MRI Master Class 2009/2010:
Numerical Bifurcation Analysis of Dynamical Systems
WEEK 13, Mar 30 and Apr 01: Willy Govaerts (Gent)
"Mathematical evolution models in the life sciences"

Course Project

Implement the budding yeast cell cycle model (4.1) with parameter values other than m as given in Table 4.1 in the course syllabus.

1. Compute a branch of equilibria under variation of m, starting from a stable equilibrium with $m = 0.3$ and $[CycB]_T \approx 0.038485$, and leaving when $m \geq 1.5$. For all limit points and Hopf points on this branch give the m and $[CycB]_T$ values. This should include three Hopf points, which we might successively call $H1$, $H2$, and $H3$. Give the first Lyapunov values of these Hopf points and the conclusions that you can draw from these values.

   Note. For $H1$ you should find $m \approx 0.654631$, for the first limit point you should find $m \approx 0.672568$.

2. Compute the branch of limit cycles born at $H1$. Show that it ends in an orbit homoclinic to saddle. Compute as accurately as possible the m-value of the saddle equilibrium and the eigenvalues at the saddle. Compute also the saddle quantity (sum of the real parts of the leading stable and unstable eigenvalues).

3. Compute the branch of limit cycles born at $H2$. When (value of m?) and how do they lose stability?

4. Study the branch of limit cycles born at $H3$. Do you find bifurcation points? How does the branch end?

5. Comment on any relation that you might see between the outcomes of the studies of the three branches of limit cycles.

6. Now start again with $k'_{13} = 0.2$. Compute the curve of equilibria, find the first Hopf point and continue it in the two parameters m, k_1. You should find a BT point for $k_1 \approx 0.050847515$. For the parameter values k'_{13}, k_1 of the BT point study again the bifurcation behaviour of the model under variation of m. Report on your findings.