant-White]:

	Estimate	Std.err	Z-value	P (> z )
Latent variables:				
visual =~				
x1	1.000			
x2	0.736	0.155	4.760	0.000
x3	0.925	0.166	5.583	0.000
textual =~				
x4	1.000			
x5	0.990	0.087	11.418	0.000
x6	0.963	0.085	11.377	0.000
speed =~				
x7	1.000			
x8	1.226	0.187	6.569	0.000
x9	1.058	0.165	6.429	0.000

Covariances:

visual ~~				
textual	0.408	0.098	4.153	0.000
speed	0.276	0.076	3.639	0.000
textual ~~				
speed	0.222	0.073	3.022	0.003

Intercepts:

x1	4.930	0.095	51.696	0.000
x2	6.200	0.092	67.416	0.000
x3	1.996	0.086	23.195	0.000
x4	3.317	0.093	35.625	0.000
x5	4.712	0.096	48.986	0.000
x6	2.469	0.094	26.277	0.000
x7	3.921	0.086	45.819	0.000
x8	5.488	0.087	63.174	0.000
x9	5.327	0.085	62.571	0.000
visual	0.000			
textual	0.000			
speed	0.000			

Variances:

x1	0.715	0.126
x2	0.899	0.123
x3	0.557	0.103
x4	0.315	0.065
x5	0.419	0.072
x6	0.406	0.069
x7	0.600	0.091
x8	0.401	0.094
x9	0.535	0.089
visual	0.604	0.160
textual	0.942	0.152

speed                    0.461    0.118

If you want to fix parameters, or provide starting values, you can use the same pre-multiplication techniques, but the single argument is now replaced by a vector of arguments, one for each group. If you use a single element instead of a vector, that element will be applied for all groups (note: this is NOT true for labels, since this would imply equality constraints). For example:

lavaan syntax

```
HS.model <- ' visual   =~ x1 + 0.5*x2 + c(0.6, 0.8)*x3
             textual  =~ x4 + start(c(1.2, 0.6))*x5 + a*x6
             speed    =~ x7 + x8 + x9 '
```

In the definition of the latent factor `visual`, we have fixed the factor loading of the `x3` indicator to the value '0.6' in the first group, and to the value '0.8' in the second group, while the factor loading of the `x2` indicator is fixed to the value '0.5' in both groups. In the definition of the `textual` factor, two different starting values are provided for the `x5` indicator; one for each group. In addition, we have labeled the factor loading of the `x6` indicator as 'a', but this label is only given to the parameter of the first group. If you want to provide labels to each of the two groups, you can write something like `c(a1,a2)*x6`. Be careful: if you write `c(a,a)*x6`, both parameters (in the first and second) group will get the same label, and hence they will be treated as a single parameter. To verify the effects of these modifiers, we refit the model:

```
> fit <- cfa(HS.model, data=HolzingerSwineford1939, group="school")
> summary(fit)
```

lavaan (0.5-12) converged normally after 58 iterations

Number of observations per group	
Pasteur	156
Grant-White	145
Estimator	ML
Minimum Function Test Statistic	118.976
Degrees of freedom	52
P-value (Chi-square)	0.000

Chi-square for each group:

Pasteur	64.901
Grant-White	54.075

Parameter estimates:

Information	Expected
Standard Errors	Standard

Group 1 [Pasteur]:

	Estimate	Std.err	Z-value	P(> z )
Latent variables:				
visual =~				
x1	1.000			
x2	0.500			
x3	0.600			
textual =~				
x4	1.000			
x5	1.185	0.102	11.598	0.000
x6	(a) 0.876	0.077	11.409	0.000
speed =~				
x7	1.000			
x8	1.129	0.279	4.055	0.000
x9	0.931	0.227	4.103	0.000

Covariances:

visual ~~				
textual	0.460	0.103	4.479	0.000
speed	0.182	0.076	2.408	0.016
textual ~~				
speed	0.181	0.069	2.625	0.009

Intercepts:				
x1	4.941	0.094	52.379	0.000
x2	5.984	0.100	59.945	0.000
x3	2.487	0.092	26.983	0.000
x4	2.823	0.092	30.689	0.000
x5	3.995	0.105	38.183	0.000
x6	1.922	0.079	24.321	0.000
x7	4.432	0.087	51.181	0.000
x8	5.563	0.078	71.214	0.000
x9	5.418	0.079	68.440	0.000
visual	0.000			
textual	0.000			
speed	0.000			

Variances:		
x1	0.388	0.129
x2	1.304	0.155
x3	0.965	0.120
x4	0.427	0.069
x5	0.454	0.086
x6	0.289	0.050
x7	0.824	0.124
x8	0.510	0.116
x9	0.677	0.105
visual	1.001	0.172
textual	0.892	0.150
speed	0.346	0.125

Group 2 [Grant-White]:

	Estimate	Std.err	Z-value	P (> z )
Latent variables:				
visual =~				
x1	1.000			
x2	0.500			
x3	0.800			
textual =~				
x4	1.000			
x5	0.990	0.087	11.425	0.000
x6	0.963	0.085	11.374	0.000
speed =~				
x7	1.000			
x8	1.228	0.188	6.539	0.000
x9	1.081	0.168	6.417	0.000

Covariances:				
visual ~~				
textual	0.454	0.099	4.585	0.000
speed	0.315	0.079	4.004	0.000
textual ~~				
speed	0.222	0.073	3.049	0.002

Intercepts:				
x1	4.930	0.097	50.688	0.000
x2	6.200	0.089	69.616	0.000
x3	1.996	0.086	23.223	0.000
x4	3.317	0.093	35.625	0.000
x5	4.712	0.096	48.986	0.000
x6	2.469	0.094	26.277	0.000
x7	3.921	0.086	45.819	0.000
x8	5.488	0.087	63.174	0.000
x9	5.327	0.085	62.571	0.000
visual	0.000			
textual	0.000			
speed	0.000			

Variances:		
x1	0.637	0.115
x2	0.966	0.120
x3	0.601	0.091
x4	0.316	0.065
x5	0.418	0.072
x6	0.407	0.069

x7	0.609	0.091
x8	0.411	0.094
x9	0.522	0.089
visual	0.735	0.132
textual	0.942	0.152
speed	0.453	0.117

### 6.2.1 Constraining a single parameter to be equal across groups

If you want to constrain one or more parameters to be equal across groups, you need to give them the same label. For example, to constrain the factor loading of the x3 indicator to be equal across (two) groups, you can write:

```
> HS.model <- ' visual =~ x1 + x2 + c(v3,v3)*x3
+              textual =~ x4 + x5 + x6
+              speed  =~ x7 + x8 + x9 '
```

Again, identical labels imply identical parameters, Both within and across groups.

### 6.2.2 Constraining groups of parameters to be equal across groups

Although providing identical labels is a very flexible method to specify equality constraints for a few parameters, there is a more convenient way to impose equality constraints on a whole set of parameters (for example: all factor loadings, or all intercepts). We call these type of constraints *group equality constraints* and they can be specified by the `group.equal` argument in the `cfa` or `sem` function calls. For example, to constrain (all) the factor loadings to be equal across groups, you can proceed as follows:

```
> HS.model <- ' visual =~ x1 + x2 + x3
+              textual =~ x4 + x5 + x6
+              speed  =~ x7 + x8 + x9 '
> fit <- cfa(HS.model, data=HolzingerSwineford1939, group="school",
+           group.equal=c("loadings"))
> summary(fit)
```

lavaan (0.5-12) converged normally after 46 iterations

Number of observations per group	
Pasteur	156
Grant-White	145
Estimator	
	ML
Minimum Function Test Statistic	124.044
Degrees of freedom	54
P-value (Chi-square)	0.000

Chi-square for each group:

Pasteur	68.825
Grant-White	55.219

Parameter estimates:

Information	Expected
Standard Errors	Standard

Group 1 [Pasteur]:

	Estimate	Std.err	Z-value	P(> z )
Latent variables:				
visual =~				
x1	1.000			
x2	0.599	0.100	5.979	0.000
x3	0.784	0.108	7.267	0.000
textual =~				
x4	1.000			
x5	1.083	0.067	16.049	0.000
x6	0.912	0.058	15.785	0.000
speed =~				
x7	1.000			
x8	1.201	0.155	7.738	0.000
x9	1.038	0.136	7.629	0.000

```

Covariances:
  visual ~~
    textual      0.416    0.097    4.271    0.000
    speed        0.169    0.064    2.643    0.008
  textual ~~
    speed        0.176    0.061    2.882    0.004

Intercepts:
  x1      4.941    0.093    52.991    0.000
  x2      5.984    0.100    60.096    0.000
  x3      2.487    0.094    26.465    0.000
  x4      2.823    0.093    30.371    0.000
  x5      3.995    0.101    39.714    0.000
  x6      1.922    0.081    23.711    0.000
  x7      4.432    0.086    51.540    0.000
  x8      5.563    0.078    71.087    0.000
  x9      5.418    0.079    68.153    0.000
  visual  0.000
  textual 0.000
  speed   0.000

Variances:
  x1      0.551    0.137
  x2      1.258    0.155
  x3      0.882    0.128
  x4      0.434    0.070
  x5      0.508    0.082
  x6      0.266    0.050
  x7      0.849    0.114
  x8      0.515    0.095
  x9      0.658    0.096
  visual  0.805    0.171
  textual 0.913    0.137
  speed   0.305    0.078

```

Group 2 [Grant-White]:

	Estimate	Std.err	Z-value	P(> z )
Latent variables:				
visual =~				
x1	1.000			
x2	0.599	0.100	5.979	0.000
x3	0.784	0.108	7.267	0.000
textual =~				
x4	1.000			
x5	1.083	0.067	16.049	0.000
x6	0.912	0.058	15.785	0.000
speed =~				
x7	1.000			
x8	1.201	0.155	7.738	0.000
x9	1.038	0.136	7.629	0.000
Covariances:				
visual ~~				
textual	0.437	0.099	4.423	0.000
speed	0.314	0.079	3.958	0.000
textual ~~				
speed	0.226	0.072	3.144	0.002
Intercepts:				
x1	4.930	0.097	50.763	0.000
x2	6.200	0.091	68.379	0.000
x3	1.996	0.085	23.455	0.000
x4	3.317	0.092	35.950	0.000
x5	4.712	0.100	47.173	0.000
x6	2.469	0.091	27.248	0.000
x7	3.921	0.086	45.555	0.000
x8	5.488	0.087	63.257	0.000
x9	5.327	0.085	62.786	0.000
visual	0.000			
textual	0.000			
speed	0.000			

```

Variances:
  x1          0.645    0.127
  x2          0.933    0.121
  x3          0.605    0.096
  x4          0.329    0.062
  x5          0.384    0.073
  x6          0.437    0.067
  x7          0.599    0.090
  x8          0.406    0.089
  x9          0.532    0.086
  visual      0.722    0.161
  textual     0.906    0.136
  speed       0.475    0.109

```

More ‘group equality constraints’ can be added. In addition to the factor loadings, the following keywords are currently supported:

- **"intercepts"**: the intercepts of the observed variables
- **"means"**: the intercepts/means of the latent variables
- **"residuals"**: the residual variances of the observed variables
- **"residual.covariances"**: the residual covariances of the observed variables
- **"lv.variances"**: the (residual) variances of the latent variables
- **"lv.covariances"**: the (residual) covariances of the latent variables
- **"regressions"**: all regression coefficients in the model

If you omit the `group.equal` arguments, all parameters are freely estimated in each group (but the model structure is the same).

But what if you want to constrain a whole group of parameters (say all factor loadings and intercepts) across groups, except for one or two parameters that need to stay free in all groups. For this scenario, you can use the argument `group.partial`, containing the names of those parameters that need to remain free. For example:

```

> fit <- cfa(HS.model, data=HolzingerSwineford1939, group="school",
+           group.equal=c("loadings", "intercepts"),
+           group.partial=c("visual=~x2", "x7~1"))

```

### 6.2.3 Measurement Invariance

If you are interested in testing the measurement invariance of a CFA model across several groups, you can use the `measurementInvariance` function which performs a number of multiple group analyses in a particular sequence, with increasingly more restrictions on the parameters. (Note: from the 0.5 series onwards, the `measurementInvariance()` function has been moved to the `semTools` package) Each model is compared to the baseline model and the previous model using chi-square difference tests. In addition, the difference in the `cfi` fit measure is also shown. Although the current implementation of the function is still a bit primitive, it does illustrate how the various components of the `lavaan` package can be used as building blocks for constructing higher level functions (such as the `measurementInvariance` function), something that is often very hard to accomplish with commercial software.

```

> library(semTools)
> measurementInvariance(HS.model, data=HolzingerSwineford1939, group="school")

```

Measurement invariance tests:

```

Model 1: configural invariance:
  chisq    df  pvalue    cfi  rmsea    bic
 115.851  48.000  0.000    0.923  0.097  7706.822

```

```

Model 2: weak invariance (equal loadings):
  chisq    df  pvalue    cfi  rmsea    bic
 124.044  54.000  0.000    0.921  0.093  7680.771

```

```

[Model 1 versus model 2]
  delta.chisq    delta.df  delta.p.value    delta.cfi

```

```

      8.192      6.000      0.224      0.002

Model 3: strong invariance (equal loadings + intercepts):
  chisq      df  pvalue      cfi      rmsea      bic
164.103  60.000   0.000   0.882   0.107 7686.588

[Model 1 versus model 3]
  delta.chisq  delta.df delta.p.value  delta.cfi
      48.251      12.000      0.000      0.041

[Model 2 versus model 3]
  delta.chisq  delta.df delta.p.value  delta.cfi
      40.059      6.000      0.000      0.038

Model 4: equal loadings + intercepts + means:
  chisq      df  pvalue      cfi      rmsea      bic
204.605  63.000   0.000   0.840   0.122 7709.969

[Model 1 versus model 4]
  delta.chisq  delta.df delta.p.value  delta.cfi
      88.754      15.000      0.000      0.083

[Model 3 versus model 4]
  delta.chisq  delta.df delta.p.value  delta.cfi
      40.502      3.000      0.000      0.042

```

By adding the `group.partial` argument, you can test for partial measurement invariance by allowing a few parameters to remain free.

## 7 Growth curve models

Another important type of latent variable models are latent growth curve models. Growth modeling is often used to analyze longitudinal or developmental data. In this type of data, an outcome measure is measured on several occasions, and we want to study the change over time. In many cases, the trajectory over time can be modeled as a simple linear or quadratic curve. Random effects are used to capture individual differences. The random effects are conveniently represented by (continuous) latent variables, often called *growth factors*. In the example below, we use an artificial dataset called `Demo.growth` where a score (say, a standardized score on a reading ability scale) is measured on 4 time points. To fit a linear growth model for these four time points, we need to specify a model with two latent variables: a random intercept, and a random slope:

lavaan syntax

```

# linear growth model with 4 timepoints
# intercept and slope with fixed coefficients
i =~ 1*t1 + 1*t2 + 1*t3 + 1*t4
s =~ 0*t1 + 1*t2 + 2*t3 + 3*t4

```

In this model, we have fixed all the coefficients of the growth functions. To fit this model, the `lavaan` package provides a special `growth` function:

```

> model <- ' i =~ 1*t1 + 1*t2 + 1*t3 + 1*t4
+           s =~ 0*t1 + 1*t2 + 2*t3 + 3*t4 '
> fit <- growth(model, data=Demo.growth)
> summary(fit)

lavaan (0.5-12) converged normally after 44 iterations

Number of observations              400

Estimator                          ML
Minimum Function Test Statistic     8.069
Degrees of freedom                   5
P-value (Chi-square)                0.152

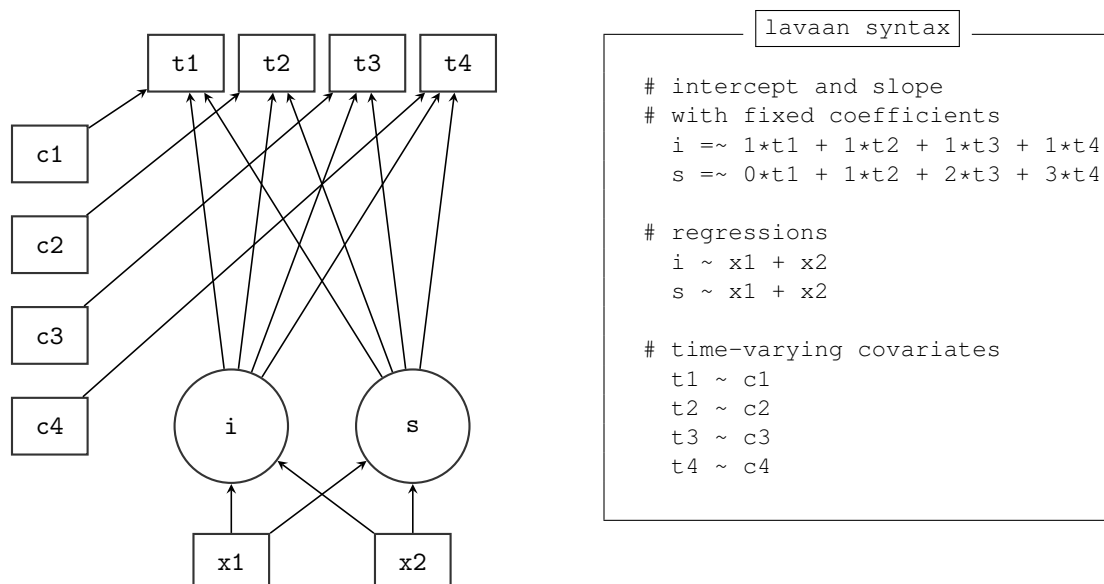
Parameter estimates:

Information                          Expected
Standard Errors                      Standard

```

	Estimate	Std.err	Z-value	P(> z )
Latent variables:				
i =~				
t1	1.000			
t2	1.000			
t3	1.000			
t4	1.000			
s =~				
t1	0.000			
t2	1.000			
t3	2.000			
t4	3.000			
Covariances:				
i ~~				
s	0.618	0.071	8.686	0.000
Intercepts:				
t1	0.000			
t2	0.000			
t3	0.000			
t4	0.000			
i	0.615	0.077	8.007	0.000
s	1.006	0.042	24.076	0.000
Variances:				
t1	0.595	0.086		
t2	0.676	0.061		
t3	0.635	0.072		
t4	0.508	0.124		
i	1.932	0.173		
s	0.587	0.052		

Technically, the `growth` function is almost identical to the `sem` function. But a mean structure is automatically assumed, and the observed intercepts are fixed to zero by default, while the latent variable intercepts/means are freely estimated. A slightly more complex model adds two regressors (`x1` and `x2`) that influence the latent growth factors. In addition, a time-varying covariate that influences the outcome measure at the four time points has been added to the model. A graphical representation of this model together with the corresponding `lavaan` syntax is presented below.



For ease of copy/pasting, the complete R code needed to specify and fit this linear growth model with a time-varying covariate is printed again below:



```

# a linear growth model with a time-varying covariate

model <- '
# intercept and slope with fixed coefficients
i =~ 1*t1 + 1*t2 + 1*t3 + 1*t4
s =~ 0*t1 + 1*t2 + 2*t3 + 3*t4

# regressions
i ~ x1 + x2
s ~ x1 + x2

# time-varying covariates
t1 ~ c1
t2 ~ c2
t3 ~ c3
t4 ~ c4
'

fit <- growth(model, data=Demo.growth)
summary(fit)

```

## 8 Using categorical variables

Binary, ordinal and nominal variables are considered categorical (not continuous). It makes a big difference if these categorical variables are exogenous (independent) or endogenous (dependent) in the model.

### 8.1 Exogenous categorical variables

If you have a binary exogenous covariate (say, gender), all you need to do is to recode it as a dummy (0/1) variable. Just like you would do in a classic regression model. If you have an exogenous ordinal variable, you can use a coding scheme reflecting the order (say, 1,2,3,...) and treat it as any other (numeric) covariate. If you have a nominal categorical variable with  $K > 2$  levels, you need to replace it by a set of  $K - 1$  dummy variables, again, just like you would do in classical regression.

### 8.2 Endogenous categorical variables

The lavaan 0.5 series can deal with binary and ordinal (but not nominal) endogenous variables. Only the three-stage WLS approach is currently supported, including some ‘robust’ variants. To use binary/ordinal data, you have two choices:

1. declare them as ‘ordered’ (using the `ordered()` function, which is part of base R) in your data.frame before you run the analysis; for example, if you need to declare four variables (say, item1, item2, item3, item3) as ordinal in your data.frame (called ‘Data’), you can use something like:

```

> Data[,c("item1", "item2", "item3", "item4")] <-
+   lapply(Data[,c("item1", "item2", "item3", "item4")], ordered)

```

2. used the `ordered=` argument when using one of the fitting functions (`cfa/sem/growth/lavaan`), for example, if you have four binary or ordinal variables (say, item1, item2, item3, item4), you can use:

```

> fit <- cfa(myModel, data=myData, ordered=c("item1", "item2", "item3", "item4"))

```

In both cases, lavaan will automatically switch to the WLSMV estimator: it will use diagonally weighted least squares (DWLS) to estimate the model parameters, but it will use the full weight matrix to compute robust standard errors, and a mean- and variance-adjusted test statistic.

A few examples (including a multiple group example) are shown in the Appendix.

## 9 Additional information

### 9.1 Using a covariance matrix as input

If you have no full dataset, but you do have a sample covariance matrix, you can still fit your model. If you wish to add a mean structure, you need to provide a mean vector too. Importantly, if only sample statistics are provided, you must specify the number of observations that were used to compute the sample moments. The following example illustrates the use of a sample covariance matrix as input:

```
> lower <- '
+ 11.834
+ 6.947 9.364
+ 6.819 5.091 12.532
+ 4.783 5.028 7.495 9.986
+ -3.839 -3.889 -3.841 -3.625 9.610
+ -21.899 -18.831 -21.748 -18.775 35.522 450.288 '
> # classic wheaton et al model
> wheaton.cov <- getCov(lower, names=c("anomia67", "powerless67", "anomia71",
+ "powerless71", "education", "sei"))
> wheaton.model <- '
+ # latent variables
+ ses =~ education + sei
+ alien67 =~ anomia67 + powerless67
+ alien71 =~ anomia71 + powerless71
+
+ # regressions
+ alien71 ~ alien67 + ses
+ alien67 ~ ses
+
+ # correlated residuals
+ anomia67 ~~ anomia71
+ powerless67 ~~ powerless71
+ '
> fit <- sem(wheaton.model, sample.cov=wheaton.cov, sample.nobs=932)
> summary(fit, standardized=TRUE)
```

lavaan (0.5-12) converged normally after 82 iterations

Number of observations	932
Estimator	ML
Minimum Function Test Statistic	4.735
Degrees of freedom	4
P-value (Chi-square)	0.316

Parameter estimates:

Information	Expected					
Standard Errors	Standard					
	Estimate	Std.err	Z-value	P(> z )	Std.lv	Std.all
Latent variables:						
ses =~						
education	1.000				2.607	0.842
sei	5.219	0.422	12.364	0.000	13.609	0.642
alien67 =~						
anomia67	1.000				2.663	0.774
powerless67	0.979	0.062	15.895	0.000	2.606	0.852
alien71 =~						
anomia71	1.000				2.850	0.805
powerless71	0.922	0.059	15.498	0.000	2.628	0.832
Regressions:						
alien71 ~						
alien67	0.607	0.051	11.898	0.000	0.567	0.567
ses	-0.227	0.052	-4.334	0.000	-0.207	-0.207
alien67 ~						
ses	-0.575	0.056	-10.195	0.000	-0.563	-0.563

Covariances:

```

  anomia67 ~~
    anomia71      1.623    0.314    5.176    0.000    1.623    0.356
  powerless67 ~~
    powerless71  0.339    0.261    1.298    0.194    0.339    0.121

Variances:
  education      2.801    0.507                                2.801    0.292
  sei            264.597  18.126                                264.597  0.588
  anomia67      4.731    0.453                                4.731    0.400
  powerless67   2.563    0.403                                2.563    0.274
  anomia71      4.399    0.515                                4.399    0.351
  powerless71   3.070    0.434                                3.070    0.308
  ses           6.798    0.649                                1.000    1.000
  alien67       4.841    0.467                                0.683    0.683
  alien71       4.083    0.404                                0.503    0.503

```

The `getCov()` function makes it easy to create a full covariance matrix (including variable names) if you only have the lower-half elements (perhaps pasted from a textbook or a paper). Note that the lower-half elements are written between two single quotes. Therefore, you have some additional flexibility. You can add comments, and blank lines. If the numbers are separated by a comma, or a semi-colon, that is fine too. For more information about the `getCov()`, see the online manual page.

If you have multiple groups, the `sample.cov` argument must be a list containing the sample variance-covariance matrix of each group as a separate element in the list. If a mean structure is needed, the `sample.mean` argument must be a list containing the sample means of each group. Finally, the `sample.nobs` argument can be either a list or a integer vector containing the number of observations for each group.

## 9.2 Estimators, standard errors and missing values

### 9.2.1 Estimators

The default estimator in the **lavaan** package is maximum likelihood (`estimator = "ML"`). Alternative estimators currently available in lavaan are:

- "GLS" for generalized least squares. For complete data only.
- "WLS" for weighted least squares (sometimes called ADF estimation). For complete data only.
- "MLM" for maximum likelihood estimation with robust standard errors and a Satorra-Bentler scaled test statistic. For complete data only.
- "MLF" for maximum likelihood estimation with standard errors based on the first-order derivatives, and a conventional test statistic. For both complete and incomplete data.
- "MLR" maximum likelihood estimation with robust (Huber-White) standard errors and a scaled test statistic that is (asymptotically) equal to the Yuan-Bentler test statistic. For both complete and incomplete data.

If maximum likelihood estimation is used ("ML", "MLM", "MLF" or "MLR"), the default behavior of **lavaan** is to base the analysis on the so-called *biased* sample covariance matrix, where the elements are divided by  $n$  instead of  $n - 1$ . This is done internally, and should not be done by the user. In addition, the chi-square statistic is computed by multiplying the minimum function value with a factor  $n$  (instead of  $n - 1$ ). This is similar to the Mplus program. If you prefer to use an unbiased covariance, and  $n - 1$  as the multiplier to compute the chi-square statistic, you need to specify the `likelihood="wishart"` argument when calling the fitting functions. For example:

```

> fit <- cfa(HS.model, data=HolzingerSwineford1939, likelihood="wishart")
> fit

lavaan (0.5-12) converged normally after 41 iterations

Number of observations                    301

Estimator                                ML
Minimum Function Test Statistic          85.022
Degrees of freedom                        24
P-value (Chi-square)                     0.000

```

The value of the test statistic will be closer to the value reported by programs like EQS, LISREL or AMOS, since they all use the 'Wishart' approach when using the maximum likelihood estimator. The program Mplus, on the other hand, uses the 'normal' approach to maximum likelihood estimation.

## 9.2.2 Missing values

If the data contain missing values, the default behavior is listwise deletion. If the missing mechanism is MCAR (missing completely at random) or MAR (missing at random), the **lavaan** package provides case-wise (or ‘full information’) maximum likelihood estimation. You can turn this feature on, by using the argument `missing="ml"` when calling the fitting function. An unrestricted (`h1`) model will automatically be estimated, so that all common fit indices are available.

## 9.2.3 Standard errors

Standard errors are (by default) based on the expected information matrix. The only exception is when data are missing and full information ML is used (via `missing="ml"`). In this case, the observed information matrix is used to compute the standard errors. The user can change this behavior by using the `information` argument, which can be set to `"expected"` or `"observed"`. If the estimator is simply `"ML"`, you request robust standard errors by using the `se` argument, which can be set to `"robust.mlm"`, `"robust.mlr"` or `"first.order"`. Or simply to `"none"` if you don't need them. This will not affect the test statistic. In fact, you can choose the test statistic independently by using the `test` argument, which can be set to `"standard"`, `"Satorra-Bentler"` or `"Yuan-Bentler"`.

## 9.2.4 Bootstrapping

There are two ways for using the bootstrap in **lavaan**. Either you can set `se="boot"` or `test="boot"` when fitting the model (and you will get bootstrap standard errors, and/or a bootstrap based p-value respectively), or you can use the `bootstrapLavaan()` function, which needs an already fitted lavaan object.

## 9.2.5 Indirect effects and mediation analysis

Consider a classical mediation setup with three variables:  $Y$  is the dependent variable,  $X$  is the predictor, and  $M$  is a mediator. For illustration, we create a toy dataset containing these three variables, and fit a path analysis model that includes the direct effect of  $X$  on  $Y$  and the indirect effect of  $X$  on  $Y$  via  $M$ .

```
> set.seed(1234)
> X <- rnorm(100)
> M <- 0.5*X + rnorm(100)
> Y <- 0.7*M + rnorm(100)
> Data <- data.frame(X = X, Y = Y, M = M)
> model <- ' # direct effect
+           Y ~ c*X
+           # mediator
+           M ~ a*X
+           Y ~ b*M
+           # indirect effect (a*b)
+           ab := a*b
+           # total effect
+           total := c + (a*b)
+           '
> fit <- sem(model, data=Data)
> summary(fit)

lavaan (0.5-12) converged normally after 13 iterations

Number of observations                    100

Estimator                                ML
Minimum Function Test Statistic          0.000
Degrees of freedom                        0
P-value (Chi-square)                     0.000

Parameter estimates:

Information                                Expected
Standard Errors                          Standard

Regressions:
      Estimate Std.err Z-value P(>|z|)
Y ~
  X      (c)    0.036   0.104   0.348   0.728
```

```

M ~
X      (a)    0.474    0.103    4.613    0.000
Y ~
M      (b)    0.788    0.092    8.539    0.000

Variances:
Y      0.898    0.127
M      1.054    0.149

Defined parameters:
ab     0.374    0.092    4.059    0.000
total  0.410    0.125    3.287    0.001

```

The example illustrates the use of the "==" operator in the lavaan model syntax. This operator ‘defines’ new parameters which take on values that are an arbitrary function of the original model parameters. The function, however, must be specified in terms of the parameter *labels* that are explicitly mentioned in the model syntax. By default, the standard errors for these defined parameters are computed using the Delta method. As with other models, bootstrap standard errors can be requested simply by specifying `se="bootstrap"` in the fitting function.

### 9.3 Modification Indices

Modification indices can be requested by adding the `modindices=TRUE` argument in the `summary` call, or by calling the `modindices` function directly. The `modindices` function returns a data frame. For example, to see only the modification indices for the factor loadings, you can use something like this:

```

> fit <- cfa(HS.model, data=HolzingerSwineford1939)
> mi <- modindices(fit)
> mi[mi$op == "=~",] # $

```

	lhs	op	rhs	mi	epc	sepc.lv	sepc.all	sepc.nox
1	visual	=~	x1	NA	NA	NA	NA	NA
2	visual	=~	x2	0.000	0.000	0.000	0.000	0.000
3	visual	=~	x3	0.000	0.000	0.000	0.000	0.000
4	visual	=~	x4	1.211	0.077	0.069	0.059	0.059
5	visual	=~	x5	7.441	-0.210	-0.189	-0.147	-0.147
6	visual	=~	x6	2.843	0.111	0.100	0.092	0.092
7	visual	=~	x7	18.631	-0.422	-0.380	-0.349	-0.349
8	visual	=~	x8	4.295	-0.210	-0.189	-0.187	-0.187
9	visual	=~	x9	36.411	0.577	0.519	0.515	0.515
10	textual	=~	x1	8.903	0.350	0.347	0.297	0.297
11	textual	=~	x2	0.017	-0.011	-0.011	-0.010	-0.010
12	textual	=~	x3	9.151	-0.272	-0.269	-0.238	-0.238
13	textual	=~	x4	NA	NA	NA	NA	NA
14	textual	=~	x5	0.000	0.000	0.000	0.000	0.000
15	textual	=~	x6	0.000	0.000	0.000	0.000	0.000
16	textual	=~	x7	0.098	-0.021	-0.021	-0.019	-0.019
17	textual	=~	x8	3.359	-0.121	-0.120	-0.118	-0.118
18	textual	=~	x9	4.796	0.138	0.137	0.136	0.136
19	speed	=~	x1	0.014	0.024	0.015	0.013	0.013
20	speed	=~	x2	1.580	-0.198	-0.123	-0.105	-0.105
21	speed	=~	x3	0.716	0.136	0.084	0.075	0.075
22	speed	=~	x4	0.003	-0.005	-0.003	-0.003	-0.003
23	speed	=~	x5	0.201	-0.044	-0.027	-0.021	-0.021
24	speed	=~	x6	0.273	0.044	0.027	0.025	0.025
25	speed	=~	x7	NA	NA	NA	NA	NA
26	speed	=~	x8	0.000	0.000	0.000	0.000	0.000
27	speed	=~	x9	0.000	0.000	0.000	0.000	0.000

Modification indices are printed out for each nonfree (or nonredundant) parameter. The modification indices are supplemented by the expected parameter change values (column `epc`). The last two columns contain the standardized, and completely standardized EPC values, respectively.

### 9.4 Extracting information from a fitted model

The `summary` function gives a nice overview of a fitted model, but is for display only. If you need the actual numbers for further processing, you may prefer to use one of several ‘extractor’ functions. We have already seen the `coef` function which extracts the estimated parameters of a fitted model. Other extractor functions are discussed below.

### 9.4.1 parameterEstimates

The `parameterEstimates` function extracts not only the values of the estimated parameters, but also the standard errors, the z-values, the standardized parameter values, as a convenient data frame. For example

```
> fit <- cfa(HS.model, data=HolzingerSwineford1939)
> parameterEstimates(fit)
```

	lhs	op	rhs	est	se	z	pvalue	ci.lower	ci.upper
1	visual	=~	x1	1.000	0.000	NA	NA	1.000	1.000
2	visual	=~	x2	0.553	0.100	5.554	0	0.358	0.749
3	visual	=~	x3	0.729	0.109	6.685	0	0.516	0.943
4	textual	=~	x4	1.000	0.000	NA	NA	1.000	1.000
5	textual	=~	x5	1.113	0.065	17.014	0	0.985	1.241
6	textual	=~	x6	0.926	0.055	16.703	0	0.817	1.035
7	speed	=~	x7	1.000	0.000	NA	NA	1.000	1.000
8	speed	=~	x8	1.180	0.165	7.152	0	0.857	1.503
9	speed	=~	x9	1.082	0.151	7.155	0	0.785	1.378
10	x1	~~	x1	0.549	0.114	4.833	0	0.326	0.772
11	x2	~~	x2	1.134	0.102	11.146	0	0.934	1.333
12	x3	~~	x3	0.844	0.091	9.317	0	0.667	1.022
13	x4	~~	x4	0.371	0.048	7.779	0	0.278	0.465
14	x5	~~	x5	0.446	0.058	7.642	0	0.332	0.561
15	x6	~~	x6	0.356	0.043	8.277	0	0.272	0.441
16	x7	~~	x7	0.799	0.081	9.823	0	0.640	0.959
17	x8	~~	x8	0.488	0.074	6.573	0	0.342	0.633
18	x9	~~	x9	0.566	0.071	8.003	0	0.427	0.705
19	visual	~~	visual	0.809	0.145	5.564	0	0.524	1.094
20	textual	~~	textual	0.979	0.112	8.737	0	0.760	1.199
21	speed	~~	speed	0.384	0.086	4.451	0	0.215	0.553
22	visual	~~	textual	0.408	0.074	5.552	0	0.264	0.552
23	visual	~~	speed	0.262	0.056	4.660	0	0.152	0.373
24	textual	~~	speed	0.173	0.049	3.518	0	0.077	0.270

### 9.4.2 standardizedSolution

The `standardizedSolution` function is similar to the `parameterEstimates` function, but only shows the unstandardized and standardized parameter estimates.

### 9.4.3 fitted.values

The `fitted` and `fitted.values` functions return the implied (fitted) covariance matrix (and mean vector) of a fitted model.

```
> fit <- cfa(HS.model, data=HolzingerSwineford1939)
> fitted(fit)
```

```
$cov
  x1  x2  x3  x4  x5  x6  x7  x8  x9
x1 1.358
x2 0.448 1.382
x3 0.590 0.327 1.275
x4 0.408 0.226 0.298 1.351
x5 0.454 0.252 0.331 1.090 1.660
x6 0.378 0.209 0.276 0.907 1.010 1.196
x7 0.262 0.145 0.191 0.173 0.193 0.161 1.183
x8 0.309 0.171 0.226 0.205 0.228 0.190 0.453 1.022
x9 0.284 0.157 0.207 0.188 0.209 0.174 0.415 0.490 1.015

$mean
 x1 x2 x3 x4 x5 x6 x7 x8 x9
 0  0  0  0  0  0  0  0  0
```

### 9.4.4 residuals

The `resid` or `residuals` functions return (unstandardized) residuals of a fitted model. This is simply the difference between the observed and implied covariance matrix and mean vector. If the estimator is maximum likelihood, it is also possible to obtain the normalized and the standardized residuals.

```
> fit <- cfa(HS.model, data=HolzingerSwineford1939)
> resid(fit, type="standardized")
```

```

$cov
  x1      x2      x3      x4      x5      x6      x7      x8      x9
x1      NA
x2 -2.196      NA
x3 -1.199  2.692  0.000
x4  2.465 -0.283 -1.948      NA
x5 -0.362 -0.610 -4.443  0.856      NA
x6  2.032  0.661 -0.701      NA  0.633      NA
x7 -3.787 -3.800 -1.882  0.839 -0.837 -0.321  0.000
x8 -1.456 -1.137 -0.305 -2.049 -1.100 -0.635  3.804      NA
x9  4.062  1.517  3.328  1.237  1.723  1.436 -2.772      NA      NA

$mean
 x1 x2 x3 x4 x5 x6 x7 x8 x9
  0  0  0  0  0  0  0  0  0

```

### 9.4.5 vcov

The `vcov` function returns the estimated covariance matrix of the parameter estimates.

### 9.4.6 AIC and BIC

The `AIC` and `BIC` functions return the AIC and BIC values of a fitted model.

### 9.4.7 fitMeasures

The `fitMeasures` function returns all the fit measures computed by lavaan as a named numeric vector. If you only want the value of a single fit measure, say, the CFI, you give the name (in lower case) as the second argument:

```

> fit <- cfa(HS.model, data=HolzingerSwineford1939)
> fitMeasures(fit, "cfi")

      cfi
0.931

```

### 9.4.8 inspect

If you want to peek inside a fitted lavaan object (the object that is returned by a call to `cfa`, `sem` or `growth`), you can use the `inspect` function, with a variety of options. By default, calling `inspect` on a fitted lavaan object returns a list of the model matrices that are used internally to represent the model. The free parameters are nonzero integers.

```

> inspect(fit)

$lambda
  visual textul speed
x1      0      0      0
x2      1      0      0
x3      2      0      0
x4      0      0      0
x5      0      3      0
x6      0      4      0
x7      0      0      0
x8      0      0      5
x9      0      0      6

$theta
  x1 x2 x3 x4 x5 x6 x7 x8 x9
x1  7
x2  0  8
x3  0  0  9
x4  0  0  0 10
x5  0  0  0  0 11
x6  0  0  0  0  0 12
x7  0  0  0  0  0  0 13
x8  0  0  0  0  0  0  0 14
x9  0  0  0  0  0  0  0  0 15

$psi
  visual textul speed

```

```

visual 16
textual 19 17
speed 20 21 18

```

To see the starting values of parameters in each model matrix, type

```

> inspect(fit, what="start")

$lambda
  visual textual speed
x1 1.000 0.000 0.000
x2 0.778 0.000 0.000
x3 1.107 0.000 0.000
x4 0.000 1.000 0.000
x5 0.000 1.133 0.000
x6 0.000 0.924 0.000
x7 0.000 0.000 1.000
x8 0.000 0.000 1.225
x9 0.000 0.000 0.854

$theta
  x1  x2  x3  x4  x5  x6  x7  x8  x9
x1 0.679
x2 0.000 0.691
x3 0.000 0.000 0.637
x4 0.000 0.000 0.000 0.675
x5 0.000 0.000 0.000 0.000 0.830
x6 0.000 0.000 0.000 0.000 0.000 0.598
x7 0.000 0.000 0.000 0.000 0.000 0.000 0.592
x8 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.511
x9 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.508

$psi
  visual textual speed
visual 0.05
textual 0.00 0.05
speed 0.00 0.00 0.05

```

To see how lavaan internally represents a model, you can type

```

> inspect(fit, what="list")

  id  lhs op  rhs user group free  ustart  exo label eq.id unco
1  1  visual ==  x1  1  1  0  1  0  0  0
2  2  visual ==  x2  1  1  1  NA  0  0  1
3  3  visual ==  x3  1  1  2  NA  0  0  2
4  4  textual ==  x4  1  1  0  1  0  0  0
5  5  textual ==  x5  1  1  3  NA  0  0  3
6  6  textual ==  x6  1  1  4  NA  0  0  4
7  7  speed ==  x7  1  1  0  1  0  0  0
8  8  speed ==  x8  1  1  5  NA  0  0  5
9  9  speed ==  x9  1  1  6  NA  0  0  6
10 10  x1 ==  x1  0  1  7  NA  0  0  7
11 11  x2 ==  x2  0  1  8  NA  0  0  8
12 12  x3 ==  x3  0  1  9  NA  0  0  9
13 13  x4 ==  x4  0  1  10 NA  0  0  10
14 14  x5 ==  x5  0  1  11 NA  0  0  11
15 15  x6 ==  x6  0  1  12 NA  0  0  12
16 16  x7 ==  x7  0  1  13 NA  0  0  13
17 17  x8 ==  x8  0  1  14 NA  0  0  14
18 18  x9 ==  x9  0  1  15 NA  0  0  15
19 19  visual == visual 0  1  16 NA  0  0  16
20 20 textual == textual 0  1  17 NA  0  0  17
21 21 speed == speed 0  1  18 NA  0  0  18
22 22 visual == textual 0  1  19 NA  0  0  19
23 23 visual == speed 0  1  20 NA  0  0  20
24 24 textual == speed 0  1  21 NA  0  0  21

```

For more inspect options, see the help page for the lavaan class which you can find by typing the following:

```

> class?lavaan

```

## A Examples from the Mplus User's Guide

Below, we provide some examples of **lavaan** model syntax to mimic the examples in the Mplus User's guide. The datafiles can be downloaded from <http://www.statmodel.com/ugexcerpts.shtml>.



## A.1 Chapter 3: Regression and Path Analysis

```
# ex3.1
Data <- read.table("ex3.1.dat")
names(Data) <- c("y1", "x1", "x2")

model.ex3.1 <- ' y1 ~ x1 + x2 '
fit <- sem(model.ex3.1, data=Data)
summary(fit, standardized=TRUE, fit.measures=TRUE)

# ex3.4
Data <- read.table("ex3.4.dat")
names(Data) <- c("u1", "x1", "x3")
Data$u1 <- ordered(Data$u1)

model <- ' u1 ~ x1 + x3 '
fit <- sem(model, data=Data)
summary(fit, fit.measures=TRUE)

# ex3.11
Data <- read.table("ex3.11.dat")
names(Data) <- c("y1", "y2", "y3",
                "x1", "x2", "x3")

model.ex3.11 <- ' y1 + y2 ~ x1 + x2 + x3
                 y3 ~ y1 + y2 + x2 '

fit <- sem(model.ex3.11, data=Data)
summary(fit, standardized=TRUE, fit.measures=TRUE)

# ex3.12
Data <- read.table("ex3.12.dat")
names(Data) <- c("u1", "u2", "u3", "x1", "x2", "x3")
Data$u1 <- ordered(Data$u1)
Data$u2 <- ordered(Data$u2)
Data$u3 <- ordered(Data$u3)

model <- ' u1 + u2 ~ x1 + x2 + x3
           u3 ~ u1 + u2 + x2 '

fit <- sem(model, data=Data)
summary(fit, fit.measures=TRUE)

# Mplus example 3.14
Data <- read.table("ex3.14.dat")
names(Data) <- c("y1", "y2", "u1", "x1", "x2", "x3")
Data$u1 <- ordered(Data$u1)

model <- ' y1 + y2 ~ x1 + x2 + x3
           u1 ~ y1 + y2 + x2 '

fit <- sem(model, data=Data)
summary(fit, fit.measures=TRUE)
```

## A.2 Chapter 5: Confirmatory factor analysis and structural equation modeling

```
# ex5.1
Data <- read.table("ex5.1.dat")
names(Data) <- paste("y", 1:6, sep="")

model.ex5.1 <- ' f1 =~ y1 + y2 + y3
                 f2 =~ y4 + y5 + y6 '

fit <- cfa(model.ex5.1, data=Data)
```

```

summary(fit, standardized=TRUE, fit.measures=TRUE)

# ex5.2
Data <- read.table("ex5.2.dat")
names(Data) <- c("u1","u2","u3","u4","u5","u6")
# declare all variables as 'ordered factors':
Data <- as.data.frame(lapply(Data, ordered))

model <- ' f1 =~ u1 + u2 + u3; f2 =~ u4 + u5 + u6 '
fit <- cfa(model, data=Data)
summary(fit, fit.measures=TRUE)

# ex5.3
Data <- read.table("ex5.3.dat")
names(Data) <- c("u1","u2","u3","y4","y5","y6")
Data$u1 <- ordered(Data$u1)
Data$u2 <- ordered(Data$u2)
Data$u3 <- ordered(Data$u3)

model <- ' f1 =~ u1 + u2 + u3
           f2 =~ y4 + y5 + y6 '
fit <- cfa(model, data=Data)
summary(fit, fit.measures=TRUE)

# ex5.6
Data <- read.table("ex5.6.dat")
names(Data) <- paste("y", 1:12, sep="")

model.ex5.6 <- ' f1 =~ y1 + y2 + y3
                f2 =~ y4 + y5 + y6
                f3 =~ y7 + y8 + y9
                f4 =~ y10 + y11 + y12
                f5 =~ f1 + f2 + f3 + f4 '

fit <- cfa(model.ex5.6, data=Data, estimator="ML")
summary(fit, standardized=TRUE, fit.measures=TRUE)

# ex5.8
Data <- read.table("ex5.8.dat")
names(Data) <- c(paste("y", 1:6, sep=""), paste("x", 1:3, sep=""))

model.ex5.8 <- ' f1 =~ y1 + y2 + y3
                f2 =~ y4 + y5 + y6
                f1 + f2 ~ x1 + x2 + x3 '

fit <- cfa(model.ex5.8, data=Data, estimator="ML")
summary(fit, standardized=TRUE, fit.measures=TRUE)

# ex5.9
Data <- read.table("ex5.9.dat")
names(Data) <- c("y1a","y1b","y1c","y2a","y2b","y2c")

model.ex5.9 <- ' f1 =~ 1*y1a + 1*y1b + 1*y1c
                f2 =~ 1*y2a + 1*y2b + 1*y2c
                y1a + y1b + y1c ~ i1*1
                y2a + y2b + y2c ~ i2*1 '

fit <- cfa(model.ex5.9, data=Data)
summary(fit, standardized=TRUE, fit.measures=TRUE)

# ex5.11

```

```

Data <- read.table("ex5.11.dat")
names(Data) <- paste("y", 1:12, sep="")

model.ex5.11 <- ' f1 =~ y1 + y2 + y3
                 f2 =~ y4 + y5 + y6
                 f3 =~ y7 + y8 + y9
                 f4 =~ y10 + y11 + y12
                 f3 ~ f1 + f2
                 f4 ~ f3 '

fit <- sem(model.ex5.11, data=Data, estimator="ML")
summary(fit, standardized=TRUE, fit.measures=TRUE)

# ex5.14
Data <- read.table("ex5.14.dat")
names(Data) <- c("y1","y2","y3","y4","y5","y6", "x1","x2","x3", "g")

model.ex5.14 <- ' f1 =~ y1 + y2 + y3
                 f2 =~ y4 + y5 + y6
                 f1 + f2 ~ x1 + x2 + x3 '

fit <- cfa(model.ex5.14, data=Data, group="g", meanstructure=FALSE,
           group.equal=c("loadings"), group.partial=c("f1=~y3"))
summary(fit, standardized=TRUE, fit.measures=TRUE)

# ex5.15
Data <- read.table("ex5.15.dat")
names(Data) <- c("y1","y2","y3","y4","y5","y6", "x1","x2","x3", "g")

model.ex5.15 <- ' f1 =~ y1 + y2 + y3
                 f2 =~ y4 + y5 + y6
                 f1 + f2 ~ x1 + x2 + x3 '

fit <- cfa(model.ex5.15, data=Data, group="g", meanstructure=TRUE,
           group.equal=c("loadings", "intercepts"),
           group.partial=c("f1=~y3", "y3~1"))
summary(fit, standardized=TRUE, fit.measures=TRUE)

# ex5.16
Data <- read.table("ex5.16.dat")
names(Data) <- c("u1","u2","u3","u4","u5","u6", "x1","x2","x3", "g")
Data$u1 <- ordered(Data$u1)
Data$u2 <- ordered(Data$u2)
Data$u3 <- ordered(Data$u3)
Data$u4 <- ordered(Data$u4)
Data$u5 <- ordered(Data$u5)
Data$u6 <- ordered(Data$u6)

model <- ' f1 =~ u1 + u2 + c(13,13b)*u3
          f2 =~ u4 + u5 + u6
          # mimic
          f1 + f2 ~ x1 + x2 + x3
          # equal thresholds, but free u3|1 in second group
          u3 | c(u3,u3b)*t1
          # fix scale of u3* to 1 in second group
          u3 ~*~ c(1,1)*u3
          ,

fit <- cfa(model, data=Data, group="g", group.equal=c("loadings","thresholds"))
summary(fit, fit.measures=TRUE)

# ex5.20
Data <- read.table("ex5.20.dat")
names(Data) <- paste("y", 1:6, sep="")

```

```

model.ex5.20 <- ' f1 =~ y1 + lam2*y2 + lam3*y3
                f2 =~ y4 + lam5*y5 + lam6*y6
                f1 =~ vf1*f1 + start(1.0)*f1    ## otherwise, neg vf2
                f2 =~ vf2*f2 + start(1.0)*f2    ##
                y1 =~ ve1*y1
                y2 =~ ve2*y2
                y3 =~ ve3*y3
                y4 =~ ve4*y4
                y5 =~ ve5*y5
                y6 =~ ve6*y6

                # constraints
                lam2^2*vf1/(lam2^2*vf1 + ve2) ==
                  lam5^2*vf2/(lam5^2*vf2 + ve5)
                lam3*sqrt(vf1)/sqrt(lam3^2*vf1 + ve3) ==
                  lam6*sqrt(vf2)/sqrt(lam6^2*vf2 + ve6)
                ve2 > ve5
                ve4 > 0
                ,

```

```

fit <- cfa(model.ex5.20, data=Data, estimator="ML")
summary(fit, standardized=TRUE, fit.measures=TRUE)

```

### A.3 Chapter 6: Growth modeling

```

# ex6.1
Data <- read.table("ex6.1.dat")
names(Data) <- c("y11", "y12", "y13", "y14")

model.ex6.1 <- ' i =~ 1*y11 + 1*y12 + 1*y13 + 1*y14
                s =~ 0*y11 + 1*y12 + 2*y13 + 3*y14 '

```

```

fit <- growth(model.ex6.1, data=Data)
summary(fit, standardized=TRUE, fit.measures=TRUE)

```

```

# ex6.8
Data <- read.table("ex6.8.dat")
names(Data) <- c("y11", "y12", "y13", "y14")

model.ex6.8 <- ' i =~ 1*y11 + 1*y12 + 1*y13 + 1*y14
                s =~ 0*y11 + 1*y12 + start(2)*y13 + start(3)*y14 '

```

```

fit <- growth(model.ex6.8, data=Data)
summary(fit, standardized=TRUE, fit.measures=TRUE)

```

```

# ex6.9
Data <- read.table("ex6.9.dat")
names(Data) <- c("y11", "y12", "y13", "y14")

model.ex6.9 <- ' i =~ 1*y11 + 1*y12 + 1*y13 + 1*y14
                s =~ 0*y11 + 1*y12 + 2*y13 + 3*y14
                q =~ 0*y11 + 1*y12 + 4*y13 + 9*y14 '

```

```

fit <- growth(model.ex6.9, data=Data)
summary(fit, standardized=TRUE, fit.measures=TRUE)

```

```

# ex6.10
Data <- read.table("ex6.10.dat")
names(Data) <- c("y11", "y12", "y13", "y14", "x1", "x2", "a31", "a32", "a33", "a34")

model.ex6.10 <- ' i =~ 1*y11 + 1*y12 + 1*y13 + 1*y14

```

```

s =~ 0*y11 + 1*y12 + 2*y13 + 3*y14
i + s ~ x1 + x2
y11 ~ a31
y12 ~ a32
y13 ~ a33
y14 ~ a34 '

fit <- growth(model.ex6.10, data=Data)
summary(fit, standardized=TRUE, fit.measures=TRUE)

# ex6.11
Data <- read.table("ex6.11.dat")
names(Data) <- c("y1", "y2", "y3", "y4", "y5")

modelex6.11 <- ' i =~ 1*y1 + 1*y2 + 1*y3 + 1*y4 + 1*y5
s1 =~ 0*y1 + 1*y2 + 2*y3 + 2*y4 + 2*y5
s2 =~ 0*y1 + 0*y2 + 0*y3 + 1*y4 + 2*y5 '

fit <- growth(modelex6.11, data=Data)
summary(fit, standardized=TRUE, fit.measures=TRUE)

```